Experimental demonstration of thermoacoustic energy conversion in a resonator

Biwa Tetsushi, Tashiro Yusuke, Mizutani Uichiro, Kozuka Motoki, Yazaki Taichi

Physical Review. E

volume 69

number 6

page range 066304

year 2004

URL http://hdl.handle.net/10097/52942

doi: 10.1103/PhysRevE.69.066304
I. INTRODUCTION

The acoustic intensity $I$ represents the time-averaged dynamic power flux sustained by oscillating motion of gas parcels in an acoustic wave. In ordinary speech, the acoustic intensity $I$ is only on the order of $10^{-7}$ W/m$^2$ at most, but in a thermoacoustic heat engine, an $I$ reaching $10^5$ W/m$^2$ has been achieved using thermally induced spontaneous gas oscillations without the use of mechanical drivers [1]. Here the acoustic intensity $I$ for a wave in a tube is written as

$$I = \langle PU \rangle = \frac{1}{2} pu \cos \Phi,$$

where the pressure $P = p e^{i\omega t}$, the cross sectional mean velocity of the oscillating gas $U = u e^{i(\omega t + \Phi)} = u \sin \Phi e^{i(\omega t + \Phi)}$, and $\Phi$ are angular brackets represent time average, and $\omega$ and $\Phi$ are an angular frequency and the phase lead of $U$ relative to $P$, respectively. Production of higher acoustic intensity $I$ is necessary to increase the potential utility of thermoacoustic engines in applications such as the liquefaction of natural gas [2].

Thermoacoustic energy conversion [3–5] between the axial heat flux $Q$ [6] and $I$ results from thermal interactions between gas parcels and solid walls of flow channels. As we will show in more detail in the next section, the traveling wave component (TWC) $u \cos \Phi$, in phase with $P$, performs Stirling cycles through the reversible heat exchange process between gas parcels and solid walls in the regenerator [1.7–11]. On the other hand, the standing wave component (SWC) $u \sin \Phi$ of $U$, out of phase with $P$ by $\pi/2$, contributes to the energy conversion through irreversible heat exchange process with the stack of plates [5,12,13].

Ceperley [7] attempted to amplify $I$ using thermoacoustic energy conversion based on the TWC. He installed a regenerator in a way such that the traveling wave field in a long tube having an acoustic driver at the end, and heated one end of the regenerator from the cold end to the hot end. The energy conservation law assures the relation

$$\Delta(Q+I) = 0,$$

for $Q$ and $I$ in the regenerator, where $\Delta$ represents the difference in respective quantities between both ends of the regenerator. Therefore, when the output power $\Delta I > 0$ is produced through thermoacoustic energy conversions, the outgoing intensity $I_{out}$ from the regenerator becomes larger than the incoming intensity $I_{in}$ by the amount of $\Delta I$. However, Ceperley only observed lower damping of $I$ than that without heat flow $Q$, not the amplification of $I$. He attributed it to a large viscous loss caused by a low specific acoustic impedance $z = p/u = \rho_0 c$ inherent in a pure traveling wave, where $\rho_0$ and $c$ represent mean density and sound speed, respectively.

In this paper, we demonstrate the occurrence of both thermoacoustic amplification and damping of $I$, depending on the direction of the temperature gradient imposed on the regenerator inserted in an acoustic resonator. We show that a high specific acoustic impedance reaching $15 \rho_0 c$ and a traveling wave phase $\Phi = 0$ are achieved at a velocity node in the present resonator. By installing the regenerator at different positions near the velocity node, we varied the phase difference $\Phi$ between $P$ and $U$ in the regenerator to change the ratio of the TWC to the SWC responsible for the energy conversion. As a result of the dominant contribution due to the TWC in the regenerator, we obtained the gain $I_{out}/I_{in}$ of 1.7 when a positive temperature gradient was applied, and 0.3 with a negative one. When the stack was used instead of the regenerator, the gain of $I$ reached 2.3, exceeding the temperature ratio at both ends of the stack. This is brought about by the addition of the SWC contribution to the traveling wave energy conversion.
FIG. 1. The thermodynamic process for a gas parcel. The gas parcel experiences the TWC (a) and SWC (b) based displacements along with $P$, which are shown as an ellipse (a), solid ($\Phi > 0$) and dashed ($\Phi < 0$) lines (b) and on a $P$-$\xi$ diagram, respectively. The temperature of a gas parcel (c) in the regenerator with good thermal contact is given by the tilted single line, but that with poor thermal contact traces an ellipse due to the relaxation time $\tau$. The thin tilted line represents the wall temperature.

II. THERMOSYNOUS ENERGY CONVERSION

We briefly describe here the thermoacoustic energy conversion executed by the gas parcels. The displacement $\xi$ of a gas parcel from its equilibrium position is simply written from the expression of $U$ as

$$
\xi = \frac{\mu}{\omega} \cos \Phi e^{i(\omega t - \pi/2)} + \frac{\mu}{\omega} \sin \Phi e^{i\omega t}.
$$

We denote the first and the second terms on the right-hand side of Eq. (3) as TWC-based and SWC-based displacement, respectively. They trace a clockwise ellipse and a straight line with a finite slope on a $P$-$\xi$ plane during one cycle of the acoustic wave, as shown in Figs. 1(a) and 1(b). A gas parcel on the $P$-$\xi$ plane experiences cyclic compression and expansion processes while exchanging heat with the walls in the flow channel.

The nature of the heat exchange process is well represented in terms of a nondimensional parameter $\omega \tau$ [4], where $\tau$ represents the thermal relaxation time given by using the characteristic transverse length $r_0$ and the thermal diffusivity $\alpha$ of the gas as $r_0^2/(2\alpha)$ [14]. If $\omega \tau < \pi$, the gas in the channel moves reversibly in equilibrium with the local temperature at the wall in contact with it, whereas if $\omega \tau \approx \pi$, the gas motion becomes isentropic but still reversible. The gas oscillation becomes thermodynamically irreversible near $\omega \tau \approx \pi$ due to incomplete heat transfer to the wall. A porous medium having flow channels with $\omega \tau < \pi$ is usually called a “regenerator.” On the other hand, a porous medium with $\omega \tau \approx \pi$ is called a “stack.”

Figure 1(c) presents the axial distribution of the wall temperature (thin line). Also shown is the variation of the absolute temperature $T$ of the gas parcels due to the displacement $\xi$ in the temperature gradient. When $\omega \tau < \pi$, $T$ always traces the local wall temperature, because the gas parcel is instantaneously heated and cooled because of its good thermal contact with the wall. On the other hand, the gas parcel does not exchange heat with the walls when $\omega \tau \approx \pi$. Near $\omega \tau \approx \pi$, the finite relaxation time $\tau$ causes a time delay in heating and cooling over the cross section of the flow channel, thus the $T$-$\xi$ diagram traces an ellipse.

The thermodynamic cycle executed in the course of the TWC-based displacement is given by combining Figs. 1(a) and 1(c). Starting from the position near the left end labeled “A” in Fig. 1(a), the gas parcel repeats a cyclic motion consisting of compression (A-B)-heating(B-C)-expansion (C-D)-cooling(D-A), when $\omega \tau < \pi$. As Ceperley has noted [7], this is a thermodynamic cycle similar to the Stirling one, thereby enabling an amplification of the acoustic intensity $I$. The TWC also contributes to the energy conversion even when $\omega \tau \approx \pi$, but poor thermal contact with the wall leads to heat losses.

During the SWC-based displacement shown in Fig. 1(b), the gas parcel experiences heating and compression at the same time when it moves to the hot end (B-C), while it simultaneously experiences cooling and expansion when it moves back to the cold end (D-A). Therefore, the SWC does not contribute to the energy conversion when $\omega \tau < \pi$. However, as is discussed by Wheatley [5], a finite $\tau$ in the heat exchange process causes a phase lag in the heating and cooling of the gas parcel. This establishes a thermodynamic cycle of compression (B-C)-heating(C-D)-expansion (D-A)-cooling(A-B). Thus, the SWC energy conversion takes place when $\omega \tau \approx \pi$.

III. EXPERIMENT

A. Experimental apparatus

The present experimental apparatus is schematically illustrated in Fig. 2(a). A middle section of the cylindrical acoustic resonator is constructed of Pyrex glass tube with a 21 mm inner diameter to allow the measurement of the velocity of the gas by means of a laser Doppler velocimeter (LDV). The remaining part of the resonator is constructed of stainless steel tubing. The total length of the resonator is 3.3 m. One
end of the resonator is closed by a woofer speaker through dynamic bellows, and the other is closed by a solid plate. The speaker is driven at 103 Hz. Air at local atmospheric pressure is used as the working gas. Either a regenerator or a stack, each being 20 mm long, is inserted near the middle of the resonator.

Hot and cold heat exchangers, consisting of brass strips aligned in parallel, are placed at both sides of a regenerator and a stack in order to produce temperature gradients along them. The hot heat exchanger is heated by a resistance heater up to \( T_H = 560 \) K, and the cold heat exchanger is kept at room temperature \( T_C = 296 \) K using running water. Positive and negative temperature gradients are formed by changing the location of the two heat exchangers with each other.

We use a pile of stainless screen meshes (60 mesh) with \( r_0 = 0.15 \) mm as the regenerator and a ceramic substrate with \( r_0 = 0.77 \) mm as the stack. The values of \( \omega r \) are estimated as 0.13 for the regenerator and 3.5 for the stack, respectively, where \( \alpha \) at the mean temperature \( \approx (T_H + T_C)/2 \) is used for the evaluation. As we will see later, the TWC contributes almost exclusively to the energy conversion in the present regenerator, while the SWC has the major contribution in the stack.

**B. Pressure and velocity measurements**

The pressure \( P = Pe^{i\omega t} \) along the glass tube was measured using small pressure transducers (Toyoda Koki, DD102-1F), which were mounted on the tube wall via short ducts of 10 mm in length and 1 mm in inner radius. We measured the axial core velocity at the center of the cylinder with an LDV, and determined both amplitude \( u \) and phase \( \Phi \) in the cross-sectional mean velocity \( U = u e^{i(\omega t + \Phi)} \) from the measured values by applying laminar flow theory. Pressure and velocity signals were collected by a 24-bit spectrum analyzer. Using power and phase spectra, we determined \( p, u, \) and \( \Phi \). It was important to determine precisely the instrumental time delay of the LDV \((= 2.7 \times 10^{-5} \) s) for the accurate evaluation of \( \Phi \) and hence \( I \) [13,15].

**C. Acoustic field in a resonator**

Before showing the experimental data on the acoustic intensity \( I \), we present the acoustic field excited in the resonator and its advantages for this experiment. In this experiment, we used the second resonance mode having a velocity node at the middle of the resonator. We found that this mode occurs at 103 Hz with a quality factor \( Q_r \) of 30 from the measurement of frequency response curves.

Figure 2(b) shows the axial distribution of \( \Phi \) in the vicinity of the room temperature regenerator at a position 1.69 m away from the speaker. If a nondissipative standing wave were formed in the resonator with a very high \( Q_r \), \( \Phi \) would take 90° or 0°, depending on whether the sign of the gradient of \( p \) is positive or negative. However, in the present resonator with the finite \( Q_r = 30 \), it was found that \( \Phi \) continuously varied in the range \(-90° < \Phi < 90°\) and passed zero at the velocity node. This continuous variation shows that the ratio of TWC to SWC varies as a function of the axial position along the resonator. In Fig. 2(b), we see that the phase \( \Phi_m \) in the center of the regenerator is 20°. Thus, we can choose a desired ratio of TWC to SWC in the center of the regenerator by shifting its axial position. This is the clear advantage of the use of the resonator in this experiment.

We have found that the specific acoustic impedance \( z \) given as \( p/u \) reaches 15\( \rho_0 \) near the velocity node in the present resonator. Hence, the gas parcel at the velocity node oscillates with both a high acoustic impedance \( z \) and a traveling wave phase \( \Phi = 0 \). This is another advantage of the use of the acoustic field in the resonator [11] over the pure traveling wave propagating through a long tube, in which \( z \) is given by \( \rho_0 \) [7]. Thus, we can demonstrate the thermoacoustic energy conversion without suffering from strong viscous damping by installing the regenerator close to the velocity node.

**IV. RESULTS AND DISCUSSION**

**A. Axial distribution of acoustic intensity**

We report here the axial distribution of acoustic intensity \( I \), when \( \Phi_m = 20° \) and the temperature gradient is zero along the regenerator as shown in Fig. 2(b). We inserted the measured \( p, u, \) and \( \Phi \) into Eq. (1), and plotted \( I \) thus obtained in Fig. 3. The origin of \( x \) is taken at the center of the regenerator. The error in \( I \), mostly due to the uncertainty in \( \Phi \), is within the size of the markers. The positive value of \( I \) indicates that \( I \) flows through the resonator from the acoustic driver to the closed end. The negative slope of \( I \) represents the dissipative energy per unit volume due to the viscosity and the thermal conductivity of the working gas. The value of the slope is found to be approximately \(-12 \) W/m\(^3\) in the region outside the regenerator and \(-120 \) W/m\(^3\) inside of it.

We supplied heat power to the regenerator to create temperature gradients on it. Here we kept \( \Phi_m = 20° \) and also adjusted the incoming acoustic intensity \( I_m \) to the regenerator to be the same as that without the heat power. When a positive temperature gradient \( \nabla T > 0 \) is formed along the regenerator, \( I \) propagates through the regenerator from the cold to the hot end. We clearly see the formation of a steep positive slope in \( I \) in the regenerator. This demonstrates the thermoacoustic amplification of \( I \), which Ceperley had attempted to
moacoustic silencer uses energy conversions between the regenerator and an \( I \) erator. It is also found that the gain lies below unity, which shows that \( I \) of 0.45. This result shows the strong thermoacoustic damping region than that with decreases more significantly for this case in the regenerator when a negative \( \nu \) is present, the decrease of \( I \) along the regenerator becomes significantly larger than that with \( \nabla T = 0 \). As a result, the gain \( I_{out}/I_{in} \) lies far below that with \( \nabla T = 0 \) and becomes 0.3 with \( \Phi_m = 70^\circ \). as shown in Fig. 4. When a negative \( \nabla T \) is present along the regenerator, the gas parcels experience cooling when it moves from \( B \) to \( C \) in Fig. 1(a), and heating when it moves from \( D \) to \( A \). As a result, the thermodynamic cycle of compression (\( A-B \))--cooling (\( B-C \))--expansion (\( C-D \))--heating (\( D-A \)) is executed in the course of the TWC-based displacement. This is the reversed Stirling cycle that makes \( \Delta I \) negative. Therefore, the thermoacoustic energy conversion is responsible for the strong damping of \( I \). We see that the gain \( I_{out}/I_{in} \) shows a peak at \( \Phi_m = -40^\circ \). This means that minimum damping is obtained when the velocity node comes to the hot end of the regenerator for the same reason as that described above when \( \nabla T \) is positive.

C. Stack (\( \omega \tau = 3.5 \))

Next, we used a stack of \( \omega \tau = 3.5 \) instead of the regenerator. Results are shown in Fig. 5. When \( \nabla T \) is zero, \( I_{out}/I_{in} \) for the stack lies below unity as in the regenerator, but shows a broader peak than that for the regenerator. While viscous losses dominated in the regenerator, the dissipation due to the thermal conduction over the cross section of flow channels, which is proportional to \( \rho^2 \) [4], becomes large in the present stack. The stack is always located around the pressure antinode (velocity node) in this experiment, and hence the value of \( \rho \) in the stack is not affected much by the value of \( \Phi_m \). Thus, we see that the thermal conduction causes the broad peak centered around \( \Phi_m = 0^\circ \).

When a positive \( \nabla T \) is set up along the stack, the gain \( I_{out}/I_{in} \) for the stack is clearly different from that for the regenerator; it monotonically increases with increasing \( \Phi_m \). Eventually, \( I_{out}/I_{in} \) reached 2.3 at \( \Phi_m = 80^\circ \), which is larger than \( T_H/T_C = 1.9 \), when the center of the stack is placed 10 cm away from the velocity node toward the acoustic driver. The contribution of the TWC to the energy conversion can be seen at \( \Phi_m = 0^\circ \), since the SWC is absent there.
When a negative $\nabla T$ is made along the stack, the gain $I_{\text{out}}/I_{\text{in}}$ increases with decreasing $\Phi_m$, and it exceeds unity when $\Phi_m<-60^\circ$. Since the TWC with a negative $\nabla T$ always contributes to the damping of $I$ regardless of the sign of $\Phi_m$, the amplification of $I$ must be due to the SWC. As described above, a negative $\Phi_m$ reverses the order of compression and expansion in the cycle. On the other hand, in the presence of a negative $\nabla T$, the order of heating and cooling is also reversed. Consequently, gas parcels experience the cycle yielding a positive $\Delta T$ during the SWC-based displacement, when both $\Phi_m$ and $\nabla T$ are negative. On the other hand, with a positive $\Phi_m$, both the SWC and TWC contribute to the damping of $I$. Thus, the gain $I_{\text{out}}/I_{\text{in}}$ lies far below unity with a positive $\nabla T$.

It is clear from the argument above that $\Phi_m$ must be positive in order to obtain a gain as large as possible in a thermoacoustic power amplifier, because both the TWC and SWC can contribute to the amplification of $I$, when $\nabla T>0$. The maximum $I_{\text{out}}/I_{\text{in}}$ exceeding $T_{\text{H}}/T_{\text{C}}$ at $\Phi_m=80^\circ$ obtained in this experiment, apparently results from the energy conversion both due to the TWC and SWC. However, while the efficiency of the energy conversion due to the TWC can ideally reach Carnot’s efficiency, that due to the SWC is essentially lower than that due to the TWC, since the SWC energy conversion intrinsically relies on the irreversible heat exchange process [3–5,7]. Thus, in order to achieve both high efficiency and gain under a given temperature ratio, one has to choose an optimum $\Phi_m$ to give the best combination of the TWC and SWC. As shown in Figs. 4 and 5, the optimum $\Phi_m$ should depend on the value of $\omega \tau$ of a given regenerator. By using a series of regenerators and stacks inserted at positions with proper $\Phi_m$, we could obtain very high acoustic intensity $I$ such as those used in practical engines.

V. SUMMARY

We demonstrated thermoacoustic energy conversions by making full use of the acoustic field induced in the resonator. Through the simultaneous measurements of $P$ and $U$, we determined the gain of the acoustic intensity $I$ and the phase angle $\Phi$ between $P$ and $U$. We found that the regenerator worked as a thermoacoustic power amplifier with a gain of 1.7 when $\nabla T$ was positive, but as a silencer with a gain of 0.3 when $\nabla T$ was negative. These results obtained for the regenerator originates from the energy conversion due to the TWC. Furthermore, the SWC was found to play the major role in the energy conversion in the stack. Especially when $\Phi_m=80^\circ$ and $\nabla T>0$, the gain $I_{\text{out}}/I_{\text{in}}$ reached 2.3 resulting from the SWC energy conversion. This represents that the addition of the SWC to the TWC makes it possible to increase the gain above the temperature ratio $T_{\text{H}}/T_{\text{C}}$, the ideal gain for the Stirling engine. By placing the regenerators and stacks near the velocity nodes in a long acoustic resonator and amplifying $I$ successively, we would be able to obtain a very high acoustic intensity $I$ even from waste heat having a low energy density. The present results will contribute to the development of new acoustic devices using thermoacoustic power amplifiers and silencers.

FIG. 5. The amplification rate $I_{\text{out}}/I_{\text{in}}$ as a function of $\Phi_m$ determined for the stack. Three different symbols represent the data with positive (solid squares), negative (open squares) temperature gradient and without a temperature gradient (solid circles). A horizontal line represents the temperature ratio $T_{\text{H}}/T_{\text{C}}$. The error of the data does not exceed the size of the markers.

We see that $I_{\text{out}}/I_{\text{in}}$ is nearly equal to unity. This represents that the amplification due to the TWC is so small that it is used up to compensate for the dissipative energy in the stack. Therefore, the observed large enhancement of $I_{\text{out}}/I_{\text{in}}$ should reflect the thermoacoustic energy conversion due to the SWC. Indeed, as we described in Sec. II, the SWC with a positive $\Phi_m$ contributes to the amplification of $I$ in the presence of a positive $\nabla T$. Thus, the present results clearly demonstrate that amplification of $I$ exceeding $T_{\text{H}}/T_{\text{C}}$, the gain expected for an ideal regenerator, is made possible by using the SWC energy conversion in the stack.

In a standing wave thermoacoustic prime mover, the acoustic intensity $I$ comes out of both sides of the stack [13] placed between the velocity node and antinode. When a $\nabla T$ is positive in the stack, the acoustic intensity $I$ at the cold end is negative, while that at the hot end is positive. Therefore, $I_{\text{in}}$ in the standing wave prime mover is zero, because $I$ is generated within the stack. This represents that the gain $I_{\text{out}}/I_{\text{in}}$ can be very large when the SWC mostly contributes to the energy conversion. Therefore, as the present results indicate, the use of the SWC energy conversion is of great importance when one attempts to make the gain $I_{\text{out}}/I_{\text{in}}$ as high as possible in a given temperature ratio.

When the stack is placed beyond the velocity node, $\Phi_m$ becomes negative and the gain $I_{\text{out}}/I_{\text{in}}$ decreases below unity in spite of a positive $\nabla T$. This is because the sign of the slope in the $P$-$\xi$ diagram is reversed when $\Phi_m$ is negative. As shown by the dashed line in Fig. 1(b), the gas parcels with negative $\Phi_m$ experience expansion when they move from left($B'$) to right($C'$). Hence, the gas parcel undergoes the thermodynamic process in the order of expansion($B'\rightarrow C'$)–heating($C'\rightarrow D'$)–compression($D'\rightarrow A'$)–cooling($A'\rightarrow B'$), which leads to the reversed cycle discussed in Sec. II. As a result, the SWC contributes to the reduction of $I$ when $\Phi_m$ is negative in the presence of a positive $\nabla T$.
Heat flux due to the oscillating gas is defined as $Q = T_m S$ using the mean temperature $T_m$ and entropy flow $S$. The entropy flow $S$ is given as $S = \rho_s \langle s \cdot U \rangle$, where angular brackets and a bar represent time and radial averages, $\rho_s$, $s$, and $U$ are, respectively, the mean mass density, entropy per unit mass, and axial velocity for the oscillating gas.