

Handling of a Constrained Flexible Object by a Robot

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Abstract

In this paper, we deal with the handling of a constrained flexible object by a robot. We regard the robot as rigid manipulator and the object as its one side being free and the other side fixed in the wall. Our purpose is, for robot, to follow up the open end of the flexible object and to suppress its vibration. As a technique to achieve our purpose, we propose a new numerical modeling to handle the object by the robot. This model reveals the force relationship in the handling point. Next, we constitute the control system to make possible achieve two performances, following up the object and vibration suppression.

1 Introduction

Some of the problems taken up in the handling of the flexible object by robot include the necessity of dynamical identification of the object, restriction on sensor selection, and mode uncertainty on account of the object being a distributed parameter system *etc.* Therefore, realization of the high-performance controller does not only depend on the handling of the object actively, because the object is a distributed parameter system in contrast to the rigid object. A set of manipulation forces applied to the object does not correspond to a set of forces reacted from the object, resulting in production of vibration.

Next, we note about the historical background of handling strategy for the flexible object. Recently some researchers have started the study of control of the flexible object using a manipulator. Zheng *et al* [1] and Arai *et al* [2] have realized the position control of the flexible object. Their purpose is to insert the flexible object's one end into a hole in concrete while holding the other end. In this paper, unlike using above strategy, we deal with a problem of handling an end of the flexible object by robot while the other end is fixed in the wall. For this we consider the dynamical characteristics of the object. Further-

more, the robot and the object together have made up a unit system, so that whole system builds up a closed loop. Generally, the analysis of thus constrained dynamical system has been done with the unknown multiplier method [3], however, we apply a new method to calculate the interfering force in the handling point based on the numerical management which does not demand any consideration of the constraint condition. For developing this method, we also propose a new handling method which simultaneously follows up the object and suppresses its vibration using only one control system structure.

A brief summary of our results and the organization of the paper is as follows: In Section 2, we note the formulation of our study. In Section 3, we present the robot's equation of motion and the modeling of the object. Section 4 gives the linearization of the robot's equation of motion. Section 5 gives how to approach the object. Section 6 shows the constrained forces in the handling position. Section 7 gives the handling system design simultaneously satisfied under contact and non-contact conditions. Section 8 gives the simulation effects of proposed modeling. Finally, in Section 9, the conclusions of this work are given.

2 Problem formulation

The robot and the object move in the same plane. We distinguish the robot's work as observation, following, handling, and manipulation. Suppose that the robot is capable of observing the object's vibration. The object can be expressed by its dynamic characteristics.

3 Kinematics and dynamics

3.1 Robot's equation of motion

Using Lagrange's formulation, the robot's equation of motion is written as follows:

$$\mathbf{J}(\theta)\ddot{\theta} + \mathbf{C}(\theta, \dot{\theta}) + \mathbf{D}\dot{\theta} + \mathbf{P}(\theta) = \boldsymbol{\tau} \quad (1)$$

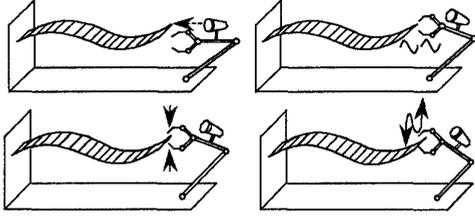


Figure 1: Observation, following, handling, manipulation.

where $\theta \in \mathbb{R}^{3 \times 1}$ is the joint angle vector. $J \in \mathbb{R}^{3 \times 3}$ is the inertial force coefficient matrix, $C \in \mathbb{R}^{3 \times 3}$ is centrifugal force term, $D \in \mathbb{R}^{3 \times 3}$ is the damping frictional force coefficient, $P \in \mathbb{R}^{3 \times 1}$ is the gravity term and $\tau \in \mathbb{R}^{3 \times 1}$ is the torque input vector.

3.2 Modeling of the object

We regard the object as a flexible beam. Because the object constrained on the wall is handled by the robot, we assume that one end of the beam is free, while the other is fixed. We also assume that the robot handles free edge of the object and suppresses vibration there. The overview of the robot and the object is shown in Figure 2. It is important to constitute the model from two points of view; to analyze the object dynamics and to design the active handling system.

For the modeling technique of the distributed parameter system, hypothetic method[4] and finite-element method[5], [6] are well known. Recently, due to simplicity of Holzer's method[7], practical usage has been proposed. In this paper, since we already know the shape of the flexible object and deal with the most simplified system whose density and mass distribution are constant, we produce the model of the object using the Galerkin's method with a precise mode function.

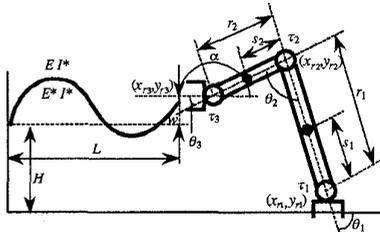


Figure 2: Situation of the robot and the flexible object.

3.2.1 The state-space description of the fundamental equation

The fundamental equation of the beam when external forces and torques are also present is given as follows:

$$EI^* \frac{\partial^4 w(x, t)}{\partial x^4} + E^* I^* \frac{\partial^5 w(x, t)}{\partial x^4 \partial t} + \rho A \frac{\partial^2 w(x, t)}{\partial t^2} = f_{by}(x_u, t) \delta(x - x_u) + \tau_b(x_v, t) \delta'(x - x_v) \quad (2)$$

$$w(x, t) = \sum_{i=1}^{\infty} \phi_i(x) \eta_i(t) \quad (3)$$

where $w_j(x_j, t)$ is the bending displacement at $x = x_j$, A is the cross-sectional-area of the beam, E it's vertical elastic coefficient, ρ is density, E^* the damping coefficient, I^* the area moment of inertia; $f_{by}(x_u, t)$ is the force input at $x = x_u$, $\tau_b(x_v, t)$ is the torque input at $x = x_v$, and $\eta_i(t)$ is an unknown function, $\phi_i(x)$ is the mode function, δ is the delta function, x_j is the measured position, L is the beam length, and t is time. i stands for i th order mode, and j stands for the number of sensor. We suppose that the force $f_{bx}(x_w, t)$ in the x -direction does not generate bending of the beam.

The mode function and the boundary condition of the free end and the fixed end are given by:

$$\phi_i(x) = \frac{\cosh\left(\frac{k_i x}{L}\right) - \cos\left(\frac{k_i x}{L}\right)}{\cosh(k_i) + \cos(k_i)} - \frac{\sinh\left(\frac{k_i x}{L}\right) - \sin\left(\frac{k_i x}{L}\right)}{\sinh(k_i) + \sin(k_i)} \quad (4)$$

$$(\phi_i(x))_{x=0} = \left(\frac{\partial \phi_i(x)}{\partial x}\right)_{x=0} = \left(\frac{\partial^2 \phi_i(x)}{\partial x^2}\right)_{x=L} = \left(\frac{\partial^3 \phi_i(x)}{\partial x^3}\right)_{x=L} = 0. \quad (5)$$

k_i can be approximated by:

$$1 + \cos(k_i) \cosh(k_i) = 0. \quad (6)$$

Using the Galerkin's method, the state-space description of the beam relative to the unknown function $\eta_i(t)$ is obtained as:

$$\dot{z}_b = A_b z_b + B_b u_b, \quad y_b = C_b z_b \quad (7)$$

where

$$\begin{aligned} z_b &= [\eta_1(t) \quad \dot{\eta}_1(t) \quad \cdots \quad \eta_i(t) \quad \dot{\eta}_i(t)]^T \in \mathbb{R}^{2i \times 1} \\ y_b &= [w_1(x_1, t) \quad w_2(x_2, t) \quad \cdots \quad w_j(x_j, t)]^T \in \mathbb{R}^{j \times 1} \\ u_b &= [f_{by}(x_u, t) \quad \tau_b(x_v, t)]^T \in \mathbb{R}^{2 \times 1} \\ A_b &= \text{block diag} [A_1, A_2, A_3, \dots, A_i] \in \mathbb{R}^{2i \times 2i} \\ B_b &= [B_1^T \quad B_2^T \quad B_3^T \quad \cdots \quad B_i^T]^T \in \mathbb{R}^{2i \times 2} \\ C_b &= \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1i} \\ C_{21} & C_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ C_{j1} & \cdots & \cdots & C_{ji} \end{bmatrix} \in \mathbb{R}^{j \times 2i} \end{aligned}$$

$$\begin{aligned} \mathbf{A}_i &= \begin{bmatrix} 0 & 1 \\ -\frac{EI^*}{\rho A} \left(\frac{k_i}{L}\right)^4 & -\frac{E^*I^*}{\rho A} \left(\frac{k_i}{L}\right)^4 \end{bmatrix} \in \mathfrak{R}^{2 \times 2} \\ \mathbf{B}_i &= \begin{bmatrix} 0 & 0 \\ \frac{1}{\rho A} \phi_i(x_u) & -\frac{1}{\rho A} \frac{d}{dx} \phi_i(x_v) \end{bmatrix} \in \mathfrak{R}^{2 \times 2} \\ \mathbf{C}_{ji} &= \begin{bmatrix} \phi_i(x_j) & 0 \end{bmatrix} \in \mathfrak{R}^{1 \times 2}. \end{aligned}$$

3.2.2 Reduction of the object into a finite dimensional system

In this section, we describe the reduction of the object beam. The control theory for the linear finite dimensional system cannot be applied to the distributed-parameter system. If the original system is approximated by some lower order modes neglecting the higher order modes, the control system may generate a spill over and unstabilize. So, we reduce the original model of the elastic vibrating system using the stabilization method[8], then the reduced model can be stabilized in spite of its modeling errors. The method is as follows:

Let the state variable vector \mathbf{z}_b^n be the state response of the system to the input \mathbf{u}_b^n as:

$$\mathbf{u}_b^n = \begin{bmatrix} \mathbf{0}^{1 \times (n-1)} & \delta(t) & \mathbf{0}^{1 \times (2-n)} \end{bmatrix}^T \in \mathfrak{R}^{2 \times 1} \quad (8)$$

$(n = 1 \sim 2).$

In the beam's state-space equations (7) the r dimensional vector \mathbf{Rz}_b^n is given as the linear combination of the state response generated by the impulse input. Let $\tilde{\mathbf{z}}_b$ be the new state variable for the reduced system. We apply the vector \mathbf{Rz}_b^n to the following reduced model:

$$\dot{\tilde{\mathbf{z}}}_b = \mathbf{A}_r \tilde{\mathbf{z}}_b + \mathbf{B}_r \mathbf{u}_b. \quad (9)$$

The error between the original model and the reduced model is given as follows:

$$\mathbf{d} = \mathbf{R}\dot{\mathbf{z}}_b - (\mathbf{A}_r \mathbf{Rz}_b + \mathbf{B}_r \mathbf{u}_b). \quad (10)$$

From the above equation (10), the system matrices of the reduced model are given as follows:

$$\mathbf{A}_r = \mathbf{R}\mathbf{A}_b \mathbf{W}_c \mathbf{R}^T \left(\mathbf{R}\mathbf{W}_c \mathbf{R}^T \right)^{-1}, \quad \mathbf{B}_r = \mathbf{R}\mathbf{B}_b$$

which minimize the mean value of the state response by the r linearly independent impulse inputs. \mathbf{W}_c is the controllability Gramian matrix of eq. (7). Choosing \mathbf{R} as follows:

$$\mathbf{R} = \begin{bmatrix} \phi_1(x_1) \mathbf{I}^{2 \times 2} & \cdots & \phi_i(x_1) \mathbf{I}^{2 \times 2} \\ \vdots & \ddots & \vdots \\ \phi_1(x_j) \mathbf{I}^{2 \times 2} & \cdots & \phi_i(x_j) \mathbf{I}^{2 \times 2} \end{bmatrix} \in \mathfrak{R}^{2j \times 2i}. \quad (11)$$

The state variables (unknown function $\eta_i(t)$) of the system (7) change to the state variables (sensor displacement $w_j(x_j, t)$) of the reduced model (9). This is desirable in the control system design, because the displacement detected by the sensor can be applied directly to the state feedback control.

4 Linearization of the robot

In this section, we note down the linearization of the robot's equation of motion. Set the position-posture variable of the end-effector as $\mathbf{y} = [x_{r3} \ y_{r3} \ \alpha]^T$. Define the relationship between the position-posture variable of the end-effector and the joint angle of the robot as $\mathbf{y} = \mathbf{f}_y(\boldsymbol{\theta})$. Set the force and moment effect on the end-effector as $\mathbf{f} = [f_{bx} \ f_{by} \ \tau_b]^T$. The relationship between $\dot{\boldsymbol{\theta}}$ and $\dot{\mathbf{y}}$ can be held as $\dot{\mathbf{y}} = \mathbf{J}_y(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}$, where $\mathbf{J}_y(\boldsymbol{\theta})$ is Jacobian matrix. Define a new input \mathbf{u}_y whose physical sense is acceleration, and using this parameter, let's input the following non-linear feedback to eq. (1),

$$\begin{aligned} \boldsymbol{\tau} &= \{ \mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \mathbf{D}\dot{\boldsymbol{\theta}} + \mathbf{P}(\boldsymbol{\theta}) - \mathbf{J}(\boldsymbol{\theta})\mathbf{J}_y^{-1}(\boldsymbol{\theta})\dot{\mathbf{J}}_y(\boldsymbol{\theta})\dot{\boldsymbol{\theta}} \} \\ &\quad + \{ \mathbf{J}(\boldsymbol{\theta})\mathbf{J}_y^{-1}(\boldsymbol{\theta}) \} \mathbf{u}_y \\ &\equiv \mathbf{C}_{r1}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \mathbf{C}_{r2}(\boldsymbol{\theta})\mathbf{u}_y \end{aligned} \quad (12)$$

then eq. (1) becomes for linearized and non-interfered system as:

$$\ddot{\mathbf{y}} = \mathbf{u}_y. \quad (13)$$

The control system to linearize the object is shown in Figure 3. Block \mathbf{M}_a represents the robot's equation of motion eq. (1). Each of the blocks \mathbf{C}_{r1} and \mathbf{C}_{r2} represents the elements of the linearizing compensator, respectively as in eq. (12).

5 Following up the object

In handling of the object, robot's work demands these performances; observation, approaching and furthermore, following up the object. For the observed bending displacement, let's substitute the following new input into the non-linear feedback (12),

$$\mathbf{u}_y = \begin{bmatrix} 0 \\ \ddot{w} \\ 0 \end{bmatrix} - \mathbf{K}_1 \begin{bmatrix} \dot{x}_{r3} \\ \dot{y}_{r3} - \dot{w} \\ \dot{\alpha} \end{bmatrix} - \mathbf{K}_2 \left(\begin{bmatrix} x_{r3} \\ y_{r3} - w \\ \alpha \end{bmatrix} - \mathbf{H} \right) \quad (\mathbf{K}_1, \mathbf{K}_2 > \mathbf{0}, \mathbf{H} = [L \ H \ \pi]^T) \quad (14)$$

then y_{r3} equals $w + H$, where \mathbf{K}_1 , \mathbf{K}_2 are positive gain matrices. So movement of end-effector responds

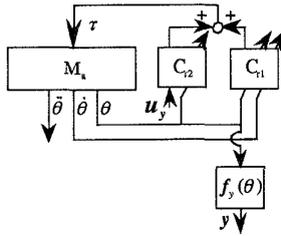


Figure 3: Linearizing control of the robot.

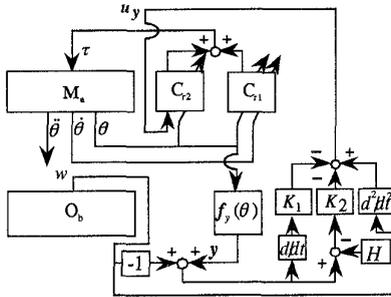


Figure 4: Following up the vibrating flexible object.

the motion of the object. The control system is shown in Figure 4. Block O_b represents the reduced model (9) of the object.

6 Constrained force in the handling position

After the robot handles the flexible object, both become a combined system then. For this system, we propose the model which can reveal the mutual relationship of torque and force in the handling position. *Interference of the torque (force)* can be calculated by being piled up repeating numerical simulation of this model continuously.

Overview of the model is shown in Figure 5. Kinematics of the robot, constrained as in our situation, has been studied using the unknown multiplier method[9]. However, this method reduces the order of the formulation of the model, so demands a change in control method, otherwise the control system design would be more complex. In our study, we suppose that the robot is capable of measurement of actual force from the object, so propose the method which is able to express positively the constraint force without using the unknown multiplier method.

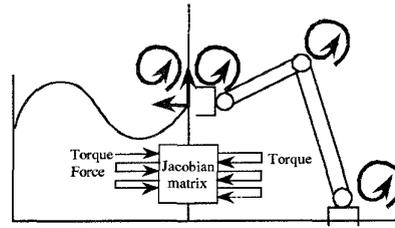


Figure 5: A summary of the combined system.

6.1 The constraint force relationship

Above-mentioned, the method to linearize the robot equation, for the position-posture variable \mathbf{y} can also be applied directly even if the external force acts on the end-effector. In our problem formulation, as dynamic reaction force from flexible object also exists, so we also consider it in our problem formulation.

In eq. (12), defining $\tau_c \equiv C_{r2}(\theta)u_y$, the feedback torque τ in the joint actuator is given by:

$$\tau = C_{r1}(\theta, \dot{\theta}) + \tau_c. \quad (15)$$

Next, we define the additional torque τ_a as the part of the feedback torque τ . The difference between the additional torque τ_a and the reaction torque τ_r from the object becomes the actual torque τ_c in the joint actuator. So, the torque relationship is defined as:

$$\tau_c = \tau_a - \tau_r. \quad (16)$$

Consequently, the relationship between τ_c and \mathbf{y} is given by:

$$\ddot{\mathbf{y}} = C_{r2}^{-1}(\theta)\tau_c. \quad (17)$$

6.2 Algorithm to calculate the interference force

In this section, we note the constitution of the control system to calculate the actual force in each joint of the robot. The combined model derived from the point of the interfere torque (force) is shown in Figure 6.

Next, we explain the details of this block diagram.

1. Calculate the difference between the additional partial element τ_a of the non-linear feedback τ in the joint actuator of the robot, and the reaction torque τ_r from the object, which becomes the actual torque τ_c .
2. Pass from τ to θ , $\dot{\theta}$, and $\ddot{\theta}$ can be given by the numerical integral method for the robot's non-linear equation (1) during $\Delta t (= \Sigma t_s)$, for every sampling interval t_s .

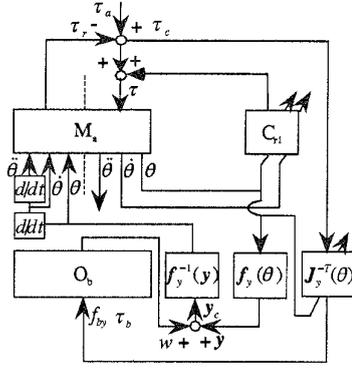


Figure 6: Algorithm to calculate the interfere torque (force).

3. Pass from θ and $\dot{\theta}$ through the block C_{r1} gives a non-linear feedback term, used to compensate the non-linearity of robot's equation.
4. Pass from τ_c to the force f_{by} and the torque τ_b is the input to the object by the robot and is transformed by the Jacobian matrix $J_y(\theta)$.
5. From θ to y is transformed by $y = f_y(\theta)$.
6. Pass from f_{by} and τ_b to the beam's bending displacement w is obtained by the numerical integral method for the reduced model (9) of the object during Δt ($= \sum t_s$), for every sampling interval t_s .
7. The sum of the position-posture variable y and the beam's displacement w becomes a new position-posture variable y_c of the end-effector.
8. The transformation from y_c to a new joint angle vector θ is given by the inverse trigonometric function $f_y^{-1}(y)$ of $f_y(\theta)$.
9. Substituting θ and its differential value $\dot{\theta}$, $\ddot{\theta}$ into the robot's equation (1), the reaction torque τ_r is obtained.
10. Repeating the process from 1 to 9, calculate again the difference between the additional torque τ_a in joint actuator of the robot and the reaction torque τ_r from the object, which becomes the actual torque τ_c , where τ_r is delayed by τ_a by integral time $\Delta t + \Delta t$.

7 The handling system satisfying under contact and non-contact conditions

We expand the control system (Figure 6) to make it possible to be applied even in the non-contact con-

dition of robot with the object. So we modify the control input of eq. (15) as follows:

$$\tau_a = C_{r2}(\theta)u_a + \tau_r. \quad (18)$$

then for the linearized and non-interfered system, new control input u_a alike eq. (13) becomes:

$$\ddot{y} = u_a. \quad (19)$$

In the control system of Figure 6, reaction torque τ_r is delayed by τ_a by an integral time $\Delta t + \Delta t$. Since the robot does not produce any effect of the reaction force of object when they are isolated from each other, so to equalize $\tau_c(t)$ to $\tau_a(t)$, switch the related terms of the control system using two dual-contact switches S_{w1} and S_{w2} in Figure 7. The sensors are mounted on the end-effectors. If the robot and the object have contact, both sensor's switch forward to side 1, else forward to side 2. To compensate time-delay in numerical integration interval in the simulation, include the temporary memory at two places in the condition of non-contact to make $\tau_c(t) = \tau_a(t)$. Consequently, eq. (19) equals eq. (13). When the robot is in contact with the object before handling, relative velocity becomes zero. At that time, suppose that each movement of the robot and the object does not produce any effect by a shock.

As one of the control input to suppress the vibration of the object, we input to eq. (19) as:

$$u_a = -K_1\dot{y} - K_2(y - H) \quad (K_1, K_2 > 0). \quad (20)$$

Even if the feedback term (eq. (20)) is also added in the control system of Figure 6 to obtain the interference force, the control system in Figure 7 equals the one in Figure 4.

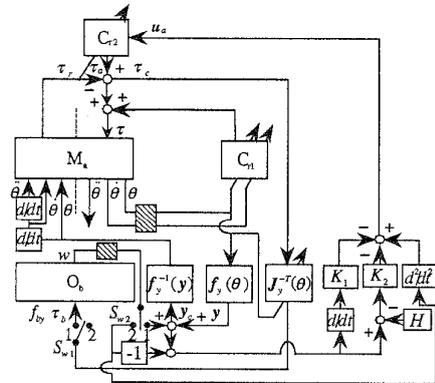


Figure 7: The handling system satisfying under contact and non-contact conditions.

Table 1: Link parameters of the robot.

k	1	2	3
m_k [kg]	5.0	5.0	-
r_k [m]	0.6	0.5	-
s_k [m]	0.3	0.2	-
J_k [Nm ²]	0.001	0.001	0.001

Table 2: Parameters of the object.

EI^* [Nm ²]	E^*I^* [Nm ² s]	ρA [kg/m]	L [m]
$\frac{2}{\pi}$	$\frac{0.02}{\pi}$	$\frac{0.02}{\pi}$	$\frac{\pi}{2}$

Table 3: Initial conditions.

w [m]	θ_1 [rad]	θ_2 [rad]	θ_3 [rad]
-0.01	$-\frac{110}{180}\pi$	$-\frac{40}{180}\pi$	$-\frac{30}{180}\pi$

Table 4: Handling point.

x_{r3} [m]
$0.95L$

Table 5: Simulation interval.

t_s [s]	Δt [s]
0.0010	0.0100

8 Simulation example

To illustrate the performance of the proposed handling system design, we present simulation results. Parameters of the robot and the object used in the simulation are presented in Table 1 and Table 2, respectively. k represents the number of link. The initial value of bending displacement and the initial posture of the manipulator are given in Table 3. The handling point description is also given in Table 4. Set the sampling interval t_s and the section of numerical integration Δt ($= \Sigma t_s$) as in Table 5. Angle of the end-effector is constant ($\alpha = \pi$).

Response of the bending displacement of the object before the robot handles the object, and appearance of the robot when follows the vibrating object, are shown in Figure 8. The effect of the vibration suppression of the object after the robot handles the object is shown in Figure 9. Next, a series of the robot's movements in following, handling, and vibration suppression of the object after releasing it are shown in Figure 10. At that time, Figure 11 shows the part u_a of control input τ to the joint of the robot.

9 Conclusions

We have proposed a method of following and vibration suppression of the free end of the flexible object constrained on the wall using a robot manipulator. We regard the robot and the object together as the entire system in handling position, thus we proposed the method derived by the numerical simulation so as to obtain the torque relationship in handling point.

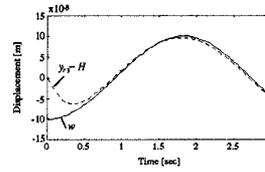


Figure 8: Following up the object.

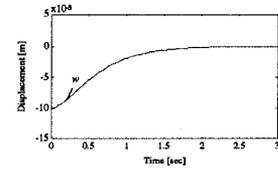


Figure 9: Vibration suppression of the object.

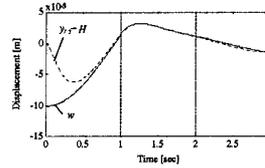


Figure 10: Following, handling, vibration suppression, separation, following.

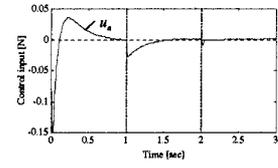


Figure 11: Control input.

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