Microscopic structure of a quantized vortex core in atomic Fermi gases

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Microscopic structure of a quantized vortex core in atomic Fermi gases

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I. INTRODUCTION

The observation of a quantized vortex is a key experimental issue to conclude the controversy about whether the novel quantum many-body system really shows superfluidity or not [1,2]. Very recently, the MIT group has succeeded in visualizing not only a singly quantized vortex but also a triangular vortex lattice in a two-component atomic Fermi gas [3]. The singly quantized vortex in a Fermi superfluid is characterized by several inherent spatial lengths [4,5]. The most distinct and well-known length among them is the coherence length, \( \xi_0 = v_F/\Delta_0 \), where \( v_F \) and \( \Delta_0 \) are the Fermi velocity and the Meissner superfluid gap, respectively, and this length scale is comparable to the vortex-core radius. The length \( \xi_0 \) also gives a spatial scale below which the macroscopic description of superfluidity such as the Ginzburg-Landau theory breaks down in Fermi superfluids. Accordingly, the vortex-core structure in the length scale less than \( \xi_0 \) reflects the microscopic structure of the fermionic system. In fact, information inside the vortex core, which can be obtained by a scanning tunneling microscope in superconductors, has played an important role in the studies of pairing mechanism and the electronic structure in high-\( T_c \) cuprate [6] and other superconductors [7]. Thus, we emphasize that from the observation of a vortex core in atomic Fermi gases definite information may be extracted on the microscopic feature of the BCS-BEC (Bose-Einstein Condensate) crossover. In addition, we point out that the atomic gas has a great advantage in the study of a vortex core since it is an almost idealistic clean system.

In contrast to superconductors the two-component atomic Fermi gases are remarkable for their tunability. It is well known that the interaction can be tuned by using a Feshbach resonance. As a result, the BCS superfluidity can be realized from the BEC of tightly bound molecules by tuning the interaction via the Feshbach resonance. The tunability makes it possible to study an interesting problem of how the structure of a vortex core, e.g., the core radius [8,9], the matter-density profile [10–14], the excitation spectra [12–14], and so on, depends on the strength of the pairing interaction [3]. Such a study about the systematics for the vortex-core structures will find, to the best of our knowledge, new frontiers of vortex physics that has a long history. In this paper, we systematically investigate the structure variation of the singly quantized vortex core by solving the Bogoliubov–de Gennes equation [4] in the fermion-boson (FB) model [15], which takes account directly of the conversion process from two atoms to a molecule and vice versa. We demonstrate that the core structure fully reflects the microscopic fermionic structure.

In the singly quantized superconducting vortex we have another well-known characteristic length \( \xi_1 = 1/\Delta_0 \lim_{r \to 0} \Delta(r)/r \) which is related to the slope of the gap function at the center of a vortex (see Fig. 1) [16]. In the low-temperature regime, \( \xi_1 \) shows a linear temperature dependence [the Kramer-Pesch (KP) effect [16]], i.e., \( \xi_1 \propto T \), which is sharply contrasted with the behavior of the coherence length \( \xi_0 \) being nearly independent of temperature. The KP effect originates from the Andreev bound states confined in the core region. Many experimental investigations have been performed to detect the KP effect in superconductors, but the experimental confirmation has not yet been complete [5,17]. This is because the impurity scattering, which is not avoidable in superconductors, easily smears out this effect [5,17]. This fact indicates a possibility that the KP effect may

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![FIG. 1. (Color online) Schematic view of the vortex-core structure in the fermionic superfluid gap function, \( \Delta^F \), and the molecular condensate function, \( \phi^B \). \( \xi_0 \) is the coherence length. \( g_i \) (i = 1, 2, and B) is a characteristic gradient for \( \Delta^F \) or \( \phi^B \) which is numerically investigated in this paper and \( \xi_i \) (i = 1, 2, and B) is the related length scale.](attachment:fig1.png)
be clearly observed in atomic fermion gases since these systems are essentially impurity-free. In this paper, motivated by such a historical view of vortex physics, we study how the gradient $g_1$=$\lim_{r\to 0} \Delta (r)/r$ [16], which dominates the behavior of the length scale $\xi_1$, changes at the BCS-BEC crossover and clarify that $g_1$ is dependent on microscopic parameters such as the Fermi velocity. In addition, we consider another characteristic gradient $g_2$ in the fermionic gap function, which is separately defined outside the region in which $g_1$ shows a linear slope (see Fig. 1) function. Our calculations show that the gradients $g_1$ and $g_2$ are well defined and reveal their characteristic behaviors, depending on the interaction strength. In addition, we investigate the spatial gradient in the molecular condensate $g_B$ (see Fig. 1). We demonstrate that these three gradients clearly reflect the essential feature of the superfluid fermion atoms exhibiting the BCS-BEC crossover. Furthermore, the profile of the matter-density depletion [10–14] and the fermionic excitation spectrum [12-14] are revisited in the broad resonance case, which are observable in the atomic-density profile imaging [3] and the rf-tunneling spectrum, respectively [18]. These quantities also show the distinct difference between BCS and BEC regimes.

II. FORMULATION AND NUMERICAL METHOD

From the Hamiltonian of the FB model [15], we have the generalized Bogoliubov–de Gennes equation as

$$
\begin{align*}
H_\sigma & = \Delta^\sigma (r) + \phi^\sigma_0 (r) \\
\Delta^\sigma (r) + \phi^\sigma_0 (r) & = E_\sigma (n_{\sigma \sigma}) = E_{\sigma} (n_{\sigma \sigma}), \\
-\frac{1}{4m} \nabla^2 + \nu \phi^\sigma_0 (r) & = \frac{g^2}{U} \Delta^\sigma (r),
\end{align*}
$$

(1)

where $H_\sigma = -1/2m \nabla^2 - \mu_\sigma, \Delta^\sigma$, and $\phi^\sigma_0$ are the mean-field vacuum expectation values for the fermionic gap and the molecular condensate functions, respectively, and $\mu$ and $\nu$ are the chemical potential and the Feshbach resonance threshold energy, respectively. These equations are solved self-consistently together with the fermionic gap equation, $\Delta^\sigma (r) = U \Sigma_{\alpha \beta} (r) u^*_{\alpha \sigma} (r)$ and the chemical potential given by the constraint for the total number of particles, $N = N_F + 2N_B$ =const, where $N_F = \int \sigma^2 |\psi (r)|^2$ and $N_B = \int \sigma^2 |\phi (r)|^2$.

In this paper, we focus on an isolated vortex solution in a two-dimensional (2D) s-wave case [19] without the trapping potential for simplicity. The calculation method using the cylindrical coordinates is described in Ref. [19] and its extension to the FB model is seen in Refs. [12,20]. In the 2D cylindrical coordinates $r= (r, \theta)$, the gap functions are expressed as $\Delta^\sigma (r) = \Delta^\sigma (r)e^{-i \theta}$ and $\phi^\sigma_0 (r) = \phi^\sigma_0 (r)e^{-i \theta}$ and the radial components of the eigenfunctions $u_{\sigma \sigma} (r)$ and $u_{\sigma \sigma} (r)$ can be expanded by the function $\phi_{\eta \eta} (r) = [\sqrt{2}/R J_{\eta}(\alpha_\eta)] f_{\eta}(\alpha_\eta r/R), \; |\eta| = 1, 2, 3, 2/3, 5/2, ...$, [19]. Thus, the BdG equation can be solved as an eigenvalue problem for $2M(\eta) \times 2M(\eta)$ matrices whose dimension $M$ is dependent on $\eta$ [19], and the radial dependence of all the functions are self-consistently determined. In the following calculations, the energy and the radial distance are normalized, respectively, in units of $E_F$ and $1/k_F$. In this paper, the value of $g$ is fixed to 5, which is in the region of the broad Feshbach resonance ($g \gg 1$), differing from those in the previous work that lead to only the narrow resonance ($g < 1$) [12,20]. The broad resonance covers the systems experimentally available at present, but much more computational resources than in the narrow resonance case are required to solve the BdG equation in this region, since the range of energy to be calculated is wider. To our knowledge, this paper is the first in which the self-consistent calculation for the single-vortex state has been successfully performed for such a large value of $g$. For the coupling constant $U$ and the cutoff radius $R$ we choose the values $U=0.5$ and $R=40/k_F$.

III. NUMERICAL RESULTS AND INTERPRETATION

A. Radial profile of fermionic superfluid gaps

Let us present numerical solutions for the single-vortex state. In Fig. 2(a), we plot the chemical potential $\mu$ as a function of $\nu$. In the inset, $1/k_F a$ vs $\nu$ is also shown, where $a$ is the scattering length. To obtain $1/k_F a$, we substitute the cutoff energy $g_{\nu} = 5E_F$ and the effective pair coupling constant $g_{\nu} = U + g^2/2\nu - 2 \mu$ into Eq. (2) in Ref. [21]. The behavior shown in this figure is essentially the same as that in both trapped and nontrapped uniform systems (nonvortex states) as seen in Ref. [22]. Figures 2(b) and 2(c) represent the profiles of the fermionic superfluid gap functions $\Delta^\sigma$ along the radial direction, respectively, in the BCS ($1/k_F a < 0$) and the BEC ($1/k_F a > 0$) sides. The profiles show a clear difference in these two sides. In the BCS side the gradient $g_1$ is almost independent of $\nu$. This tendency is more clearly seen in the enlarged view as shown in the inset of Fig. 2(b). On the other hand, in the BEC side, $g_1$ strongly depends on $\nu$ as seen in Fig. 2(c). Here, we note $g_1 \sim 1/k_F$ in the BCS region and $g_1 \sim \nu - \mu$ in the BEC region. The former relation was suggested in the superconducting vortex [23] and the Fermi-gas vortex [13] in the quantum limit. From these relations one understands that the Fermi edge of the fermionic atomic gas is nearly invariant in the BCS regime, whereas it is strongly dependent on $\nu$ in the BEC regime [see also the sharp $\nu$ variation of $\mu$ in Fig. 1(a)]. In other words, from the observation of $g_1$, one may detect a signal of the crossover from fermionic to bosonic superfluid states. It is also noted that the linear slope of $g_1$ is restricted in the region of $0 < r < 1/k_F$ [See Fig. 2(b)]. This is because $g_1$ is determined only by the Andreev localized-bound states as shown again in Ref. [13,23].

Let us next consider the gradient $g_2$ (see Fig. 1). As seen in Fig. 2(b), the gap function $\Delta^\sigma$ in $r > 1/k_F$ is dependent on $\nu$ even in the BCS side. The value of $g_2$ increases with decreasing $\nu$, i.e., increasing the pairing strength in the BCS side ($1/k_F a < 0$). This behavior is sharply contrasted to $g_1$. On the other hand, $g_2$ decreases with decreasing $\nu$, i.e., increasing the pairing strength in the BEC side contrary to the BCS side. Thus, one understands that $g_2$ is correlated with the fermionic superfluid gap $\Delta^\sigma$ whose Meissner absolute
value shows the maximum at the crossover. Here, we also notice that the coherence length $\xi_0$ has the approximate relation as $\xi_0 \sim \xi_1 + \xi_2$ [24]. This is a remarkable feature that our calculation predicts in the fermionic superfluid vortex. From these observations one concludes that $g_1$ and $g_2$ behave as $g_1 \sim 1/kF$ (BCS) and $\sim v - \mu$ (BEC) and $g_2 \sim \Delta^F$ (BCS and BEC).

B. Radial profile of a molecular condensate

Figures 3(a) and 3(b) show the self-consistent solutions for the molecular condensate function $\rho^B(r)$ in the BCS ($1/k_Fa < 0$) and the BEC ($1/k_Fa > 0$) sides, respectively. As seen in these figures, the amplitude of the molecular condensate increases with decreasing $v$, i.e., increasing the pairing strength in both regions. However, the gradient at
the center of the core, i.e., \( g_R \) (see Fig. 1), shows clear different behaviors between these two regions, namely it increases only when approaching the crossover from the BCS region, while \( g_R \) is constant for the BEC region. This behavior comes from the fact that \( \phi^B \sim 1/(\nu - \mu) \Delta^B \) inside the core from Eq. (5) and \( \Delta^B \sim \nu - \mu \) in the BEC region as discussed above in terms of \( g_1 \). This prediction can be checked by observing the diatom density profile inside the core in the BEC side. It is also noted that the region in which \( \phi^B \) is depressed expands with approaching the BEC limit. This result is equivalent with previous work [8,9]. From Figs. 2 and 3, we find that there are three length scales that characterize the vortex core and reflect different microscopic information.

C. Radial profile of fermionic matter-density

Figures 4(a) and 4(b) show the radial variation of the fermionic matter density \( n_F(r) \) in the BCS \((1/k_Fa < 0)\) [10–12] and the BEC sides \((1/k_Fa > 0)\) [9], respectively. As seen in Fig. 4(a), \( n_F(r) \) is nearly independent of \( \nu \) in the BCS side, while it rapidly decreases with decreasing \( \nu \) in the BEC side [9]. These features clearly reflect the behavior of the fermionic structure discussed above in terms of \( g_1 \), i.e., the stability of the Fermi edge in the BCS region and the collapse of the fermionic structure in the BEC one. Since \( n_F(r) \) is experimentally observable, the change in the vortex-core structure may be definitely detected by means of the density profile imaging [3]. It is also noted that \( n_F \) is finite at the center of the vortex [10–12], though the diatom \( \phi^B \) vanishes at \( r=0 \) as pointed out in the previous work. This result comes from the fact that the fermionic matter density also includes a large contribution from the noncondensed fermions due to the Fermi statistics. We would like to emphasize that elaborate studies of the matter-density profile will provide useful microscopic information about atomic Fermi gases [3]. This is in marked contrast to superconductors, in which the change in the matter-density profile is too difficult to detect.

D. Quasiparticle excitation spectrum

Figure 5 shows typical quasiparticle excitation spectra \( (E_\nu, \eta) \) in the BCS (a) and the BEC (b) sides, respectively. As seen in Figs. 5(a) and 5(b), one notices that the excitation spectra in the BCS and BEC regions show remarkable difference in their dispersion of the first excitation level. In the BCS region, it is clearly seen that a uniform gap opens in the range of angular momentum from \( \eta = -50 \) to \( \eta = 50 \). The lowest one-fermion excitation energy in this region is roughly given by \( \nu = \sqrt{(\epsilon_{\text{kin}} - \mu)^2 + \Delta^B^2} \). This flat gap structure creates the BCS coherence peak just above the gap energy in the density of states [14,22]. In the BEC region, as seen in Fig. 5(b), the uniform gap structure vanishes and the gap energy increases parabolically with \( \eta \). As a result, the coherence peak in the density of states is expected to diminish in the BEC side [14,22]. This result indicates, as expected, that the BCS energy gap shifts to the dissociation energy of a diatomic molecule composed of two fermion atoms in the BEC limit. The difference in the excitation energy spectra shown above may be easily observed by the rf-tunneling spectroscopy [18,22].

Finally, we mention that the Andreev bound states located inside the energy gap, which give the well-known low-energy excitations in the vortex state of fermionic superfluids and are clearly seen in Fig. 5(a), still remain even in the BEC limit as seen in Fig. 5(b). This result is consistent with the result in Ref. [13] in which the so-called single channel model is employed. This indicates that both models still maintain the fermionic character even in the BEC region.

IV. SUMMARY AND CONCLUSION

In summary, we systematically studied the difference in the core structure of a singly quantized vortex in the BCS and BEC regions, using the fermion-boson model. Our numerical calculations revealed that the core structure, i.e., radial variation of the gap functions, which can be characterized by three length scales, \( g_1, g_2, \) and \( g_B \), carry different microscopic information, respectively. Furthermore, we clarified the profile of the matter-density depletion and the character of the quasiparticle excitation spectrum in both the
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