Short Papers

A Noise-Adaptive Discriminant Function and Its Application to Blurred Machine-Printed Kanji Recognition

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Abstract—Accurate recognition of blurred images is a practical but previously to mostly overlooked problem. In this paper, we quantify the level of noise in blurred images and propose a new modification of discriminant functions that adapts to the level of noise. Experimental results indicate that the proposed method actually enhances the existing statistical methods and has impressive ability to recognize blurred image patterns.

Index Terms—Discriminant function, Mahalanobis distance, Bayes classifier, distribution of feature vectors, noise, blurred character recognition.

1 Introduction

Significant achievements are made by statistical pattern recognition methods considering distribution of sample patterns in feature spaces [1], [2], [3]. In most conventional pattern recognition methods, the first step is extracting features from objects. These features are always expressed in the form of feature vectors. Then, the distribution of feature vectors is estimated for each category. Finally, an unknown input pattern is assigned to the category with the maximum probability.

Usually, different types of noise may be present and no one can predict what kind of noise an unknown input pattern will carry. Because noise may change the appearance of a pattern, the feature vector extracted from a noisy image will be very different from that from a clean image. If the distribution is estimated with only noiseless samples, whereas the unknown input pattern is noisy, the recognition result is often unsatisfactory. On the other hand, if the distribution is estimated with noisy samples, there is no guarantee that the type of noise of an unknown input pattern is included in the training samples. For these reasons, selecting training samples is not the most essential element of constructing a dictionary for recognizing both clean and noisy patterns.

For noisy pattern recognition, many regularization methods of discriminant functions are proposed. These are done by adding a regularization term or by a noise injection to input signals [4], [5]. However, as is known, distribution of feature vectors will change according to noise that occurs irregularly and accidentally. Therefore, how to quantify noise and formulate the relationship between noise and discriminant function is extremely important. In this paper, by introducing the concept of level of noise, a new modification method of existing discriminant functions is proposed and a new discriminant function, called *Adaptive Mahalanobis distance*, is presented. In the proposed method, elements of feature vectors of an unknown input pattern are investigated as to whether they are with or without noise. Furthermore, distribution of feature vectors of each category is changed according to the noise detected from the unknown input pattern. It can quantify changes in noisy unknown samples and dynamically rectify the original distribution of categories according to the detected noise.

Although the research of Chinese character and Japanese character recognition has been continued [6], [7], [8] since Casey and Nagy opened up the field [9], methods for blurred character recognition still need to be developed. Compared to numerals and alphabet characters, the structure of Chinese characters is quite complex and there is a large amount of structurally similar characters. Since Chinese characters have complex structures, if the character images are copied or transferred with facsimile, blurring can make the appearances quite different from the originals and, certainly, feature vectors that absolutely depend on the shapes of images will change according to noise. For these reasons, blurring is a serious problem in recognizing noisy Chinese characters.

In this paper, as a practical application of the Adaptive Mahalanobis distance, it is adopted to recognize blurred Kanji (Chinese characters used in Japan) images. With the experimental results, it is shown that the new discriminant function is extremely effective for blurred character recognition and also has satisfactory performance on clean pattern recognition. All the results indicate that the proposed method supplements the existing statistical methods.

2 Related Work

Some methods for recognizing poor quality characters have been proposed. Hobby and Ho [10] have developed a method to enhance degraded document images by finding and averaging bitmaps of the same kind of symbols. It improves the display appearance and recognition accuracy. Chou and Chang have proposed a flexible matching method between template images and unknown character images [11]. A vector field, called character deformation field, is used to represent deformation. Rodríguez et al. have exploited a two-stage classifier [12]. First, a multifont classifier is applied. Then, a specialized classifier rerecognizes the ambiguous patterns using the patterns whose certainty of correct classification is high.

With the widespread use of digital cameras, some studies on recognizing poor quality characters that exist in the images taken by digital cameras have been done [13], [14], [15]. Sawa et al. use the Gaussian Laplacian filter to emphasize images [13]. Then, segmentation and recognition of characters are accomplished with dynamic programming. The moving subtraction method has been proposed by Kosai et al. [14]. It uses plural images by swinging a camera vertically and horizontally to supplement the bad influence caused by the lowness of resolution. The method developed by Sawaki et al. prepares a multiple-dictionary to deal with the images under any conditions [15]. The environmental condition of an image is estimated and a relevant dictionary reflecting the condition is used for recognition.

All these methods focus on how to construct an optimal reference pattern or a dictionary from training samples. However, it is more important to detect noise and to rectify discriminant function according to the noise for blurred image recognition. Moreover, some of these methods deal with multiple-valued images. However, thickness of character images that are copied or transferred with facsimile is mostly binarized to white or black, with no intermediate thickness. Therefore, these methods are not proper for this case.

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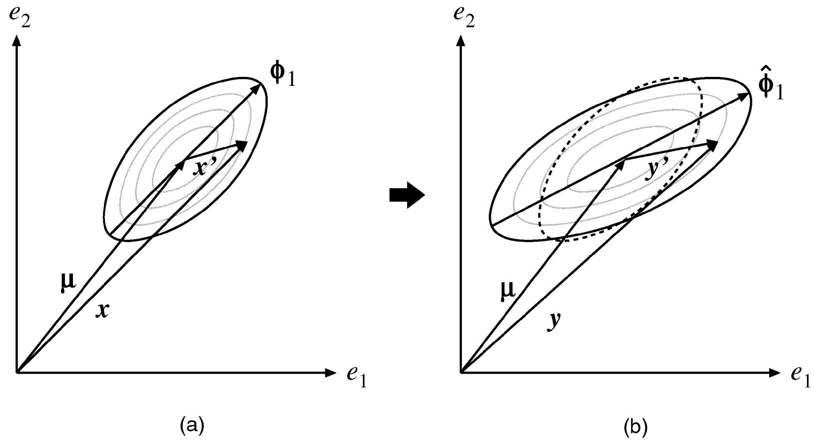

Fig. 1. Change in distribution. (a) Original distribution and (b) changed distribution.

3 Discriminant Function Reflecting Change in Distribution

During an observation process, it is very difficult to avoid the occurrence of noise. Some kinds of noise, such as noise that occurs in character images, is neither uniform nor continuous and it often only appears on certain parts of an object. The features of an object are expressed as a feature vector in most pattern recognition methods. As a result, noise will only appear on some elements of a feature vector, whereas the other elements will keep the essential values. In this case, the standard deviation of the elements with noise will become larger corresponding to degree of noise.

Because noise happens irregularly and accidentally, it is impossible to make a complete dictionary that can include all kinds of noise. Even if it were possible, that dictionary might not be valid for clean images. The main purpose of this study is to quantify the relationship between noise and distribution of category and to rectify distribution dynamically according to degree of noise. To quantify the relationship between degree of noise and change in distribution, an appropriate way is to calculate the ratio of standard deviation of elements with b degrees of noise to those of noiseless elements. The ratio can be denoted as r_b. Obviously, the ratio of standard deviation takes the role of intermediation between noise and distribution of category. From this point of view, a new discriminant function that reflects the change in distribution can be proposed by considering the ratio of standard deviations.

Here, the Mahalanobis distance and the Bayes classifier of multivariate normal distribution are considered. Let $\boldsymbol{\mu}$ and Σ be the mean vector and the $n \times n$ covariance matrix, respectively. The squared Mahalanobis distance from $\boldsymbol{\mu}$ to $\boldsymbol{x}$ is defined as

$$d^2 = (\boldsymbol{x}-\boldsymbol{\mu})^t \Sigma^{-1} (\boldsymbol{x}-\boldsymbol{\mu}). \tag{1}$$

The squared Mahalanobis distance is abbreviated as the Mahalanobis distance below. The discriminant function for the Bayes classifier with equal prior probabilities of all categories is defined as

$$g = (\boldsymbol{x}-\boldsymbol{\mu})^t \Sigma^{-1} (\boldsymbol{x}-\boldsymbol{\mu}) + \log|\Sigma|. \tag{2}$$

For simplicity, the case of two-dimensional normal distribution is discussed first. As shown in Fig. 1a, e_1 and e_2 are the axes of the original coordinate. Let $\boldsymbol{\phi}_1$ be the eigenvector that corresponds to the first principal component. When b degrees of noise is added to the e_1-element while the e_2-element is noiseless, it is observed that only the standard deviation of e_1-element becomes r_b times larger. Then, the change in the distribution can be illustrated as Fig. 1b.

Let $b(j)$ be the degree of noise added to the jth element of n-dimensional feature vector $\boldsymbol{x}$. Suppose that the mean vector is not changed and the standard deviation of jth element of $\boldsymbol{x}$ becomes $r_{b(j)}$ times larger, where $r_{b(j)}$ is determined depending on the value $b(j)$. For noiseless elements, say j, $r_{b(j)} = 1$.

Let $\boldsymbol{x}_i$ be a noiseless sample and $\boldsymbol{y}_i$ be a sample with noise ($i = 1, 2, ..., N$). The observation on the change in deviation by noise may be described as the following: Let $\boldsymbol{x}_i = \boldsymbol{\mu} + \boldsymbol{x}'_i$ and $\boldsymbol{y}_i$ be the corresponding noisy data such that $\boldsymbol{y}_i = \boldsymbol{\mu} + \boldsymbol{y}'_i$. If the jth element of $\boldsymbol{y}'_i$ is changed from $\boldsymbol{x}'_i$ as $y'_{ij} = r_{b(j)} x'_{ij}$, then the standard deviation $\hat{\sigma}_j$ of y'_{ij} is $r_{b(j)}$ times larger than that of x'_{ij}. If a diagonal matrix K is defined as

$$K = \begin{bmatrix} r_{b(1)} & & & 0 \\ & r_{b(2)} & & \\ & & \ddots & \\ 0 & & & r_{b(n)} \end{bmatrix}, \tag{3}$$

which is called *revision matrix*, then, $\boldsymbol{y}'_i (= \boldsymbol{y}_i - \boldsymbol{\mu})$ can be written as $\boldsymbol{y}'_i = K\boldsymbol{x}'_i$.

Using the above equations, the covariance matrix of noisy samples $\hat{\Sigma}$ is calculated as,

$$\begin{aligned} \hat{\Sigma} &= \frac{1}{N-1} \sum_{i=1}^{N} (\boldsymbol{y}_i - \boldsymbol{\mu})(\boldsymbol{y}_i - \boldsymbol{\mu})^t \\ &= \frac{1}{N-1} \sum_{i=1}^{N} (K\boldsymbol{x}'_i)(K\boldsymbol{x}'_i)^t \\ &= \frac{1}{N-1} \sum_{i=1}^{N} K(\boldsymbol{x}_i - \boldsymbol{\mu})(\boldsymbol{x}_i - \boldsymbol{\mu})^t K^t \\ &= K\Sigma K. \end{aligned} \tag{4}$$

Note that $K = K^t$, in order to reflect the change in distribution, the following discriminant functions that include the revision matrix K is proposed:

$$\begin{aligned} \hat{d}^2 &= (\boldsymbol{x}-\boldsymbol{\mu})^t \hat{\Sigma}^{-1} (\boldsymbol{x}-\boldsymbol{\mu}) \\ &= (K^{-1}(\boldsymbol{x}-\boldsymbol{\mu}))^t \Sigma^{-1} (K^{-1}(\boldsymbol{x}-\boldsymbol{\mu})), \end{aligned} \tag{5}$$

$$\begin{aligned} \hat{g} &= (\boldsymbol{x}-\boldsymbol{\mu})^t \hat{\Sigma}^{-1} (\boldsymbol{x}-\boldsymbol{\mu}) + \log|\hat{\Sigma}| \\ &= (K^{-1}(\boldsymbol{x}-\boldsymbol{\mu}))^t \Sigma^{-1} (K^{-1}(\boldsymbol{x}-\boldsymbol{\mu})) + \log|\Sigma| + 2\log|K|. \end{aligned} \tag{6}$$

Here, $\boldsymbol{x}$ is a noisy observation. Equations (5) and (6) are called *Adaptive Mahalanobis distance* and *Adaptive Bayes classifier*.

4 Application to Blurred Kanji Recognition

Although noisy Kanji recognition is very necessary and important in a practical character recognition system, there have been few attempts that give expressive performance. In this paper, a new discriminant function for noisy pattern recognition is proposed. As one of its practical applications, (5) is rectified to fit the characteristic of character images. Recognition experiments are performed with blurred machine printed Kanji images to confirm the effectiveness of the proposed method.

The Directional Element Feature [16] is used as the feature vector here. The effectiveness of this feature is shown with clean machine printed Kanji images [16]. It is calculated as follows: First, input image is normalized to 64×64 dots and thinned. Then, it is divided into 49 areas of 16×16 dots where each area overlaps eight dots of the adjacent area. For each area, a four-dimensional vector is defined to represent the quantities of the four orientations: vertical, horizontal, and two oblique lines slanted at $\pm 45^\circ$. Thus, the total vector for one character has 196 ($= 49 \times 4$) dimensions.

4.1 Discriminant Function

According to the simulation results [17], [18], the Adaptive Mahalanobis distance is superior to the Adaptive Bayes classifier, in the case that noise is terrible, i.e., the value of $b(j)$ is large. Therefore, only the Mahalanobis distance is investigated below.

In the case of recognizing character patterns, it is known that the Mahalanobis distance has disadvantages. One big disadvantage is caused by small number of samples [19], [20], [21]. To decrease this influence, many regularization methods of discriminant functions are proposed [4], [5]. One typical method is to add a regularization term to Σ. In order to examine this kind of method, (1) is modified as follows:

$$d_R^2 = (\boldsymbol{x} - \boldsymbol{\mu})^t (\Sigma + \alpha I)^{-1} (\boldsymbol{x} - \boldsymbol{\mu}). \tag{7}$$

Here, I is an identity matrix and α is a small positive constant. In the following experiments, $\alpha = 0.1$. In this paper, (7) is called the regularized Mahalanobis distance, or RMD.

Another big problem of the Mahalanobis distance is the expensive computation cost, especially when it is used for Kanji recognition that uses usually very high dimensional feature vectors. To reduce the computation cost, various modifications of the Mahalanobis distance are proposed. Three typical methods are the Quasi-Mahalanobis distance [22] (QMD), the Modified Mahalanobis distance [23] (MMD), and the Simplified Mahalanobis distance [24] (SMD). The statistical properties of the SMD are most similar to the Mahalanobis distance among the three functions and its effectiveness has been shown [24] with ETL9B [25], which is the largest handwritten character database in Japan.

In order to explain the SMD, (1) is rewritten as

$$d^2 = \sum_{i=1}^{n} \frac{1}{\lambda_i} ((\boldsymbol{x} - \boldsymbol{\mu})^t \boldsymbol{\phi}_i)^2. \tag{8}$$

Here, λ_i is the ith eigenvalue of Σ sorted by descending order, and $\boldsymbol{\phi}_i$ is the eigenvector that corresponds to λ_i. The SMD replaces $\lambda_i (i > m)$ in (8) with the mean value α_m of λ_i $(i = m + 1, ..., n)$, and it is written as

$$\begin{aligned} d_S^2 &= \sum_{i=1}^{m} \frac{1}{\lambda_i} ((\boldsymbol{x} - \boldsymbol{\mu})^t \boldsymbol{\phi}_i)^2 + \frac{1}{\alpha_m} \sum_{i=m+1}^{n} ((\boldsymbol{x} - \boldsymbol{\mu})^t \boldsymbol{\phi}_i)^2 \\ &= \sum_{i=1}^{m} \frac{1}{\beta_i} ((\boldsymbol{x} - \boldsymbol{\mu})^t \boldsymbol{\phi}_i)^2 + \frac{1}{\alpha_m} \|\boldsymbol{x} - \boldsymbol{\mu}\|^2, \end{aligned} \tag{9}$$

where

$$\alpha_m = \frac{\mathrm{tr}\Sigma - \sum_{i=1}^{m} \lambda_i}{n - m}, \tag{10}$$

$$\frac{1}{\beta_i} = \frac{1}{\lambda_i} - \frac{1}{\alpha_m}. \tag{11}$$

The revision matrix K proposed in Section 3 can be introduced to many discriminant functions. In this paper, to investigate the effectiveness of the revision matrix K to noisy pattern recognition, K is introduced to RMD and SMD and the expressions are expressed as

$$\hat{d}_R^2 = (K^{-1}(\boldsymbol{x} - \boldsymbol{\mu}))^t (\Sigma + \alpha I)^{-1} (K^{-1}(\boldsymbol{x} - \boldsymbol{\mu})), \tag{12}$$

$$\hat{d}_S^2 = \sum_{i=1}^{m} \frac{1}{\beta_i} ((\boldsymbol{x} - \boldsymbol{\mu})^t K^{-1} \boldsymbol{\phi}_i)^2 + \frac{1}{\alpha_m} \|K^{-1}(\boldsymbol{x} - \boldsymbol{\mu})\|^2. \tag{13}$$

Equation (12), which modifies the RMD by the addition of the revision matrix K, is called Adaptive RMD. Similarly, (13) is called Adaptive SMD. Note that the computational cost of the Adaptive RMD is $O(n^2)$ and it is much more expensive than that of the Adaptive SMD which is $O(nm)$ for $m \ll n$.

4.2 Degree of Blur

4.2.1 *Definition*

In order to quantify level of noise, the concept of *degree of blur* is introduced. Degree of blur is defined for each area of a character image and it is calculated in the thinning process. Thinning is a repeating process of erasing a black pixel from boundaries of black pixels of a character image. By scanning neighbor pixels around each black pixel, stroke width of a character image is finally erased to one-pixel [26], [27]. Fig. 2a and Fig. 2e are examples of a normalized blurred image and a clean image. The erasing process of these images are shown by Fig. 2b, Fig. 2c, and Fig. 2d and Fig. 2f, Fig. 2g, and Fig. 2h, respectively. In order to get a completely thinned image of Fig. 2d, 14 times of repetitions are needed to erase pixels on boundaries, while Fig. 2h requires only four repetitions.

If the repetition times in the thinning process is limited to the number that a clean image is completely thinned, obviously it will not be enough for a blurred image. To decide the optimum number of times for thinning a clean image, preexperiments are carried out with 2,964 different clean Kanji images[1] scanned at 400 dots per inch. The size of each image is 128×128. For different numbers of repetition times, the number of completely thinned character images is counted. Two mainly used fonts, *Mincho* and Gothic, are examined here. The amounts and percentages of completely thinned character images are shown in Table 1. The tendencies of two fonts are similar and it has been shown that over 96 percent of the character images are completely thinned after six times erased. Obviously, six times is not the optimum number for any font. The optimum number depends on font and complexity of characters. Fortunately, no matter what kind of font is used, for each kind of font, the maximum width of line segments that are used for constructing a character is usually fixed. Therefore, calculating an optimum number of a certain kind of font is not a difficult task. In the above case, a Kanji image can be regarded as a blurred one if any line width is more than one pixel after being thinned six times.

Since noise in character images is neither uniform nor continuous, an image usually includes both clean parts and blurred parts. It is thought to be more feasible to examine blurred parts of image area-by-area. Here, for every one of the 49 areas, the number of black pixels that are not located at boundaries is counted and the quantized value of $\lfloor (\text{Number of black pixels})/\text{M} \rfloor$ is defined as *degree of blur*. In the experiments, $M = 32$. Since the largest number of black pixels in one area is $16 \times 16 = 256$ pixels,

1. The number of Kanji that are included in the first class of Japanese Industrial Standard (JIS) is 2,965. Among these characters, "—," which seems to have a special structure, is not used.

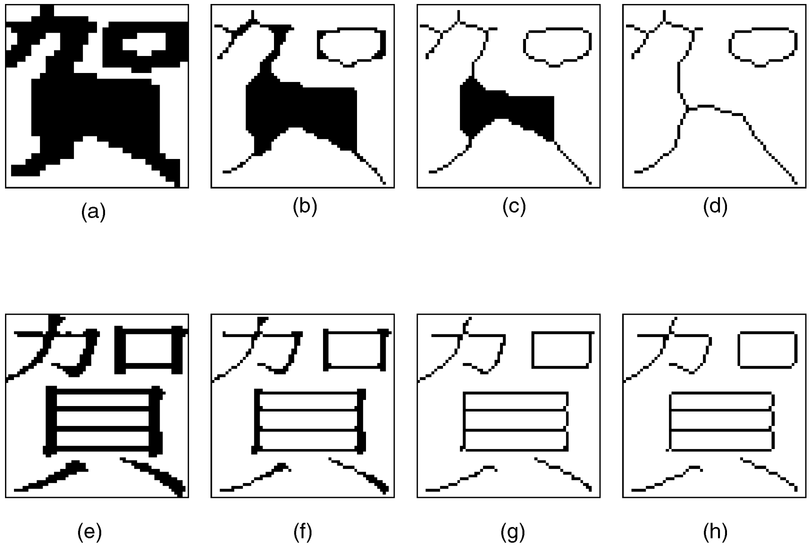

Fig. 2. Thinning process of top: blurred image and bottom: clean image. (a) Normalized image, (b) three times erased, (c) six times erased, and (d) completely thinned. (e) Normalized image, (f) one time erased, (g) two times erased, and (h) completely thinned.

the value of degree of blur is either zero or a positive integer not greater than eight. Hence, the greater value of degree means the blur in that area is more severe. Fig. 3a shows the contour image (pixels that are located at boundaries) of Fig. 2c, and Fig. 3b gives the detected blurred region.

4.2.2 Characteristics

If a character image has been damaged by photocopying or facsimile, the appearance of the image is often blurred. Degree of blur is defined to describe the state of each area of an image. In order to quantize the change in the feature distribution of a category, it is necessary to investigate how much the standard deviation of each element of feature vector changes according to the degree of blur. For this purpose, 2,965 kinds of Kanji of 10 sizes (from 6 point to 22 point) are used to carry out a preexperiment. Small font character images are damaged easily. All these sample patterns are scanned at 400dpi by an optical image scanner and are transformed to feature vectors.

TABLE 1
The Number of Completely Thinned Character Images at Each Number of Times of Thinning

Number of Times of Thinning	*Mincho*	Gothic
3	8 (0.3%)	0 (0.0%)
4	946 (31.9%)	605 (20.4%)
5	2346 (79.1%)	2420 (81.6%)
6	2869 (96.7%)	2920 (98.5%)
7	2961 (99.8%)	2963 (99.9%)
8	2964 (100.0%)	2964 (100.0%)

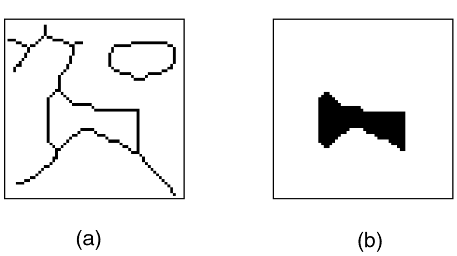

Fig. 3. Detection of blur. (a) Contour image and (b) detected blurred region.

Let x_{ij}^k be the ith element of a feature vector of jth sample pattern of category k and b_{ij}^k be the degree of blur of x_{ij}^k. First, the mean value $\bar{x}_i^k$ and the standard deviation σ_i^k are calculated from the areas whose degree of blur is zero. That is,

$$\bar{x}_i^k = \frac{1}{|J_i^k|} \sum_{j \in J_i^k} x_{ij}^k, \tag{14}$$

$$\sigma_i^k = \sqrt{\frac{1}{|J_i^k|} \sum_{j \in J_i^k} (x_{ij}^k - \bar{x}_i^k)^2}, \tag{15}$$

where the set

$$J_i^k = \left\{ j | b_{ij}^k = 0 \right\}, \tag{16}$$

and $|J_i^k|$ is the number of elements in the set J_i^k. Each value x_{ij}^k is normalized as

$$\hat{x}_{ij}^k = \frac{x_{ij}^k - \bar{x}_i^k}{\sigma_i^k}.$$

Then, r_b $(0 \leq b \leq 8)$ is determined as follows:

TABLE 2
Relationship bewteen Degree of Blur and Ratio of Standard Deviation

Degree of blur	Ratio
0	1.0
1	5.0
2, 3	8.3
$4 \sim 6$	10.0
7	12.5
8	20.0

(a) (b)

Fig. 4. Data for experiments. (a) Eight point, copied with thin mode and (b) six point, copied with thick mode.

TABLE 3
Experimental Results

Method	Thin mode			Thick mode		
	6pt	7pt	8pt	6pt	7pt	8pt
RMD	4.3%	0.9%	0.1%	16.7%	8.3%	0.8%
Adaptive RMD	3.4%	0.7%	0.1%	8.5%	4.2%	0.7%
SMD	11.4%	2.5%	0.3%	28.6%	15.2%	1.7%
Adaptive SMD	7.2%	1.5%	0.3%	12.8%	5.5%	1.0%

$$r_b = \begin{cases} 1 & \text{if } b = 0 \\ \sqrt{\frac{1}{|D_b|}\sum_{(i,j,k)\in D_b}(\hat{x}_{ij}^k - m_b)^2} & \text{otherwise,} \end{cases} \tag{17}$$

where

$$D_b = \left\{(i,j,k) | b_{ij}^k = b\right\}, \tag{18}$$

$$m_b = \frac{1}{|D_b|}\sum_{(i,j,k)\in D_b}\hat{x}_{ij}^k. \tag{19}$$

It means the ratio r_b of standard deviation of the areas with b degrees of blur to the areas with zero degree of blur is calculated.

The results are summarized in Table 2. The value of the ratio of standard deviation r_b increases as the value b of the degree of blur becomes larger. This verifies that the distribution of the feature vectors is changed by blur and the amount of change in distribution can be described by the degree of blur.

4.3 Experiments

For the experiments, 2,965 kinds of Kanji that are commonly used in Japan are adopted. Machine printed single font characters of 10 sizes (from 6 point to 22 point) are prepared as training data. All these sample patterns are scanned at 400dpi by an optical image scanner and are transformed to feature vectors. Test data includes three sizes of printed characters (6, 7, and 8 points) photocopied with two modes (thin mode and thick mode) by a copying machine. The appearances of the copied images are very blurred, especially the ones copied with thick mode. Examples of sample character images are displayed in Fig. 4. Fig.4a shows 8 point characters copied with thin mode, while Fig. 4b shows 6 point characters copied with thick mode. The qualities of these sets are quite different.

As discriminant functions, RMD, Adaptive RMD, SMD, and Adaptive SMD are used. The value of m in (13) is five. The revision matrix K based on the degree of blur in each area is estimated for each individual unknown character.

Experimental results (error rates) are shown in Table 3. The error rates of RMD are smaller than that of SMD. However, computational cost of the RMD is about forty times larger than that of the SMD. In every case of our experiments, the error rates of the Adaptive RMD and the Adaptive SMD are smaller (or has no change) compared with the results of the RMD and the SMD, respectively. Most of conventional methods have focused on how to construct an optimum dictionary for noisy image recognition; the error rate, unfortunately, tends to increase if recognition objects are clean. Here, in the experiments of recognizing the comparatively clean character images, such as 7 and 8 points copied with thin mode, the ability of the Adaptive discriminant functions are as good as, or even better than, the original ones which give fine performance on clean character recognition. Furthermore, the recognition results of the poor quality images, for instance, 6 and 7 points copied with thick mode, reveal that the Adaptive discriminant functions can decrease the error rates to about half compared to the original ones. All of these results confirm that the proposed modification method can inspect the state of an unknown input image and adapt the discriminant functions. The experimental results show that the Adaptive discriminant functions can deal with both clean and noisy patterns simultaneously and effectively.

Table 4 shows some examples that are correctly recognized by the Adaptive SMD while they are missed by the original SMD. Original images, correct answers, and candidates selected by these two discriminant functions are displayed in the table. Apparently, each combination of answer and candidate of the original SMD are quite similar and the distinctive parts among these similar characters are exactly the blurred parts. In the case of column (a), because the right part of the image is clean, the right parts of both candidates of the Adaptive SMD and the SMD have the same structure. However, since the left part of the image is blurred, the SMD selects a similar, but incorrect, candidate. For the Adaptive SMD, the information of the blurred part, especially the relatively clean area in the blurred part like the upper left area of the image, helped to achieve success in recognition process.

Our experiments have tested only one font. However, if the data is a multifont document, the problem will be troublesome. How to find an optimum number of times of thinning for a multifont document or characters with different structural complexities is an arduous task for improving our method. Moreover,

TABLE 4
Examples of Images That Are Correctly Recognized by the Adaptive SMD

		(a)	(b)	(c)
Image		飴	闇	憶
Answer		飴	闇	憶
Candidate	Adaptive SMD	飴	闇	憶
	SMD	胎	簡	撹

there are some peculiarities of Kanji that should be concerned. For example, for some kinds of designs of Kanji, such as *Mincho* font, the width of the vertical stroke is much wider than the width of the horizontal stroke. If blurring occurs between two vertical strokes, the noise will be easily detected by our method. However, if the space between two horizontal strokes is blurred, the noise is almost impossible to find since the width of the two joint horizontal strokes may be smaller than the width of one vertical stroke.

5 CONCLUSIONS

For most statistical methods of pattern recognition, achieving the exact expression of distribution of feature vectors is the first step of accurate recognition. However, in the case that noise is included in an image, the feature vector will be quite different from that of a clean image. Since noise occurs irregularly and accidentally, it is difficult to create a dictionary that can cope with all kinds of noise, regardless of using numerous kinds of noisy training patterns.

In this paper, by analyzing the characteristics of noise, a new modification of discriminant functions for recognizing noisy patterns has been presented. Blurred Kanji recognition is adopted to examine the usefulness of the proposed discriminant function in solving practical problems. In order to quantify the relationship between noise and change in distribution, the ratio of the standard deviation of the elements with a certain degree of blur to noiseless elements is introduced. Then, a revision matrix is constructed using these ratios. By introducing the revision matrix to the Mahalanobis distance, Adaptive Mahalanobis distance has been proposed. Since the Adaptive Mahalanobis distance always considers the information from an unknown pattern and can adapt correspondingly to the condition of individual patterns, it is a more suitable discriminant function in coping with various quality patterns simultaneously. The effectiveness of this discriminant function is confirmed by the experimental results of blurred character recognition.

The proposed discriminant function can be used for other practical, noisy pattern recognition, such as speech recognition and face recognition, provided that a way can be found to calculate the revision matrix. Also, the revision matrix can be easily incorporated into other discriminant functions. To make our model effective for multifont document and other kinds of noise is the subject of future work.

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