Adaptive Filtering With Averaging-Based Algorithm for Feedforward Active Noise Control Systems

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***Abstract*—This letter proposes an adaptive filtering with averaging-based algorithm for active noise control (ANC) systems. This algorithm uses a similar structure as that of the FxLMS-based ANC system. The proposed algorithm, called Filtered-x Adaptive Filtering with Averaging (FxAFA) algorithm, uses averages of both data and correction terms to find the updated values of the tap weights of the ANC controller. The computer simulations are conducted for single-channel feedforward ANC systems. It is shown that the proposed algorithm gives fast convergence as compared with the FxLMS algorithm and achieves better performance in the presence of the measurement noise. The comparison with the FxRLS algorithm shows that the proposed FxAFA algorithm is a better choice for low computational complexity and stable performance.**

***Index Terms*—Active noise control, adaptive filters, averaging, FxLMS algorithm.**

I. INTRODUCTION

ACTIVE NOISE CONTROL (ANC) [1] is based on the simple principle of destructive interference of propagating acoustic waves. The most popular adaptation algorithm used for ANC applications is the FxLMS algorithm which is a modified version of the LMS algorithm [2]. The schematic diagram for a single-channel feedforward ANC system using the FxLMS algorithm is shown in Fig. 1(a). Here, $P(z)$ is primary acoustic path between the reference noise source and the error microphone. The reference noise signal is filtered through $P(z)$ and appears as a primary noise signal at the error microphone. The objective of the adaptive filter $W(z)$ is to generate an appropriate antinoise signal $y(n)$ propagated by the secondary loudspeaker. This antinoise signal combines with the primary noise signal to create a zone of silence in the vicinity of the error microphone. The error microphone measures the residual noise $e(n)$, which is used by $W(z)$ for its adaptation to minimize the sound pressure at error microphone. Here $\hat{S}(z)$ accounts for the model of the secondary path $S(z)$ between the output $y(n)$ of the controller and the output $e(n)$ of the error microphone. The filtering of the reference signal $x(n)$ through $\hat{S}(z)$ is demanded by the fact that the output $y(n)$ of the adaptive filter is filtered through $S(z)$ [2].

The FxLMS algorithm is computationally simple, but its convergence speed is slow. Different ANC algorithms, with

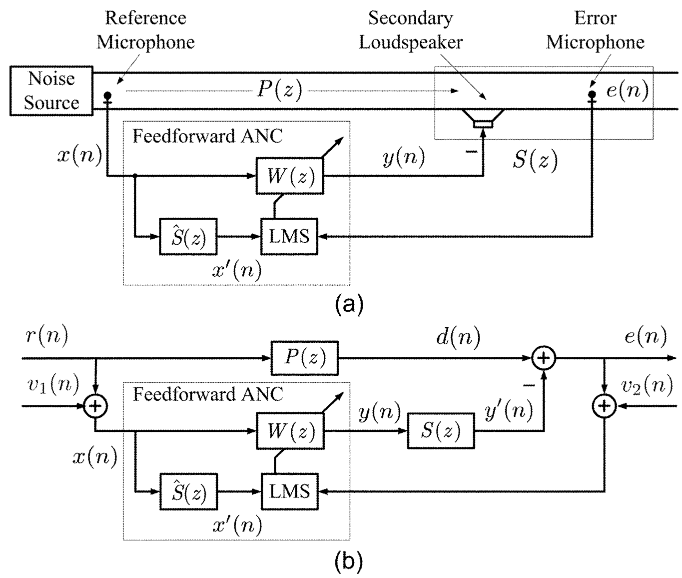

Fig. 1. FxLMS algorithm based single-channel feedforward ANC system. (a) Schematic and (b) block diagram.

improved convergence properties, have been proposed, viz., 1) lattice-ANC systems [3]; 2) IIR-filter-based LMS algorithms called Filtered-u Recursive LMS (FuRLMS) [4], and filtered-v algorithms [5]; 3) RLS-based algorithms called Filtered-x RLS (FxRLS) [1] and Filtered-x Fast-Transversal-Filter (FxFTF) [6]; and 4) frequency-domain-ANC systems (see [7] and references there in). There are the following problems with these approaches. 1) IIR-based structures have inherent stability problems; 2) other approaches mentioned above increase the computational burden substantially; and 3) RLS-based ANC systems have numerical instability problems. These reasons make FxLMS still a good choice for ANC applications.

Another important issue that often arises in the ANC systems is measurement noise in the reference signal (and error signal as well) present due, for example, to airflow over the microphone in the duct. It can be shown [1] that due to the measurement noise in the reference signal, the controller does not converge to the optimal solution. Further more, due to the measurement noise in the error signal, the overall convergence speed is degraded. Therefore, it is necessary to find an efficient method to improve the performance of the ANC systems in the presence of the measurement noise.

The main idea in this letter is to accelerate the convergence speed of the FxLMS algorithm, and to improve the performance in the presence of the measurement noise. The method, we use for improving the performance, is *averaging*. The idea of using

Manuscript received July 16, 2003; revised October 3, 2003. This work was supported by the Monbokagakusho Government of Japan. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Darren B. Ward.

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Digital Object Identifier 10.1109/LSP.2004.827958

averaging to accelerate the convergence of the stochastic gradient algorithms was originally proposed in the stochastic and optimization literature [8], [9]. Later, this idea was extended to adaptive filters [10], [11]. Recently, averaging-based adaptive algorithms have been proposed for blind multiuser detections in Direct Sequence/Code division multiple access (DS/CDMA) systems [12], [13].

In this letter, we explore the realization of an ANC algorithm based on adaptive filtering with averaging (AFA). The proposed algorithm is called filtered-x AFA, FxAFA, algorithm. The FxAFA algorithm has slightly increased computational complexity as compared with the FxLMS algorithm. The computer simulations are conducted for single-channel feedforward ANC systems. The simulation results show that the proposed FxAFA algorithm provides better performance than the FxLMS algorithm.

The organization of this letter is as follows. In Section II, the proposed FxAFA algorithm is explained in connection with the FxLMS algorithm and some properties of the proposed algorithm are discussed. In Sections III details of the computer experiments are given, and in Section IV concluding remarks are presented.

II. Averaging Based Filtered-x Algorithm

A. Algorithm Development

Fig. 1(b) shows the block diagram of the feedforward ANC system of Fig. 1(a). Here, $v_1(n)$ and $v_2(n)$ are measurement noise signals associated with the reference and error microphones, respectively. We make the following assumptions for $v_1(n)$ and $v_2(n)$. A.1) They are uncorrelated with each other and with the reference and error signals as well. A.2) They are zero mean white Gaussian noise signals with variances $\sigma_{v_1}^2$ and $\sigma_{v_2}^2$.

Here, A.1) is evident from the nature of the system [Fig. 1(a)] and A.2) comes from the fact that $v_1(n)$ and $v_2(n)$ are produced by the turbulent air flow (random in nature) over the microphones [14].

In Fig. 1, $\hat{S}(z)$ is obtained offline and kept fixed during the online operation of ANC. Assuming that $W(z)$ is a FIR filter of tap-weight length L, the secondary signal $y(n)$ is expressed as

$$y(n) = \boldsymbol{w}^T(n)\boldsymbol{x}_L(n). \tag{1}$$

where $\boldsymbol{w} = [w_0(n),\, w_1(n),\, \cdots, w_{L-1}(n)]^T$ is the tap-weight vector and $\boldsymbol{x}_L(n) = [x(n),\, x(n-1),\, \cdots, x(n-L+1)]^T$ is an L sample reference signal vector. The residual error signal $e(n)$ is given as

$$e(n) = d(n) - y'(n) \tag{2}$$

where $d(n) = p(n) * r(n)$ is the primary disturbance signal, $r(n)$ is the reference noise signal, $y'(n) = s(n) * y(n)$ is the secondary canceling signal $y(n)$, $*$ denotes linear convolution, and $p(n)$ and $s(n)$ are impulse responses of the primary path $P(z)$ and secondary path $S(z)$, respectively. The FxLMS update equation for the coefficients of $W(z)$ is given as

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu\{e(n) + v_2(n)\}\boldsymbol{x}'(n) \tag{3}$$

where μ is the step size parameter $\boldsymbol{x}'(n) = [x'(n)x'(n-1)\cdots x'(n-L+1)]^T$, $x'(n)$ is the reference signal $x(n)$ filtered through the modeling filter $\hat{S}(z)$, and $x(n) = r(n) + v_1(n)$ is the reference signal picked by reference microphone. Equation (3) shows that the adaptation process is perturbed by an undesired term $\mu v_2(n)\boldsymbol{x}'(n)$. Assuming that $\hat{S}(z)$ is represented by a FIR filter of tap-weight length M, the filtered reference signal $x'(n)$ is obtained as

$$x'(n) = \hat{\boldsymbol{s}}^T \boldsymbol{x}_M(n). \tag{4}$$

where $\hat{\boldsymbol{s}} = [\hat{s}_0\hat{s}_1\hat{s}_2\cdots\hat{s}_{M-1}]^T$ is the impulse response of the modeling filter $\hat{S}(z)$ and $\boldsymbol{x}_M(n) = [x(n)x(n-1)\cdots x(n-M+1)]^T$ is an M—sample reference signal vector. Taking the z-transform of (2), we get

$$E(z) = P(z)R(z) - S(z)W(z)\{R(z) + V_1(z)\}. \tag{5}$$

When the adaptive filter converges, the residual error is (ideally) zero, i.e., $E(z) = 0$, which requires $W(z)$ to realize the optimal transfer function

$$W^{\circ}(z) = \frac{P(z)}{S(z)\{1 + V_1(z)/R(z)\}}. \tag{6}$$

This equation shows that the optimal solution $W^{\circ}(z)$ is independent of the measurement noise $v_2(n)$ associated with the error microphone. However the optimal solution for ideal case ($P(z)/S(z)$ for $v_1(n) = v_2(n) = 0$) is distorted by the reference input measurement noise, $v_1(n)$.

Since we have assumed that the both $v_1(n)$ and $v_2(n)$ are white Gaussian noise signals, we can use *averaging* to remove their effects. In [11] two averaging-based adaptive filtering algorithms are proposed. The first algorithm uses averaging in iterates only and in the second algorithm averaging is incorporated with both iterates and observations. Motivated by the second approach, we incorporate averaging with both the iterates, $\boldsymbol{w}(n)$, and the correction term (the observation vector), $\mu\{e(n) + v_2(n)\}\boldsymbol{x}'(n)$, of the FxLMS algorithm and propose the following algorithm:

$$\boldsymbol{w}(n+1) = \overline{\boldsymbol{w}(n)} + \overline{\boldsymbol{g}(n)} \tag{7}$$

where

$$\overline{\boldsymbol{w}(n)} = \frac{1}{n}\sum_{k=1}^{n}\boldsymbol{w}(k) \tag{8}$$

$$\overline{\boldsymbol{g}(n)} = \frac{1}{n^{\gamma}}\sum_{k=1}^{n}\mu\{e(k) + v_2(k)\}\boldsymbol{x}'(k); \quad 1/2 < \gamma < 1. \tag{9}$$

Here, computing the running average of the data does not put so much computational burden since averages can be calculated recursively. For example, (8) can be recursively computed as

$$\overline{\boldsymbol{w}(n)} = \frac{1}{n}[(n-1)\overline{\boldsymbol{w}(n-1)} + \boldsymbol{w}(n)]. \tag{10}$$

Similarly, the averaged gradient vector $\overline{\boldsymbol{g}(n)}$ (9) can be computed as

$$\overline{\boldsymbol{g}(n)} = \frac{1}{n^{\gamma}}[(n-1)^{\gamma}\overline{\boldsymbol{g}(n-1)} + \mu\{e(n) + v_2(n)\}\boldsymbol{x}'(n)]. \tag{11}$$

Equations (1), (4), (7), (10), and (11) are combined to give the proposed FxAFA algorithm. We see that the introduction of averaging in the FxLMS update equation results in a multistep algorithm (proposed FxAFA algorithm). Hence, an increased computational burden is expected as discussed later in this section.

B. Choice of the Parameter γ

Here, we present some comments on the choice of the parameter γ in the proposed algorithm. For convenience, we rewrite the proposed algorithm in a compact form

$$\boldsymbol{w}(n+1) = \overline{\boldsymbol{w}(n)} + \alpha(n)\sum_{k=1}^{n}\mu\{e(k)+v_2(k)\}\boldsymbol{x}'(k) \quad (12)$$

where $\alpha(n) = 1/n^{\gamma}$ is a slowly varying gain parameter. In adaptive algorithms, it is desirable to have a large step gain at the startup for fast convergence. As the time increases, the gain is desirable to slowly decrease so that misadjustment is small. The time varying gain $\alpha(n)$ indeed exhibits these properties and $\lim_{n\to\infty}\alpha(n) \to 0$. It is seen that $\gamma = 1$ will rapidly decrease the gain parameter, and hence adaptation process may be very slow. Therefore, one may wish to choose $\gamma < 1$ [11]. On contrary if γ close to zero is selected then $\alpha(n)$ is very slowly decreasing. This is also not desirable for large mismatch. Hence $1/2 < \gamma < 1$ is the recommended range for the values of γ.

C. Comparison of Computational Complexity

In Table I, the computational complexity analysis, on the basis of computations required for completion of operations per iteration, is presented. It is seen that the computational burden of the proposed FxAFA algorithm ($6L+M+1$ multiplications/iteration) is greater than that of the FxLMS algorithm ($2L+M+1$ multiplications/iteration), but it is far less than that of the FxRLS algorithm ($3L^2+5L+M+1$ multiplications/iteration [6]).

D. Comments on Convergence Behavior

1) Ideal Condition ($\sigma_{v_1}^2 = \sigma_{v_2}^2 = 0$): We know that the method of steepest descent computes a tap-weight vector that moves down the ensemble-average error-performance surface along a deterministic trajectory that terminates on the Wiener solution (it takes infinite number of iterations, n, to do so). The LMS algorithm, on the other hand behaves differently because of the presence of the gradient noise: Rather than terminating on the Wiener solution, the tap-weight vector computed by the LMS algorithm executes a random motion around the minimum point of the error performance surface [15, p. 234]. Furthermore, by assigning a small value to the step size parameter, the adaptation is made to progress slowly, and the effects of the gradient noise on the tap weights are largely filtered out [15, p. 235].

In the proposed algorithm, the aim is to have the iterations move to the Wiener Solution reasonably fast. With the averaging approach of (9), with $\gamma < 1$ the estimates from (12) are allowed to approach the vicinity of the true value faster. At the same time, averaging removes the random fluctuations in the gradient vector and ensures that the iterations move toward the optimal (Wiener) solution (refer to simulation case study Case 1).

2) Measurement Noise Condition ($\sigma_{v_1}^2 \neq 0, \sigma_{v_2}^2 \neq 0$): In the presence of the measurement noise signals, $v_1(n)$ and $v_2(n)$, the first round of averaging is expected to remove the effect of $v_2(n)$ from the gradient vector. The second round of the averaging attempts to remove the effect of $v_1(n)$ from $W(z)$. From (6), it is clear that the $v_1(n)$ is affecting the optimal solution in a nonlinear fashion, and *averaging* cannot *eliminate* it. Nevertheless we can expect better results as compared with the standard FxLMS algorithm (see the simulation results presented in Case 2).

TABLE I
COMPUTATIONAL COMPLEXITY COMPARISON BETWEEN FxLMS AND FxAFA ALGORITHMS

Operation	Computation	Total
FxLMS Algorithm		
(1)	Mul $= L$; Add $= L-1$	
(4)	Mul $= M$; Add $= M-1$	Mul $= 2L+M+1$
(3)	Mul $= L+1$; Add $= L$	Add $= 2L+M-2$
FxAFA Algorithm		
(1)	Mul $= L$; Add $= L-1$	
(4)	Mul $= M$; Add $= M-1$	Mul $= 6L+M+1$
(10)	Mul $= 2L$; Add $= L+1$; Inv $= 1$	Add $= 4L+M$
(11)	Mul $= 3L+1$; Add $= L+1$	Inv $= 2$
	Inv $= 1$; P $= 2$	P$= 2$
(7)	Add $= L$	

(Here Mul=Multiplication, Add=Addition, Inv=Scalar inversion, and P=Power computation)

III. COMPUTER SIMULATIONS

In this section, the performance of the proposed FxAFA algorithm is demonstrated using computer simulation. Two parameters are used for performance evaluations. They are 1) the noise reduction, $\Re$, achieved at the error microphone, which is defined as $\Re = -10\log_{10}(\Sigma e^2(n)/\Sigma d^2(n))$; and 2) the estimation error of $\boldsymbol{w}(n), \Delta w(n)$, which is defined as $\Delta w(n) = 10\log_{10}(\Sigma_{l=0}^{L-1}\{w_{(\mathrm{ideal})l} - w_l(n)\}^2/\Sigma_{l=0}^{L-1} w_{(\mathrm{ideal})l}^2)$, where $\boldsymbol{w}_{\mathrm{ideal}}$ is the optimal value of the tap weight vector obtained under ideal conditions when there is no measurement noise, and L is the order of the control filter $W(z)$.

The data for acoustical paths is adopted from [1] where both the primary acoustical path $P(z)$ and the secondary path $S(z)$ are modeled by IIR filters of order 25 (the data is provided on a disk included with [1]). Since industrial noise often has significant power in the frequency range between 50–250 Hz [16], simulations are carried with signals having frequency falling in this range. The sampling frequency of 2 kHz is used. The secondary-path model $\hat{S}(z)$ is an FIR filter of order (M) 128, and is identified offline. The ANC controller $W(z)$ is also an FIR filter of length (L) 128.

A. Case 1

It has been shown in [6] and [17] that FxRLS algorithm becomes numerically unstable toward the higher number of iterations; hence, it is not included in the simulations presented here. The reference noise is a broadband signal and is a sinusoid containing five harmonics (of equal power) with the fundamental frequency of 50Hz. First we consider the ideal case when there is no measurement noise present in the system, i.e., $\sigma_{v_1}^2 = \sigma_{v_2}^2 = 0$. The parameters for the FxLMS algorithm and the FxAFA algorithm are adjusted for fast and stable convergence and (by trial-and-error) are found to be $\mu_{\mathrm{FxLMS}} = 5\times10^{-5}, \mu_{\mathrm{FxAFA}} = 1\times10^{-3}$ and $\gamma = 0.6$. The noise reduction curves for the two algorithms are shown in Fig. 2(a). We see that the proposed FxAFA algorithm achieves

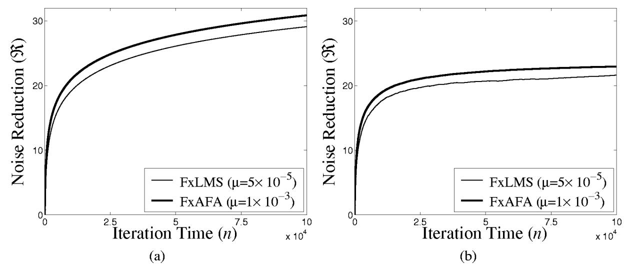

Fig. 2. Performance comparison between FxAFA and FxLMS algorithms. (a) Noise reduction $\Re$ versus iteration time n in Case 1. (b) Noise reduction $\Re$ versus iteration time n in Case 2.

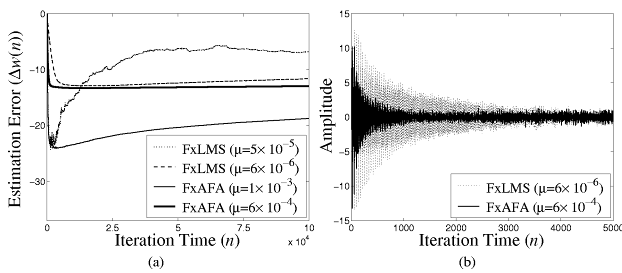

Fig. 3. Performance comparison between FxAFA and FxLMS algorithms. (a) Estimation error $\Delta w(n)$ versus iteration time n in Case 2. (b) Residual error signal $e(n)$ in Case 2.

faster convergence and higher noise reduction than the FxLMS algorithm. This performance difference is in agrement with the comments presented earlier.

B. Case 2

In the next experiment, we consider the situation when both the reference microphone and the error microphone introduce measurement noise signals $v_1(n)$ and $v_2(n)$, respectively, with the variances $\sigma_{v_1}^2 = \sigma_{v_2}^2 = 0.05$. In order to have a fair comparison, the parameters for the two algorithms are kept same as before. The noise reduction, $\Re$, curves are shown in Fig. 2(b). We see that the performance of the two algorithms is degraded as compared with that shown in Fig. 2(a). The curves for $\Delta w(n)$, the estimation error of $\boldsymbol{w}(n)$, are shown in Fig. 3(a). We see that for the same step size ($\mu_{\text{FxLMS}} = 5 \times 10^{-5}, \mu_{\text{FxAFA}} = 1 \times 10^{-3}$) the $\Delta w(n)$ for the FxLMS algorithm decreases initially very fast, but later it starts increasing and then settles at a value higher than that of the FxAFA algorithm.

As discussed earlier in Section II, due to measurement-noise signals, the performance of the two algorithms is degraded. Nevertheless the FxAFA algorithm, due to averaging process, gives better performance than the FxLMS algorithm. Since, in both algorithms, the estimation error, $\Delta w(n)$, is still negative, hence the overall performance of the ANC system is stable, even at $n = 10 \times 10^4$.

One way to solve this divergence problem is to use small step size so that adaptation takes place slowly. The step size of the two algorithms is reduced on trial-and-error basis. The resulting curves for estimation error are shown in Fig. 3(a). The corresponding curves for residual error signal, $e(n)$, are shown in Fig. 3(b). We see that to make FxLMS algorithm converge all the time, a very small step size is needed and hence convergence speed is very slow. In case of FxAFA algorithm, on the other hand, a large step size can be selected and hence fast convergence can be realized.

IV. Concluding Remarks

This letter proposes a new ANC algorithm based on adaptive filtering with averaging. The main limitation of the proposed algorithm is its poor tracking properties, which is due to the running-length averaging process. This problem can be overcome, for example, by using weighted averaging with exponential forgetting factor [12], or by re-initializing the averaging process at regular intervals. It would be interesting to do theoretical analysis of the proposed algorithm on the similar lines as done for the FxLMS algorithm in [18]. The development of an ANC system with online secondary-path modeling, incorporating adaptive filtering with averaging is a task for future work.

References

[1] S. M. Kuo and D. R. Morgan, *Active Noise Control Systems-Algorithms and DSP Implementation*. New York: Wiley, 1996.
[2] B. Widrow and S. D. Stearns, *Adaptive Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1985.
[3] Y. C. Park and S. D. Sommerfeldt, "A fast adaptive noise control algorithm based on lattice structure," *Appl. Acoust.*, vol. 47, no. 1, pp. 1–25, 1996.
[4] L. J. Eriksson, M. C. Allie, and R. A. Greiner, "The selection and application of an IIR adaptive filter for use in active sound attenuation," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-35, pp. 433–437, 1987.
[5] D. H. Crawford and R. W. Stewart, "Adaptive IIR filtered-v algorithms for active noise control," *J. Acoust. Soc. Amer.*, vol. 101, no. 4, pp. 2097–2103, 1997.
[6] M. Bouchard and S. Quednau, "Multichannel recursive-least-squares algorithms and fast-transversal-filter algorithms for active noise control and sound reproduction systems," *IEEE Trans. Speech Audio Processing*, vol. 8, pp. 606–618, Sept. 2000.
[7] S. M. Kuo and M. Tahernezhadi, "Frequency-domain periodic active noise control and equalization," *IEEE Trans. Speech Audio Processing*, vol. 5, pp. 348–358, July 1997.
[8] B. T. Polyak and A. B. Juditsky, "Accelaration of stochastic approximation by averaging," *SIAM J. Contr. Optim.*, vol. 30, no. 4, pp. 838–355, July 1992.
[9] H. J. Kushner and J. Yang, "Stochastic approximation with averaging of iterates: Optimal symptotic rate of convergence for general process," *SIAM J. Contr. Optim.*, vol. 31, no. 4, pp. 1045–1062, 1993.
[10] G. G. Yin and Y. Zhu, "Averaging procedures in adaptive filtering: An efficient approach," *IEEE Trans. Automat. Contr.*, vol. 37, pp. 466–475, Apr. 1992.
[11] G. Yin, "Adaptive filtering with averaging," in *Adaptive Control, Filtering, and Signal Processing*, K. J. Astrom, G. C. Goodwin, and P. R. Kumar, Eds. New York: Springer-Verlog, 1995.
[12] V. Krishnamurthy, "Averaged stochastic gradient algorithms for adaptive blind multiuser detection in DS/CDMA systems," *IEEE Trans. Commun.*, vol. 48, pp. 125–134, Jan. 2000.
[13] G. G. Yin, V. Krishnamurthy, and C. Ion, "Iterate-averaging sign algorithms for adaptive filtering with applications to blind multiuser detection," *IEEE Trans. Inform. Theory*, vol. 49, pp. 657–671, Mar. 2003.
[14] A. Roure, "Self-adaptive broadband active sound control system," *J. Sound Vibr.*, vol. 101, no. 3, pp. 429–441, 1985.
[15] S. Haykin, *Adative Filter Theory*, 4th ed. Englewood Cliffs, NJ: Prentice-Hall, 2002.
[16] K. K. Shyu and C. Y. Chang, "Modified FIR filter with phase compensation technique to feedforward active noise controller design," *IEEE Trans. Ind. Electron.*, vol. 47, pp. 444–453, Apr. 2000.
[17] M. T. Akhtar, M. Abe, and M. Kawamata, "On use of averaging in FxLMS algorithm for single-channel feedforward ANC systems," in *Proc. IEEE ISCAS 2003*, vol. IV, 2003, pp. 389–392.
[18] E. Bjarnason, "Analysis of the filtered-x LMS algorithm," *IEEE Trans. Speech Audio Processing*, vol. 3, pp. 504–514, Nov. 1995.