Cooperative Control of a Vibrating Flexible Object by a Rigid Dual-Arm Robot

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Abstract

In this paper, we deal with the handling of a vibrating flexible object by a rigid dual-arm robot. Each arm consists of two links and an end-effector. The aim of this work is to realize position control of the flexible object while suppressing its vibration. In particular, the problem taken up here is about the control system design to realize the active handling of the flexible object. For this purpose, we propose a model for both robot and the object in the connected position so as to guarantee the whole handling system's stability. We also express a new mathematical modeling of the flexible object using model reduction theory in the same connected position. The model while the robot and the object are in contact, is derived by the positional requirements at the handling point. Next, we consider the design of controller to satisfy the constraint condition at the static equilibrium point. Designing of the combined system consisting of the robot and the object is useful for the analysis of whole handling system's stability.

1 Introduction

The study of handling of the flexible object using a single-arm manipulator has been done (Arai *et al* [1] and Zheng *et al* [2]). The purpose of their research is to devise the position control strategy for insertion of the flexible bar into a hole. On the other hand, also the study of coordinated control of the rigid object using the rigid dual-arm robot has been already done (Uchiyama *et al* [3]). They have given theoretical derivation of workspace coordinates for the dual-arm robot handling the rigid object. Recently, the study of coordinated control of the flexible object using the rigid dual-arm robot has started. Svinin *et al* [4] have applied the geometrical analysis to perform the position control and vibration suppression of the flexible object, and in these researches, the flexible object consists of lumped masses and springs. While in this paper, we consider the object to be a flexible beam.

The problem addressed in this paper is shown in Figure 1. We consider both the robot and the object together as the entire system, and make a mathematical model for the combined system. This model derives from the positional and velocity considerations in the holding position. For the combined system's stability, we need to understand both the robot system's stability, and the mutual relationship between the robot system's stability and the full assembly system's stability. If the object's parameters are larger than the robot's one, it is important to consider the cooperative control for the combined system and make it possible that a micro robot can manipulate a macro object. For the control system design, we assume that the control input derived by the control law for the combined model can guarantee the stability of the dual-arm robot and the flexible object. So it is clear that the stabilization problem for the combined system is not the same as the stabilization of either the object or the dual-arm robot. We also regard the controller design from the point of view of the handling characteristics at the static equilibrium point, as a cooperative handling system design.

A brief summary of our results and the organization of the paper is as follows: In Section 2, we note the formulation of our study. In Section 3, we present the robot's equation of motion and the modeling of the object. Section 4 gives the modeling of the handling system. Section 5 gives the control scheme for the combined model presented in Section 4. Section 6 gives the simulation results of the proposed handling system design. Finally, in Section 7, the conclusions of this work are given.

2 Problem formulation

The dual-arm robot and the object move in the same plane. Suppose that robot can observe the vi-

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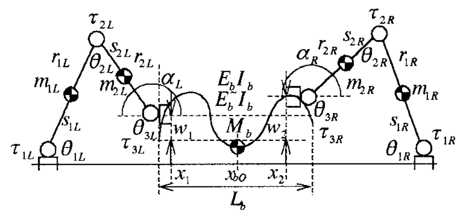

Figure 1: Situation of the dual-arm robot and the flexible object in contact with each other.

brating object and the object can be expressed by its dynamic characteristics. Parameters of the dual-arm robot and the object are shown in Figure 1.

3 Kinematics and dynamics

3.1 Robot's equation of motion

Using Lagrange's formulation, the dual-arm robot's equations of motion are written as follows:

$$\boldsymbol{J}_L(\boldsymbol{\theta}_L)\ddot{\boldsymbol{\theta}}_L + \boldsymbol{C}_L(\boldsymbol{\theta}_L,\ \dot{\boldsymbol{\theta}}_L) + \boldsymbol{D}_L\dot{\boldsymbol{\theta}}_L + \boldsymbol{P}_L(\boldsymbol{\theta}_L) = \boldsymbol{\tau}_L \tag{1}$$

$$\boldsymbol{J}_R(\boldsymbol{\theta}_R)\ddot{\boldsymbol{\theta}}_R + \boldsymbol{C}_R(\boldsymbol{\theta}_R,\ \dot{\boldsymbol{\theta}}_R) + \boldsymbol{D}_R\dot{\boldsymbol{\theta}}_R + \boldsymbol{P}_R(\boldsymbol{\theta}_R) = \boldsymbol{\tau}_R \tag{2}$$

where $\boldsymbol{\theta}_{L,R} \in \Re^{3\times 1}$ are the joint angle vectors. $\boldsymbol{J}_{L,R} \in \Re^{3\times 3}$ are the inertial force coefficient matrices. $\boldsymbol{C}_{L,R} \in \Re^{3\times 1}$ are centrifugal force terms. $\boldsymbol{D}_{L,R} \in \Re^{3\times 3}$ are the damping frictional force coefficients. $\boldsymbol{P}_{L,R} \in \Re^{3\times 1}$ are the gravity terms, and $\boldsymbol{\tau}_{L,R} \in \Re^{3\times 1}$ are the torque input vectors. The symbols L and R represent the left arm and the right arm.

3.2 Modeling of the object

We choose the object to be a flexible beam. While handling the object by using the dual-arm manipulator, we assume that both ends of the beam are free, and control the both arms to suppress its vibration.

3.2.1 The state-space description of the fundamental equation

The fundamental equation of the beam when external forces and torques are also present is given as follows:

$$E_b I_b \frac{\partial^4 w(x,t)}{\partial x^4} + E_b^{\sim} I_b \frac{\partial^5 w(x,t)}{\partial x^4 \partial t} + \rho_b S_b \frac{\partial^2 w(x,t)}{\partial t^2} = \sum_{\xi=1}^{n_f} f_{b\xi}(x_{u\xi},t)\delta(x - x_{u\xi}) + \sum_{\zeta=1}^{n_t} \tau_{b\zeta}(x_{v\zeta},t)\delta'(x - x_{v\zeta}) \tag{3}$$

$$w(x,t) = \sum_{i=1}^{\infty} \phi_i(x)\eta_i(t) \tag{4}$$

where $w_j(x_j,t)$ is the bending displacement at $x = x_j$, S_b is the cross-sectional-area of the beam, E_b it's vertical elastic coefficient, ρ_b is density, $E_b^{\sim}$ the damping coefficient, I_b the area moment of inertia; $f_{b\xi}(x_{u\xi},t)$ is the force input at $x = x_{u\xi}$, $\tau_{b\zeta}(x_{v\zeta},t)$ is the moment input at $x = x_{v\zeta}$, $\eta_i(t)$ is an unknown function, $\phi_i(x)$ is a mode function, δ is the delta function, x_j is the measured position (handling position), L_b is the beam length, and t is time. i stands for order of mode, and j stands for the number of sensor.

The mode function and the boundary conditions of the free ends of beam are given by:

$$\phi_i(x) = \frac{\cosh\left(\frac{k_i x}{L_b}\right) + \cos\left(\frac{k_i x}{L_b}\right)}{\cosh(k_i) - \cos(k_i)} - \frac{\sinh\left(\frac{k_i x}{L_b}\right) + \sin\left(\frac{k_i x}{L_b}\right)}{\sinh(k_i) - \sin(k_i)} \tag{5}$$

$$\left(\frac{\partial^2 \phi_i(x)}{\partial x^2}\right)_{x=0,L_b} = \left(\frac{\partial^3 \phi_i(x)}{\partial x^3}\right)_{x=0,L_b} = 0\ . \tag{6}$$

k_i can be approximated by $1 - \cos(k_i)\cosh(k_i) = 0$. Using the Galerkin's method, the state space description of the beam relative to the unknown function $\eta_i(t)$ (in case of dual-arm, n_f and n_t both equal to 2) is obtained as:

$$\dot{\boldsymbol{x}}_b = \boldsymbol{A}_b\boldsymbol{x}_b + \boldsymbol{B}_b\boldsymbol{u}_b,\ \boldsymbol{y}_b = \boldsymbol{C}_b\boldsymbol{x}_b \tag{7}$$

where

$$\boldsymbol{x}_b = \left[\ \eta_1(t)\quad \dot{\eta}_1(t)\quad \cdots\quad \eta_i(t)\quad \dot{\eta}_i(t)\ \right]^T \in \Re^{2i\times 1}$$

$$\boldsymbol{y}_b = \left[\ w_1(x_1,t)\quad w_2(x_2,t)\ \cdots\ w_j(x_j,t)\ \right]^T \in \Re^{j\times 1}$$

$$\boldsymbol{u}_b = \left[f_{b1}(x_{u1},t)\,\tau_{b1}(x_{v1},t)\,f_{b2}(x_{u2},t)\,\tau_{b2}(x_{v2},t)\right]^T \in \Re^{4\times 1}$$

$$\boldsymbol{A}_b = \text{block diag}\left[\boldsymbol{A}_{b1},\ \boldsymbol{A}_{b2},\ \boldsymbol{A}_{b3},\ \cdots,\ \boldsymbol{A}_{bi}\right] \in \Re^{2i\times 2i}$$

$$\boldsymbol{B}_b = \left[\ \boldsymbol{B}_{b1}^T\quad \boldsymbol{B}_{b2}^T\quad \boldsymbol{B}_{b3}^T\quad \cdots\quad \boldsymbol{B}_{bi}^T\ \right]^T \in \Re^{2i\times 4}$$

$$\boldsymbol{C}_b = \begin{bmatrix} \boldsymbol{C}_{b11} & \boldsymbol{C}_{b12} & \cdots & \boldsymbol{C}_{b1i} \\ \boldsymbol{C}_{b21} & \boldsymbol{C}_{b22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{C}_{bj1} & \cdots & \cdots & \boldsymbol{C}_{bji} \end{bmatrix} \in \Re^{j\times 2i}$$

$$\boldsymbol{A}_{bi} = \begin{bmatrix} 0 & 1 \\ -\frac{E_b I_b}{\rho_b S_b}\left(\frac{k_i}{L_b}\right)^4 & -\frac{E_b^{\sim} I_b}{\rho_b S_b}\left(\frac{k_i}{L_b}\right)^4 \end{bmatrix} \in \Re^{2\times 2}$$

$$\boldsymbol{B}_{bi} = \frac{1}{\rho_b S_b}\begin{bmatrix} 0 & 0 & 0 & 0 \\ \phi_i(x_{u1}) & -\frac{d}{dx}\phi_i(x_{v1}) & \phi_i(x_{u2}) & -\frac{d}{dx}\phi_i(x_{v2}) \end{bmatrix} \in \Re^{2\times 4}$$

$$\boldsymbol{C}_{bji} = \left[\ \phi_i(x_j)\quad 0\ \right] \in \Re^{1\times 2}\ .$$

3.2.2 The state-space transformation of the flexible object to get the boundary conditions

In this section, we describe the state-space transformation of the object. Beam's equations (7) are manipulated with model reduction theory to derive the constraint condition in the handling point. The control theory for the linear finite dimensional system cannot be applied to the distributed-parameter system. If the original system is approximated by some lower order modes neglecting the higher order modes, the control system may generate a spill over and unstabilize. So we reduce the original system using the model reduction theory [5] considering the stabilization for the elastic vibration of the system, which can also be considered as the stability of the reduced system inspite of its modeling errors. In the holding position, to combine the relationship of the robot's and the object's state-space, we need to change the state-space of the object from unknown function to the bending displacement. To rearrange the state-space equations with respect to same physical property in the vector form is useful for the analysis of the system, as the dynamic differential equations have also the static boundary conditions mixed. So we describe them as an assembly differential equation. The method is as follows:

Let the state variable vector $\boldsymbol{y}_b^n$ be the state response of the system to the input $\boldsymbol{u}_b^n$ as:

$$\begin{aligned}\boldsymbol{u}_b^n &= \left[\begin{array}{ccc} \mathbf{0}^{1\times(n-1)} & \delta(t) & \mathbf{0}^{1\times(n_f+n_t-n)} \end{array}\right]^T \\ &\in \Re^{(n_f+n_t)\times 1} \quad (n = 1 \sim (n_f+n_t)) \ . \end{aligned} \tag{8}$$

In the beam's state-space equations given as (7), r $(= 2j)$, dimensional vector $\boldsymbol{R}_b\boldsymbol{y}_b^n$ is given as the linear combination of the state response generated by the impulse input $\boldsymbol{u}_b^n$. Let $\boldsymbol{y}_{br}$ be the new state variable for the reduced system. We apply the vector $\boldsymbol{R}_b\boldsymbol{y}_b^n$ to the following reduced model:

$$\dot{\boldsymbol{y}}_{br} = \boldsymbol{A}_{br}\boldsymbol{y}_{br} + \boldsymbol{B}_{br}\boldsymbol{u}_b \ . \tag{9}$$

The error between the original model and the reduced model is given as follows:

$$\boldsymbol{d}_{br} = \boldsymbol{R}_b\dot{\boldsymbol{y}}_b - (\boldsymbol{A}_{br}\boldsymbol{R}_b\boldsymbol{y}_b + \boldsymbol{B}_{br}\boldsymbol{u}_b) \ . \tag{10}$$

In the above equation (10), the system matrices of the reduced model are given as follows:

$$\begin{aligned}\boldsymbol{A}_{br} &= \boldsymbol{R}_b\boldsymbol{A}_b\boldsymbol{W}_{bc}\boldsymbol{R}_b^T\left(\boldsymbol{R}_b\boldsymbol{W}_{bc}\boldsymbol{R}_b^T\right)^{-1} \\ \boldsymbol{B}_{br} &= \boldsymbol{R}_b\boldsymbol{B}_b \equiv [\ \boldsymbol{B}_{brL} \ \vdots \ \boldsymbol{B}_{brR}\ ]\end{aligned}$$

which minimize the mean value of the state response by r linearly independent impulse inputs. $\boldsymbol{W}_{bc}$ is the controllability Gramian matrix of eq. (7). Choose $\boldsymbol{R}_b$ as follows:

$$\boldsymbol{R}_b = \begin{bmatrix} \phi_1(x_1)\boldsymbol{I}^{2\times 2} & \cdots & \phi_i(x_1)\boldsymbol{I}^{2\times 2} \\ \vdots & \ddots & \vdots \\ \phi_1(x_j)\boldsymbol{I}^{2\times 2} & \cdots & \phi_i(x_j)\boldsymbol{I}^{2\times 2} \end{bmatrix} \in \Re^{2j\times 2i} \ . \tag{11}$$

The state variables (unknown function $\eta_i(t)$) of the system (7) change to the state variables (sensor displacement $\widehat{w}_j(x_j,t)$) of the reduced model. This is desirable in the control system design, because the displacement detected by the sensors can be applied directly to the state feedback control. In other words, a new state-space is given by:

$$\begin{aligned}\boldsymbol{y}_{br} &= \left[\ \widehat{w}_1(x_1,t) \ \ \dot{\widehat{w}}_1(x_1,t) \ \cdots \ \widehat{w}_j(x_j,t) \ \ \dot{\widehat{w}}_j(x_j,t)\ \right]^T \\ &\in \ \Re^{r\times 1} \ . \end{aligned} \tag{12}$$

To get the precise reduced model, it is important to select appropriate values for the elements of $\boldsymbol{R}_b(\phi_i(x_j))$ (handling position x_j).

3.2.3 An expression including the characteristics of the axial direction of the object

We regard the flexible object as rigid in the horizontal (x) direction. The rigid body is usually acted directly by force and torque generated by the robot. Since the constraint conditions in each direction are independent of the other directions, there is no correlation between them, so we can combine both the equation of bending displacement and the one of rigid characteristics without considering the correlation in the both directions.

The mass of the object is given by $M_b = S_bL_b$. The formulation of balance of force $f_{xL,R}$ at the handling point in x direction is given by $M_b\ddot{x}_{bo} = -f_{xL} + f_{xR}$, where x_{bo} stands for the center of gravity. Transforming this equation into the first order differential equation, we get

$$\dot{\boldsymbol{x}}_{bd} = \boldsymbol{A}_{bd}\boldsymbol{x}_{bd} + \boldsymbol{B}_{bd}\boldsymbol{u}_{bd} \tag{13}$$

where

$$\begin{aligned}&\boldsymbol{x}_{bd} = \left[\begin{array}{cc} x_{bo} & \dot{x}_{bo} \end{array}\right]^T, \ \boldsymbol{u}_{bd} = \left[\begin{array}{cc} f_{xL} & f_{xR} \end{array}\right]^T \\ &\boldsymbol{A}_{bd} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad \boldsymbol{B}_{bd} = \begin{bmatrix} 0 & 0 \\ -M_b^{-1} & M_b^{-1} \end{bmatrix} \ .\end{aligned}$$

Next combining these with eq. (9) and eq. (13), we get the equation which involves the characteristics in both x and y directions. As a result, a set of input functions from the joint is injected to the corresponding output functions of the object.

Defining the input force and torque to the object as $\boldsymbol{f}_{oL,R} = [\ f_{xL,R} \quad f_{b1,2} \quad \tau_{b1,2}\ ]^T$. The relationship between the generalized forces and torques at the end effectors and those at each of the joints is transferred by $\boldsymbol{\tau}_{L,R} = \boldsymbol{J}_{aL,R}^T(\boldsymbol{\theta}_{L,R})\boldsymbol{f}_{oL,R}$, where $\boldsymbol{J}_{aL,R}(\boldsymbol{\theta}_{L,R})$ are the Jacobian matrices which stand for the relationship between $\dot{\boldsymbol{\theta}}_{L,R}$ and $\dot{\boldsymbol{y}}_{L,R}$, and where $\boldsymbol{y}_{L,R}$ stand for the position-posture variables of the end-effector. We also assume the region of controllability as $\boldsymbol{J}_{aL,R}(\boldsymbol{\theta}_{L,R})$, are regular matrices. Consequently, the equation of the object is given by:

$$\dot{\boldsymbol{z}}_{ob} = \boldsymbol{A}_{ob}\boldsymbol{z}_{ob} + \boldsymbol{B}_{ob}\boldsymbol{u}_{ob} \tag{14}$$

where

$$\begin{aligned}
\boldsymbol{z}_{ob} &= [\ \boldsymbol{x}_{bd}^T \quad \boldsymbol{y}_{br}^T\ ]^T \in \Re^{(2+r)\times 1}\\
\boldsymbol{u}_{ob} &= [\ \boldsymbol{\Delta\tau}_L^T \quad \boldsymbol{\Delta\tau}_R^T\ ]^T \in \Re^{(3+3)\times 1}\\
\boldsymbol{A}_{ob} &= \text{block diag}[\ \boldsymbol{A}_{bd}, \quad \boldsymbol{A}_{br}\ ] \in \Re^{(2+r)\times(2+r)}\\
\boldsymbol{B}_{ob} &=
\end{aligned}$$

$$\left[\begin{array}{c|c}
0^{1\times 3} & 0^{1\times 3}\\
[-M_b^{-1} \mid 0 \quad 0]\boldsymbol{J}_{aL}^{-T}(\boldsymbol{\theta}_L) & [\ M_b^{-1} \mid 0 \quad 0]\boldsymbol{J}_{aR}^{-T}(\boldsymbol{\theta}_R)\\
\hline
[\ 0^{r\times 1} \mid \boldsymbol{B}_{brL}]\boldsymbol{J}_{aL}^{-T}(\boldsymbol{\theta}_L) & [\ 0^{r\times 1} \mid \boldsymbol{B}_{brR}]\boldsymbol{J}_{aR}^{-T}(\boldsymbol{\theta}_R)
\end{array}\right]$$

$$\equiv [\ \boldsymbol{B}_{obL} \mid \boldsymbol{B}_{obR}\ ] \in \Re^{(2+r)\times(3+3)}$$

and for the sake of convenience, $\boldsymbol{\tau}_{L,R}$ is changed to $\boldsymbol{\Delta\tau}_{L,R}$.

4 Handling model

4.1 Combined model's equation

In this section, we consider the constitution of the full assembly system including the dual-arm robot and the object while satisfying the positional boundary conditions at the handling point. Transforming eq. (1) and eq. (2) into first order differential equations, and combining these with eq. (14), we get

$$\dot{\boldsymbol{x}}_m = \boldsymbol{A}_m(\boldsymbol{x}_m)\boldsymbol{x}_m + \boldsymbol{B}_m(\boldsymbol{x}_m)\boldsymbol{u}_m + \boldsymbol{W}_m(\boldsymbol{x}_m) \tag{15}$$

where

$$\begin{aligned}
\boldsymbol{x}_m &= \left[\ \boldsymbol{\theta}_L^T \quad \dot{\boldsymbol{\theta}}_L^T \quad \boldsymbol{z}_{ob}^T \quad \boldsymbol{\theta}_R^T \quad \dot{\boldsymbol{\theta}}_R^T\ \right]^T\\
&\equiv \left[\ x_{m1} \quad x_{m2} \quad \cdots \quad x_{mo_s}\ \right]^T \in \Re^{o_s\times 1}\\
\boldsymbol{u}_m &= \left[\ \boldsymbol{\tau}_L^T \quad \boldsymbol{\tau}_R^T\ \right]^T \in \Re^{o_u\times 1}\\
\boldsymbol{A}_m &= \text{block diag}[\boldsymbol{A}_L, \boldsymbol{A}_{ob}, \boldsymbol{A}_R] \in \Re^{o_s\times o_s}\\
\boldsymbol{B}_m &= \left[\begin{array}{c|c}
\boldsymbol{B}_L & 0^{6\times 3}\\ \hline
\boldsymbol{B}_{obL} & \boldsymbol{B}_{obR}\\ \hline
0^{6\times 3} & \boldsymbol{B}_R
\end{array}\right] \in \Re^{o_s\times o_u}\\
\boldsymbol{W}_m &= \left[\ \boldsymbol{W}_L^T \quad -(\boldsymbol{B}_{ob}\,\overline{\boldsymbol{u}}_m)^T \quad \boldsymbol{W}_R^T\ \right]^T \in \Re^{o_s\times 1}\\
\boldsymbol{A}_{L,R} &= \begin{bmatrix} 0^{3\times 3} & \boldsymbol{I}^{3\times 3}\\ 0^{3\times 3} & -\boldsymbol{J}_{L,R}^{-1}(\boldsymbol{\theta}_{L,R})\boldsymbol{D}_{L,R}\end{bmatrix} \in \Re^{6\times 6}\\
\boldsymbol{B}_{L,R} &= \begin{bmatrix} 0^{3\times 3}\\ \boldsymbol{J}_{L,R}^{-1}(\boldsymbol{\theta}_{L,R})\end{bmatrix} \in \Re^{6\times 3}\\
\boldsymbol{W}_{L,R} &=
\end{aligned}$$

$$\begin{bmatrix} 0^{3\times 1}\\ -\boldsymbol{J}_{L,R}^{-1}(\boldsymbol{\theta}_{L,R})\left\{\boldsymbol{C}_{L,R}(\boldsymbol{\theta}_{L,R}, \dot{\boldsymbol{\theta}}_{L,R}) + \boldsymbol{P}_{L,R}(\boldsymbol{\theta}_{L,R})\right\}\end{bmatrix} \in \Re^{6\times 1}$$

$$\overline{\boldsymbol{u}}_m = \left[\ \overline{\boldsymbol{\tau}}_L^T \quad \overline{\boldsymbol{\tau}}_R^T\ \right]^T \in \Re^{o_u\times 1}.$$

$\overline{\boldsymbol{u}}_m$ is the joint torque vector at the equilibrium point when the bending displacement is zero, where as the elements of vector $\boldsymbol{u}_m$ are given by:

$$\boldsymbol{\tau}_{L,R} = \overline{\boldsymbol{\tau}}_{L,R} + \boldsymbol{\Delta\tau}_{L,R} \tag{16}$$

where

$$\begin{aligned}
\overline{\boldsymbol{\tau}}_{L,R} &= \boldsymbol{P}_{L,R}(\overline{\boldsymbol{\theta}}_{L,R})\\
&+ \boldsymbol{J}_{aL,R}(\overline{\boldsymbol{\theta}}_{L,R})\ [\ 0 \quad \varphi_{L,R}M_b \quad 0\ ]^T\\
&\qquad (0 < \varphi_{L,R} < 1,\ \varphi_L + \varphi_R = 1)\ .
\end{aligned}$$

$\varphi_{L,R}$ is the force distribution coefficient to sustain the mass of the object by each manipulator. If the object is uniform in density and its cross-sectional-area is constant then the center of gravity is in the center of the object resulting $\varphi_{L,R}$ to be chosen as 1/2. o_s and o_u respectively stand for the dimensions of the state-space and the input vector for the combined model. Defining the positional constraints to combine the equations of the dual-manipulators with that of the object:

$$\boldsymbol{f}_c(\boldsymbol{x}_m) \equiv \text{col}(\ f_{c1}, \quad f_{c2}, \quad \cdots, \quad f_{co_c}\ ) = \boldsymbol{0} \in \Re^{o_c\times 1}\ . \tag{17}$$

Next, an inequality equation:

$$o_c < o_u \tag{18}$$

is a necessary condition for the existence of the unobservable space capable of setting all the poles of the combined model. The state-space equation of the combined system with the boundary conditions is given by:

$$\dot{\boldsymbol{x}}_m = \boldsymbol{A}_m(\boldsymbol{x}_m)\boldsymbol{x}_m + \boldsymbol{B}_m(\boldsymbol{x}_m)\boldsymbol{u}_m + \boldsymbol{W}_m(\boldsymbol{x}_m) + \boldsymbol{F}_c^T(\boldsymbol{x}_m)\boldsymbol{\lambda} \tag{19}$$

where $\boldsymbol{F}_c(\boldsymbol{x}_m) = \partial\boldsymbol{f}_c(\boldsymbol{x}_m)/\partial\boldsymbol{x}_m \in \Re^{o_c\times o_s}$, and $\boldsymbol{\lambda}$ is the unknown multiplier vector. Suppose eq. (19) exists when the following condition is satisfied:

$$\text{rank}(\boldsymbol{F}_c(\boldsymbol{x}_m)) = o_c\ . \tag{20}$$

4.2 Design of the controller satisfying the boundary conditions

For a reference posture of the dual-arm robot, define the equilibrium point as $\boldsymbol{x}_m = \boldsymbol{x}_m^*$ that satisfies the boundary condition $\boldsymbol{f}_c(\boldsymbol{x}_m) = \boldsymbol{0}$. All the system matrices of eq. (19) are time-variant. Define the notation of the system matrices and vectors at the reference equilibrium point as "$*$". Assume that the following conditions can be held at the reference equilibrium point:

$$\dot{\boldsymbol{\theta}}^*_{L,R} = \ddot{\boldsymbol{\theta}}^*_{L,R} = 0^{3\times 1},\ \dot{\boldsymbol{y}}^*_{br} = 0^{r\times 1},\ \dot{\boldsymbol{x}}^*_{bd} = 0^{2\times 1} .$$

When the constraint conditions have been held, $\boldsymbol{F}_c^* \dot{\boldsymbol{x}}_m = 0^{o_s\times 1}$ can be satisfied. Consequently, the reference multiplier vector $\boldsymbol{\lambda}^*$ is given by:

$$\boldsymbol{\lambda}^* = -\boldsymbol{F}_c^{*+}\left(\boldsymbol{B}_m^* \boldsymbol{u}_m^* + \boldsymbol{W}_m^*\right) \equiv \left[\ \lambda_1^*\ \ \lambda_2^* \cdots \ \lambda_{o_c}^*\ \right]^T . \tag{21}$$

Let $\boldsymbol{F}_c^{*+}$ be the Moore-Penrose inverse of $\boldsymbol{F}_c^*$, and exists if and only if eq. (20) holds. Paying the attention that the first term of eq. (19) equals a zero vector, the reference constraint force $\boldsymbol{\lambda}^*$ is derived. Next, the state-space expression separating the part of static equilibrium point and that of the error is given by:

$$\begin{aligned}\dot{\boldsymbol{x}}_m^* + \boldsymbol{\Delta}\dot{\boldsymbol{x}}_m =& \left(\boldsymbol{A}_m^* + \boldsymbol{\Delta A}_m\left(\boldsymbol{\Delta x}_m\right)\right)\left(\boldsymbol{x}_m^* + \boldsymbol{\Delta x}_m\right)\\ &+ \left(\boldsymbol{B}_m^* + \boldsymbol{\Delta B}_m\left(\boldsymbol{\Delta x}_m\right)\right)\left(\boldsymbol{u}_m^* + \boldsymbol{\Delta u}_m\right)\\ &+ \boldsymbol{W}_m^*\left(\boldsymbol{x}_m^* + \boldsymbol{\Delta x}_m,\ \overline{\boldsymbol{u}}_m\right)\\ &+ \boldsymbol{F}_c^{*T}\left(\boldsymbol{\lambda}^* + \boldsymbol{\Delta\lambda}\right) + \boldsymbol{F}_\lambda^* \boldsymbol{\Delta x}_m \end{aligned}\tag{22}$$

where

$$\boldsymbol{F}_\lambda^* = \operatorname{diag}\left.\left(\sum_{\iota=1}^{o_c} \frac{\partial^2 f_{c\iota}}{\partial x_{m1}^2}\lambda_\iota, \cdots, \sum_{\iota=1}^{o_c} \frac{\partial^2 f_{c\iota}}{\partial x_{mo_s}^2}\lambda_\iota\right)\right|_{x_{m\bullet}=x^*_{m\bullet},\ \lambda_\iota=\lambda^*_\iota} .$$

Setting $\boldsymbol{A}_m^* + \boldsymbol{\Delta A}_m\left(\boldsymbol{\Delta x}_m\right) \cong \boldsymbol{A}_m^*$ and $\boldsymbol{B}_m^* + \boldsymbol{\Delta B}_m\left(\boldsymbol{\Delta x}_m\right) \cong \boldsymbol{B}_m^*$, the error equation in the vicinity of the equilibrium point is given by:

$$\boldsymbol{\Delta}\dot{\boldsymbol{x}}_m = \boldsymbol{A}_m^{\sim} \boldsymbol{\Delta x}_m + \boldsymbol{B}_m^* \boldsymbol{\Delta u}_m + \boldsymbol{F}_c^{*T} \boldsymbol{\Delta\lambda} \tag{23}$$

where

$$\boldsymbol{A}_m^{\sim} = \text{block diag}\left[\ \boldsymbol{A}_L^{\sim},\ \ \boldsymbol{A}_{ob},\ \ \boldsymbol{A}_R^{\sim}\ \right] + \boldsymbol{F}_\lambda^* ,$$

and where

$$\boldsymbol{A}_{L,R}^{\sim} = \left.\begin{bmatrix} 0^{3\times 3} & \boldsymbol{I}^{3\times 3} \\ -\boldsymbol{J}_{L,R}^{-1}(\boldsymbol{\theta}_{L,R})\left[\dfrac{\partial \boldsymbol{P}_{L,R}}{\partial \boldsymbol{\theta}_{L,R}}\right] & -\boldsymbol{J}_{L,R}^{-1}(\boldsymbol{\theta}_{L,R})\boldsymbol{D}_{L,R} \end{bmatrix}\right|_{\boldsymbol{\theta}_{L,R}=\boldsymbol{\theta}^*_{L,R}} .$$

Assume that $(\boldsymbol{A}_m^{\sim},\ \boldsymbol{B}_m^*)$ is controllable and rank $(\boldsymbol{B}_m^*) = o_u$. When the constraint conditions hold, $\boldsymbol{F}_c^* \boldsymbol{\Delta x}_m = \boldsymbol{F}_c^* \boldsymbol{\Delta}\dot{\boldsymbol{x}}_m = 0^{o_s\times 1}$ can be satisfied. So the constraint force in the vicinity of the reference equilibrium point is given by:

$$\boldsymbol{\Delta\lambda} = -\boldsymbol{F}_c^{*+}\left(\boldsymbol{A}_m^{\sim}\boldsymbol{\Delta x}_m + \boldsymbol{B}_m^* \boldsymbol{\Delta u}_m\right) . \tag{24}$$

Substituting eq. (24) into eq. (23), the error equation of the combined model can be computed by:

$$\boldsymbol{\Delta}\dot{\boldsymbol{x}}_m = \boldsymbol{F}_c^{\sim} \boldsymbol{A}_m^{\sim} \boldsymbol{\Delta x}_m + \boldsymbol{F}_c^{\sim} \boldsymbol{B}_m^* \boldsymbol{\Delta u}_m \tag{25}$$

where $\boldsymbol{F}_c^{\sim} = \boldsymbol{I}^{o_s\times o_s} - \boldsymbol{F}_c^{*T}\boldsymbol{F}_c^{*+} \geq \boldsymbol{0}$, which is a positive semidefinite matrix. If, for example, both robot and the object are already stabilized with gravity compensators *etc.*, we can say that the stability of the combined system is guaranteed.

5 Control method for the handling system

5.1 The coordinates transformation

The necessary condition to realize the handling is to satisfy the constraint condition in the equilibrium position. For this purpose, we consider the decomposition of the state-space mode in eq. (25), transforming the state-space using $\boldsymbol{T}_c$, as $\boldsymbol{\Delta x}_m = \boldsymbol{T}_c \boldsymbol{\Delta}\widehat{\boldsymbol{x}}_m$, then the transformed equation is given by:

$$\begin{aligned}\boldsymbol{\Delta}\dot{\widehat{\boldsymbol{x}}}_m &= \boldsymbol{T}_c^T \boldsymbol{F}_c^{\sim} \boldsymbol{A}_m^{\sim} \boldsymbol{T}_c \boldsymbol{\Delta}\widehat{\boldsymbol{x}}_m + \boldsymbol{T}_c^T \boldsymbol{F}_c^{\sim} \boldsymbol{B}_m^* \boldsymbol{\Delta u}_m \\ &\equiv \widehat{\boldsymbol{A}}_m \boldsymbol{\Delta}\widehat{\boldsymbol{x}}_m + \widehat{\boldsymbol{B}}_m \boldsymbol{\Delta u}_m \end{aligned}\tag{26}$$

where $\boldsymbol{\Delta}\widehat{\boldsymbol{x}}_m$ constitutes the unobservable space $\boldsymbol{\mathcal{S}}_{pu}$ and its complement $\boldsymbol{\mathcal{S}}_{po}$. $\boldsymbol{\mathcal{S}}_{pu}$ is the space capable of constraining the dual-arm robot and the object. In eq. (26), the partitioning space can be represented as:

$$\begin{bmatrix} \boldsymbol{\Delta}\dot{\widehat{\boldsymbol{x}}}_{mu} \\ \boldsymbol{\Delta}\dot{\widehat{\boldsymbol{x}}}_{mo} \end{bmatrix} = \begin{bmatrix} \widehat{\boldsymbol{A}}_{muc} & 0^{o_c\times(o_s-o_c)} \\ \widehat{\boldsymbol{A}}_{mu} & \widehat{\boldsymbol{A}}_{mc} \end{bmatrix}\begin{bmatrix} \boldsymbol{\Delta}\widehat{\boldsymbol{x}}_{mu} \\ \boldsymbol{\Delta}\widehat{\boldsymbol{x}}_{mo} \end{bmatrix} + \begin{bmatrix} 0^{o_c\times o_u} \\ \widehat{\boldsymbol{B}}_{mc} \end{bmatrix} \boldsymbol{\Delta u}_m \tag{27}$$

where $\boldsymbol{\Delta}\widehat{\boldsymbol{x}}_{mu} \in \Re^{o_c\times 1}$ and $\boldsymbol{\Delta}\widehat{\boldsymbol{x}}_{mo} \in \Re^{(o_s-o_c)\times 1}$. If $(\boldsymbol{A}_m^{\sim},\ \boldsymbol{B}_m^*)$ is controllable, then $(\widehat{\boldsymbol{A}}_{mc},\ \widehat{\boldsymbol{B}}_{mc})$ constitutes the controllable space $\boldsymbol{\mathcal{S}}_{pc}$. If the rank of the above condition is o_z then the combined system has $o_s - o_u - o_z$ uncontrollable poles. If any pole in the combined system is unstable, the constraint condition cannot be held, in consequence its stabilization requires an additional controller. If the unstable pole exists in space $\boldsymbol{\mathcal{S}}_{pu}$ and the constraint condition has not been satisfied in the beginning, the stability of space $\boldsymbol{\mathcal{S}}_{po}$ has to overcome to stabilize all state-space in eqs. (25), (26), and (27). In the formulation of our study, since the dual-arm robot handles the object at the initial time, the boundary condition has already held, so we don't have to consider the stability of $\boldsymbol{\mathcal{S}}_{pu}$. Realization of the cooperative control is possible by stabilizing $\boldsymbol{\mathcal{S}}_{pc}$ space.

5.2 Stabilization at the equilibrium point

The state-space equation corresponding to controllability space $\boldsymbol{\mathcal{S}}_{pc}$ in eq. (27) can be written in the following form:

$$\boldsymbol{\Delta}\dot{\widehat{\boldsymbol{x}}}_{mo} = \widehat{\boldsymbol{A}}_{mc}\boldsymbol{\Delta}\widehat{\boldsymbol{x}}_{mo} + \widehat{\boldsymbol{B}}_{mc}\boldsymbol{\Delta u}_m \ . \tag{28}$$

To stabilize the system presented in above equation, we apply the LQR theory to minimize the following performance index:

$$\mathcal{J} = \int_0^\infty \left(\boldsymbol{\Delta}\widehat{\boldsymbol{x}}_{mo}^T \boldsymbol{W}_x \boldsymbol{\Delta}\widehat{\boldsymbol{x}}_{mo} + \boldsymbol{\Delta u}_m^T \boldsymbol{W}_u \boldsymbol{\Delta u}_m\right) dt \tag{29}$$

where $\boldsymbol{W}_x \geq \boldsymbol{0}^{(o_s-o_c)\times(o_s-o_c)}$ and $\boldsymbol{W}_u > \boldsymbol{0}^{o_u\times o_u}$ are the weighting matrices. Suppose that $(\boldsymbol{W}_x^{1/2},\ \widehat{\boldsymbol{A}}_{mc})$ is observable. $\boldsymbol{\Lambda}_{mc}$ is a positive definite solution of Riccati equation:

$$\widehat{\boldsymbol{A}}_{mc}^T\boldsymbol{\Lambda}_{mc}+\boldsymbol{\Lambda}_{mc}\widehat{\boldsymbol{A}}_{mc}-\boldsymbol{\Lambda}_{mc}\widehat{\boldsymbol{B}}_{mc}\boldsymbol{W}_u^{-1}\widehat{\boldsymbol{B}}_{mc}^T\boldsymbol{\Lambda}_{mc}+\boldsymbol{W}_x=\boldsymbol{0} \ . \tag{30}$$

The optimal control input for the error equation (25), in the vicinity of the equilibrium point, is given by:

$$\begin{aligned}\boldsymbol{\Delta u}_m &= \left[\ \boldsymbol{0}^{o_u\times o_c} \quad -\boldsymbol{W}_u^{-1}\widehat{\boldsymbol{B}}_{mc}^T\boldsymbol{\Lambda}_{mc}\ \right]\boldsymbol{T}_c^T\boldsymbol{\Delta x}_m \\ &\equiv \boldsymbol{H}\boldsymbol{T}_c^T\boldsymbol{\Delta x}_m \ .\end{aligned} \tag{31}$$

It is possible to control the flexible object using the control input $\boldsymbol{\Delta u}_m$ and the static control input $\boldsymbol{u}_m^*$ at the equilibrium point.

5.3 Stabilization at the intermediate point of the object

Using the control input (31), we can also examine the bending displacement response at the intermediate point of the object and can conclude whether the vibrations can be suppressed or not. Defining a new state-space $\boldsymbol{\Delta x}_{m\infty}$ of the combined model considering the bending displacement at the intermediate point in the object. Giving the state-space relationship between $\boldsymbol{\Delta x}_m$ and $\boldsymbol{\Delta x}_{m\infty}$ as $\boldsymbol{\Delta x}_m = \boldsymbol{T}_e\boldsymbol{\Delta x}_{m\infty}$. The expanded error equation of the combined model at the reference equilibrium point becomes:

$$\boldsymbol{\Delta}\dot{\boldsymbol{x}}_{m\infty} = \boldsymbol{F}_{c\infty}^{\sim}\boldsymbol{A}_{m\infty}^{\sim}\boldsymbol{\Delta x}_{m\infty} + \boldsymbol{F}_{c\infty}^{\sim}\boldsymbol{B}_{m\infty}^{*}\boldsymbol{\Delta u}_m \ . \tag{32}$$

The closed loop system, when the control input (31) is applied to the system (32) is given as follows:

$$\boldsymbol{\Delta}\dot{\boldsymbol{x}}_{m\infty} = \boldsymbol{F}_{c\infty}^{\sim}(\boldsymbol{A}_{m\infty}^{\sim}+\boldsymbol{B}_{m\infty}^{*}\boldsymbol{H}\boldsymbol{T}_c^T\boldsymbol{T}_e)\boldsymbol{\Delta x}_{m\infty} \ . \tag{33}$$

Note that $\boldsymbol{F}_{c\infty}^{\sim}$ is a symmetric and positive semidefinite matrix, and its eigenvalues are 0 or 1. The necessary condition to satisfy the stability at the intermediate point of the object is given by:

$$\mathrm{Re}\{\lambda_i(\boldsymbol{A}_{m\infty}^{\sim} + \boldsymbol{B}_{m\infty}^{*}\boldsymbol{H}\boldsymbol{T}_c^T\boldsymbol{T}_e)\} \leq 0 \quad (\ \forall\ i\ ) \ . \tag{34}$$

6 Simulation example

To illustrate the performance of the proposed handling system design, we present a simulation result. The parameters of the dual-arm robot and the object used in the simulation are presented in Table 1 and Table 2 respectively. k represents the number of link. The initial values of bending displacement and the initial posture of the dual-arm robot are shown in Table 3. The position of the handling point is shown in Table 4. We set the initial posture of the dual-arm robot as reference considering the bending displacement as zero. Thus, we set the static torque input $\boldsymbol{u}_m^*$ equal to $\overline{\boldsymbol{u}}_m$. A simulation study has been carried out for the combined model (15) with the controller (31) derived by the error system (25), and the static input $\boldsymbol{u}_m^*$. For the following simulation, reference input is a unit step function. The dynamic behavior of

Table 1: Link parameters of the dual-arm robot.

k	1	2	3	k	1	2	3
m_{kL} [kg]	5.0	5.0	-	m_{kR} [kg]	5.0	5.0	-
r_{kL} [m]	0.6	0.5	-	r_{kR} [m]	0.6	0.5	-
s_{kL} [m]	0.3	0.2	-	s_{kR} [m]	0.3	0.2	-
J_{kL} [Nm^2]	0.001	0.001	0.001	J_{kR} [Nm^2]	0.001	0.001	0.001

Table 2: Parameters of the object.

E_bI_b[Nm^2]	$E_b^{\sim}I_b$[Nm^{2}s]	ρ_bS_b[kg/m]	L_b[m]	M_b[kg]
$\frac{2}{\pi}$	$\frac{0.02}{\pi}$	$\frac{0.02}{\pi}$	$\frac{\pi}{2}$	1.0

Table 3: Initial conditions of the robot and the object.

$\widehat{w}_1$[m]	θ_{1L}[rad]	θ_{2L}[rad]	θ_{3L}[rad]	$\widehat{w}_2$[m]	θ_{1R}[rad]	θ_{2R}[rad]	θ_{3R}[rad]
-0.001	$-\frac{70}{180}\pi$	$\frac{40}{180}\pi$	$\frac{30}{180}\pi$	-0.01	$-\frac{110.7}{180}\pi$	$-\frac{37.8}{180}\pi$	$-\frac{31.5}{180}\pi$

Table 4: Handling points.

x_1 [m]	x_2 [m]
$0.05L_b$	$0.95L_b$

the combined system is computed by the fourth order Runge-Kutta method.

Figure 2 shows the responses of the bending displacement of the object at the handling point when the robot handles the object. At that time, Figure 3 shows the responses of the joint angle of each robot, and Figure 4 shows the control input in the vicinity of the equilibrium point. Figure 5 shows the response of the bending displacement when the characteristics of the object are changed to $\rho_b S_b = 0.002/\pi$ from $\rho_b S_b = 0.02/\pi$. We can see that the controllability of the object depends upon its density. Figure 6 shows the responses of the bending displacement when each of the handling points change to $0.30L_b$ and $0.70L_b$ from $0.05L_b$ and $0.95L_b$, respectively. As sensor is mounted on each end-effector, changing of the handling point depends upon the dynamics of the object. Next, we examine the robustness of the controller. Figure 7 shows the responses of the bending displacement. Let the controller designed with $\rho_b S_b = 0.02/\pi$, be applied to the combined system with $\rho_b S_b = 0.01/\pi$. We can see that the convergence velocity of the response curves is slower than that when the control input is applied to the nominal combined system, but stabilization is accomplished. Figure 8 shows the responses of the bending displacement at each handling point $x_{1,\,2} = 0.05L_b,\ 0.95L_b$ using the controller derived for the handling points $x_{1,\,2} = 0.07L_b,\ 0.93L_b$. We can see that stabilization is achieved even though worse than that acheived with nominal control input. Figure 9 shows the responses of the bending displacement $\hat{w}_{3,\,4,\,5}$ at the intermediate handling points $x_{3,\,4,\,5} = 0.25L_b,\ 0.50L_b,\ 0.75L_b$. We can see that stabilization is achieved with LQR robustness.

7 Conclusions

We have proposed a handling method of a flexible object using a rigid dual-arm robot. We regard both the arms and the object together as the entire system and use the positional constraints as the boundary conditions. We have also designed the position control system to make the entire system stable, and thus have realized the handling of the object using the dual-arm robot. The proposed handling method can also be accomplished for vibration suppression at the intermediate points on the object.

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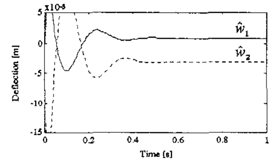

Figure 2: The bending displacement.

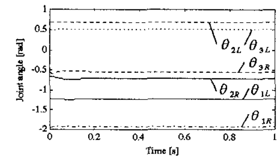

Figure 3: The joint angle of the robot.

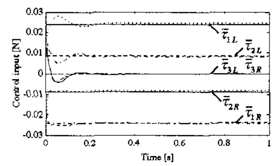

Figure 4: Control input in the vicinity of the equilibrium point.

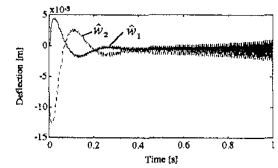

Figure 5: Responses of the object when its density is changed.

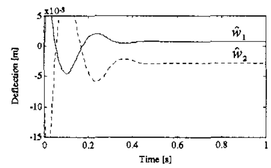

Figure 6: Responses of the object when the handling points are changed.

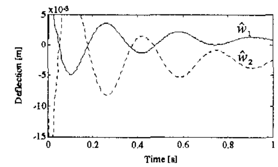

Figure 7: Responses of the object when its density is changed, using nominal controller.

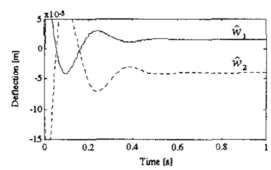

Figure 8: Responses of the object when the handling points are changed, using nominal controller.

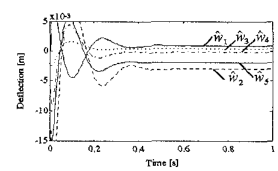

Figure 9: Responses at the intermediate point of the object.

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