Unified Analysis of Postdetection Diversity for Binary Digital FM Mobile Radio

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Abstract—**An analysis is presented of postdetection diversity using both selection combining and general combining for the reception of a binary digital FM signal in a Rayleigh fading environment. Noncoherent (differential and frequency) demodulation is assumed. In the general combiner, the output of each branch demodulator is weighted by the υth power of the demodulator input signal envelope (weighting factor). The optimum weighting factor is found to be $\upsilon = 2$. It is shown that postdetection general combiners using weighting factors of $\upsilon = 1$ and 2 correspond to predetection equal-gain and maximal-ratio combiners, respectively. A closed-form solution and a fairly simple expression are derived for the average bit error rate. Numerical calculations show that the postdetection two-branch diversity gain is only about 0.9 dB inferior to the predetection system when minimum shift keying (MSK) is used.**

I. Introduction

RECENTLY, digital FM transmission has been a growing interest in the mobile radio field. A narrow-band modulation scheme is needed to make the most efficient use of the limited frequency resource and a constant envelope property is also necessary because class C RF amplifiers are used in mobile transmitters. Digital FM (or continous phase frequency shift keying) is a desirable modulation scheme for digital land mobile radio [1], [2].

Since the received signal is subject to multipath fading [3], which severely degrades the signal transmission performance, some auxiliary techniques are necessary to reduce the multipath fading effects. One of the most efficient techniques is diversity reception [4]. The literature contains many papers [5]–[9] devoted to the effect of predetection diversity on digital FM reception, but recently, postdetection diversity, where all branch outputs are combined after demodulation, has also attracted much attention [10]–[13]. The advantage of postdetection diversity is that the cophasing function is not required since after demodulation all baseband signals are in phase. Furthermore, postdetection selection combining may not cause switching noise (as predetection does) and is applicable to narrow-band digital FM reception.

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When the signal fades, the demodulator output is corrupted by noise. In these circumstances, it is advantageous to reduce the contribution of the fading branch to the combiner output, and this can be achieved by weighting the output of each demodulator before combining. In general, the υth power of the signal envelope at the input of the demodulator is used as the weighting factor, and this technique is termed "general combining." Assuming weighting factors of $\upsilon = 1$ and 2, Adachi and Hattori [12] analyzed the average bit error rate (BER) of two-branch diversity systems using digital FM with frequency demodulation (FD). For a weighting factor $\upsilon = 2$, Suzuki [13] analyzed the average BER of digital FM with differential demodulation (DD). The effect of postdetection selection combining has been also analyzed for digital FM with DD [10] and experimentally validated [11].

The aim of this paper is to present a unified analysis of multiple-branch postdetection diversity for binary FM with DD and FD in a fast Rayleigh fading environment. Premodulation filtering of the input to the FM modulator as used in Gaussian minimum shift keying (GMSK) [1] and generalized tamed FM (GTFM) [2] is not considered and hence conventional digital FM using a nonreturn-to-zero (NRZ) data sequence as an input to the FM modulator is assumed. While the intersymbol interference due to the receiver predetection filter bandwidth restriction is not taken into account, both cochannel interference and random FM noise are considered. A mathematical expression for the postdetection combiner output is derived in Section II and the optimum weighting factor which yields the maximum diversity improvement is found in Section III. The average BER analysis is presented in Section IV and a fairly simple expression is derived.

Digital FM with a modulation index of 0.5 is called minimum shift keying (MSK) [14] and is considered to be a reference modulation scheme for partial response digital FM schemes such as GMSK and GTFM when a performance comparison is made. Furthermore, the best BER performance with DD is achieved when MSK is used. Therefore, numerical calculations for the average BER and the diversity gain using MSK are presented in Section V. The diversity gain is compared with that of a predetection diversity system.

II. Postdetection Combiner Output Representation

A. Demodulator Output Representation

Let us assume that both desired and cochannel interference signals, having a bandwidth less than the coherence bandwidth

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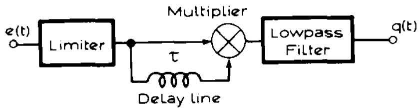

Fig. 1. Differential demodulator.

of the multipath channel, are received. Both desired and cochannel interference signals are subject to mutually independent multiplicative fading. Further, we assume that the predetection filter bandwidth of the receiver is wide enough not to distort the received signal.

Since we are assuming Rayleigh fading, the input signal to the demodulator can be represented as

$$e(t) = \mathrm{Re}\,\{z(t)\exp[j\omega_c t]\} = R(t)\cos[\omega_c t + \Psi(t)] \tag{1}$$

where Re $\{\cdot\}$ is the real part of the complex value, ω_c is the carrier angular frequency, $R(t)$ and $\Psi(t)$ are the time-varying envelope and signal phase plus phase noise contributed by cochannel interference and additive white Gaussian noise (AWGN), respectively. $z(t)$ is the complex envelope given by

$$z(t) = z_s(t)\exp[j\Phi_s(t)] + z_i(t)\exp[j\Phi_i(t)] + z_n(t) = R(t)\exp[j\Psi(t)]. \tag{2}$$

In (2), the subscripts s, i, and n denote desired signal, cochannel interference and AWGN, respectively, $z_s(t)$, $z_i(t)$, and $z_n(t)$ are mutually independent zero-mean complex Gaussian processes, and $\Phi_s(t)$ and $\Phi_i(t)$ are the modulation phases. $\Phi_s(t)$ is given by

$$\Phi_s'(t) = \frac{d}{dt}\Phi_s(t) = \frac{m\pi}{T}\sum_{n=-\infty}^{\infty} a_n g(t-nT) \tag{3}$$

where $a_n (= \pm 1)$ is an NRZ data sequence with bit duration T, m is the modulation index and $g(t) = 1$ for $-T/2 < t \leq T/2$ and 0 elsewhere.

Digital FM signals can be frequency demodulated, or alternatively differentially demodulated [15]. The output $q(t)$ for 1-bit differential demodulation and for frequency demodulation considered in this paper are the sine function of the phase difference over one bit duration and the time derivative of $\Psi(t)$, respectively, and hence $q(t) = \sin[\Psi(t) - \Psi(t-T)]$ for DD and $q(t) = \Psi'(t)$ for FD. In the following, a complex representation will be used for succeeding BER analysis. DD is illustrated in Fig. 1, the input signal and its delayed replica being multiplied to yield the baseband signal output. The time delay τ of the delay line is chosen as $\tau \approx T$ and $\omega_c\tau = \pi(2n - 1/2)$. The complex representation for the output of DD is given by

$$q(t) = \mathrm{Im}\left\{\frac{z(t)z^*(t-T)}{|z(t)z(t-T)|}\right\} \tag{4}$$

where Im $\{\cdot\}$ denotes the imaginary part of the complex value and the asterisk denotes complex conjugate.

If τ is much shorter than one bit period T in DD, the differential demodulator output approaches the instantaneous angular frequency of its input. Since $z(t-\tau) \approx z(t) - z'(t)\cdot\tau$ for $\tau \ll T$, the complex representation for the FD output is given by

$$q(t) \approx \tau \cdot \mathrm{Im}\left\{\frac{-z(t)z'^*(t)}{|z(t)|^2}\right\}. \tag{5}$$

B. Diversity Combiner Output Representation

When the received signal fades, the demodulator output is corrupted by noise due to AWGN and cochannel interference. Therefore, it seems that there is little to be gained by simply combining all demodulator outputs. For N-branch diversity it is necessary either to select the demodulator output associated with the largest envelope of the demodulator input (selection combining) or to weight each demodulator output before combining (general combining).

Selection Combining: Switching before demodulation can cause an abrupt phase change in the input signal to the demodulator, thereby producing a decision error in the data. Switching after demodulation does not cause such errors, but except for switching noise effects, no inherent difference exists between postdetection and predetection selection diversity.

We note that the denominators of (4) and (5) are always positive, and therefore, as far as binary data transmission is concerned, the output of a selection combiner (SC) can be represented as

$$Q(t) = \mathrm{Im}\,\{z_k(t)\xi_k^*(t)\} \tag{6}$$

for simplification of the BER calculation, where $\xi_k(t) = z_k(t-T)$ for DD and $\xi_k(t) = -z_k'(t)$ for FD and $z_k(t)$ is the kth branch complex envelope given by (2). We have assumed that kth branch has the largest envelope.

General Combining: Each demodulator output is weighted before combination as indicated above. Let the weighting factor for the kth branch be

$$w_k(t) = \begin{cases} |z_k(t)|^{\nu-1}|z_k(t-T)|, & \text{for DD} \\ |z_k(t)|^{\nu}, & \text{for FD} \end{cases} \tag{7}$$

where $|z_k(t)| = R_k(t)$ is the envelope of the kth branch. For slow Rayleigh fading, the envelope is nearly constant over the duration of 1 bit. If the signal/noise-plus-interference ratio is sufficiently high, $|z_k(t)| \approx |z_k(t-T)|$ and (7) shows that the weighting factor for DD is then approximately the same as that for FD. Using (4), (5), and (7), a representation for the output of the general combiner can be obtained, as

$$Q(t) = \sum_{k=1}^{N} w_k(t)q_k(t) = \mathrm{Im}\left[\sum_{k=1}^{N} z_k(t)\xi_k^*(t)|z_k(t)|^{\nu-2}\right]. \tag{8}$$

III. Optimum Weighting Factor for General Combining

In this section we derive the optimum weighting factor which yields the minimum BER (thus maximum diversity improvement) assuming independent fading and that the power spectral of the complex envelopes of fading and AWGN are symmetrical. To find the optimum weighting factor, we first derive the conditional BER when all branch complex envelopes are given.

A. Derivation of Conditional BER

We consider the nth data bit a_n to be detected, without loss of generality. The combiner output Q_n at the decision instant t_n can be written as

$$Q_n = \mathrm{Im}\left\{\sum_{k=1}^{N} z_k \xi_k^* |z_k|^{v-2}\right\} \tag{9}$$

where z_k and ξ_k are the values of $z_k(t_n)$ and $\xi_k(t_n)$, respectively, and $t_n = (n + 1/2)T$ for DD (the end of the bit) and nT for FD (the middle of the bit). If the conditional probability density function (pdf) $p(\xi_k|z_k)$ of ξ_k with z_k given is found, then we can calculate the BER under the condition that all z_k are given. Since z_k and ξ_k are complex zero-mean Gaussian variable in Rayleigh fading, ξ_k is also a complex Gaussian variable. From Appendix I, we have

$$p(\xi_k|z_k) = \frac{1}{2\pi\sigma_2^2(1-|\rho|^2)} \exp\left[-\frac{|\xi_k - (\sigma_2/\sigma_1)\rho^* z_k|^2}{2\sigma_2^2(1-|\rho|^2)}\right] \tag{10}$$

where $\sigma_1^2 = 1/2\langle z_k \cdot z_k^*\rangle$, $\sigma_2^2 = 1/2\langle \xi_k \xi_k^*\rangle$, and $\sigma_1\sigma_2\rho = 1/2\langle z_k \cdot \xi_k^*\rangle$.

It is apparent from (10) that with z_k given, ξ_k is a complex Gaussian variable with mean $(\sigma_2/\sigma_1)\rho^* z_k$ and variance $\sigma_2^2(1 - |\rho|^2)$. Therefore, the postdetection combiner output Q_n becomes a Gaussian variable with mean $(\sigma_2/\sigma_1)\rho_s \Sigma |z_k|^v$ and variance $\sigma_2^2(1 - |\rho|^2)\Sigma|z_k|^{2v-2}$ when all z_k are given, where ρ_s is the imaginary part of ρ. Since a decision error in data bit a_n is produced when $a_n \cdot Q_n < 0$, the conditional BER is

$$\begin{aligned} p_e &= \frac{1}{2}\int_{-\infty}^{0} p(a_n Q_n | z_1, z_2, \cdots, z_N)\, da_n Q_n \\ &= \frac{1}{2}\,\mathrm{erfc}\left\{\frac{a_n \rho_s}{\sqrt{1-|\rho|^2}} \frac{R}{\sqrt{2}\sigma_1}\right\} \end{aligned} \tag{11}$$

where erfc $\{\cdot\}$ is the complementary error function, and

$$R = \frac{\sum_{k=1}^{N} R_k^v}{\sqrt{\sum_{k=1}^{N} R_k^{2v-2}}}, \tag{12}$$

with $R_k = |z_k|$ being the envelope of the kth branch.

Putting $N = 1$ in (11) gives the conditional BER for single branch reception, i.e., without diversity. In this case R is the envelope of the input to the demodulator. Therefore, R can be termed the equivalent signal envelope when postdetection diversity is employed.

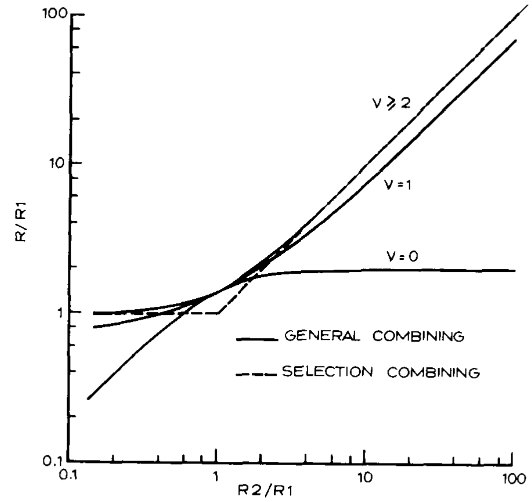

Fig. 2. Equivalent signal envelope for two-branch diversity (N = 2).

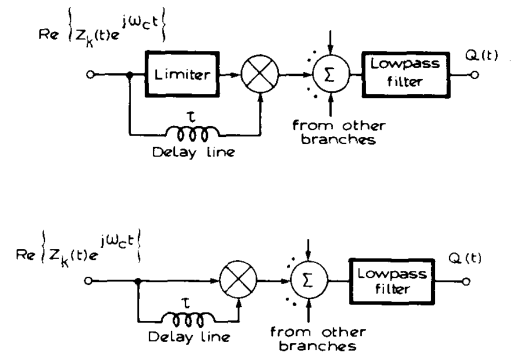

Fig. 3. Postdetection diversity combiner model. (a) v = 1. (b) v = 2.

B. Finding the Optimum Weighting Factor

It is easily understood that the minimum BER can be obtained when v is chosen so that R is maximized. By differentiating (12) with respect to v, it can be shown that R is maximized when $v = 2$. Fig. 2 shows the equivalent signal envelope R as a function of the envelope ratio R_2/R_1 for two-branch diversity ($N = 2$). The result for selection combining is also plotted for comparison (dashed line). It can be seen that while $v = 2$ is the optimum value, the equivalent envelopes for $v > 2$ are almost identical with that for $v = 2$. Choosing $v = 1$ results in only a small degradation in R.

For the case of $v = 0$, as R_2/R_1 becomes very small (the envelope R_2 fades while the other R_1 remains large), R/R_1 approaches zero. This represents the case when all the branch outputs are simply combined, and it may therefore be concluded that v should not be zero. This is easily understood from the fact that when the signal fades, the demodulator output is corrupted by noise whose power is comparable with the demodulator output with a high received signal level.

In the following, weighting factors of $v = 1$ and 2 are considered. The combiner models for $v = 1$ and 2 are shown in Fig. 3. Note that the combiner with the weighting factor $v = 2$ is identical with that proposed for DPSK reception [17], [18, ch. 6]. The combiner with the weighting factor $v = 1$ is more practical since the multiplier in Fig. 1 can be replaced with a mixer.

C. Comparison with Predetection Diversity

The comparison is made from the viewpoint of the equivalent signal envelope. For high signal/noise-plus-interference ratios, the demodulator input is predominated by its desired signal component. Therefore, the equivalent signal envelope given by (12) can be approximated as

$$R \approx \begin{cases} \sum_{k=1}^{N} R_{sk}/\sqrt{N}, & \text{for } v=1 \\ \sqrt{\sum_{k=1}^{N} R_{sk}^2}, & \text{for } v=2 \end{cases} \tag{13}$$

where $R_{sk} = |z_{sk}|$ is the kth branch desired signal envelope.

The predetection combiner output signal-to-noise ratio (SNR) γ is given by [18]

$$\gamma = \begin{cases} \left(\sum_{k=1}^{N} \sqrt{\gamma_k}\right)^2 \Big/ N, \\ \qquad \text{for equal-gain combining (EGC)} \\ \sum_{k=1}^{N} \gamma_k, \\ \qquad \text{for maximal-ratio combining (MRC)} \end{cases} \tag{14}$$

where $\gamma_k = R_{sk}^2/2\sigma_n^2$ is the individual SNR and σ_n^2 is the AWGN power of each branch. Since the equivalent signal envelope R can be defined so that $\gamma = R^2/2\sigma_n^2$, the equivalent signal envelopes of the postdetection combiners with $v = 1$ and 2 are approximately identical with those of predetection EGC and MRC, respectively, for high signal/noise-plus-interference ratios. Therefore, in this paper, postdetection combining using the weighting factors $v = 1$ and 2, respectively, are called postdetection EGC and MRC for convenience.

IV. Average BER Analysis

A. Expression for Average BER

The conditional BER is a function of the equivalent envelope R and is given by (11) for postdetection EGC ($v = 1$) and MRC ($v = 2$). The conditional BER for postdetection SC (selection combining) can also be represented by the same equation with R being the maximum envelope of the N branches. The average BER can be obtained by averaging the conditional BER with the pdf of R. From (12), R becomes

$$R \approx \begin{cases} \max\,[R_1, R_2, \cdots, R_N], & \text{for SC} \\ \sum_{k=1}^{N} R_k/\sqrt{N}, & \text{for EGC.} \\ \sqrt{\sum_{k=1}^{N} R_k^2}, & \text{for MRC} \end{cases} \tag{15}$$

Realizing that the expression for R is identical to that for the corresponding predetection combiner (see Section III-C), the pdf of R is exactly given by

$$p(R) = \begin{cases} \sum_{k=1}^{N} \binom{N}{k} (-1)^{k+1} k (R/\sigma_1^2) \exp\left[-k(R^2/2\sigma_1^2)\right], \\ \qquad \text{for SC} \\ \dfrac{R}{\sigma_1^2} \dfrac{(R^2/2\sigma_1^2)^{N-1}}{(N-1)!} \exp\left[-\dfrac{R^2}{2\sigma_1^2}\right], \\ \qquad \text{for MRC.} \end{cases} \tag{16}$$

For EGC, the approximate pdf is obtained by replacing σ_1^2 by $\alpha \cdot \sigma_1^2$, where $\alpha = [(2N-1)!!]^{1/N}/N$ [18, eq. 5.4-33]. Thus, averaging the conditional BER with the pdf of R and performing partial integration leads to the average BER given by

$$\begin{aligned} P_e &= \int_0^\infty p_e \cdot p(R)\, dR \\ &= \frac{1}{2}\left[1 - \sum_{k=1}^{N} \binom{N}{k} (-1)^{k+1} \cdot \frac{a_n \rho_s}{\sqrt{\rho_s^2 + k(1-|\rho|^2)}}\right], \quad \text{for SC} \\ &= \frac{1}{2}\left[1 - \frac{a_n \rho_s}{\sqrt{\rho_s^2 + (1-|\rho|^2)/\alpha}} \sum_{k=0}^{N-1} \frac{(2k-1)!!}{(2k)!!} \cdot \left(1 - \frac{\rho_s^2}{\rho_s^2 + (1-|\rho|^2)/\alpha}\right)^k\right], \quad \text{for EGC} \\ &= \frac{1}{2}\left[1 - \frac{a_n \rho_s}{\sqrt{1-\rho_c^2}} \sum_{k=0}^{N-1} \frac{(2k-1)!!}{(2k)!!} \cdot \left(1 - \frac{\rho_s^2}{1-\rho_c^2}\right)^k\right], \quad \text{for MRC.} \end{aligned} \tag{17}$$

It is apparent that the conditional BER decreases rapidly as the equivalent signal envelope R increases. It is therefore clear that almost all errors are caused by deep fades. Thus the average BER can also be calculated using an approximate formula for the pdf of R at small R. We have

$$P_e \approx k_N \left(\frac{1 - |a_n \rho|^2}{2\rho_s^2}\right)^N \tag{18}$$

where

$$k_N = \begin{cases} \dfrac{1}{2}(2N-1)!!, & \text{for SC} \\ \dfrac{1}{2}\dfrac{N^N}{N!}, & \text{for EGC.} \\ \dfrac{1}{2}\dfrac{(2N-1)!!}{N!}, & \text{for MRC} \end{cases} \tag{19}$$

A comparison of the average BER achieved using the three types of postdetection combiner can be made by examining the coefficient k_N. The smallest average BER is obtained using MRC, and when $N = 2$, the BER is half that obtainable using SC. The average BER with EGC is 2/3 that of SC.

To calculate the average BER, finding the value of ρ is necessary. As shown in Appendix I, ρ is derived from $\sigma_1\sigma_2\rho = 1/2\,\langle z_k \cdot \xi_k^*\rangle$. Let $\sigma_s^2\rho_s(\tau) = 1/2\,\langle z_s(t)\cdot z_s^*(t-\tau)\rangle$, $\sigma_i^2\rho_i(\tau) = 1/2\,\langle z_i(t)\cdot z_i^*(t-\tau)\rangle$, and $\sigma_n^2\rho_n(\tau) = 1/2\,\langle z_n(t)\cdot z_n^*(t-\tau)\rangle$. σ_s^2, σ_i^2, and σ_n^2 are the average powers of desired fading signal, cochannel interference, and AWGN, respectively. $\rho_s(\tau)$, $\rho_i(\tau)$, and $\rho_n(\tau)$ are the normalized autocorrelation functions of the complex envelopes $z_s(t)$, $z_i(t)$, and $z_n(t)$ and are derived by Fourier transform of the respective power spectra. Since the fading signal is composed of many multipath waves reflected from buildings surrounding the mobile, the shapes of $\rho_s(\tau)$ and $\rho_i(\tau)$ are affected by the distributions of arrival angle and amplitude of the multipath waves. The spectral density of AWGN at the input to the receiver predetection filter can be assumed to be constant, and therefore $\rho_n(\tau)$ can be determined from the bandpass characteristics of the predetection filter. We can assume that the predetection filter has symmetrical characteristics and that, in urban areas, the building are uniformly distributed around the mobile. Therefore, the power spectra of $z_s(t)$, $z_i(t)$, and $z_n(t)$ are symmetrical. Since $\rho_s(\tau)$, $\rho_i(\tau)$, and $\rho_n(\tau)$ become all real functions, $\rho_s'(0) = \rho_i'(0) = \rho_n'(0) = 0$, and ρ is given by

$$\rho = \begin{cases} \dfrac{\Gamma\rho_s(T)\exp[ja_n m\pi] + \dfrac{\Gamma}{\Lambda}\rho_i(T)\exp[j\Delta\Phi_i] + \rho_n(T)}{\dfrac{\Gamma}{\Lambda}+1}, & \text{for DD} \\[2ex] j\dfrac{\left(a_n\dfrac{m\pi}{T}\right)\Gamma + \Phi_i'\dfrac{\Gamma}{\Lambda}}{\sqrt{\Gamma+\dfrac{\Gamma}{\Lambda}+1}\sqrt{\Gamma\left\{\left(\dfrac{m\pi}{T}\right)^2 - \rho_s''(0)\right\} + \dfrac{\Gamma}{\Lambda}\{\Phi_i'^2 - \rho_i''(0)\} - \rho_n''(0)}}, & \text{for FD} \end{cases} \tag{20}$$

where $\Delta\Phi_i = \Phi_i(t_n) - \Phi_i(t_n - T)$, and $\Gamma = \sigma_s^2/\sigma_n^2$ and $\Lambda = \sigma_s^2/\sigma_i^2$ are the average signal-to-noise ratio and average signal-to-interference ratio (SIR), respectively.

The BER expressions derived as (17) and (20) are general formulas and can be applied to any digital FM signal with arbitrary modulation index. a_n, $\Delta\Phi_i$, and Φ_i' are random variables. It can be assumed that a_n takes values of ± 1 with equal probability. Obviously, the statistical properties of $\Delta\Phi_i$ and Φ_i' depend on the modulation scheme employed for the cochannel interference signal. Hence the calculation of overall average BER requires knowledge of the pdfs of $\Delta\Phi_i$ and Φ_i'. Also, the average BER depends on the power spectrum of AWGN at the predetection filter output.

B. Average BER's due to AWGN, Random FM Noise, and Cochannel Interference

For slow Rayleigh fading, errors are caused by AWGN and cochannel interference. For fast Rayleigh fading and large average SNR and SIR, most errors are caused by random FM noise. In the following, expressions for the average BER's due to AWGN, cochannel interference and random FM noise are derived using (18) and (20).

Average BER due to AWGN: Letting $\Lambda \to \infty$, $\rho_s(T)$ and $\rho_i(T) \to 1$, and $\rho_s''(0)$ and $\rho_i''(0) \to 0$ (slow fading assumption), the average BER due to AWGN is given by

$$Pe_1 \approx \begin{cases} k_N\left\{\dfrac{1}{\Gamma}\dfrac{1-\rho_n(T)\cos(m\pi)}{\sin^2(m\pi)}\right\}^N, & \text{for DD} \\[2ex] k_N\left\{\dfrac{1}{2\Gamma}\dfrac{(m\pi)^2 - \rho_n''(0)T^2}{(m\pi)^2}\right\}^N, & \text{for FD} \end{cases}. \tag{21}$$

It can be seen that when DD is employed, a modulation index of 0.5 (MSK) provides the smallest average BER irrespective of the number of diversity branches and the type of diversity combiner. On the other hand, in the case of FD, a larger modulation index provides a smaller average BER. However, for large modulation indices bandwidth restriction due to the predetection filter causes intersymbol interference, which is ignored in this paper, and thus there exists an optimum modulation index [21]. It can also be seen that the average BER decreases proportionately to the Nth power of the average SNR as in the case of predetection diversity.

Average BER due to Random FM Noise: Letting Γ and $\Lambda \to \infty$, the average BER due to random FM noise is given by

$$Pe_2 \approx \begin{cases} k_N\left\{\dfrac{1-\rho_s^2(T)}{2\rho_s^2(T)\sin^2(m\pi)}\right\}^N, & \text{for DD} \\[2ex] k_N\left\{\dfrac{-\rho_s''(0)T^2}{2(m\pi)^2}\right\}^N, & \text{for FD} \end{cases}. \tag{22}$$

For land mobile radio using a 900-MHz carrier and a data bit rate greater than several hundred per second, the fading can be considered to be very slow compared with the bit rate. In this case, $\rho_s(T)$ can be well approximated by $1 + \rho_s''(0)T^2/2$ (since $\rho_s'(0) = 0$). Therefore, the average BER is clearly a function of $-\rho_s''(0)$ for both DD and FD cases. $-\rho_s''(0)$ is the second-order moment of the fading power spectrum and is proportional to the square of the maximum Doppler frequency, given by vehicle speed/carrier wavelength [3]. If the vehicle speed is doubled, the average BER increases 16 times, when $N = 2$.

Average BER due to Cochannel Interference: We assume that the cochannel interference signal is also frequency modulated with the same modulation index as the desired signal. The signal timing and binary data of the cochannel interference can be assumed to be independent of those of the desired signal. Averaging (18) over $\Delta\Phi_i$ and Φ_i' gives us the average BER. Assuming that marks and spaces are sent with equal probability, we obtain the pdf's of $\Delta\Phi_i$ and Φ_i' by

$$p(\Delta\Phi_i) = \frac{1}{4}\,\delta(\Delta\Phi_i - m\pi) + \frac{1}{4}\,\delta(\Delta\Phi_i + m\pi) + \frac{1}{4m\pi}, \quad \text{for } |\Delta\Phi_i| \leqq m\pi$$

$$p(\Phi_i') = \frac{1}{2}\,\delta\left(\Phi_i' - \frac{m\pi}{T}\right) + \frac{1}{2}\,\delta\left(\Phi_i' + \frac{m\pi}{T}\right) \tag{23}$$

where $\delta(\cdot)$ is the delta function. Letting $\Gamma \to \infty$, $\rho_s(T)$ and $\rho_i(T) \to 1$, and $\rho_s''(0)$ and $\rho_i''(0) \to 0$ (slow fading assumption) and averaging (18) using (23), we find that the average BER due to cochannel interference becomes

$$Pe_3 \approx \begin{cases} \dfrac{k_N}{4}\left(\dfrac{2}{\Lambda}\right)^N \left[1 + \dfrac{\sum_{k=0}^{N-1} (-1)^{N+k} \binom{2N}{k} \dfrac{\sin[2m\pi(N-k)]}{m\pi(N-k)} + \binom{2N}{N}}{2^{2N-1}\sin^{2N}(m\pi)}\right], & \text{for DD} \\ \dfrac{k_N}{2}\left(\dfrac{2}{\Lambda}\right)^N, & \text{for FD} \end{cases} \tag{24}$$

The average BER decreases proportionately to the Nth power of the average SIR, as seen for the average BER due to AWGN. This is not a surprising result because the cochannel interference suffers from Rayleigh fading characterized by multiplicative complex Gaussian process (see (2)) and can be considered as Gaussian noise.

V. Numerical Calculations

A. Average BER

We assume that equal amplitude multipath waves arrive from all directions with equal probability and that the receiver predetection filter has a rectangular bandpass characteristic with bandwidth-time product BT = 1.0. Autocorrelation functions $\rho_s(\tau)$, $\rho_i(\tau)$, and $\rho_n(\tau)$ are given by

$$\rho_s(\tau) = \rho_i(\tau) = J_0(2\pi f_D \tau)$$

$$\rho_n(\tau) = \frac{\sin(\pi B\tau)}{\pi B\tau} \tag{25}$$

where $J_0(\cdot)$ is the zero-order Bessel function and f_D is the maximum Doppler frequency.

The expressions for average BER's due to AWGN, cochannel interference, and random FM noise are given by (21), (22), and (24), respectively, and are applicable for an arbitrary modulation index. MSK (digital FM with modulation index 0.5) is a popular modulation scheme and is considered a reference for other digital FM schemes such as GMSK and GTFM. Furthermore, when DD is used, MSK gives the best BER performance. In the following numerical calculations, we assume MSK transmission.

Assuming that a mark is sent with a probability of 1/2, the average BER's, Pe_1, Pe_2, and Pe_3 become

$$Pe_1 \approx \begin{cases} k_N \dfrac{1}{\Gamma^N}, & \text{for DD} \\ k_N \dfrac{1}{\left(\dfrac{6}{7}\Gamma\right)^N}, & \text{for FD} \end{cases} \tag{26}$$

$$Pe_2 \approx \begin{cases} k_N(\pi f_D T)^{2N}, & \text{for DD} \\ k_N(2 f_D T)^{2N}, & \text{for FD} \end{cases} \tag{27}$$

and

$$Pe_3 \approx \begin{cases} \dfrac{k_N}{4(\Lambda/2)^N}\left\{1 + \dfrac{\binom{2N}{N}}{2^{2N-1}}\right\}, & \text{for DD} \\ \dfrac{k_N}{2(\Lambda/2)^N}, & \text{for FD} \end{cases} \tag{28}$$

Numerical results calculated from (26)–(28) are plotted in Figs 4–6 for two-branch diversity ($N = 2$). Although MRC is superior to EGC and SC, the difference in diversity improvement is small. Note that FD is superior to DD as far as the random FM noise is concerned.

B. Diversity Gain

Diversity gain is defined as the reduction in the average SNR or SIR required to achieve a certain average BER. From (26) and (28), diversity gains in decibels for average SNR and SIR are calculated and are shown in Figs. 7 and 8 for $Pe = 10^{-5}$. Almost the same diversity gain is obtained for DD and FD. Diversity gain for average SNR is slightly larger than that for average SIR. When two-branch diversity ($N = 2$) is used, a diversity gain of approximately 19–22 dB is obtained for $Pe = 10^{-5}$. MRC has the largest gain, but the difference in gain between MRC and SC is less than 2 dB for $N = 2$.

C. Comparison of Diversity Gain with Predetection Diversity

Since the diversity gain of postdetection SC is identical with that of predetection SC, MRC and EGC are considered here.

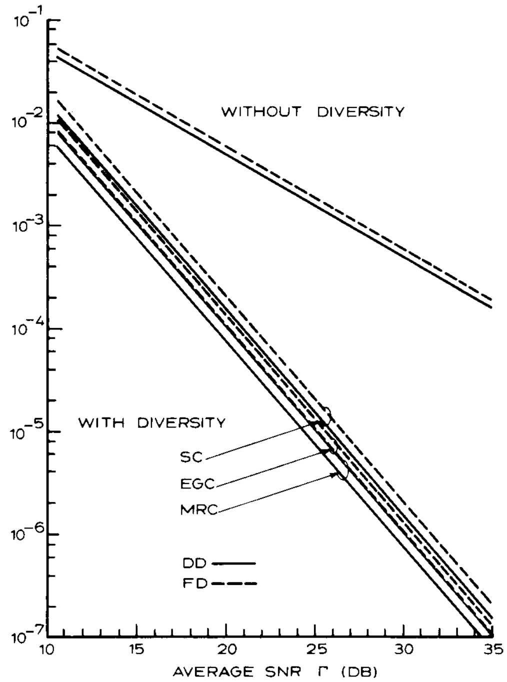

Fig. 4. Average BER of MSK due to additive Gaussian noise using two-branch diversity ($N = 2$).

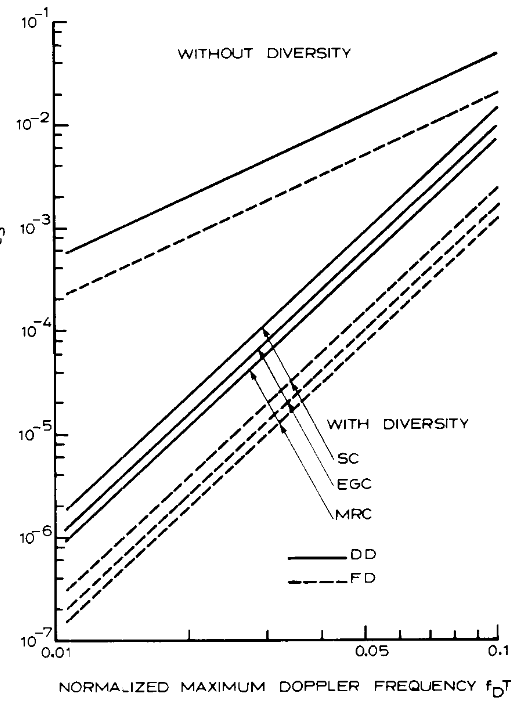

Fig. 6. Average BER of MSK due to random FM noise using two-branch diversity ($N = 2$).

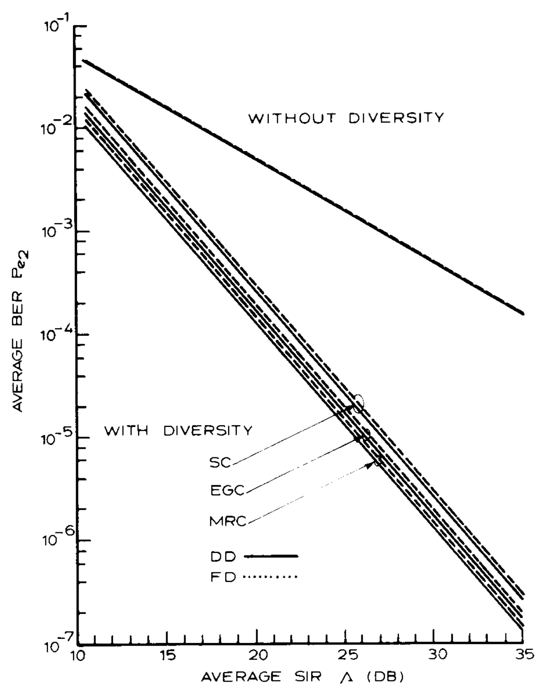

Fig. 5. Average BER of MSK due to cochannel interference using two-branch diversity ($N = 2$)

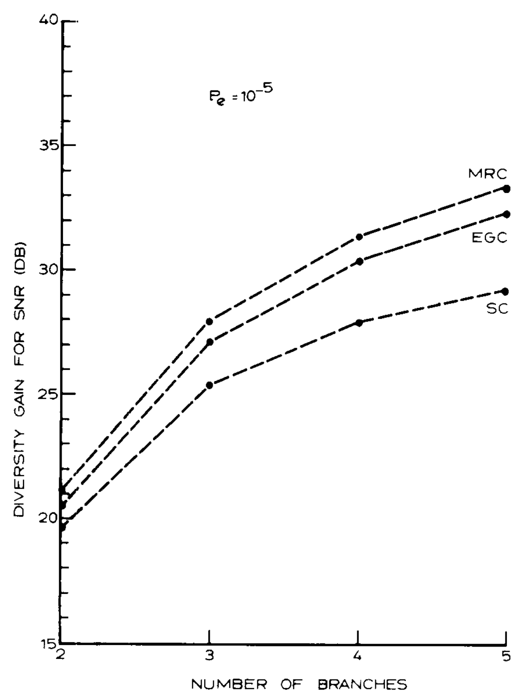

Fig. 7. Diversity gain for average SNR. Same diversity gain is obtained for DD and FD.

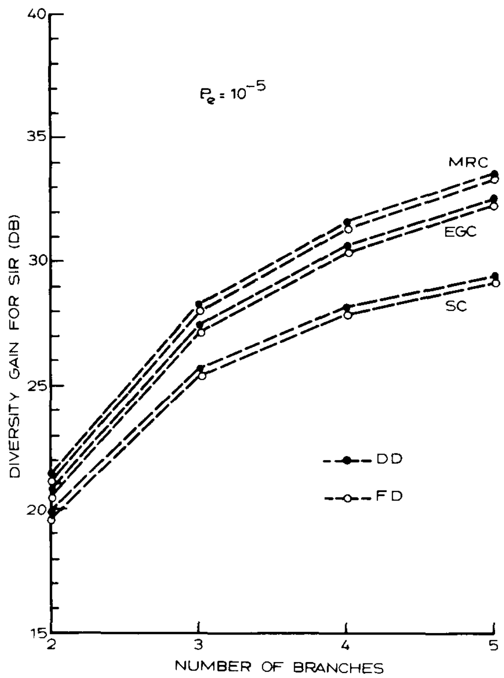

Fig. 8. Diversity gain for average SIR.

We assume ideal predetection MRC and EGC which perfectly cophase all desired signals and thus eliminate the random FM noise. The average BER due to AWGN with predetection MRC becomes (from Appendix II)

$$Pe_1 \approx \begin{cases} \dfrac{1}{2\Gamma^N}, & \text{for DD} \\[2ex] \dfrac{1}{2\Gamma^N} \displaystyle\sum_{k=0}^{N} \binom{N}{k} \frac{(2k-1)!!}{(2k)!!} \frac{1}{3^k}, & \text{for FD} \end{cases} \quad (29)$$

For EGC, Γ should be replaced by $\alpha \cdot \Gamma$.

Comparing (26) and (29), the diversity gain difference between postdetection and predetection diversity can be easily calculated. As the number of diversity branches increases, the increase in diversity gain becomes less and the receiver structure becomes more complex. Therefore, two-branch diversity ($N = 2$) is considered to be the most practical. In the postdetection MRC ($\upsilon = 2$) and EGC ($\upsilon = 1$) systems considered in this paper, all demodulator outputs are used; the contribution of the fading branch to the combiner output is reduced by weighting the output of each demodulator before combining, and consequently, the diversity gain must be larger than that of SC. It is known that predetection MRC is 1.5 dB superior to SC (this can be shown by examining the pdf of the predetection combiner output SNR) and that there is no inherent difference in the diversity gain between predetection and postdetection SC. Therefore, the difference in diversity gain between predetection and postdetection diversity using EGC or MRC must be smaller than 1.5 dB. It can be shown from (26) and (29) that when $N = 2$, the difference is only 0.88 dB for DD and 0.86 dB for FD.

VI. Conclusion

This paper has presented a unified analysis of postdetection diversity for binary digital FM with differential demodulation and frequency demodulation in a Rayleigh fading environment. Both selection and general combining have been considered. The weighting factor for general combining is the υth power of the demodulator input signal envelope. The optimum weighting factor to yield the maximum diversity improvement has been found to be $\upsilon = 2$. Postdetection diversity combiners using weighting factors of $\upsilon = 1$ and 2 correspond to the predetection equal-gain combiner and maximal-ratio combiner, respectively. A closed-form solution and a fairly simple approximation have been derived for the average BER. Assuming MSK reception, numerical calculations of the diversity gain show that two-branch postdetection diversity is only 0.9 dB inferior to predetection. Because of this small difference in diversity gain compared with predetection diversity and the absence of the requirement for a cophasing function, postdetection diversity can be applied successfully to digital communication systems.

Since differential demodulation of MSK signals provides the same BER as that of differential phase-shift keyed (DPSK) signals [20], the results obtained for MSK in this paper can be applied to a DPSK system. Postdetection diversity for DPSK has been analyzed [18, ch. 6] and is equivalent to the postdetection diversity ($\upsilon = 2$) of this paper. Using the expression for BER [18, eq. 6.7-26], it can be shown that the diversity gain difference between the postdetection and predetection cases is 0.9 dB, which is identical with the MSK case.

Partial response digital FM systems are attractive for mobile radio use because a much narrower power spectrum than MSK is realized by introducing premodulation filtering of the baseband signal before application to the FM modulator at the transmitter [1], [2]. In the BER analysis in this paper, premodulation filtering was not considered. However, the analysis can be easily extended to the case of partial response digital FM by modifying $g(t)$ in (3) which is now the pulse response of the premodulation filter.

Appendix I

The conditional pdf of ξ_k can be derived from

$$p(\xi_k | z_k) = \frac{p(z_k, \xi_k)}{p(z_k)} \tag{30}$$

where $p(z_k, \xi_k)$ is the joint pdf of z_k and ξ_k and $p(z_k)$ is the pdf of z_k. Let the covariance matrix $\boldsymbol{H}$ of $z = (z_k, \xi_k)^T$ be defined by

$$\boldsymbol{H} = \frac{1}{2}\langle z^* \cdot z^T \rangle = \begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho^* \\ \sigma_1\sigma_2\rho & \sigma_2^2 \end{pmatrix} \tag{31}$$

where $(\cdot)^T$ is a transposed matrix. Assuming that each branch has an identical covariance matrix $\boldsymbol{H}$, but that the fading is independent, then $p(z_k, \xi_k)$ and $p(z_k)$ are given by [16]

$$p(z_k, \xi_k) = \frac{1}{(2\pi)^2 \det \boldsymbol{H}} \exp\left[-\frac{1}{2} z^T \boldsymbol{H}^{-1} z^*\right]$$

$$p(z_k) = \frac{1}{(2\pi)^2 \sigma_1^2} \exp\left[-\frac{|z_k|^2}{2\sigma_1^2}\right]. \tag{32}$$

Substitution of (32) into (30) gives (10).

Appendix II

The average BER due to AWGN with predetection MRC will be derived. The conditional BER, when the demodulator input SNR is γ, is given by [15], [19]

$$\begin{aligned} p_e(\gamma) &= \frac{1}{2\pi}\int_0^{\pi} \exp\left[-\gamma \frac{c^2}{1+d^2\cos\Theta}\right] d\Theta \\ &= \frac{1}{\pi}\int_0^{\pi/2} \exp\left[-\gamma \frac{c^2}{1-d^2+2d^2\cos^2\Theta}\right] d\Theta \end{aligned} \tag{33}$$

where

$$\begin{aligned} c^2 &= \frac{\sin^2(m\pi)}{1-\rho_n(T)\cos(m\pi)} \\ d^2 &= \frac{\rho_n(T)-\cos(m\pi)}{1-\rho_n(T)\cos(m\pi)}, \qquad \text{for DD} \end{aligned} \tag{34}$$

and

$$\begin{aligned} c^2 &= \frac{2(m\pi)^2}{(m\pi)^2 - T^2\rho_n''(0)} \\ d^2 &= \frac{(m\pi)^2 + T^2\rho_n''(0)}{(m\pi)^2 - T^2\rho_n''(0)}, \qquad \text{for FD} \end{aligned} \tag{35}$$

where $\rho_n(\tau)$ and m are again the normalized autocorrelation function of the complex envelope of AWGN and the modulation index, respectively. The pdf of γ for MRC is given by [18]

$$p(\gamma) = \frac{1}{\Gamma^N} \frac{\gamma^{N-1}}{(N-1)!} \exp\left[-\frac{\gamma}{\Gamma}\right]. \tag{36}$$

Averaging the conditional BER with the pdf of γ, the average BER due to AWGN with predetection MRC is given by

$$Pe_1 \approx \frac{1}{2}\left(\frac{1-d^2}{c^2}\frac{1}{\Gamma}\right)^N \sum_{k=0}^{N} \binom{N}{k} \frac{(2k-1)!!}{(2k)!!} \left(\frac{2d^2}{1-d^2}\right)^k. \tag{37}$$

We used the following integrations

$$\int_0^{\pi} e^{-ax} x^b \, dx = \frac{\Gamma(b+1)}{a^{b+1}}$$

$$\int_0^{\pi} \cos^{2n} x \, dx = \frac{\sqrt{\pi}}{2}\frac{\Gamma(n+0.5)}{\Gamma(n+1)} = \frac{\sqrt{\pi}(2n-1)!!}{2(2n)!!} \tag{38}$$

where $\Gamma(\cdot)$ is the Gamma function.

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