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Hydromagnetic Disturbances in the Earth's Core*

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Abstract

In the simple case, the small hydromagnetic disturbances in the conducting fluid are discussed in connection with the time-dependent parts of the inner geomagnetic field (namely, the non-dipole fields, secular variation and the westward drift). Owing to the strong stabilizations due to Coriolis factor and to the large toroidal field in the earth's core, it seems likely to be concluded that the direct sources of these time-dependent phenomena lie in the upper boundary layer of the core, even if we take account of the fluidity of the earth's core.

1 Introduction

With relation to the dynamo theory of the earth's main magnetic field (Elsasser [1], Bullard [2], Takeuchi-Shimazu [3]), it may be certainly an adequate interpretation that the origin of the secular variations and the non-dipole fields are associated with the eddy motions in the conducting earth's core (for example, Elsasser [4], [5], Runcorn [6], Inglis [7]).

Bullard [8, 9] has studied quantitatively the secular variation near the South Africa in terms of spinning-induction of a solid conducting sphere in the non-conducting matter and in applied magnetic field. From this investigation, the existence of strong inducing (toroidal) field in the earth's core was concluded.

Recently, this problem was attacked again by Herzenberg and Lowes [10]. Considering the leakage of current into surroundings, they discussed the non-dipole field and the secular variation on the standpoint of solid induction in the various cases. Due to the screening effect of the solid conductor, however, their results might not be successfully developed even in the assumption of considerable strong inducing field. In such circumstances, they suggested that the fluidity of conductor increases the skin depth of the disturbance extraordinarily since the motional induction acts in such a way as to reduce the self-induction.

With association of the stability of the stationary dynamo model, small hydromagnetic oscillations in the spherical core were studied by Rikitake [11, 12] and the importance of Coriolis force in the core was pointed.

In the present preliminary paper, the hydromagnetic disturbance in the crude model is discussed with relations to the time-dependent field in the earth's core.

2 Small Hydromagnetic Disturbances in the Semi-infinite Conducting Layer

Provided that the fluid is incompressible and the constants of matter (namely, $\sigma$

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the electrical conductivity, \( \nu \) the viscosity, \( \rho \) the density) are uniform, the equations relating \( \mathbf{H} \), the magnetic field, to \( \mathbf{V} \), the velocity in the uniformly rotating system, are

\[
\frac{\partial \mathbf{H}}{\partial t} = \nu_m \nabla^2 \mathbf{H} + \text{rot} \left[ \mathbf{V} \times \mathbf{H} \right],
\]

\[
\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} + 2 (\Omega \times \mathbf{V}) = (4 \pi \rho)^{-1} \text{rot} \mathbf{H} \times \mathbf{H} - \frac{1}{\rho} \nabla \left( \frac{v}{\rho} \right) \nabla^2 \mathbf{V},
\]

where \( \nu_m = (4 \pi \sigma)^{-1} \) is the magnetic viscosity, \( \Omega \) the angular velocity of rotating system, \( \Phi \) a scalar function including the pressure and the gravitational potential.

Representing \( \mathbf{H} = \mathbf{H}^0 + \mathbf{h} \) and \( \mathbf{V} = \mathbf{V}^0 + \mathbf{v} \) in which \( \mathbf{H}^0, \mathbf{V}^0 \) is the stationary solution corresponding to the dynamo field and \( \mathbf{h}, \mathbf{v} \) is the small deviation from the stationary field, and substituting to eq. (1), (2), the perturbation equations are

\[
\frac{\partial \mathbf{h}}{\partial t} = \nu_m \nabla^2 \mathbf{h} + \text{rot} \left( \mathbf{v} \times \mathbf{H} \right) + \text{rot} \left( \mathbf{V} \times \mathbf{h} \right),
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{V} + (\mathbf{V} \cdot \nabla) \mathbf{v} + 2 \Omega \times \mathbf{v} = (4 \pi \rho)^{-1} \left[ \text{rot} \mathbf{h} \times \mathbf{H} + \text{rot} \mathbf{H} \times \mathbf{h} \right] - \frac{1}{\rho} \nabla \left( \frac{v}{\rho} \right) \nabla^2 \mathbf{h},
\]

where \( \mathbf{h}, \mathbf{v} \) are the solenoidal vector, respectively.

We shall apply the perturbation equations to the case of the small disturbances in the earth's core under the simple model.

In Cartesian-coordinates \( x, y, z \), suppose that the region \( z > 0 \) is occupied by the conducting fluid (the earth's core), while the region \( z < 0 \) consists in non-conducting solid (the mantle). Taking account the BULLARD' s model of the stationary dynamo in which \( T \times \mathbf{H} > \mathbf{v}, S \), \( \pi \mathbf{H} \times \mathbf{H} \) (BULLARD [2, 13], HIDE [14]), we have

\[
\mathbf{H} = (H_s, 0, H_T) \quad \text{and} \quad \mathbf{V} = (\varepsilon, 0, 0),
\]

where \( H_s \) stands for the radial component of the dipole field, \( H_T \) the \( T \times \mathbf{H} \)-toroidal field and \( \varepsilon \) the westward drift motion.

For the simplicity, in the subsequent discussion we restrict the problem to the case that all the field quantities are uniform along the \( y \)-direction. In such a case, we have the following solenoidal vector solutions

\[
\mathbf{h} = \left\{ \begin{array}{l}
\frac{\partial P}{\partial x} \frac{dS}{dz} \\
- \frac{\partial P}{\partial x} \frac{T}{\lambda^2 P S} \\
\lambda^2 P S \end{array} \right\}
\]

\[
\mathbf{v} = \left\{ \begin{array}{l}
\frac{\partial P}{\partial x} \frac{dU}{dz} \\
- \frac{\partial P}{\partial x} V \\
\lambda^2 P U \end{array} \right\}
\]

where \( P(x, t) \) is satisfied by the equation

\[
\frac{\partial^2 P}{\partial x^2} + \lambda^2 P = 0
\]

and \( \lambda \) is a positive real number and \( S, T, U \) and \( V \) are scalar functions of \( z \) corresponding to the poloidal, toroidal part respectively.

Taking

\[
P(x, t) = e^{i\lambda(x+ct)}
\]
and substituting the elementary solutions (6), (7) to eq. (3), (4), and after some reductions, we have the following simultaneous differential equation satisfied by $S$, $T$, $U$ and $V$

$$[\lambda (c+\epsilon) + i \nu_m \left( \frac{d^2}{dz^2} - \lambda^2 \right)] \left( \begin{array}{c} S \\ T \\ U \\ V \end{array} \right) + i H_s \left( \begin{array}{c} \frac{d}{dz} + i \lambda H \\ \frac{d}{dz} + i \lambda H \end{array} \right) \left( \begin{array}{c} U \\ V \end{array} \right) = 0, \quad (10)$$

$$\left(4\pi \rho \right)^{-1} H_s \left( \frac{d}{dz} + i \lambda H \right) T - 2 \Omega \frac{dU}{dz} - i \left[ \lambda (c+\epsilon) + i \nu \left( \frac{d^2}{dz^2} - \lambda^2 \right) \right] V = 0, \quad (11)$$

$$\left(4\pi \rho \right)^{-1} H_s \left( \frac{d}{dz} + i \lambda H \right) \left( \frac{d^2}{dz^2} - \lambda^2 \right) - i \lambda H'' S + 2 \Omega \frac{dV}{dz} - i \left[ \lambda (c+\epsilon) + i \nu \left( \frac{d^2}{dz^2} - \lambda^2 \right) \right] \left( \frac{d^2}{dz^2} - \lambda^2 \right) v'' U = 0, \quad (12)$$

where $H = H_T/H_s$ and $H''$, $\epsilon''$ stands for the second derivatives of $H$, $\epsilon$ with respect to $z$, respectively.

Although the rotation in the core is non-uniform (then also toroidal field) from the standpoint of the dynamo theory, only the case in which the stationary fields are uniform is treated here for the convenience of the treatment. Then, eliminating $S$, $T$ and $V$ from eq. (10), (11) and (12), we have

$$[V_S^2 \left( D + i \lambda H \right)^2 + \lambda (c+\epsilon) \left( \frac{d^2}{dz^2} - \lambda^2 \right)] \left( \lambda (c+\epsilon) \right) + i \nu_m \left( D^2 - \lambda^2 \right) U \left( \frac{d^2}{dz^2} - \lambda^2 \right) U = 0, \quad (13)$$

where $V_S^2 = H_s^2 / 4 \pi \rho$, $D = \frac{d}{dz}$.

Since the viscous dissipation is negligibly small compared with the Joule loss (Bullard [15], Runcorn [16], Hide [14]) in the earth's core, we neglect the viscous term in the subsequent discussion. Furthermore, it appears likely that the radial velocity is considerably smaller than the lateral components in the upper boundary region of the core, then we have approximately $\left| D^2 \right| > \lambda^2$ from eq. (7). Therefore, eq. (13) becomes

$$[\left( V_S^2 \left( D + i \lambda H \right)^2 + \lambda (c+\epsilon) \left( \frac{d^2}{dz^2} - \lambda^2 \right) \right) \left( \lambda (c+\epsilon) \right) + i \nu_m \left( D^2 - \lambda^2 \right) \right] U \left( \frac{d^2}{dz^2} - \lambda^2 \right) U = 0, \quad (14)$$

where $K_1$, $K_2$ is constant determined from the boundary conditions, but by the condition at infinity, $K_1$ must vanish. The solution of eq. (14) in the semi-infinite region $z > 0$ is

$$U \left( z \right) = A e^{kt} + B e^{kt} + C, \quad (15)$$

where $A$, $B$ and $C$ are complex numbers and $R \lambda_{1,2}$ are negative. Similarly, there are

$$V \left( z \right) = \overline{A} e^{kt} + B e^{kt}, \quad \overline{A} = -i k_1 A, \quad B = ik_2 B, \quad (16)$$

$$T \left( z \right) = \overline{A} e^{kt} + B e^{kt}, \quad \overline{A} = -i k_1 A, \quad B = ik_2 B,$n

$$S \left( z \right) = A^* e^{kt} + B^* e^{kt} + C^*,$n

and the every constant coefficient may be represented with $A$, $B$ and $C$ by eqs. (10),
$$H = D e^{\lambda z} \left( \frac{\partial P}{\partial x}, 0, \lambda P \right),$$

where $D$ is a constant.

From the boundary conditions that the magnetic field is continuous and the vertical motion vanishes at $z=0$, we have the following relation

$$\left\{ \begin{array}{l}
(k_1 - \lambda) A^* + (k_2 - \lambda) B^* + \lambda C^* = 0, \\
\vec{A} + \vec{B} = 0, \\
A + B + C = 0.
\end{array} \right.$$  \hfill (18)

Representing $A^*$, $B^*$, etc. with $A$, $B$ and $C$, then the determinant equation is

$$\begin{vmatrix}
(k_1 + i \lambda H) & (k_2 + i \lambda H) & \lambda H \\
\lambda (c+\epsilon) + i \nu_m k^2_1 & \lambda (c+\epsilon) + i \nu_m k^2_1 & \lambda H \\
\lambda (c+\epsilon) + i \nu_m k^2_1 & \lambda (c+\epsilon) + i \nu_m k^2_1 & 0
\end{vmatrix} = 0$$  \hfill (19)

that is,

$$\frac{(k_1 + i \lambda H) (k_2 + i \lambda H)}{\lambda (c+\epsilon) + i \nu_m k^2_1} \begin{vmatrix}
(k_1 + i \lambda H) & \lambda H \\
\lambda (c+\epsilon) + i \nu_m k^2_1 & \lambda H
\end{vmatrix} - \frac{\lambda H}{c+\epsilon} \begin{vmatrix}
k_1 (k_1 + i \lambda H) & k_2 (k_2 + i \lambda H) \\
\lambda (c+\epsilon) + i \nu_m k^2_1 & \lambda (c+\epsilon) + i \nu_m k^2_1
\end{vmatrix} = 0.$$  \hfill (20)

Since the condition $2Q > \lambda |c+\epsilon|$, may be valid for the time rate of the secular variation and for the lateral dimension of the non-dipole field, $k_1$ and $k_2$ are approximately

$$k_1 = -\frac{\lambda H (\delta + i)}{\delta^2 + 1} \left\{ 1 + \left[ \left( 1 - \frac{2Q (c+\epsilon)}{\lambda V^2} \right) (1+i\delta) - 1 \right]^{1/2} \right\},$$

$$k_2 = -\frac{\lambda H (-\delta + i)}{\delta^2 + 1} \left\{ 1 + \left[ \left( 1 + \frac{2Q (c+\epsilon)}{\lambda V^2} \right) (1-i\delta) - 1 \right]^{1/2} \right\},$$  \hfill (21)

by eq. (14), where

$$\delta = \nu_m \gamma/V^2, \quad \gamma^2 = 4Q^2 + \lambda^2 (c+\epsilon)^2.$$  \hfill (22)

Then principally, we may be to obtain an eigen value $\epsilon$ in terms of the parameters $H$, $\delta$ and $\lambda$ from eqs. (20) and (21), that is

$$\epsilon + \epsilon = f (\lambda, H, \delta) + i g (\lambda, H, \delta),$$  \hfill (23)

but in fact, there is left a tedious numerical calculations, particularly in the conditions of the earth's core. As it is shown in eq. (21) that $I_{n,k_{1,2}}$ does not equal to $\lambda H$ in general, the direction of propagation differs from that of the lines of force.

3 Discussion

As the appropriate constants in the earth's core, we adopt the following values now, $Q=7.3 \times 10^{-5}$ sec$^{-1}$, $\rho=10$ gr/c.c., $\sigma=3 \times 10^{-6}$ e.m.u., $H_z=4$ gauss, $\epsilon_{\max}=4 \times 10^{-2}$.
cm/sec and apply the results of the last section to the secular variations with the time rate of 200 years which is a typical one in the spectrum of the time variation of the secular changes. Then, from eq. (21) the attenuation depth of the small disturbances in the vertical direction are determined and are shown in table 1 for the case in which the disturbance has the lateral dimensions of 1000 km and 500 km, respectively. For references, the skin depth corresponding to the case of solid induction and of that $Q=0$ are also shown.

<table>
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<th>$H$</th>
<th>$L=1000$ km</th>
<th>$L=500$ km</th>
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<td>$k_1$-wave</td>
<td>$k_2$-wave</td>
</tr>
<tr>
<td>1000</td>
<td>1.7 km</td>
<td>1.0 km</td>
</tr>
<tr>
<td>100</td>
<td>17</td>
<td>9.9</td>
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<tr>
<td>10</td>
<td>95</td>
<td>84</td>
</tr>
<tr>
<td>1</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>0</td>
<td>75</td>
<td>72</td>
</tr>
<tr>
<td>$(Q=0)$ (metal)</td>
<td>$4 \times 10^7$ km</td>
<td>$70$ km</td>
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From this table, it may be expected that the hydromagnetic disturbances with the time rate longer than several hundred years can propagate through the fluid core without any appreciable attenuations, provided that there is no predominance of Coriolis force (HERZENBERG and LOWES [10]). But Coriolis force is the most dominant one in the fluid motions of the core (for exp., RUNCION [16], HIDE [14]) and its effect to the hydromagnetics cannot be neglected.

In the simple case (the rotational axis coincide with that of uniform field), the attenuation depth of plane wave propagating to the direction of field becomes 75 km and is the same order of that of solid induction. This decrease of the skin depth in the predominance of Coriolis factor ($\omega$: angular frequency of the wave $\leq Q$: that of the rotation of the whole body) is to be attributed to the stabilizing effect of Coriolis force (CHANDRASEKHAR [17]). In the above-mentioned simplest case, the disturbance is a transverse plane wave (velocity vector associated with the disturbance is normal to $Q$), while in the limit of zero viscosity, the fluid particles attach to the vortex line (the axis of $Q$) permanently, then motions at right angles to $Q$ become increasingly difficult. In other words, Coriolis force possesses an effect deflecting the motion to the direction at right angles to both $Q$ and the original motion, and as pointed out by LEHNERT [18], when the axis of the rotation and the direction of field coincide, the wave consists in two circularly polarized, transverse waves with the field vectors rotating in opposite sense (as appeared in eqs. (16) and (21), two elliptically polarized waves in the present case). On the viewpoint of the induction effect, therefore, the rotational period of this polarization being shorter than that of the disturbance ($\omega$), is apparently replaced in the formula.

Taking account the $T_a$-field (in the present case, the oblique, uniform external field), furthermore, the attenuation depth associated with the longitudinal infinite rolls with
wave length of 1000~500 km becomes of the order of 1 km under the strong field $H_T \approx 4 \times 10^3$ gauss and then, the disturbance cannot almost propagate in the vertical direction.

When the disturbances propagate at right angles to the field, field acts only in terms of the induction drag. The larger the field intensity and the smaller the lateral dimension of disturbance are, the stronger the stabilizing effect of field (if the disturbance is uniform in the direction of filed, no stabilizing effects of filed). The measure of this stabilizing effect of field is given in terms of the following conditions; $(L/T)^2/V_T^2 > 1$ or $< 1$, where $L$ is the lateral dimension of the disturbance, $T$ its period and $V_T = H_T/(4\pi \rho)^{1/2}$ the velocity of Alfvén wave propagating to the direction of field $H_T$. In the former case, the effects of the induction drag due to field are negligible and the phenomena are similar to the diffusion in the metal conductor. While in the latter case, there is a strong drag. If we take the values of $H \approx 1$, $L \approx 1000$ km and $T = 200$ years, then $(L/T)^2/V_T^2$ becomes of the order of $10^{-4}$ ($< 1$).

Considering the longitudinal rolls with the wave length of 1000 km in eq. (20), under the conditions that $|\lambda (c + \epsilon)| < 2\Omega$ and $2\Omega |c + \epsilon| < V_T^2$, we have approximately $c + \epsilon \approx -(\xi_1 + \xi_2) \nu_m(\lambda H)^{3/2}/(1 + i)\xi_1 + i\xi_2$ where $\xi_{1,2} = k_{1,2}/\lambda H$ are complex non-dimensional numbers and the order of $10^{-4}$. As roughly shown that $I_m(c + \epsilon)$ is the order of $2\xi^2 \nu_m(\lambda H)^{3/2}/(1 + 4\xi^2) > 0$, where $\xi \approx (\delta - 1)(\delta/2)^{1/2}(1 + \delta)^{-1}$, in such circumstances, it appears that the small disturbances are substantially stable under the strong toroidal field.

From the above-mentioned point of view, it cannot be impossible to interpret that the source of the secular variations, the non-dipole fields observed at the surface lie directly in the deep interior of the core where the strong toroidal field might be exist, even if we take into account the fluidness of the core. Therefore, such eddies smaller than to stationary dynamo pattern must be considered as the disturbances in the upper boundary layer of the core where toroidal field is rather weak and the large velocity shears exist. It must, however, not be to consider that there is no relations between the disturbances in the upper boundary layer and the large convective pattern in the core, since it seems that in the boundary layer, there are the sources (the upward flow from the interior) and the sinks (the return current to the interior) of the flow associated with the convective large pattern if the core is in the state of convective equilibrium, and there is a left possibility that the disturbance will develop near such regions. In this connection, it is interesting to study quantitatively the stability of small disturbances in the boundary layer of the core.

Analysing VESTINE's data, BULLARD et all [15] obtained the mean velocity of the westward drift of the order of $0.18^\circ$/year (non-dipole field), $0.32^\circ$/year (secular variation) and interpreted this phenomena in terms of their three body model. If the direct source of the secular variations lies in the boundary layer of the core as discussed above, then $z_0/L > 1$ where $z_0$ is the depth of the boundary layer (say, 100 km), $L$ the lateral dimension of the eddy (say, 1000 km) and the ratio $z_0 \epsilon / \nu_m L$ of the first to the second term in the right in eq. (1) is the order of $10^{-4}$ for $\epsilon \approx 10^{-2}$ cm/sec. It is, therefore, not likely that the eddies associated with the secular change is drifted with the main flow permanently. Furthermore, because of the hypothesis (VESTINE [18], TAKEUCHI-
Elsasser [19] who attribute the negative correlation between the fluctuations of the westward drift and the rate of the earth's rotation to the electromagnetic torque acted on the boundary between the mantle and core seems likely to be adequate interpretation, there are some questions in the process connecting the order of the velocity of the westward drift to that of the fluid motions in the core directly.

Thus far, it has been concluded that the small hydromagnetic disturbances cannot grow to the unstable eddies under the strong toroidal field. But if we introduce the so strong unstable factor (say, thermal origin ?), it appears that the disturbance overcomes the stabilizing effect due to the field and drags the part of the field with it. The state of the fluid motions of the core might be probably intermediate between the large scale convection as Bullard's model and the random turbulent eddies, since no eddies smaller than the critical dimensions can exist in the core under the strong stabilization due to the toroidal field and Coriolis force.

There is a possibility that the eddy smaller than the stationary dynamo pattern overcomes the rigidity of the toroidal field and rises to the upper boundary of the core under the aid of the action due to the local concentration of the heat sources. However, such a disturbance has essentially a character of non-linear and cannot be discussed quantitatively. Here, we shall only estimate the approximate order of the quantities for the simple case.

If the part of the conducting fluid (say, a disk of the diameter $L$ and the thickness $d$) suffers local heating and has the temperature difference $\delta T$ compared with surroundings under the horizontally stratified field, this part will be accelerated at right angles to the field by the buoyancy force $\rho g \alpha \delta T$ per unit volume, where $\rho$ is the density, $g$ the acceleration of gravity and $\alpha$ the coefficient of thermal expansion, but the upward motion is prevented by the rigidity of field until the temperature difference between the heating part and its surroundings becomes so sufficiently large that the buoyancy force overcomes the stabilization of field. Thus, such upward motion not arise continuously but suddenly, that is, the thermal convection in Wale's term [21, 22].

If there is the flow of conducting fluid across the field, the distribution of the lines of force is changed since the flow carries more or less the part of the lines of force with it. When the part of the lines of force with the lateral dimension $L$ is distorted by the flow across to it and displaces to upward by distance $D$, the electromagnetic force, of which order of magnitude is approximately $H^2D/4\pi L^2$ by Wale [22], is produced and has a tendency to prevent the further flow again.

The possibility of the thermal convection in the earth's core was studied by Bullard [23, 13] in connection with the dynamo theory, and its results (including the other physical constants of the core) settled in his paper (Bullard and Gellman [21]). After his investigation, the electromagnetic force is same order with Coriolis force and the order of $10^{-6}$ dyne/c.c. in the stationary dynamo process due to the large scale convection. Since if the conservation of energy is confirmed, these forces are equal to the buoyancy force, we have the temperature difference $3 \times 10^{-4}$C between the up and down ward flow in the stationary stage, when $\rho$ is 10.7 gr/c. c., $\alpha = 4.5 \times 10^{-4}$ deg$^{-1}$ and $g = 800$ cm/sec$^2$. In order to be unstable the small pattern of fluid mass in the
equilibrium state of the large scale convection, the temperature difference $\delta T$ between the local heating mass and its surroundings must be larger than this order at least. When the part of the fluid rises upward by a distance $D$ in the process of local heating and then balances in the state of the existence of electromagnetic force due to its displacement, we have approximately $H^2 = 4\pi pg L^3 \delta T / D$. Take the typical values of $L = 10^8$ cm, $D = 10^7$ cm and $\delta T = 3 \times 10^{-3}$°C, then $H \approx 500$ gauss.

From this estimation, there may be a possibility that the strong field pattern is carried into the boundary layer of the core by the thermal convection. Since Coriolis force may play an important role in this case, the upward flow may be accompanied with the local rotational motion and then, it may be expected that the poloidal part of the distorted field is produced.

From the relation of that $2\Omega v_{\max} \approx g z \delta T$, the maximum rising velocity $v_{\max}$ is the order of $8.6 \times 10^{-2}$ cm/sec and the mean velocity $v = v_{\max} / 2$ is $4.3 \times 10^{-3}$ cm/sec. The time required to rising upward by distance $D$ is approximately $D / v \approx 40$ years. The deformed pattern of field associated with local rising diffuses to the uniform field by the process of Joule dissipation. If the disturbed region of field lies over that of the sphere of radius $a$, the decay time of the distorted field is the order of $\tau = 4\sigma a^2 / \pi \approx 300$ years, where $a = L / 2 = 500$ km and $\sigma = 3 \times 10^{-6}$ e.m.u.

In the above discussions, the heat transfer due to the process of conduction was not considered. If we take account this process, the rate of the heat-production necessary to produce the temperature difference as previously mentioned, may be estimated approximately as follows. The heat flow $F$ transferred from the disk with the radius $L / 2$, thickness $d$, to its surroundings by the conduction is $F = F_i + F_v$ where $F_i = q \beta_i S_i$ and $F_v = q \beta_v S_v$ are the flows from the side of and the end of the disk and $S_i$ the area of the side, $S_v$ the crosssection of the end of the disk, $\beta_i$ the temperature difference between the disk and its surroundings, $\beta_v$ the mean vertical lapse rate of the temperature in the disk and $q$ the conduction coefficient of heat. If we replace $\beta_v$ to the mean adiabatic lapse rate $\beta_{ad}$ in the core and use the values of $q = 0.1$ cal/cm. deg. sec., $\beta_{ad} = 2 \times 10^{-6}$ deg./cm after Bullard, and take the typical values of $L = 1000$ km, $d = 10$ km, then the total flow $F$ from the disk is roughly $1.2 \times 10^{11}$ cal/sec. The minimum rate of heat production producing the thermal convection is $F / M$ and $1.5 \times 10^{-14}$ cal/gr. sec., where $M$ is the mass of the disk. This value of heat production is about 10 times that obtained by Bullard $1.6 \times 10^{-15}$ cal/gr. sec. for the whole core.

4 Conclusion

Under the simple case, the small hydromagnetic disturbances in the earth's core were discussed with connect to the secular variations. From this results, it was shown that such small disturbance cannot be unstable under the strong toroidal field, and the attenuation of small vertically propagating hydromagnetic wave is unexpectedly large in the condition of the predominance of Coriolis force, and furthermore, little disturbance can propagate across to the strong $T_2$-field.

Considering such circumstances, it seems likely that the direct source of the secular change of field lies in the upper boundary layer of the core. In this connection, the
problem determining the stability-criterion of the hydromagnetic disturbances in such layer seems to be very interesting.

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