修士学位論文要約（平成29年３月）

２量子ビット一般化量子測定のためのハイパー量子もつれ光子源の開発

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Design of an hyperentangled photon source for two-qubit generalized quantum measurements

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Long thought to be nothing more than mere Gedankenexperiment, variable-strength quantum measurements and weak measurements have been successfully implemented experimentally for a single qubit in a wide range of situations in the recent years. Yet, the extension to the two-qubit case proved to be a challenge, due to nonlocality and causality restrictions. In this work, we demonstrate progress towards deterministic generalized 2-qubit measurement with a scheme for measuring nonlocal spin product operators. This novel method only requires an additional pair of maximally-entangled qubits, and yields the expected statistics for a genuine generalized two-qubit measurement. A polarization/time-bin hyperentangled photon pair source suitable for the linear-optical implementation of this scheme is proposed along with results obtained so far.

1. Introduction

The notion of measurement plays an important role in quantum theory as it lies at the heart of the problem of its interpretation. As a mean to solving these fundamental issues while broadening experimental possibilities, the theory of generalized quantum measurements was developed, effectively extending the notion of quantum measurements beyond standard textbook projective measurements.

In this new framework, a measurement is defined as an interaction between two quantum states, respectively called the system $|\psi\rangle$ (a property of which we wish to measure) and the meter (initially prepared in a specific state $|m_0\rangle$, which we will use to measure the system), followed by a projective measurement on the meter, in order to read out the result of the measurement.

In order to quantitatively analyze such a process and its impact on the measured system, some mathematical tools are required, one of the most useful being the Positive Operator Valued Measure (POVM) of the measurement. The POVM is a set of operators $\{F_\alpha\}$ which fully describe the statistics of a measurement process, since they allow one to compute probabilities for the measurement outcomes in the following way :

$$P(\text{result } \alpha) = \langle \psi | F_\alpha | \psi \rangle$$

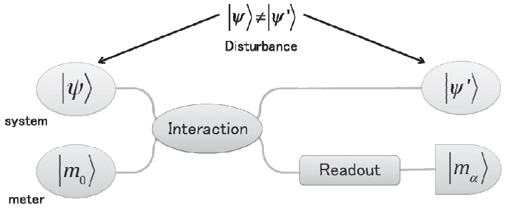


Figure 1: Schematic representation of a generalized quantum measurement.

2. A measurement scheme for the two-qubit case

This approach has been very successful in the one-qubit case, where the following POVM,

$$F_{\pm 1} = \frac{1}{2}\left(1 \pm \cos(2\theta)\sigma_z\right) \quad (1)$$

proposed by Lund and Wiseman in [1], was optically implemented to experimentally test the Error-Disturbance Relations for a polarization qubit [2].

Yet, the extension of this process to the case of a system composed of two qubits proved to be a both a theoretical and experimental challenge, due to nonlocality and causality restrictions. Causality indeed forbids a single local meter to interact with a nonlocal system spread over a space-like distance.

In this work, we show that using an additional shared maximally-entangled state as a ”nonlocal meter” and applying the following scheme yields the expected behaviour of a genuine nonlocal measurement for two qubits, in a non-instantaneous but completely deterministic fashion.

More precisely, if the meter is in the initial state :

$$\left|m_0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|00\right\rangle + \left|11\right\rangle\right) \quad \text{(Bell state)}$$

then the POVM of the process depicted in Figure 2 has the form :

$$F_{\pm 1} = \frac{1}{2}\left(1 \pm \cos\left(2\theta\right)\sigma_{z1}\sigma_{z2}\right)$$

which is the straightforward 2-qubit generalization of the POVM (1).

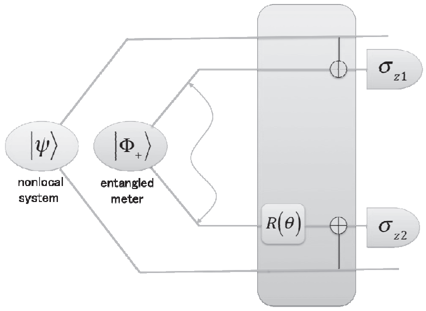
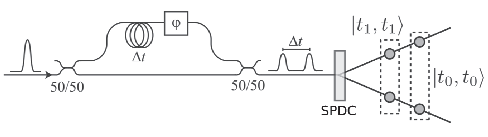


Figure 2: Proposed scheme for two-qubit generalized measurement, here for the measurement of the spin product $\sigma_{z1}\sigma_{z2}$.

3. Linear optical implementation via an hyperentangled photon source

The experimental implementation of this scheme is feasible in a linear optical fashion, using hyperentangled photonic states for instance. Here, we propose a time-bin/polarization hyprentangled photon source consisting of an unbalanced interferometer placed before a non-linear crystal in which Spontaneous Parametric Down Conversion (SPDC) occurs. As shown in Figure 3, the resulting state is expected to be entangled in both degrees of freedom and the entanglement in one degree of freedom (e.g. polarization) can then be used as a meter to measure the entanglement in the other degree of freedom (e.g. time-bin).

Such a source is currently being assembled, and results for the SPDC spectrum in a Periodically-Poled Lithium Niobate (PPLN) waveguide are presented in Figure 4.



$$\left|\Psi\right\rangle = \frac{1}{2}\left(\left|t_0, t_0\right\rangle + e^{i\theta}\left|t_1, t_1\right\rangle\right) \otimes \left(\left|H, V\right\rangle + \left|V, H\right\rangle\right)$$

nonlocal system (to be measured)　　nonlocal meter

Figure 3: *(up)* Hyperentangled time-bin/polarization state generation using an unbalanced interferometer and SPDC in a nonlinear crystal.
(down) Resulting hyperentangled final state.

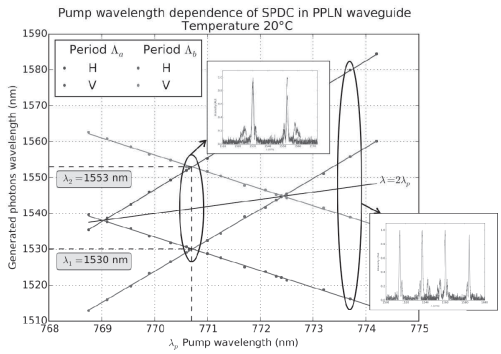


Figure 4: Pump wavelenght dependence of the SPDC spectrum in PPLN. Photons created at degenerate wavelengths (around 1530 nm and 1553 nm) are expected to be polarization-entangled.

4. Conclusion

The measurement scheme presented here, along with the proposed hyperentangled photon source pave the way towards genuine generalized two-qubit measurements. Experimental realization of such measurements would count as important technical progress, with some important nonlocal observables becoming completely measurable with arbitrary strength (e.g. Bell measurement). But this would also allow us to explore furthermore the foundations of quantum mechanics, via experiments such as testing Error-Disturbance Uncertainty Relations in the nonlocal case, a domain where causality, quantum uncertainty and nonlocality are the most intertwined.

References

1) A. P. Lund and H. M. Wiseman, *New Journal of Physics* **12**, 093011 (2010).
2) S.-Y. Baek, F. Kaneda, M. Ozawa, and K. Edamatsu, *Scientific reports* **3**, 2221 (2013).