Sound Quality of Two-tone Complex Sounds with Different Overall Loudness

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1 Introduction

Timbre of a steady-state complex sound is an attribute of multidimensional character and is closely correlated to the waveform or frequency spectrum of the steady-state complex sound (Lichte, 1941; Plomp et al., 1967; Pols et al., 1969; Plomp, 1970; Plomp and Steeneken, 1969, 1973; Bismarck, 1974a, b; Brujin, 1978; Benedini, 1980a, b; Ohgushi, 1980; Preis, 1984; Darwin and Gardner, 1986; Patterson, 1987). Even for the simplest complex sound, namely, a two-tone complex sound, its timbre changes depending on the relative amplitudes of two components and their phase difference (Craig and Jeffress, 1962; Raiford and Schubert, 1971; Hall and Schroeder, 1972; Ozawa et al., 1993). In these investigations the stimuli had almost the same loudness or overall sound pressure levels. This is due to the definition of timbre, that “attribute of auditory sensation in terms of which a listener can judge that two sounds similarly presented and having the same loudness and pitch are dissimilar” (ASA, 1960; JIS, 1986).

Even if loudness and pitch of two sounds are different, we can comment on similarity or dissimilarity between the sounds. In this case, what we perceive is not the timbre but the sound quality. In other words, timbre is considered to be the limited phase of sound quality in the case where loudness and pitch of two sounds are the same. Thus, sound quality must also be multidimensional and closely correlated to the waveform or the frequency spectrum of a sound. However, sound quality seems dependent on the loudness of a complex sound even when the waveform of the sound is kept constant. For example, when music is reproduced at relatively lower levels in our home than the original level played in a concert hall, we perceive its sound quality differently from the original one even if the waveform is preserved from modification by the reproduction system.

The objective of this study is to investigate the effect of the overall sound pressure level or loudness of a complex sound on its sound quality. The simplest two-tone complex sound was used in this paper as the first step to fully understanding the effect. Two experiments were conducted in which the stimuli were two-tone complex sounds having the same frequencies and phase differences, but different amplitudes.

2 Experiment I: Sound quality of two-tone complex sounds

2.1 Stimuli and purpose of experiment

The waveform \( f(t) \) of a two-tone complex sound is given by

\[
f(t) = A_1 \sin (2\pi f_1 t) + A_2 \sin (2\pi f_2 t + \phi),
\]

where \( f_1 \) and \( f_2 \) are the frequencies of the two components, \( A_1 \) and \( A_2 \) represent the amplitudes of \( f_1 \) and \( f_2 \) components, respectively. Phi (\( \phi \)) denotes the phase difference between the components. In this experiment similarities in sound quality were evaluated between two-tone complex stimuli consisting of \( f_1 = 500\text{-Hz} \) and \( f_2 = 750\text{-Hz} \) components with different amplitude, while \( \phi \) was fixed at zero. The results of a similar experiment using two-tone complex sounds consisting of \( f_1 = 500\text{-Hz} \) and \( f_2 = 1\text{-kHz} \) components have been reported elsewhere (Sone et al., 1989). The purpose of this experiment is twofold. One is to evaluate the effect of overall loudness on sound quality of the stimuli. The other is to examine whether the difference in frequencies of the components influences the overall loudness effect on sound quality.
2.2 Method and apparatus

The sound pressure levels of the two components used in the experiment are shown in Table 1. The stimuli were divided into four groups labeled A to D. Each group consisted of six stimuli: one was the reference stimulus common to all the groups, while the other five were comparison stimuli. The sound pressure levels of the 500-Hz component of the five comparison stimuli in a group were the same, and were lower than that of the reference stimulus—namely, 5 dB lower in group A, 10 dB lower in group B, 15 dB lower in group C, and 20 dB lower in group D. For the comparison stimuli in a group, the sound pressure levels of the 750-Hz component were changed by 3-dB decrements. Clearly, the waveforms of A2, B2, C2, and D2 were the same as that of the reference stimulus because the level differences between the components were the same. Each group experiment was carried out in separate session.

In each session similarities in sound quality among the six stimuli were obtained by using the complete method of triads. Triads (X-Y-Z) were presented sequentially. Each element had a duration of 2 s with 20 ms rise and fall times. Intervals of 0.8 s were inserted between the elements. For each triad a subject was asked to judge if stimulus Y was more similar in sound quality to stimulus X or Z. All possible triad combinations were tested three times in random order. Thus, 360 (= 3 × 4²) trials were made by a subject in a session. These trials were divided into three subsessions, each taking approximately 20 minutes.

The stimuli were digitally generated and fed to a D/A converter with an accuracy of 16 bits and a sampling rate of 50 kHz on a computer (Toshiba, DS-600). Stimuli were presented diotically via headphones (Yamaha, YHD-3) to a subject seated in a soundproof chamber.

2.3 Subjects

Twenty-four male subjects, between 21–24 years of age, participated in the experiment. They had no history of auditory disease. The MAPs (Minimum Audible Pressures) of the left and right ears of subjects were checked for normalcy using a Békésy-type audiometer from 100 Hz to 10 kHz.

2.4 Results

The judgments for each triad were gathered across all subjects to obtain dissimilarity matrices (Torgerson, 1958), and analyzed using Torgerson’s metric MDS (Multi-Dimensional Scaling) program. Two-dimensional solutions were employed for all stimuli groups because two indices of goodness of fit, $F$ and $P$ (Saito, 1980), were

<table>
<thead>
<tr>
<th>Group</th>
<th>Level (dB SPL)</th>
<th>Stimulus No.</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>500 Hz</td>
<td>750 Hz</td>
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<tr>
<td>A</td>
<td>70</td>
<td>65</td>
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<td></td>
<td>65</td>
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<td>B</td>
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<td></td>
<td></td>
<td>46</td>
</tr>
<tr>
<td>C</td>
<td>70</td>
<td>65</td>
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<td>55</td>
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<td></td>
<td></td>
<td>41</td>
</tr>
<tr>
<td>D</td>
<td>70</td>
<td>65</td>
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<td>50</td>
<td>48</td>
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<td>39</td>
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<td></td>
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<td>36</td>
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</table>
almost equal to the maximum value of one, indicating a perfect goodness of fit. Figure 1 shows the stimulus configuration for each stimuli group. Stimuli are represented by crosses, and the distances between the crosses correspond to the dissimilarities between the stimuli obtained in this experiment. The scale of distance in the configurations is shown as a line segment, which corresponds to JND (Just Noticeable Difference) calculated using the model of Thurstone’s Case V (Thurstone, 1927).

In all configurations the comparison stimuli are placed on a curved line and the reference stimulus is located apart from the line. The effect of overall loudness on sound quality is clearly seen—the larger the attenuation from the reference stimulus the larger the distance between the reference stimulus and the curved line. The difference in overall loudness resulted in a dissimilarity in sound quality even when the waveform was the same. As for the comparison stimuli on the other hand, the larger the attenuation from the reference stimulus, the shorter the length of the curved line. Therefore, the larger the attenuation the smaller the resulting dissimilarity between the comparison stimuli *1 and *5 (where * is A, B, C, or D), which exhibit the greatest spectral differences within a group. Consequently, the same degree of difference of the sound pressure level of the higher frequency component did not result in the same perceptual difference if the overall loudness of the stimuli was different.

2.5 Discussion

The number of dimensions should be one if the sound quality of a two-tone complex sound with fixed $\phi$ is determined only by the level difference between the components. As described above, however, the experimental results indicated that the number of dimensions was two. What factors created the two dimensions? Solutions of Torgerson’s MDS demonstrate freedom of rotation. If we rotate the configuration, two axes—each representing a dimension of the sound quality of our two-tone complex sounds—could be obtained as shown by the broken lines in Fig. 1(a). It is well known that sound quality is generally expressed by the three factors of “aesthetic state”, “volume”, and “brightness” (Sone et al., 1962; Kitamura et al., 1968; Bismarck, 1974a). Axis I is considered to correspond to the brightness since the level difference between the two components changes along this axis. It is known that the level of a higher frequency component affects the “brightness” (Lichite, 1941). On the other hand, Axis II’ seems to be the “volume” because the overall loudness of the stimuli changes along the axis.

The authors had carried out a similar experiment using two-tone complex sounds consisting of 500-Hz and 1-kHz components (Sone et al., 1989). The results of this experiment were very similar to those obtained here, while the consonance (Kameoka and Kuriyagawa, 1969a, b) of the reference stimulus differed between the two experiments. The consonance is an attribute of sound quality of complex sounds, and is the highest for a complex sound consisting of tones in an octave relation. The consonance of the reference stimulus used in the present
experiment was lower than that used in the previously reported experiment because the components used here were not in an octave relation. This indicates that the effect of overall loudness on sound quality is consistent irrespective of the consonance of the stimuli.

3 Consideration on the relation between sound quality and frequency spectrum in Experiment I

3.1 Similarity in sound quality and difference in frequency spectrum denoted in sound pressure level

Plomp (1970) and Plomp and Steeneken (1973) indicated that the timbre difference between two sounds with equal loudness and pitch is highly correlated with the difference between their sound spectra denoted in dB per one-third octave band. The difference, \( d_{ij} \), between the spectra of stimuli \( i \) and \( j \) is given by

\[
d_{ij} = \sqrt{\sum_{k=1}^{n} (L_{k,i} - L_{k,j})^2},
\]

where \( L_{k,i} \) is SPL (Sound Pressure Level) of the stimulus \( i \) in 1/3-octave frequency band \( k \), and \( n \) is the total number of frequency bands. Although this idea was supported by some investigations for timbre of broad-band sounds (Preis, 1984; Ozawa et al., 1995), it is not clear whether the idea holds for sound quality of narrow-band sounds with different loudness.

As for the two-tone complex sounds used in this experiment, the SPL of each component was directly applied to the equation. The spectral differences among the stimuli were calculated by Eq. (2) and analyzed by MDS. The results of these calculations are shown by open squares in Fig. 2, in which crosses from Fig. 1 are duplicated for comparison. As can be seen the configurations of the crosses and the open squares are different, especially in the reference stimulus. In each panel the difference between the two configurations is shown as a geometrical measure of the difference, \( \tau \), which is zero when two configurations are the same (Appendix A; Saito, 1980). The larger the attenuation from the reference stimulus the larger the difference \( \tau \). This indicates that the spectral differences calculated by Eq. (2) overestimate the effect of overall loudness on sound quality. Consequently, the spectral differences failed to account for the similarities in sound quality with different overall levels.

3.2 Introduction of the idea of masked frequency spectrum of a complex sound

It is assumed that some characteristics of the auditory system such as nonlinear transmission, lateral inhibition, and band-pass filtering translate the physical frequency spectrum of a sound into an “internal spectrum,”

![Fig. 2. Configurations of stimuli derived by MDS. Crosses: duplication from Fig. 1 with altered scale. Open squares: spectral difference denoted in SPL.](image)
which is the description of a sound spectrum in the auditory pathway (Buunen et al., 1974; Hirahara, 1991). The authors assume that the internal spectrum is then converted into a “subjective spectrum” in terms of subjective (psychological) magnitude. Since this “subjective spectrum” is assumed to be used as the input to the central stage of sound quality perception, the sound quality would be more closely correlated with it rather than with the physical spectrum. Furthermore, the authors suppose that the subjective spectrum could be approximated by a “masked frequency spectrum,” which is the frequency spectrum in terms of loudness of each component partially masked by each other (Sone et al., 1989; Ozawa et al., 1990; Ozawa et al., 1995).

Partially masked loudness, $S$, of a tone is approximated by

$$ S = \beta(I^n - I_0), $$

where $I$ is the intensity of the tone, $I_0$ is the intensity of the tone at threshold, $\beta$ is a dimensional constant, and the exponent $n$ is approximately 0.3 (Zwislocki and Hellman, 1960; Lochner and Burger, 1961; Ozawa et al., 1997). The thresholds at the component frequencies of 500 and 750 Hz masked by each other were measured to estimate the partially masked loudness of each component of the stimuli using Eq. (3). The levels of the masker were set at 70, 65, 60, 55, and 50 dB for the 500-Hz masker and 65, 60, 55, 50, and 45 dB for the 750-Hz masker. These levels are those of the components of the reference stimulus and the comparison stimuli of A2, B2, C2 and D2. The measurements were carried out by the method of limits. Four downward and four upward sequences were alternately tested in which the signal level was changed by 2-dB steps. The last six data values were averaged to obtain a threshold. Figure 3 shows the masked thresholds with error bars representing ±1 SD among the subjects. The amount of masking by the higher frequency masker is relatively small as shown in Fig. 3(a), while the lower frequency masker results in a large degree of masking as shown in Fig. 3(b). This tendency is consistent with the well-known masking pattern of a tone (Fletcher, 1953).

### 3.3 Representation of sound quality by the masked frequency spectrum

The authors intended to represent the similarities in sound quality based on the masked frequency spectrum. They attempted to represent the two psychological factors of the configurations described in Section 2.5. Since

![Figure 3](image)

Fig. 3. Thresholds masked by the alternative component of a two-tone complex sound in Experiment I.
the sound quality of "brightness" is related to the balance of loudness between the components, the ratio of the masked loudness, \( S_i / S_j \), is assumed to be correlated with this factor, where \( S_k \) is the partially masked loudness of the component \( k \). As the other sound quality factor of "volume" is related to the overall loudness of the stimuli, the sum of the masked loudness, \( S_i + S_j \), is considered to be correlated with this factor. The Euclidean distance between two stimuli \( i \) and \( j \) is then defined by

\[
D_{ij} = \sqrt{\left( \frac{S_{1i}}{S_{1j}} - \frac{S_{2i}}{S_{2j}} \right)^2 - c \left\{ (S_{1i} + S_{2i}) - (S_{1j} + S_{2j}) \right\}^2},
\]

where \( c \) is a dimensional constant.

The distances were analyzed by MDS, and the configurations of filled circles in Fig. 4 were obtained. The differences, \( d_{ij} \), between the configurations are generally smaller than those in Fig. 2. This means that the spectral distances denoted by the masked loudness show better agreement with the perception of the similarities in sound quality.

In order to examine the effect of the mutual masking, the simple loudness of each component was calculated by setting the threshold, \( I_0 \), in Eq. (3) to zero. Spectral distances were then calculated by Eq. (4) and the configurations of open circles in Fig. 4 were obtained by MDS. The \( d_{ij} \) are almost the same for open and filled circles. This indicates the effect of mutual masking is not clear in this experiment. This is because of the effect of recruitment in loudness. Recruitment is a phenomenon whereby the threshold, \( I_0 \), becomes negligible if \( I \gg I_0 \). In this experiment the sound pressure levels of the components were higher than the thresholds by 30 dB or more. Consequently there was little effect of mutual masking on the loudness of the component. Experiment II was designed to examine the influence of mutual masking by setting the level of components near the threshold.

4 Experiment II: Sound quality of two-tone complex sounds with component levels near threshold

4.1 Stimuli

As stated above, the effect of overall loudness on sound quality is independent of the frequencies of the components. Thus, stimuli with 500-Hz and 1-kHz components were used in this experiment as shown in Table 2. The sound pressure levels of the higher frequency component used in this experiment were relatively lower than those used in the previous experiment in order to more effectively examine the effect of mutual masking between the components on sound quality of complex sounds.

![Fig. 4. Configurations of stimuli derived by MDS. Crosses: duplication from Fig. 1. Open circles: Euclidean distance in terms of the simple loudness. Filled circles: Euclidean distance in terms of the partially masked loudness.](image-url)
4.2 Method

The experimental procedure was the same as in Experiment I, except that the duration of the stimuli was 1.6 s and the silent interval was 0.5 s. Only six stimuli were used, but every possible triad was tested 16 times by a subject. Thus the number of judgments was 1920 (= 16 × 3²) for a subject. Trials were divided into 16 sessions.

Thresholds masked by each component were measured by the constant method with A-X-B paradigm, in which a subject rated which of A or B was the same as X. Tones A, X, and B had a duration of 1.2 s and an inter-

![Diagram](image-url)

Fig. 5. Configurations of stimuli derived by MDS in Experiment II. Crosses: similarity of sound quality. Open circles: Euclidean distance in terms of the simple loudness. Filled circles: Euclidean distance in terms of the partially masked loudness.
Table 3. Difference index, \( \tau \), between two configurations derived from the experimental data and the spectral distances in Experiment II.

<table>
<thead>
<tr>
<th>Subj</th>
<th>( \tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27.1%</td>
</tr>
<tr>
<td>2</td>
<td>29.1%</td>
</tr>
<tr>
<td>3</td>
<td>13.0%</td>
</tr>
<tr>
<td>4</td>
<td>29.4%</td>
</tr>
<tr>
<td>5</td>
<td>30.2%</td>
</tr>
<tr>
<td>All</td>
<td>22.1%</td>
</tr>
</tbody>
</table>

Table 4. Masked thresholds obtained in Experiment II. Values are given in dB SPL. Values in parentheses denote Standard Deviation (SD) in dB. Threshold for “Subj. All” was determined as the average of the thresholds of the five subjects.

<table>
<thead>
<tr>
<th>Signal frequency</th>
<th>Masker frequency and level</th>
<th>500 Hz</th>
<th>1 kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>45</td>
<td>40</td>
</tr>
<tr>
<td>1 kHz</td>
<td></td>
<td>11</td>
<td>11 (1)</td>
</tr>
<tr>
<td>500 Hz</td>
<td></td>
<td>11 (2)</td>
<td>8 (5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16 (9)</td>
<td>27 (3)</td>
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<tr>
<td></td>
<td></td>
<td>23 (6)</td>
<td></td>
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<td></td>
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<td>26 (5)</td>
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<td></td>
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<td>25</td>
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</table>

val of 0.5 s. Tone X consisted of only the masker component. As to tones A and B, one of them was the same tone as X and the other was a complex tone consisting of the masker component and a signal. Seven signal levels were prepared with 3-dB steps around a center level which had been determined by a preliminary experiment for each subject. The number of trials for every signal level was forty, and a 75% correct response level was adopted as a threshold by fitting the logistic function to the experimental data.

4.3 Subjects

Five male subjects, between 22–26 years of age, participated in Experiment II. They had no history of auditory problems and MAPs of their left and right ears were checked for normalcy using the audiometer. Subjects 3 and 4 had participated in Experiment I.

4.4 Results and discussion

The data for individual subjects, as well as the pooled data for all subjects, was analyzed by MDS. Prior to the analysis pre-processing was carried out on the data concerning the triads of stimuli 1-4-5 and 1-5-4. Four subjects judged that stimuli pair 4-5 was more similar than pairs 1-4 and 1-5 for every triad. In such a case, psychological distances corresponding to stimuli pairs 1-4 and 1-5 equaled infinity by Thurston’s law (Thurstone, 1927). These distances were estimated by one-step computation (Torgerson, 1958) using finite distances for other stimuli, e.g., the distance between 1-5 was estimated as the sum of the distances 1-3 and 3-5.

Two-dimensional solutions were derived as shown by crosses in Fig. 5. The outlines of the configurations are similar to those in Fig. 1. It is reconfirmed that the effect of overall loudness on sound quality of two-tone complex sound is independent of the consonance of the stimuli because they were in an octave ratio in Experiment II, while they were not in Experiment I. Spectral differences among the stimuli were calculated by Eq. (2) and they were analyzed by MDS. The differences, \( \tau \), between the configurations were quite large as shown in Table 3. The spectral difference can never account for the similarity.

Filled circles in the figure were derived by MDS from the distances calculated by Eq. (4) with the partially masked loudness estimated by Eq. (3) using the measured thresholds shown in Table 4 as \( L_0 \). On the other hand, open circles represent distances obtained with simple loudness when \( L_0 \) was set to zero. Open and filled circles are almost overlapping for the comparison stimuli 1 to 5.

As shown in Fig. 5, the differences between configurations, \( \tau \), are small. Moreover, comparison among the three configurations indicates that the configurations based on the partially masked loudness (filled circles) show a slightly better similarity to the experimental data (crosses) than those based on simple loudness (open circles). This suggests that the Euclidean distance between the spectra denoted by the partially masked loudness is in good agreement with the dissimilarity evidenced in sound quality.
5 Conclusions

Two experiments on sound quality of two-tone complex sounds were carried out. The similarities in sound quality were analyzed by MDS and two-dimensional configurations were derived. Two factors of sound quality were considered to be “brightness” and “volume.” The effects of overall loudness on sound quality were consistent irrespective of the consonance of stimuli used in this study.

Spectral differences defined using SPL failed to account for the effect of overall loudness on sound quality. An alternative idea to explain the effect of overall loudness on sound quality was presented. Two factors calculated from the partially masked loudness of each component were introduced. Euclidean distances with respect to the partially masked loudness showed rather good agreement with the experimental data. This suggests that the masked frequency spectrum, which is the spectrum in terms of the partially masked loudness of each component, is directly correlated to sound quality of a two-tone complex sound.

REFERENCES

Appendix A: Definition of the measure of difference, $\tau$

The geometrical measure of the difference between two configurations, $\tau$, is defined as follows (Saito, 1980):

The relation between two configurations $X$ and $Y$ is given by the equation

$$Y = sY + 1_m a' + E,$$

where $X$ and $Y$ are matrices consisting of $m$-dimensional coordinates of $r$ stimuli. In the present study, the number of dimensions is two and the number of stimuli is six. Matrix $T$ is an $r \times r$ orthogonal matrix representing rotation, $s$ is a scaling constant representing expansion or contraction, and the term $1_m a'$ represents parallel translation where $a' = (a_1, a_2, \ldots, a_r)$ and $1_m = (1, 1, \ldots, 1)$. The elements of matrix $E$ corresponds to errors which cannot be represented even by the rotation, the scaling, and the parallel translation. The difference $\tau$ is defined with respect to the errors by

$$\tau = \frac{\text{tr} E' E}{\text{tr} Y' HY} = 1 - \frac{(\text{tr} T'X'HY)^2}{(\text{tr} X'HX)(\text{tr} Y'HY)},$$

where

$$H = I_m - \frac{1}{m} 1_m 1_m^T,$$

and

$$I_m = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}.$$