Atomic structures and magnetic behavior of Mn clusters

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The structures and magnetic properties of Mn clusters containing 13, 15, 19, and 23 atoms have been predicted from ab initio electronic structure calculations. The lowest energy structures of the 13-, 19-, and 23-atom clusters are icosahedral and have ordered spin configurations in which blocks of ferromagnetically coupled spins are antiferromagnetically coupled with each other. The 15-atom cluster is body-centered cubic with antiferromagnetic coupling. The net moments are small and lie in the range of 0.2–1.2 \( \mu_B \)/atom; however, the average local atomic moments have high values close to 4 \( \mu_B \). These results are discussed in light of recent Stern-Gerlach experiments.

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The study of the evolution of magnetic behavior from atoms to the bulk is important for the development of magnetic nanomaterials. The occurrence of noncrystallographic structures in small clusters could provide avenues for magnetic phenomena. While several studies have been done1 on clusters of ferromagnetic materials and the occurrence of magnetism in clusters of nonmagnetic elements, Mn offers a range of magnetic properties to excite and challenge theorists and experimentalists alike. The most stable bulk form is \( a\)-Mn, with a complicated geometric structure, a low magnetic moment of about 1 \( \mu_B \) per atom, and antiferromagnetic behavior that can be explained by spin waves.2 However, small clusters (Mn\(_3\)–Mn\(_8\)) are ferromagnetic, with high moments of 4–5 \( \mu_B \)/atom.3,4 Finally, the dimer is weakly bound, and while recent density functional calculations show ferromagnetic behavior3,5 there is still some question as to whether the coupling is actually ferromagnetic or antiferromagnetic.5

With increasing cluster size, the spin configuration should change from ferromagnetic for small clusters towards bulk behavior, but the intermediate structures are not yet understood. Elucidation of the magnetic behavior in Mn clusters of intermediate size could lead to species with magnetic properties. The 4\( s^2\)3\( d^5\) configuration of the Mn atom leads to weak bonding and high magnetic moments in small clusters. As the size increases, there should be a delocalization of both the \( s \) and \( d \) electrons, leading to metallization of the clusters as well as lowering of the magnetic moments.

A recent Stern-Gerlach experiment by Knickelbein6 on Mn\(_{11}\)–Mn\(_{99}\) suggested magnetic behavior that differs from either the bulk or the smaller clusters. Key results include relatively small magnetic moments of \( \sim 0.3–1.4 \mu_B \)/atom with notable local minima at Mn\(_{13}\) and Mn\(_{19}\) and maxima at Mn\(_{15}\) and Mn\(_{23}\)–Mn\(_{25}\). The clusters were assumed to be superparamagnetic with ferromagnetic coupling between spins. Icosahedral structures were suggested for the 13- and 19-atom clusters.

With the aim of understanding the above experimental results, we have performed first-principles calculations for clusters containing 13, 15, 19, and 23 atoms. Our results confirm the small net moments per atom observed in experiment as well as icosahedral structures for both Mn\(_{13}\) and Mn\(_{19}\). These small net moments arise out of antiferromagnetic or a combination of ferromagnetic and antiferromagnetic couplings between spins in the well-ordered ferrimagnetic clusters. The average local moments on each atom are high, however, with values of about 4 \( \mu_B \).

We used a planewave method employing ultrasoft pseudopotentials7 and the generalized gradient approximation8 for the exchange-correlation potential.3 The 3\( d \) and 4\( s \) orbitals were treated as valence states. Clusters were positioned in a cubic supercell with an edge of 15 Å, and periodic boundary conditions were imposed. The cutoff energy for the plane waves was set to 283.9 eV (\( \sim 21 \) Ry), and reciprocal space integrations were carried out at the \( \Gamma \) point. Structural optimizations were performed using quasi-Newton-Raphson and conjugate gradient methods. Structural optimizations were deemed sufficiently converged when the forces were about 1 meV/Å. This level of precision correctly reproduces9 the ferromagnetic state of the manganese dimer earlier obtained with a similar exchange-correlation functional.

Determination of the low lying spin configurations was a delicate task, as many spin isomers could lie close in energy. We used a variety of methods to find the lowest energy magnetic configurations, including freezing the net magnetic moment of the cluster and specifying the initial magnetic moments on each atom followed by unrestricted optimization.

While most initial configurations were ferrimagnetic, some were purely ferromagnetic. Both symmetric and asymmetric spin arrangements were considered. Noncollinear magnetic ordering, however, was not permitted. In all, about 50 geometric and spin combinations were calculated for each cluster size. The local magnetic moments were determined by integrating the spin density over Voronoi cells, which we refer to as “integrated spin densities” to avoid confusion with LCAO-type local moments. The thermally averaged net moment \( \langle \mu \rangle_T \) was calculated from

\[
\langle \mu \rangle_T = \frac{\sum_i \mu_i e^{-E_i/k_B T}}{\sum_i e^{-E_i/k_B T}},
\]

where \( \mu_i \) is the net moment for isomer \( i \), \( E_i \) the total energy, \( k_B \) the Boltzmann constant, and \( T \) the temperature.

In addition to the icosahedral structures for each cluster size, we also considered decahedral, cuboctahedral, and hexagonal close-packed structures for Mn\(_{13}\), hexagonal and bcc
for Mn\(_{15}\), and decahedral and cuboctahedral for Mn\(_{19}\). For Mn\(_{13}\), the decahedral and cuboctahedral structures transformed into icosahedral structures, while the hexagonal close packed structure was a local minimum. Similarly for Mn\(_{19}\), the decahedral structure transformed into a double icosahedron.\(^{10}\) Figure 1 shows the lowest energy geometry and spin configuration for each cluster size. The icosahedral structures were found to have the lowest energies, except for the case of Mn\(_{15}\), where the bcc was lower than the icosahedral by 0.28 eV. This icosahedral growth sequence for the 13-, 19-, and 23-atom clusters has also been seen in clusters of divalent metals such as Ba\(^{11}\) and Sr\(^{12}\), which exhibit nonmetal to metal transitions with increasing size. The calculated binding energy increases with increasing cluster size, and has values of 1.84, 1.89, 2.01, and 2.11 eV, for Mn\(_{13}\), Mn\(_{15}\), Mn\(_{19}\), and Mn\(_{23}\), respectively. The maximum value is about 70% of the bulk.

The average bond lengths were calculated by assuming a cutoff of 3 Å. Based on this criterion, the averages for the four cluster sizes vary from 2.62 to 2.65 Å. This is in line with average bond lengths of 2.62–2.73 Å for Mn\(_{3}\)-Mn\(_{8}\) calculated at the GGA level.\(^{3}\) For the icosahedral structures, the shortest bond lengths occur for the central atoms bonding between either themselves or with the top and bottom capping atoms (Fig. 1). The shortest bond length for Mn\(_{13}\) is 2.41 Å, and for icosahedral Mn\(_{15}\) it is 2.36 Å, both longer than the bulk value of 2.24 Å. For bcc Mn\(_{15}\), the shortest bonds (2.41 Å) occur between the central atom and its eight corner neighbors. For Mn\(_{19}\) and Mn\(_{23}\), they are respectively 2.25 and 2.29 Å, which is close to the shortest bulk value.

The lowest energy structure of each cluster is ferrimagnetic (Fig. 1). For Mn\(_{13}\), there is ferromagnetic coupling within each five-membered ring as well as between each ring and its nearest neighbor cap. There is, however, antiferromagnetic coupling between the rings. Additionally, the highly coordinated central atom is ferromagnetically coupled with one ring and antiferromagnetically coupled with the other. This leads to a net magnetic moment of only 3\(\mu_B\). We should note that the low moment is in disagreement with an earlier calculation,\(^{13}\) in which a moment of 33\(\mu_B\) was found. However, our binding energy is slightly higher, suggesting that our results show at least a deeper local minimum. Similar spin structures can be seen for Mn\(_{19}\) and Mn\(_{23}\), where the spins on the central atoms are antiferromagnetically coupled with those on the upper and lower atoms in the clusters. The ground state structures of these clusters have magnetic moments of 23 and 21\(\mu_B\), respectively. The lowest energy icosahedral isomer of Mn\(_{15}\) also has a spin configuration similar to Mn\(_{13}\). The spins on both of the face capping atoms couple ferromagnetically to one ring and antiferromagnetically to the other (Fig. 1). The net moment is 9\(\mu_B\). For the bcc isomer Mn\(_{15}\), which is the ground state structure, the spins on the central and face-centered atoms are antiferromagnetically coupled with those on the corners, resulting in a net moment of only 3\(\mu_B\).

Looking at the results for the icosahedral structures, it appears that ferromagnetic coupling within the five-membered rings is energetically favored, as is fivefold rotational symmetry for Mn\(_{13}\) and Mn\(_{19}\) and reflection symmetry for Mn\(_{19}\) and Mn\(_{23}\). Both high symmetry and ferromagnetic coupling within the five-membered rings cannot be obtained for Mn\(_{15}\), however. Therefore it seems that the more symmetric bcc structure, where nearest neighbor antiferromagnetic interactions result in ferromagnetic planes with antiferromagnetic coupling between, becomes more favorable.

Integrated spin densities are listed in Table I. For all the clusters, the highest coordinated atoms (the central atoms) have the lowest moments, while the surface atoms have the highest. Also, as expected from Pauli repulsion, we find that the ferromagnetically aligned large moments are associated with longer bond lengths than those that are antiferromagnetically aligned. In general the mean value of the integrated

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**FIG. 1.** (Color) Lowest energy structures for each cluster size. Icosahedral Mn\(_{15}\) is also shown for comparison. The relative spin alignments are marked, with dark blue indicating spin up and light pink spin down. The local moments, however, have differing values. See the text and Table I for details.
atomic spin densities decreases with increasing cluster size. The exception is bcc Mn$_{15}$, where the geometrical structure and spin configurations are quite different from the icosahedral clusters. The increase in the average spin density in this case is due to the lower coordination of atoms in the bcc structure. The trend of a decrease in the magnetic moment per atom with increasing cluster size is also in agreement with results from calculations on the smaller ferromagnetic clusters Mn$_2$-Mn$_8$ (Ref. 3) as well as the small moment of $\sim 1\mu_B$/atom in the bulk.

Examination of the total density of states (DOS) for the lowest energy structure of each system (Fig. 2) points to the relative stabilities of the different sized clusters. While none of the lowest energy clusters shows a minimum in the DOS at the Fermi energy, the curvatures are all positive. This indicates that each of the clusters is stable with respect to small perturbations in geometry and magnetic configuration. For the smaller clusters, the curvatures in the DOS are relatively large. The corresponding differences between the lowest and second lowest energy spin structures are 0.15 and 0.13 eV, respectively, for Mn$_{13}$ and Mn$_{15}$. Mn$_{19}$ and Mn$_{23}$ have smaller curvatures in their DOS, suggesting that less energy would be required for excitation to another spin state. This is seen in the small energy differences (0.06 and 0.05 eV, respectively) between spin isomers.

Projection of the local density of states (LDOS) into its $s$, $p$, and $d$ components produces interesting site-dependent results. The central, lower ring, and lower atoms in the lowest energy Mn$_{13}$ isomer are shown as an example in Fig. 3. We should note that the $s$ and $p$ components are significantly underrepresented due to expansion of the wave function into angular momentum components in an atomic sphere with a radius of 1.3 Å. However, the $d$-projected LDOS are of greatest interest here, and our method represents the gross trends properly. The $d$-projected LDOS for the lower ring and lower atoms are similar in that for each the peaks of the LDOS are rather localized and the majority spin is nearly fully occupied. The LDOS is quite low below the Fermi energy for the minority spin. This is in agreement with the

|       | $\mu_{\text{tot,theory}}$/atom | $\mu_{\text{tot,exp}}$/atom$^a$ | $|\mu|_{\text{min}}$ | $|\mu|_{\text{max}}$ | $|\mu|_{\text{ave}}$ |
|-------|-------------------------------|---------------------------------|-------------------|-------------------|-------------------|
| Mn$_{13}$ | 0.23                          | 0.56±0.13                       | 2.17              | 4.24              | 3.91              |
| Mn$_{15,\text{bcc}}$ | 0.20                          | 1.40±0.07                       | 3.22              | 4.37              | 4.11              |
| Mn$_{15,\text{ico}}$ | 0.60                          | 1.40±0.07                       | 0.95              | 4.51              | 3.81              |
| Mn$_{19}$  | 1.21                          | 0.43±0.10                       | 2.19              | 4.03              | 3.77              |
| Mn$_{23}$  | 0.91                          | 1.23±0.06                       | 0.04              | 4.35              | 3.42              |

$^a$Reference 6.

FIG. 2. (Color) Gaussian-broadened (half-width = 0.1 eV) densities of states for the lowest energy clusters. The spin up, spin down, and total density of states per atom are represented by the blue, green, and red lines, respectively. The gray vertical dashed lines indicate the Fermi energies.
relatively large moments on the lower ring and lower atoms (4.2 and 3.7\(\mu_B\), respectively). On the other hand, the d-projected LDOS for the central atom is rather broad for both the majority and minority spin states. The broadening occurs due to the high coordination of the central atom, and reflects the tendency towards metallic behavior. This tendency is also seen in the shorter bond lengths and smaller magnetic moment (2.17\(\mu_B\)). The beginning of metallic behavior is also seen from the small gaps between the highest occupied and lowest unoccupied molecular orbitals (HOMO-LUMO) gaps for all of the clusters, which range from 0.09 to 0.16 eV for the ground state structures.

The energies of the low lying isomers as a function of the magnetic moment relative to the ground state are shown in Fig. 4. Most of the lowest energy configurations are clustered near the minima, and there are several spin isomers lying close in energy. The general trend is towards lower net magnetic moments for Mn_{19} and Mn_{23}. More than one energy is shown for some moments, which is due to different spin configurations having the same total spin. This points to the delicacy of calculations in systems having antiferromagnetic coupling between spins on different atoms, where there is a drastic increase in the number of potential spin configurations for any net magnetic moment. It is found that more ordered spin arrangements are favored over the less ordered ones. Many of the less ordered icosahedral structures we explored have large ferromagnetic domains that are antiferromagnetically coupled to smaller domains. The interplay between domain size, degree of antiferromagnetic coupling, and symmetry appear to be maximized for the lowest energy structures. The second lowest energy structures for both Mn_{19} and Mn_{23} have the same basic spin arrangements as the ground state, but slightly different moments per atom. This demonstrates the stability of the ferromagnetic coupling within the rings and antiferromagnetic coupling between them.

FIG. 3. (Color) Gaussian-broadened (half-width = 0.1 eV) partial densities of states for three representative atoms in the lowest energy Mn_{13} cluster. The s, p, and d partial densities are represented by the blue, green, and red lines, respectively. The grey vertical dashed lines indicate the Fermi energy.
Interestingly, the second lowest energy configuration for Mn$_{15}$ is also bcc, but with slightly shorter bond lengths, lower moments on the central and corner atoms, and higher moments on the face-centered atoms. This illustrates the challenge in dealing with such complicated systems, in that it is difficult to absolutely guarantee that we have reached the true minima in our calculations. However, a large sampling as well as the occurrence of ordered spin configurations that are of lowest energy lend support to our results.

In spite of the small energy differences between spin isomers, the temperature dependence of the net magnetic moment is expected to be rather weak, and only the lowest energy isomer will dominantly contribute at the temperature of 68 K used by Knickelbein.\textsuperscript{6} A similar weak temperature dependence was obtained for Pd$_{13}$, which has a ferromagnetic ground state and much lower local moments.\textsuperscript{14} Several closely lying spin configurations were also found for icosahedral Pd$_{55}$ (Ref. 15) around the ground state, similar to what we have obtained for Mn$_{19}$ and Mn$_{23}$.

The net theoretical and experimental moments are listed in Table I. The theoretical values of the net moments vary from 0.20 to 1.21 $\mu_B$/atom, while the experimental values range from 0.43 to 1.40 $\mu_B$/atom; however, the latter have large uncertainties. Based on this comparison, the theoretical moments per atom for Mn$_{15}$ would then increase to 0.60 $\mu_B$, but this is still smaller than the experimental value of 1.40 $\mu_B$. There is a consistency in the theoretical spin configurations, however, in that the larger-sized icosahedral clusters naturally follow the pattern initiated with Mn$_{13}$. Another possible explanation for the discrepancy between theory and experiment is that noncollinear spin effects may be important, as has been shown for small chromium clusters.\textsuperscript{16} This should be considered in future calculations. Also, the assumption in the Stern-Gerlach experiment was for ferromagnetically aligned superparamagnetic clusters.\textsuperscript{6} Since this is not the case, we must consider if the experimental data should be reinterpreted for ferrimagnetic behavior.

In conclusion, we have studied the geometric and magnetic properties of Mn clusters with 13, 15, 19, and 23 atoms. Icosahedral structures are favored for Mn$_{13}$ and Mn$_{19}$, while the bcc structure is favored for Mn$_{15}$. The magnetic behavior of the clusters is ferrimagnetic, and the spin configurations are quite well ordered. The clusters were shown to exhibit magnetic ordering that is qualitatively different from that seen in the antiferromagnetic bulk and smaller ferromagnetic clusters in that there are well-defined ferromagnetic domains of a few atoms that are antiferromagnetically coupled together. Our results confirm the low net moments per atom seen in experiment. An interesting finding is the competition between structural and spin ordering, as seen in Mn$_{15}$. This could lead to different structures for some Mn clusters that may not follow icosahedral growth due to the lack of higher symmetry spin alignment. This aspect of magnetic clusters should be further explored.

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