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<th>著者</th>
<th>佐藤孝雄</th>
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1 Introduction

In olden times the observation of twilight was executed by FESSENKOFF (1), SMART (2), HULBURT (3). The latest one was perhaps given by LJUNGHALL (4), who introduced us much historical process and many useful references in this paper.

As far as the theoretical investigation is concerned, HULBURT computed by considering only first scattering, so that the error mounted up to 30% and he confessed that the rigorous calculation would be impossible with the sun's zenith distance greater than 98° owing to the far predominancy of secondary scattering. Hence a satisfactory explanation of twilight has not been completed until now.

Giving a paper on the scattering of the sun in the day time (5) the author has now researched the twilight as the extension of the paper by assuming also the earth's atmosphere to be composed of numberless homogeneous thin layers bounded by concentric spheres, and considering the extinction of the sun's ray and further discussing strictly the secondary scattering with respect to each wave-length, and computed the radiation of each portion of the sky dome and that falling on a horizontal plane on the earth's surface in the case of the sun's zenith distances 96°, 96°.404, 97°; but as the above papers (1), (2), (3) and that introduced in (4) are giving the observation of particular portion on the sky, and (4) is concerned to the luminosity on a vertical plane, so we have some unreliance and inconvenience in the representation of solid angle and the method of measurement.

On the contrary, the observation of Mr. OSAWA of Tokyo Astro. Obs. (6) can not only give full reliance but is concerned to the horizontal plane on the earth's surface with the unit of Lux, so his measurement is most convenient to be compared with the author's computation. This comparison has been made with good identity. Eventurely the explanation of twilight which has not been hitherto researched has become possible up to about 100° of the sun's zenith distance if we consider up to secondary scattering on the practical point of view. For the greater zenith distance, the consideration of the scattering of higher grade is necessary, but we can assert that we must conquer much difficulty. As already mentioned, this paper is an extension of the above paper of the author. Please the reader refer to it.

Let us take two points T and E in the atmosphere. Define any arbitrarily chosen set of axes of rectangular coordinates Xi Yi Zi with its center at T, with the Xi axis drawn towards the sun. Now, let i be the direct insolation reaching T, ET=r, φ the angle between Xi axis and r, so the intensity of primary scattering generated by unit volume becomes

\[\frac{3}{16} \frac{k_{\lambda r}}{\pi r^3} (1+\cos^2 \phi) i\]

in which we can substitute \(k \rho_r\) for \(k_{\lambda r}\), \(k\) being the function of wave-length \(\lambda\), \(\rho_r\) the atmospheric density at T. The wave generated at E by a plane polarized light coming from Xi
direction and oscillating in $Z_1$ direction is also a plane polarized and advances in $r$ direction and oscillates in a definite direction normal to it.

Now take the origin at $E$, $X_2$ axis in $r$, that is, TE direction, $Z_2$ axis in the direction of oscillation, $Y_2$ axis normal to $X_2 Z_2$.

Now take any point $O'$ and notate $EO'=R$ and let $\omega$ and $\omega'$ be the angle between $r$ and $Z_1$, and $Y_1$, $\Omega$ the angle between $Z_2$ and $EO'$.

The plane polarized light coming from $X_1$ and oscillating in $Y_1$ direction produces at $E$ a plane polarized light which advances in the direction $r$ and oscillates in a definite direction normal to it. Draw from $E$ the axis $X_2'$ and $Z_2'$ in the direction of $r$ and oscillation respectively and $Y_2'$ normal to them, and let $\Omega'$ be the angle between $EO'$ and $Z_2'$. Hence, when we take unit volume of atmosphere at $T$ and $E$ respectively, $T$ being exposed by solar ray, emits primarily scattered light to $E$ and again $E$, receiving this light, emits secondarily scattered light to $O'$. The intensity of this final scattered light at $O'$ becomes

$$\frac{i}{2} \left( \frac{3}{8\pi} \right)^2 \frac{1}{R^2} \left[ (\sin^2 \omega \sin^2 \Omega + \sin^2 \omega' \sin^2 \Omega') \cot \theta \right] f_{OE} k_{OE},$$

in which $k_{OE}$ is the extinction coefficient at $E$. If we consider the effect of absorption on the optical path, we must only multiply the absorption term.

### 2 First scattering

#### § 1 General aspect

Take a point $E$ in the atmosphere seen from $O'$ point on the earth's surface. Let us define a set of axes of coordinates $X, Y, Z$, with its origin at the earth's center $O$, taking $O O'$ as $Z$ axis, $X$ axis being normal to $Z$ axis in the plane determined by this and the sun's centre, and on the sun-side, and $Y$ axis being normal to the other two.

Let the coordinates of $E$ referred to $XYZ$ system be

$$X = e \sin \gamma \cos A, \quad Y = e \sin \gamma \sin A, \quad Z = e \cos \gamma,$$

where $0 \leq \gamma \leq \frac{\pi}{2}, 0 \leq A \leq 2\pi$.

Let the transformed system of $XYZ$ obtained by moving the origin to $O'$ be $X', Y', Z'$ system, and define the polar angle $\theta$ and $A$ by

$$X' = R \cos \theta \cos A, \quad Y' = R \cos \theta \sin A, \quad Z' = R \sin \theta,$$

and denote the coordinates of $E$ referred to this new system by $X', Y', Z'$, then we get

$$X = X', \quad Y = Y', \quad Z = Z' + \omega_t,$$

where $\omega_t$ is the earth's radius, now, let $H'$ be the height of a point from the earth's surface of a point distant from $E$ by $s$ on a solar ray passing through $E$, and $d$ be the sun's dip (i.e. negative altitude) so is

$$(\omega_t + H')^2 = (X + s \cos d)^2 + Y^2 + (Z - s \sin d)^2$$

$$= s^2 + 2s(X \cos d - Z \sin d) + e^2,$$

from this $H'$ can be expressed by a function of $s$. Consider a new axis of $x_t$ drawn at the angle $d$ with $X$ axis. Then the relation between the new system $x_t, y_t, z_t$ and the original are shown below

<table>
<thead>
<tr>
<th>$x_t$</th>
<th>$y_t$</th>
<th>$z_t$</th>
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<tbody>
<tr>
<td>$X$</td>
<td>$\cos d$</td>
<td>0</td>
</tr>
<tr>
<td>$Y$</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$Z$</td>
<td>$-\sin d$</td>
<td>0</td>
</tr>
</tbody>
</table>

Here, needless to say, $x_t$ is directed towards the sun. If the $x_t y_t z_t$ coordinates of $E$ be denoted by $a, b, c$, we get

$$a = X \cos d - Z \sin d, \quad b = Y,$$

$$c = X \sin d + Z \cos d.$$

Hence the optical path along the solar ray in the atmosphere between $E$ and the upper limit, is

$$\delta_t = \sqrt{(a_0 + l)^2 - b^2 - c^2} - a,$$

where $l$ is the height of upper limit. Thus, $\delta_t$
can be expressed by a function of XYZ, the
cordinate of E. So becomes the direct solar
ntensity at E

\[
\frac{I_0}{D^2} \exp \left( - \int_0^R k_p(H') \, ds \right).
\]

Now, the light scattered by this direct ray at
E will be of course similarly affected by the
atmospheric absorption on the way from E to
O'.

The height of a point, H, distant from O'
by s on the line O'E is evidently

\[
H = \sqrt{a_0^2 + s^2 + 2a_0 s \sin \theta} - a_0,
\]

and the absorption effect between O'E is

\[
\exp \left( - \int_0^R k_p(H) \, ds \right)
\]

and the angle \( \phi \), between O'E line and solar
ray passing through E is

\[
\cos \varphi = \frac{X}{R} \cos d - \frac{Z - a_0}{R} \sin d
= \cos \theta \cos A \cos d - \sin d \sin \theta.
\]

Thus the horizontal intensity \( dS_i \) at O'
due to primary scattering produced by 1 cm²
with its centre at E in a domain in the
atmosphere radiated by direct sun on the
horizon at O' (this will be called hereafter the
bright zone) is

\[
dS_i = \frac{3k \rho_K}{16\pi R^2} (1 + \cos^2 \varphi) \sin \theta \frac{I_0}{D^2}
\times \exp \left( - \int_0^R k_p(H') \, ds - \int_0^R k_p(H) \, ds \right)
\]

where \( \rho_K \) is the air density at E.

dS_i can be expressed by a function of \( R, \theta, A \)
by the above explanation. Now letting \( R_i \)
be the distance from O' to a point of intersection
between the upper limit and a line passing
through O', so

\[
R_i = \frac{\cos(\theta + \alpha)}{\cos \theta} (a_0 + l),
\]

where

\[
\sin \alpha = \frac{a_0}{a_0 + l} \cos \theta.
\]

The domain in the atmosphere besides the
bright zone is not contributory to 1st scattering
since it is never radiated by the direct sun.
Therefore, the total intensity is obtainable by
integrating \( dS_i \) all over the bright zone. Since
this zone is a fraction of the total sky above
horizon at O' outside a cylinder generated by
the sun's ray touching the earth, it is conve-
nient to express this zone by \( X' Y' Z' \) system.
The equation of cylinder referred to XYZ
system is

\[
Y^2 + (X \cos d + Z \cos d)^2 = a_0^2,
\]

converting this to \( X' Y' Z' \) system we get

\[
Y^2 + (X' \cos d + (Z' + a_0) \cos d)^2 = a_0^2.
\]

Next consider a new system \( X', Y', Z' \)
which is made from \( X' Y' Z' \) system by
rotating it by the angle \( A \) around \( Z' \) axis,
then we get evidently the next relation between
both systems.

<table>
<thead>
<tr>
<th>( X' )</th>
<th>( Y' )</th>
<th>( Z' )</th>
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<tbody>
<tr>
<td>( X' \cos A - \sin A )</td>
<td>( -\sin A \cos A )</td>
<td>0</td>
</tr>
<tr>
<td>( Y' \sin A \cos A )</td>
<td>( \cos A )</td>
<td>0</td>
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<tr>
<td>( Z' )</td>
<td>0</td>
<td>1</td>
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From this the equation will become

\[
(X' \sin A + Y' \cos A)^2 + (Z' + a_0 \cos d)^2 = a_0^2.
\]

Substituting \( Y_i = 0 \) in this, we get the section
of cylinder by plane \( OX'Z' \), i. e.

\[
(X' \sin A)^2 + (Z' + a_0 \cos d)^2 = (a_0 + 1)^2.
\]

Secendly, the equation of sphere representing
the atmospheric upper limit is

\[
X'^2 + Y'^2 = (a_0 + 1)^2,
\]

\[
X'^2 + Y'^2 + (Z' + a_0)^2 = (a_0 + 1)^2,
\]

\[
(X' \cos A - Y' \sin A)^2 + (Z' + a_0 \cos d)^2 + (Z' + a_0)^2 = (a_0 + 1)^2.
\]

The line of intersection produced by this
and \( Y_i = 0 \) is

\[
X'^2 + (Z' + a_0)^2 = (a_0 + 1)^2.
\]

Solving \( X' \) and \( Z' \) by the combination of (a)
and (b) which are curves on \( Y_i = 0 \) plane, and
getting $\Theta = t g^{-1} \frac{Z'_{i}}{X_{i}}$, then $\Theta$ is obviously the upper limit of integration with respect to $\theta$ for one given $A$. Next, since the limiting value with respect to $A$ must of course satisfy $\Theta = 0$, i.e. $Z'_{i} = 0$, we must only to eliminate $X'_{i}$ from (a), (b) and $Z' = 0$ to get it, $A'$, consequently from

$$(X_{i} \sin A)^{2} + (\alpha_{0} \cos d + X_{i} \cos A \sin d)^{2} = a_{0}^{2} X_{i}^{2} + a_{0}^{2} = (a_{0} + 1)^{2}.$$ Substituting $X = R \cos \theta \cos A$, $Y = R \cos \theta \sin A$, $Z = R \sin \theta$, in the equation of cylinder, we get

$$R^{3} \cos^{2} \theta \sin^{2} A + (\cos \theta + \alpha_{0}) \cos d + R \cos \theta \cos A \sin d)^{2} = a_{0}^{2}.$$ The solution $R_{1}$ of this equation for given $\theta$ and $A$ is naturally one limit of integration with respect to $R$. Still the integral formula representing the bright zone will take different expression according as whether the intersected point of $Z$ axis by the upper atmospheric limit (hereafter denoted by $G'$ point) is inside or out-side the cylinder.

§ 2 The case when $0 < d < d_{1} = \frac{\cos^{-1} \frac{a_{0}}{a_{0} + 1}}{L}$

In this case, the $G'$ point is within the bright zone, and yet the intensity of radiation is symmetry as to $Y' = 0$ plane, so that the total horizontal intensity due to lst scattering is

$$D_{1} = 2 \left( \int_{0}^{\pi} \int_{0}^{\pi} \int_{0}^{R_{1}} \sin \theta d \theta d \phi \int_{R_{0}}^{R_{1}} \rho d \rho \right) dS_{i}.$$ 

§ 3 The case $d_{1} < d < 2 d_{1}$

In this case, the $G'$ point is within the dark zone. Using the same notations in the same meanings as in § 1, yet $\Theta$ being applied in $0 < A < A'$, we get as the total intensity

$$D'_{1} = 2 \left( \int_{0}^{\pi} \int_{0}^{\pi} \int_{0}^{R_{1}} \cos \theta d \theta d \phi \int_{R_{0}}^{R_{1}} \rho d \rho \right) dS_{i}.$$ However, when it becomes $d < 2 d_{1}$, the lst scattering will entirely disappear since there exists no bright zone, reducing the total intensity to zero.

3. Secondary scattering

§ 4 General aspect

Now consider a cylindrical coordinate system given by

$$x = x_{1}, \quad y = y_{1} \cos \alpha, \quad z = z_{1} \sin \alpha.$$ Consider a point $T$ exposed by direct sun in the atmosphere visible from a point $E$ in the sky dome. Let $x_{1}, y_{1}, z_{1}$ be the coordinates of $T$ referred to $x_{1}, y_{1}, z_{1}$ system, then that of a point distant by $s$ from $T$ on a solar ray passing through $T$ are $x_{1} + s, y_{1}, z_{1}$ and its height $H_{2}$ is given by a function of $s$ as follows

$$(\alpha_{0} + H_{2})^{3} = (x_{1} + s)^{3} + y_{1}^{2} + z_{1}^{2}.$$ The optical path $L$ traversed by the sun's ray until it reaches $T$ is

$$L = \sqrt{(a_{0} + 1)^{2} - y_{1}^{2} - z_{1}^{2}} - x_{1},$$ thus the intensity of direct sun at $T$ is given by a function of $x_{1}, y_{1}, z_{1}$ as follows

$$\frac{I_{0}}{D_{1}} \exp \left( - \int_{0}^{L} k \rho (H_{2}) ds \right).$$ Since the coordinates of $E$ referred to $x_{1}, y_{1}, z_{1}$ system are $a, b, c$, so is the distance of a line $ET$ becomes

$$r = \sqrt{(x-a)^{2} + (y-b)^{2} + (z-c)^{2}}$$ and its direction cosines are

$$\lambda = \frac{x_{1} - a}{r}, \quad \mu = \frac{y_{1} - b}{r}, \quad \nu = \frac{z_{1} - c}{r}$$ and the height $H_{1}$ of a point distant by $s$ from $E$ on it becomes

$$a_{0} + H_{1} = \sqrt{(a + s \lambda)^{2} + (b + s \mu)^{2} + (c + s \nu)^{2}}$$ and the absorption effect of the ray between $ET$ is

$$\exp \left( - \int_{0}^{L} k \rho (H_{1}) ds \right)$$ is a function of $x_{1}, y_{1}, z_{1}$. And further that between $EO'$ is the same to 2 and can be given by a function of $R, \theta$. 

Now let $X_1, Y_1, Z_1$ be a new system of coordinates with its centre at $T$ and parallel to $x, y, z$. Let the coordinates of $E$ referred to the new system be $\gamma, \delta, \kappa$, then

$$\gamma = a - x_1, \quad \delta = b - y_1, \quad \kappa = c - z_1$$

and

$$\cos \omega = \frac{\kappa}{r}, \quad \cos \omega' = \frac{\delta}{r}.$$

Since the coordinates of $O'$ referred to $x, y, z$ system are

$$-a_0 \sin d, \quad 0, \quad a_0 \cos d,$$

then the $Z_2$ coordinate of $O'$ referred to $X_2, Y_2, Z_2$ system is

$$N = \frac{1}{r^2} \left\{ \gamma \kappa (-a_0 \sin d - x_1 - \gamma) + \delta \kappa (-y_1 - \delta) - (\gamma^2 + \kappa^2)(a_0 \cos d - z_1 - \kappa) \right\}$$

and that corresponding to $X_1, Y_1, Z_1$ is

$$N' = \frac{1}{r^2} \left\{ \gamma \delta (-a_0 \sin d - x_1 - \gamma) - (\gamma^2 + \kappa^2)(-y_1 - \delta) + \delta \kappa (a_0 \cos d - z_1 - \kappa) \right\}.$$

About it the reader must refer to Chapter 1 of (5). Then we get

$$\cos \Omega = \frac{N}{R}, \quad \cos \Omega' = \frac{N'}{R}.$$

Hence the horizontal intensity due to secondary scattering at $O'$ contributed by 1 cm$^3$ atmosphere at $E$ and $T$

$$dS_2 = \frac{1}{2} \frac{I_0}{D^2} \left( \frac{3}{8\pi} \right)^2 \frac{k^2}{R_0^2} \rho x \rho r \sin \theta \times (\sin^2 \omega \sin^2 \Omega + \sin^2 \omega' \sin^2 \Omega') \times \exp \left( - \int_0^\infty k p(H) ds - \int_0^\infty k p(H) ds \right.$$

will be a function of some kinds of coordinates of $E$ by integrating with respect to $T$ by the cylindrical coordinates $r, \alpha, \varphi$. In this function when we express $a, b, c$ by $R, \theta, A$ by coordinate transformation, we get the horizontal intensity from the whole sky by integrating it with respect to $E$ over this domain by the polar coordinates.

The integration of $E$ and $T$ are different at the sun's dip, so discussion shall be proceeded by dividing in some sections as follows.

§ 5 The integration of $T$ when $0 \leq d \leq 2d_
u$

a) The integration of $T$ when $E$ is in bright zone.

Now, consider a cone with its centre at $E$ and touching the earth (shall be called $B$ cone). The equation of a section of $B$ cone by a plane perpendicular to $x$ axis can be obtained from its equation.

Using the polar coordinates on the plane of section

$$y_1 = r \cos a, \quad z_1 = r \sin a,$$

the equation of section becomes the form

$$r = \Phi (x_1, a).$$

The section of the earth is $y_1^2 + z_1^2 = a_0^2 - x_1^2$, so it can be transformed to the form

$$r = \varphi_1 (x_1, a) = \sqrt{a_0^2 - x_1^2} = \rho (x_1).$$

Similarly that of the upper limit of the earth's atmosphere becomes

$$r = \varphi_2 (x_1, a) = \sqrt{(a_0 + I)^2 - x_1^2} = q (x_1)$$

and that of the earth's shadow

$$r = \varphi_0 (x_1, a) = a_0.$$

Let the values of $a$ of the intersecting points between
\[ \Phi \text{ and } \phi_2 \text{ be } a_2, a'_2, \]
\[ \Phi \text{ and } \phi_1 \text{ be } a_1, a'_1, \]
\[ \Phi \text{ and } \phi_0 \text{ be } a_0, a'_0. \]

Let \( x \) be the greatest value of \( x_i \) of the touching points of \( B \) cone and the earth, and \( x_i \) that of the points on the circle in which the cone intersects with the sphere representing the upper limit of the earth's atmosphere (shall be called \( G \) sphere). Then the integration of \( T \) with respect to \( n, a \) on the plane perpendicular to \( x \) axis has different conditions according as \( x \) is between \( -q(a_0) \sim 0, 0 \sim x_c \) and \( x_c \sim x \) respectively as illustrated in Fig. 1 a, b, c, and Fig. 2.

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**Fig. 1** This makes us easy to understand the integration domains of \( r_1 \) and \( a \) in three domains of \( x_i \) concerning the integration of \( T \) when \( E \) is in bright zone.

**Fig. 2** This makes us easy to understand the lower and upper limits of integration of \( x_i \) concerning the integration of \( T \) when \( E \) is in bright zone.

Then the intensity of secondary scattering by one \( E \) will be

\[
\Delta_0 = \left[ \int_0^{x_b} \left\{ \frac{d\alpha_0}{a_1} \left[ \frac{d\eta}{n_1} \right] + \frac{d\alpha_0'}{a_1'} \left[ \frac{d\eta}{n_1'} \right] \right\} \right] dS_2.
\]

b) The integration of \( T \) when \( E \) is in dark zone.

In this case \( E \) is not solarradiated. Now, let \( x_b \) be the greatest value of \( x_i \) of the points on the intersecting curve between \( B \) cone and the cylinder

\[ y_1^2 + z_1^2 = a_0^2. \]

Then the partial domains of \( x_i \) corresponding to a) become

\[ -q(a_0) \sim x_b, \quad x_b \sim x. \]
as illustrated in Fig. 3 a, b, c and Fig. 4.

Hence in this case will be

$$\Delta_l = \int \frac{d\gamma}{d\Omega} \left[ \int \frac{d\nu}{\nu} \int \frac{d\gamma'}{d\Omega'} \right] dS_2 \cos \theta d\theta \int R^2 dR + \int \frac{d\nu}{\nu} \int \frac{d\gamma}{d\Omega} \int R^2 dR \] .$$

§ 6 The integration of E when $0 \leq d \leq d_1$.

a) The integration of bright zone

The intersecting point of $Z'$ axis and $G$ sphere (shall be called $G'$ point) is within the bright zone, and so the integration domain of E is identical with § 2

$$E_0 = 2 \left[ \int A \int \frac{d\gamma}{d\Omega} \int R^2 dR + \int \frac{d\nu}{\nu} \int \frac{d\gamma}{d\Omega} \int R^2 dR \right] .$$

b) The integration of dark zone

In this case it becomes

$$E_0 = 2 \left[ \int A' \int \frac{d\gamma}{d\Omega} \int \frac{d\gamma'}{d\Omega'} \int R^2 dR + \int \frac{d\nu}{\nu} \int \frac{d\gamma}{d\Omega} \int R^2 dR \right] .$$

§ 7 The horizontal intensity due to secondary scattering when $0 \leq d \leq d_1$. Thus it becomes

$$D_2 = E_0 \Delta_0 + E_d \Delta_d .$$

§ 8 The integration of E when $d_1 \leq d \leq 2d_1$.

a) The integration of bright zone

In this case the integration of E is

$$E_0' = 2 \left[ \int A' \int \frac{d\gamma}{d\Omega} \int \frac{d\gamma'}{d\Omega'} \int R^2 dR \right] .$$

b) The integration of dark zone.

In this case

$$E_d' = 2 \left[ \int A' \int \frac{d\gamma}{d\Omega} \int \frac{d\gamma'}{d\Omega'} \int R^2 dR \right] .$$

§ 9 The horizontal intensity due to secondary scattering when $d_1 \leq d \leq 2d_1$. From above it becomes

$$D_2' = E_0' \Delta_0 + E_d' \Delta_d .$$

§ 10 The domain of T when $2d_1 \leq d \leq 4d_1$.

In this case all the portion of the total sky will not send secondary scattering.

The circle in which the shadow side of the cylinder $y^2 + z^2 = a^2$ and G sphere intersect shall be called D circle.
That is, the portion enclosed by a cone, guided by \( D \) circle and touching the earth, and total sky will not be indifferent to secondary scattering (ref. Fig. 5).

As for \( E \) outside this cone (called \( G \) cone), \( E \) will send out secondary scattering by receiving primary scattering from \( T \) in the portion (hatched portion in Fig. 5) of the solar-radiated domain bounded by the cylinder

\[
y^2 + z^2 = a_0^2
\]

and the sphere representing the atmospheric upper limit which is outside the cone touching the earth and having \( E \) as vertex. And the problem is to determine the region of \( E \) of this portion.

\( G' \) cone has the base angle

\[
\frac{\pi}{2} - 2d_1 \quad (\text{here } d_1 > 0)
\]

whose equation is

\[
y^2 + z^2 = (a_0 + (x_1 + q(a_0) \tan 2d_1))^2.
\]

Now let the axes \( x_1', y_1', z_1' \) be a new system of coordinates with its centre at \( E \) which is parallel to \( x_2, y_2, z_2 \) system (see the next figure 6 and 7).

For simplicity this being expressed by

<table>
<thead>
<tr>
<th>( x_2 )</th>
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<th>( z_2 )</th>
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<tbody>
<tr>
<td>( X )</td>
<td>( a_1 )</td>
<td>( m_1 )</td>
</tr>
<tr>
<td>( Y )</td>
<td>( a_2 )</td>
<td>( m_2 )</td>
</tr>
<tr>
<td>( Z )</td>
<td>( a_3 )</td>
<td>( m_3 )</td>
</tr>
</tbody>
</table>

Now, let \( x', y', z' \) be a new system of coordinates (see the next figure 6 and 7).

\begin{figure}
\centering
\includegraphics[width=\textwidth]{Fig5.png}
\caption{This figure makes us easy to understand the domain of integration with respect to \( T \) when the Sun's dip \( d \) is between \( 2d_1 \) and \( 4d_1 \), where \( d_1 = 6.404 \).
}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{Fig6.png}
\caption{Representation between \( X, Y, Z \) axes and \( x_2, y_2, z_2 \) axes.
}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{Fig7.png}
\caption{Representation between \( X, Y, Z \) axes, \( x_2, y_2, z_2 \) axes and \( x', y', z' \) axes.
}
\end{figure}
Define $\theta_1$, $A_1$ by

\[
\begin{align*}
x_1' &= r \sin \theta_1 \cos A_1, \\
y_1' &= r \sin \theta_1 \sin A_1, \\
z_1' &= r \cos \theta_1.
\end{align*}
\]

In this case the integration of $T$ shall be executed by this polar coordinate.

Let $x_2''$, $y_2''$, $z_2''$ system be produced by rotating $x_1'$, $y_1'$, $z_1'$ system by the angle $A_1$ around $z_2'$ axis.

Find the intersecting point of $G$ circle with $y_2'' = 0$, by solving both corresponding equations. But there is only a single point because $y_2'' = 0$ is one sided plane with it's end in $OE$, and let the value of $\theta_1$ given by this point be $\theta_1'$.

How about then the limits of $A_1$?

Find $A_1$ by expressing $D$ circle in $x_2'$, $y_2'$, $z_2'$ system and substituting

\[
\begin{align*}
\theta_1 = \theta_0 = \sin^{-1}\frac{\alpha_0}{e}, \\
r = e \cos \theta_0 + a_0 \tan(\theta_0 + \Delta_r)
\end{align*}
\]

in it and denote it by $A_1'$, $A_1''$, here it is so defined that the $y_1$ value corresponding to $A_1''$ is greater than that of $A_1'$. They are the limits required. For the domain from $A_1'$ to $A_1''$, the value of $r$, $n'$, of the intersected point of the cylinder

\[
y_1'' + z_1'' = a_0''
\]

by a line passing through $E$ can be determined by giving $\theta_1$ and $A_1$.

Then the integration with respect to $T$ for such region of $A_1$ is

\[
\Delta_3 = \int_{A_1'}^{A_1''} \int_{\theta_0}^{\theta_1'} \int_{r_0}^{r_1'} r^2 dr \, dS_z.
\]

For the rest domain of $A_1$ there is no integration.

§ 11 The integration of $E$ when $2d_1 \leq d \leq 3d_1$.

In this case $G'$ point is outside $G$ cone.

Let

\[
f(R, \theta, A) = 0
\]

be the equation of $G'$ cone expressed in $X' Y' Z'$ system by the method similar to the above mentioned. we can get $R$, the distance from $O'$ to the conical surface, in terms of $\theta$, $A$. Let it be $R_2$.

From $f(R, \theta, A) = 0$ and $Z' = 0$,

\[
\text{that is, by putting } \theta = 0 \text{ in } f = 0 \text{ and then } R = q(\alpha_0),
\]

we can get $A_0'$ representing the value $A$ of the intersecting point of horizontal line with the line in which the cone intersects with the horizontal plane. Then for $A > A_0'$, let $\theta_0'$ be the value of $\theta$ given by the intersecting point between two sections of the cone and the atmospheric upper limit by a plane containing $Z$ axis whose direction is defined by $A$, similarly to the above process.

Then the domain of $E$ is

\[
E_{\delta} = 2 \int_{A_0'}^{A_1'} \int_{\theta_0}^{\theta_1'} \frac{1}{R_2} \left( \frac{\pi}{2} \int_{0}^{\theta_1} \cos \theta \, d\theta + \frac{\pi}{2} \int_{\theta_0}^{\theta_1'} \cos \theta \, d\theta \right) \, dR_2.
\]

§ 12 The integration of $E$ when $3d_1 \leq d \leq 4d_1$.

In this case $G'$ point is inside $G$ cone. The integration of $E$ will be by using the same notation as in § 11, (but in the condition of using $\theta_0'$ for $0 < A < A_0'$).

\[
E_{\delta} = 2 \int_{A_0'}^{A_1'} \int_{\theta_0'}^{\theta_1'} \cos \theta \, d\theta \, dR_2.
\]

§ 13 Horizontal intensity due to secondary scattering when $2d_1 \leq d \leq 3d_1$.

From § 10 and § 11 the intensity will be

\[
D_{SS} = E_{\delta} \Delta_3.
\]

§ 14 Horizontal intensity due to secondary scattering when $3d_1 \leq d \leq 4d_1$.

From § 10 and § 12 the intensity will be

\[
D_{SS'} = E_{\delta'} \Delta_3.
\]

§ 15 The secondary scattering when $4d_1 \leq d$.
Δ₂ and the domain of E vanish just when d reaches 4d₁ and the intensity also vanishes.

4 The horizontal intensity of twilight

§ 16 The horizontal intensity when 0 ≤ d ≤ d₁.

From § 2 and § 7 the intensity will be

\[ D = D₁ + D₂. \]

§ 17 The horizontal intensity when \( d₁ ≤ d ≤ 2d₁. \)

From § 3 and § 9 the intensity will be

\[ D' = D₁' + D₂'. \]

§ 18 The horizontal intensity when \( 2d₁ ≤ d ≤ 3d₁. \)

In this case the first scattering will vanish. From § 13 \( D₂₁ \) is the intensity.

§ 19 The horizontal intensity when \( 3d₁ ≤ d ≤ 4d₁. \)

From § 14 \( D₂₂ \) is the intensity.

5 The method of evaluation and its result

The method was executed basing on Chapter 5 of (5), giving full attention to the lower altitude of the sun. Table 1 gives the intensity of first scattering from 1 steradian on each portion of the sky for \( d = 6.0, \ d = 6.404 \) (in which shadow cylinder passes through G' point) and \( d = 7.0. \) Here the figure in the bracket is the power of 10 to be multiplied to the left figure, for example 0.3325 \((-3)\) means 0.3325 \(\times 10^{-3}. \) As the above explanation may lead the reader to the misunderstanding as if the intensity were uniform all over that solid angle, so it is good idea to say that the above value multiplied by \( 10^{-6}. \) is the intensity of the same nature from a solid angle subtended by 1 mm² area on sphere with radius 1 meter.

Table 2 is the intensity of secondary scattering from 1 steradian, the figure in the bracket for each \( A \) being the same with that of \( A = 0. \)

Table 3 gives the sum of Table 1 and 2.

In general, secondary scattering is greater than the 1st, and the former becomes far predominant with decreasing \( p \) (the trans. coeff.) \( i, e, \) decreasing wave-length. Several conditions can be easily explained by Table 4 giving the ratio secondary : first scattering.

Table 5 is the radiation falling on a horizontal plane.

However, assuming the pure dry atmosphere there exists a relation between \( p \) and the wave-length \( \lambda \) (c.f., Linke: Meteor. Taschenbuch), so the values of \( \lambda \) corresponding \( p = 0.9, \ldots, 0.6 \) are given in the last column of the table.

The above tables except Table 4 are expressed in unit of the intensity of solar radiation for each of the corresponding wave-lengths received vertically by the upper atmospheric limit of the earth. Consider now the wave-length for \( p = 0.5, 0.9 \) represents respectively the domain

\[ 0 - 0.394, 0.394/\lambda - 0.419/\lambda, 0.419/\lambda - 0.488/\lambda, 0.488/\lambda - \infty. \]

The contributions of the intensities of above four domains to the total intensity of the sun, calculating from the table by LINKE's book, as follows 0.067, 0.030, 0.104, 0.799.

Summing up these values multiplied by the corresponding figures of Table 5, we can produce Table 6 representing the horizontal intensity in unit of the total intensity of the sun falling normally on the upper limit of the earth's atmosphere.

This is given for example by

\[ \frac{1.4 \times 10^3}{D^2} \text{ Lux}, \quad \frac{1.940}{D^2} \text{ g.cal.cm}^2 \text{.min etc.} \]

We can select any one of them as we desire. The horizontal luminosity expressed in Lux of 2nd column from 1st column of Table 6 is in good agreement with the observation by Mr. OOSAWA of 3rd column given in (6).

The energy among the wave-length range...
with 200 Å breadth with its centre at each of the above given wave-lengths (0.532…0.393/μ) is respectively 57.0, 58.0, 38.0, 36.3 (10⁻³ g. cal/cm².min), and their ratios to the sun’s total energy 1.940 g.cal/cm².min are respectively 0.0294, 0.0299, 0.0196, 0.0187.

Table 7 is the multiplication of Table 3 and these values. Therefore this is expressed in unit of the sun’s total energy, and tells us that the value is greatest at A = 0, smallest at A = 90 and secondary maximum at A = 180 for each θ and λ.

Table 8 is the value of Table 7 in unit of the value for θ = 0.9 in this table for each θ and A. According to this expression the energy decreases absolutely with decreasing wavelength for every (θ, A), and yet this value (i.e. the ratio of energy of θ = 0.8, 0.7, 0.6 to that of 0.9) increases exactly with increasing θ for each A and λ, and at θ = 0 the ratio increases with increasing A. But at other θ the ratio has the inclination of increasing with increasing A, but decreasing at A = 180 with increasing θ and decreasing λ besides the above one.

Moreover at θ = 0 the ratio decreases with increasing d for each A.

Table 9 is found from Table 3 by the same method of finding Table 6 from Table 5.

so the new table is expressed in Io/D² unit.

Fig. 8 is the graphical representation of Table 9 multiplied by 10⁷ with the curves of isophotens.

Table 1.

This gives the intensity of first scattering from one steradian on each portion of the sky dome for three dips (d) of the Sun. The figure in the blacket is the power of 10 to be multiplied to the left figure, for example 0.3325 (-3) means 0.3325 x 10⁻³. θ is the altitude and A the azimuth from the Sun, μ the transmission’s coefficient. The unit is the intensity of solar radiation received vertically at the upper atmospheric limit of the earth, for each wave-length corresponding to μ in table.

Literature

1) FössenKoFF, A. N. 220 (1923)
2) SMART, M. N. 93 (1933)
3) Hulburt, J. O. S. A. 28 (1938)
4) Arbid LjungWall, The intensity of twilight and its connection with the density of the atmosphere (Halsingborg. 1949)
This is the intensity of secondary scattering from one steradian on each portion of the sky dome for three dips (d) of the Sun. The figure in the black is the power of 10 to be multiplied to the left figure, (the value for each A being the same to that A = 0.) b) is the altitude and A the azimuth from the Sun, p the trans. coeff. The unit is the intensity of solar radiation received vertically at the upper atmospheric limit of the earth, for each wave-length corresponding to p in table.

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Table 2.
This gives the sum of table 2 and 3, i.e. the intensity of total scattering from one steradian on each portion of the sky dome for three dips (d) of the Sun. The figure in the blacket is the power of 10 to be multiplied to the left figure, the value for each wave-length corresponding to p in table. A being the same to that of A = 0, φ is the altitude and A the azimuth from the Sun, p the trans. coeff. The unit is the intensity of solar radiation received vertically at the earth's upper atmospheric limit, for each wave-length corresponding to p in table.
This gives the ratio of the value of first scattering to the secondary scattering. $\theta$ is the altitude and $A$ the azimuth from the Sun, $\Phi$ the trans. coeff., $d$ the Sun's dip.

<table>
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<tr>
<th>$\theta$</th>
<th>$\Phi \backslash d$</th>
<th>6.0</th>
<th>6.4</th>
<th>7.0</th>
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<td>0.6269(-5)</td>
<td>0.3665(-5)</td>
<td>0.1977(-6)</td>
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Table 4.

This is the radiation due to total scattering falling on a horizontal plane, in unit of the intensity of solar radiation received vertically at the earth's upper atmospheric limit, for each wave-length (\lambda) corresponding to the trans. coeff. $\Phi$ in the table. $d$ is the dip of the Sun. The figure in the blacket is the power of 10 to be multiplied to the left figure.

<table>
<thead>
<tr>
<th>$\lambda$ (in $\mu$)</th>
<th>6.0</th>
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<th>7.0</th>
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<td>0.2421(-5)</td>
<td>0.1490(-5)</td>
<td>0.0886(-5)</td>
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</table>

Table 5.

This is the intensity falling on a horizontal plane. First column is the calculated value in unit of the total intensity all over the wave-length of the Sun falling normally on the upper limit of the earth's atmosphere. Second column is the author's calculation in unit of Lux, and the 3rd column is the observed value by Oosawa in the same unit. $d$ is the Sun's dip. The figure in the blacket is the power of 10 to be multiplied to the left figure.

<table>
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<th>$d$</th>
<th>6.0</th>
<th>6.4</th>
<th>7.0</th>
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<td>1.9047(-5)</td>
<td>0.9312(-5)</td>
<td>0.3293(-5)</td>
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<td>0.8</td>
<td>1.27</td>
<td>0.74</td>
<td>0.46</td>
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<tr>
<td>0.7</td>
<td>1.36</td>
<td>0.75</td>
<td>0.43</td>
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Table 7.
This is the intensity of total scattering for 200 Å breadth with its centre at each wavelength corresponding to the value of \( \varphi \) in table from one steradian in unit of the sun's total intensity for all over the wave length falling normally at the earth's upper atmospheric limit. \( d \) is the sun's dip, \( \theta \) the altitude, \( A \) the azimuth from the sun, \( \varphi \) the trans. coeff. The figure in the bracket is the power of 10 to be multiplied to the left figure. The former value for each \( A \) being the same to that of \( A = 0 \).

<table>
<thead>
<tr>
<th>( d )</th>
<th>( \theta )</th>
<th>( \varphi )× ( A )</th>
<th>0</th>
<th>90</th>
<th>180</th>
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<tbody>
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<p>| ( \varphi \times ) | ( \theta ) | 0  | 90  | 180 |</p>
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Table 8.
This is the intensity of total scattering for 200 Å breadth with its centre at each wavelength corresponding to the value of \( \varphi \) in table from one steradian in unit of this value for \( \varphi = 0.9 \).

<table>
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<th>( \varphi \times )</th>
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<th>90</th>
<th>180</th>
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Table 9.
This is the intensity of the total scattering from one steradian on each portion of the sky dome for
the dip \( d = 6.0 \) of the sun, in unit of the total intensity of the sun all over the wave-length falling normally on the earth’s upper atmospheric limit \( \theta \) the altitude, \( A \) the azimuth from the sun. The figure in

<table>
<thead>
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<th>( A )</th>
<th>0</th>
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<th>60</th>
<th>90</th>
<th>120</th>
<th>150</th>
<th>180</th>
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<td>0.0388</td>
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<td>0.0030</td>
<td>0.0034</td>
<td>0.0045</td>
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<tr>
<td>30</td>
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<td>0.0668</td>
<td>0.0358</td>
<td>0.0238</td>
<td>0.0228</td>
<td>0.0266</td>
<td>0.0346</td>
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<tr>
<td>60</td>
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<td>0.2103</td>
<td>0.1781</td>
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<td>0.1504</td>
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<tr>
<td>90</td>
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<td>0.0034</td>
<td>0.0030</td>
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the bracket is the power of 10 to be multiplied to the
left figure, the former for every \( A \) being the same to
that of \( A = 0 \).