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Surprisal analysis of inclusive reactions

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Surprisal analysis is applied to inclusive nuclear reactions of \((e,e')\), \((\alpha,\alpha')\), and \((\text{Li},\text{Li}')\). By introducing constraints on the excitation energy and its square root, excellent fits to the experimental data are obtained. It is suggested that a substantial part of the so-called quasielastic (quasifree) scattering cross sections is due to more complex processes.

I. INTRODUCTION

The concept of surprisal was successfully introduced to nuclear physics by Levine et al.\(^1\) for the analysis of heavy ion transfer reactions to the continuum, where very complex excitations are expected and the maximum entropy process will be realized. In their analysis the most important constraint is the average excitation energy \(\langle E_x \rangle\). The linear surprisal is defined as

\[
S_1 = -\ln(\sigma/\sigma_0) = \lambda_0 + \lambda_1 E_x ,
\]

where \(\sigma\) is the measured cross section, \(\sigma_0\) is the \(a\) \(\text{priori}\) level density or the prior distribution, \(\lambda_1\) is a Lagrange multiplier for the constraint, and \(\lambda_0\) insures proper normalization. For the prior distribution \(\sigma_0\), the conventional Fermi gas model level density is used:

\[
\sigma_0 = E_x^{-2} \exp[2(\alpha E_x)^{1/2}] ,
\]

where \(\alpha\) is the level density parameter, which is usually chosen to be \(A/8\) \(\text{MeV}^{-1}\) for heavy nuclei. The kinematical constraint that the average excitation energy is close to the most probable energy \(E_x^{\text{mp}}\) leads to the equation

\[
\lambda_1^{\exp} = (\alpha / E_x^{\text{mp}})^{1/2} - 2(\exp E_x^{\text{mp}})^{-1} .
\]

Excellent agreement was obtained between the values of \(\lambda_1\) and \(\lambda_1^{\exp}\) for the reaction \(^{232}\text{Th}(^{16}\text{O},x)\) with the channels \(x = ^{16}\text{N}, ^{15}\text{N}, ^{14}\text{C}, ^{12}\text{C},\) and \(^{11}\text{B}\). Remarkable linearity was also obtained for the surprisal plot versus excitation energy and they conclude that the reaction proceeds predominantly through an assumed process. However, for the cases of transfers of a very small number of nucleons, i.e., \(x = ^{16}\text{N}\) and \(^{15}\text{N}\), linearity of the surprisal plot is not so good and they varied the state density parameter \(a\) to get better fits over the measured excitation energy range. In the next paper\(^2\) they showed that the change of the state density parameter can be explained by introducing the average exciton number \(\langle n_x \rangle\) as the second constraint.

In this paper we apply their method—surprisal analysis—to inclusive nuclear reactions of \((e,e')\), \((\alpha,\alpha')\), and \((\text{Li},\text{Li}')\). For inclusive electron scattering, the so-called “quasielastic scattering cross section” is shown to be relevant to more complex excitations than single particle knockout. For the considered cases of hadron scatterings, it is shown that alpha particles can excite complex states in the target nuclei, but Li projectiles can only excite simple 1p-1h states.

II. INCLUSIVE ELECTRON SCATTERING

Quasielastic electron scattering data have been obtained for many kinds of nuclei under various conditions.\(^3\) Usually these data are analyzed by the Fermi gas model to deduce the Fermi momentum of nuclei. Shell-model calculations have also been applied.\(^4\) Successful results of these models have been believed to reflect the single particle nature of inclusive quasielastic electron scattering. Recently, however, Horikawa et al.\(^5\) showed by us-
ing the Green's function method that about half of the inclusive quasielastic electron scattering cross section is due to more complex excitations in target nuclei. This theoretical result is consistent with the exclusive \((e,e'p)\) experimental data by Mougey et al., where deduced occupation probabilities of the single particle states are considerably less than one, e.g., for \(^{40}\text{Ca}\) around 0.7.

Below we analyze quasielastic electron scattering data by the method of surprisal. We choose \(\langle E_x \rangle\) as the first constraint and search the Lagrange multiplier \(\lambda_1\) in Eq. (1). For the prior \(\sigma_0\) we take into account the situation that one particle is in the continuum and we express it as

\[
\sigma_0 = \int_0^{E_x - E_0} \rho_{\text{cont}}(E_0) \rho_{A-1}(E_x - E_0) dE_0,
\]

where \(E_0\) is the threshold energy for the particle emission. \(\rho_{A-1}\) is the state density of the \(A-1\) nucleons system, which is assumed to be given by the exciton model

\[
\rho_{A-1}(E_x) = \sum_{n_p, n_h} \frac{g(gE_x - A_{ph})^{n_{-1}}}{n_p! n_h!(n_x - 1)!},
\]

where \(n_x = n_p + n_h\) and \(g = (\pi^2/6)a\). Introducing \(A_{ph} = n_p(n_p - 1) + n_h(n_h - 1)\) in Eq. (5) we can numerically approximate the state density of the equidistance model.\(^7\) \(\rho_{\text{cont}}\) is the state density for the one particle in the continuum,

\[
\rho_{\text{cont}}(E_0) = \sqrt{E_0}.
\]

We have analyzed several experimental data under different experimental conditions. For example, in Fig. 1 we show the obtained linear surprisal by the dashed line for the typical experimental data of 500 MeV electron scattering at 60° from \(^{181}\text{Ta}\) by Whitney et al.\(^5\) Seven experimental data points are used for the analysis and they are indicated by the closed circles. \(\lambda_1 = 0.39\) MeV\(^{-1}\) is obtained by fitting to the experimental data. Resultant parameter values of \(\lambda_1\) do not depend much on the chosen data points. In the data fitting procedure we have used cross section values as the weighting factor.\(^1\) Owing to this weighting factor, linear surprisal fits better to the data points around the quasielastic peak than in other parts. The value of the parameter \(\lambda_1\) is related to the quasielastic peak energy through Eq. (3). Using the experimental value of \(E_{\text{exp}}^{\text{pe}} \approx 150\) MeV, we get \(\lambda_1^{\text{exp}} = 0.38\) MeV\(^{-1}\). The agreement between \(\lambda_1\) and \(\lambda_1^{\text{exp}}\) is satisfactory. However, we see in Fig. 1 a significant discrepancy between the experimental data points and the linear surprisal far above and below the quasielastic peak. This discrepancy indicates that quasielastic electron scattering cannot be fully explained by the single constraint maximal entropy process predominated by the excitation energy. Therefore, surprisal with two constraints,

\[
S_2 = \lambda_0 + \lambda_1 E_x + \lambda_2 E_x^{1/2},
\]

is introduced, where \(\lambda_2\) is the second Lagrange multiplier.\(^2\) The best fit curve to the experimental data by Eq. (7) is shown in Fig. 1 by the solid line. Fitting to the data is performed by varying three parameters \(\lambda_0\), \(\lambda_1\), and \(\lambda_2\). The obtained value of \(\lambda_1\) is different from the one of the linear case. Using Eq. (2) for the theoretical level density, \(a_{\text{th}}\), the following \(a_{\exp}\) may be explained as an observed level density parameter concerning the measured process,

\[
a_{\exp} = (a_{\text{th}}^{1/2} - 1/2)(\lambda_2)\).
\]

If we use this value of \(a_{\exp}\) in Eq. (3) instead of \(a_{\text{th}}\), the obtained value of \(\lambda_1\) coincides with \(\lambda_1^{\text{exp}}\) again. The parameter \(\lambda_2\) is related to the expected exciton number excited by quasielastic electron scattering. It is well known that the distribution function of \(n_x\) has a sharp peak at \(\approx (gE_x/2)^{1/2}\).\(^7\) The estimated average exciton number at the quasielastic peak is \(\langle n_x \rangle_{QE} \approx (g_{\exp} E_x/2)^{1/2} \approx 4\), where \(g_{\exp} = \pi^2 a_{\exp}/6\). For the other cases in Ref. 3 we also got \(\langle n_x \rangle_{QE} \approx 4\) independent of target nuclei. In this way we conclude that a considerable amount of the quasielastic electron scattering cross section is due to more complex excitations than the single particle knockout process, consistent with

\[\text{FIG. 1. Surprisal for the }^{181}\text{Ta}(e,e') \text{ cross section at } E_x = 500 \text{ MeV and } \theta_e = 60° \text{ is plotted as a function of the excitation energy. The dashed line is the two parameter fit and the solid line is the three parameter fit.}\]
III. INCLUSIVE HADRON SCATTERING

Inclusive spectra of hadron scattering have also been measured for a variety of nuclei. The main aim there has been to deduce the giant quadrupole resonance cross sections. Only very recently so-called backgrounds to the giant resonances have come to attract much interest. Firstly Wu showed that these spectra, especially in the low excitation energy region—around giant resonances—can be explained mainly by the quasifree scattering process for the $^{12}$C$(p,p')$ reaction at $E_p=62$ MeV. High excitation energy parts are due to more complex excitations and preequilibrium emission processes of $[\geq (3p,2h)]$ contribute much.

Recently Godioli and Erba have shown that inclusive alpha scattering spectra in the continuum can be interpreted in terms of an incoherent sum of alpha emissions after single and multiple alpha-nucleon collisions. They have analyzed the inclusive alpha scattering data of $^{93}$Nb at $E_a=55$ MeV, $^{197}$Au at $E_a=65$ and 90 MeV, and $^{232}$Th at $E_a=130$ MeV. Their results show that in the giant resonance energy region alpha-nucleon scattering takes place almost only once, and in the higher excitation energy region more complex processes occur—for the considered cases alpha-nucleon scatterings take place up to five times.

In the following we analyze inclusive hadron scattering data by the same method applied to $(e,e')$ in the previous section. Figure 3 shows the surprisal plot for the $^{28}$Si$(a,a')$ cross section recently measured at $E_a=104$ MeV by Nitsche.
et al.\textsuperscript{10} Six data points are taken for each 5 MeV step. Again we get a clear agreement between the value of \(\lambda_1=0.27\,\text{MeV}^{-1}\) and \(\lambda_2^\text{pp}=0.26\,\text{MeV}^{-1}\) obtained by Eq. (3) with \(E_x^\text{pp}=40\,\text{MeV}\). For the two constraints case we have \(\lambda_2=-1.4\,\text{MeV}^{-1/2}\), which corresponds to the average exciton number of \(\langle n_x \rangle_QF\approx3\). This result agrees fairly well with the estimation by Gadioli and Erba.\textsuperscript{9} Nitsche et al.\textsuperscript{10} have also done the \(^{28}\text{Si}(\text{Li},\text{Li}')\) experiment under the condition of the same momentum transfer. For the inclusive Li scattering case \(\lambda_1=0.31\,\text{MeV}^{-1}\) (linear fit) and \(\lambda_2=-1.4\,\text{MeV}^{-1/2}\) are obtained. These values correspond to the most probable energy \(E_x^\text{mp}=27\,\text{MeV}\) and the average exciton number \(\langle n_x \rangle_QF\approx2\). A lower value of \(\langle n_x \rangle\) in Li scattering than alpha scattering obtained here supports the conclusion of Ref. 10, where they have shown that the inclusive spectrum of Li scattering can be explained by the sum of single particle knockout processes. For Li scattering the measured excitation function decreases very rapidly at the high excitation energy region compared to the alpha scattering. This tendency, which is also seen in heavy ion scattering data, may be because Li or heavy ions cannot penetrate deeply into the nucleus without losing their identity and so cannot excite most of the complex states which alpha particles can excite.

IV. CONCLUSION

We draw the following conclusions by comparing the results for \((e,e')\), \((\alpha,\alpha')\), and \((\text{Li},\text{Li}')\). High energy electrons excite more complex states than 1p-1h states in the so-called quasielastic electron scattering. The number of excitons at the same excitation energy, however, is estimated to be less than the case of inelastic alpha scattering at \(E_\alpha=100\,\text{MeV}\). This is simply explained by the fact that alpha particles with these energies in-

\begin{figure}[h]
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\includegraphics[width=0.5\textwidth]{si_plot.png}
\caption{Surprisal plot for the \(^{28}\text{Si}(\alpha,\alpha')\) cross section at \(E_\alpha=104\,\text{MeV}\) and \(\theta_\alpha=15\degree\). The meaning of the lines is the same as in Fig. 1.}
\end{figure}

teract very strongly with the target nuclei, and the average number of excitons at the same excitation energy is larger than the case of quasielastic electron scattering. In the cases of Li scattering and heavy ion scattering, projectiles cannot penetrate deeply into the nucleus. They are easily broken at nuclear surfaces. The inclusive cross sections are thus dominated by rather simple excitations in the target nuclei.

The excellent success of the present surprisal analysis with two constraints strongly suggests that complex processes other than the simple knockout process play a substantial role in the so-called quasielastic (quasifree) scattering.

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\begin{itemize}
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