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Test of a scaling law for quasi-elastic electron scattering

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A simple scaling law for quasi-elastic scattering is tested by deep-inelastic scattering on 40Ca. Data taken at widely different scattering angles and bombarding energies can be combined to yield a simple function of one variable. Systematic comparison of low- and high-energy data may permit isolation of the contribution from the single-particle knockout process.

\[ \text{NUCLEAR REACTIONS } 40\text{Ca}(e,e')X. \text{ Measured deep-inelastic scattering. Derived momentum distribution and scaling law.} \]

Scaling hypotheses, in which the scattering is considered as a function of only one kinematic variable, have been extensively studied in particle physics. A similar hypothesis has been used to describe the cross section of quasi-elastic scattering.\(^1\) In this paper both quite new and much older data are combined in a manner which further supports the scaling hypothesis.

Analysis of deep-inelastic electron-nucleus scattering is complicated by the presence of two-body knock-out processes. In addition to the principal channel of quasi-elastic (one-body knockout) scattering,\(^3\) these two-body processes include those introduced by meson exchange currents,\(^4\) short range correlations,\(^5\) and pion production.\(^6\)

In a nonrelativistic approximation the energy conservation relationship for deep-inelastic scattering may be written

\[
\frac{(\mathbf{p} + \mathbf{q})^2}{2m_N} = \frac{p^2}{2m_N} + \omega - \xi,
\]

where \(\mathbf{p}\) is the initial momentum of the nucleon in the target nucleus, \(\mathbf{q}\) is the momentum transfer, \(\omega\) is the electron energy loss and \(\xi\) is the average separation of the nucleons. The variable \(m_N\) is the nucleon mass. The scaling variable, \(y\), is chosen to be the initial velocity of the nucleon parallel to \(\mathbf{q}\) \((y = p_\parallel/m_N)\). Equation (1) leads to the following useful expression:

\[
y = \frac{(\omega - \xi)}{q} - \frac{q}{2m_N}.
\]

The strength of longitudinal single nucleon transitions, and transverse transitions, other than those due to convection currents, is independent of the value of \(p_\perp\), the component of \(\mathbf{p}\) perpendicular to \(\mathbf{q}\). The contribution of convection currents to the transverse part of the cross section is, fortunately, quite small.\(^1\) When convection currents are neglected, the measured quasi-elastic cross section can be written in terms of \(y\) and the kinematic variables as

\[
\frac{d^2\sigma}{d\Omega d\omega} = \alpha \sigma f(y) \frac{dy}{d\omega} F_\omega^2(q^2) K(\theta, \omega, \theta),
\]

where

\[
K(\theta, \omega, \theta) = \left[ Z \left( \frac{\mu_N}{q^2} \right)^2 \left( 1 - (2\mu_p - 1) \frac{q^2}{4m_N^2} \right) + \left( \frac{q^2}{2q^2} + \tan^2 \theta / 2 \right) \left( Z \mu_p^2 + N \mu_p^2 \right)^2 \frac{q^2}{2m_N^2} \right].
\]

The variables \(Z\) and \(A\) are the atomic number and nucleon number of the target nucleus; \(\mu_p\) and \(\mu_N\) are the proton and neutron magnetic moments, respectively. Terms of order \((q/m_N)^4\) in the Darwin-Foldy correction have been neglected for both neutrons and protons. The nucleon form factor is given by \(F_N\) and \(\alpha\) is the Mott cross section. The number distribution of the target nucleons expressed in \(y\) space is given by the function \(f(y)\) normalized so that

\[
\int f(y) dy = 1.
\]
If the scaling hypothesis fairly represents the elementary dependence of the one-nucleon knock-out part of the cross section, a direct estimate of the nucleon momentum distribution becomes feasible.

We have extracted \( f(y) \) from data obtained from \(^{40}\text{Ca}\) by various investigators at both Stanford (HEPL) and the MIT Bates Linear Accelerator Center. These data cover a broad range of kinematical conditions:

(a) \( E_0 = 500 \text{ MeV}, \theta = 60^\circ \) (Ref. 7),
(b) \( q = 250 \text{ MeV/c}, \theta = 60^\circ \) (Ref. 8),
(c) \( q = 500 \text{ MeV/c}, \theta = 120^\circ \) (Ref. 9),
(d) \( E_0 = 200 \text{ MeV}, \text{ and } 250 \text{ MeV}, \theta = 160^\circ \) (Ref. 10),
(e) \( E_0 = 300 \text{ MeV}, \theta = 160^\circ \) (Ref. 11).

It is believed that the radiative tail was over subtracted at large \( \omega \) in the data of Ref. 9; points at values of \( y > 0.06 \) were therefore excluded from this analysis.

In order to calculate \( f(y) \) the value of the average separation energy, \( \bar{E} \), must be estimated. For values of \( q \lesssim k_F \), the Fermi momentum, \( \bar{E} = 22 \text{ MeV} \) was chosen. At larger values of \( q \), \( \bar{E} \) was taken to be 33 MeV in good agreement with the results of Refs. 7, 9, and 10. Similarly, at low values of \( q \), we should use the nucleon effective mass, \( m^*_N = m_N/1.4 \), which should replace \( m_N \). At larger values \( (q \gtrsim k_F) \) we take \( m^*_N = m_N \), and, therefore, \( K(q, \omega, \theta) \) should be multiplied by \( m^*_N/m_N \), with the use of the appropriate value of \( m^*_N \) for the value of \( q \) being understood. Scaling is expected to set in only in the limit of very large \( q^2 \). The use of an effective mass at low \( q \) compensates for some of the nonscaling interactions and extends the region over which the data can be represented as a function of the variable, \( y \).

All of the data we have analyzed are displayed in Fig. 1. With the exception of a few points at values of \( y \lesssim 0.1 \), from the spectrum at \( q = 250 \text{ MeV/c}, \theta = 60^\circ \), all of the data cluster in a narrow band up to values of \( y \sim 0.2 \). An inspection of the original spectrum shows that the points which do not fall within the scaling band have energy losses placing them in the region of the giant resonance. It is not expected that collective states should follow this scaling rule. The largest energy losses, and hence values of \( y \), obtained in the \( q = 250 \text{ MeV/c} \) spectrum lie below pion threshold. Because meson exchange currents contribute primarily to the transverse response function, they should also not be significant in this spectrum, obtained at a forward angle, and points at larger values of \( y \) should scale.

All other spectra exhibit larger values of \( f(y) \) for \( y \gtrsim 0.2 \). Furthermore, these values of the scaling variable all correspond for these spectra to energy losses where strong contributions from exchange currents are expected\(^4,10\) or to energy losses which are well above pion threshold. We note that a broad distribution of points is found in this region of \( y \), and that the values of \( f(y) \) do not lie within a narrow band. It is possible that measurements of deep-inelastic scattering at \( q \leq k_F \) and \( \theta \sim 60^\circ \) might be useful to establish the one-body knockout scaling curve. Deviations from the scaling curve at large \( y \) and larger values of \( q \) and \( \theta \) could then reasonably be attributed to contributions from processes other than quasi-elastic scattering. It is also possible that the scaling curve obtained at large \( q \) and small \( y \) might be useful in the estimation of the quasi-elastic background to the giant resonance.

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1Y. Kawazoe, G. Takeda, and H. Matsuzaki, Prog. Theor. Phys. 54, 1394 (1975). We note that in the original Eq. (5) was misprinted; a correct version is given here.


11The points at $E_0 = 300$ MeV, $\theta = 160^\circ$ represent unpublished data obtained by P. D. Zimmerman, J. M. Finn, and C. F. Williamson as part of the program described in Ref. 10.