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Josephson plasma resonance in the Josephson vortex lattice in intrinsic Josephson junctions

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Small phase oscillation modes of a Josephson vortex lattice in a stack of intrinsic Josephson junctions are numerically calculated. The eigenfrequency of the lowest excitation mode \( \omega_1 \) is less than the zero-field plasma frequency \( \omega_{pl} \) and is a decreasing function of the magnetic field \( H \). The eigenfrequencies \( \omega_n \) with \( n \geq 2 \) is located above \( \omega_{pl} \) and increases with increasing \( H \). This result is consistent with the field dependence of the Josephson plasma resonance observed in Bi-2212. We also clarify the nature of the phase oscillation modes, calculating the eigenfunctions.

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I. INTRODUCTION

In Josephson-coupled layered superconductors such as Bi-2212 the Josephson plasma resonance (JPR) has been observed in microwave absorption experiments.\(^1\),\(^2\) It has been recognized that the microwave absorption experiments under an external magnetic field provide a powerful tool for studying the vortex state in the high-\( T \) superconductors. In the presence of a magnetic field perpendicular to the layers pancake vortices enter the superconducting layers. When an array of pancake vortices along the \( c \) direction is distorted by thermal fluctuations, pinning potential, etc., the phase coherence among the superconducting layers is disturbed and, as a result, the Josephson critical current density \( j_c \) is reduced.\(^3\),\(^4\)

Since the Josephson plasma frequency \( \tilde{\omega}_{pl} \) is directly related to the Josephson critical current density \( j_c \), the variation of the resonant frequencies seems complex, Kakeya et al.\(^5\) clarified that the resonant frequencies scaled as \( \omega_{pl}(T,H)/\omega_{pl}(T,0) \) form two smooth lines as shown in Fig. 1 and these lines may be interpreted as the dispersion relations of the JPR due to the magnetic field.\(^7\) As seen in this figure, the lower branch of the resonant frequencies is located below the zero-field Josephson plasma frequency and is a decreasing function of the magnetic field. On the other hand, the resonant frequencies forming the higher branch increases with increasing the magnetic field.

An external magnetic field parallel to the layers creates Josephson vortices in the insulating layers located between the superconducting layers. The JPR under an in-plane magnetic field is expected to take place when the electric field of a microwave resonantly excites a phase oscillation mode in the Josephson vortex state. Since an electric field perpendicular to the layers moves the Josephson vortices in the in-plane direction, the phase oscillation in the Josephson vortex state couples also with the vortex-vibration modes.\(^9\) Fetter and Stephen clarified the small phase oscillation modes in a 1D periodic array of Josephson vortices in a single Josephson junction system.\(^10\) A gapless sliding mode of the vortex lattice appears in the frequency region below the zero-field Josephson plasma frequency \( \omega_{pl} \) in the vortex lattice state. However, this sliding mode does not couple with an applied microwave. Then, the phase oscillation mode excited by an applied microwave is the one having a large wave number which is folded inside the first Brillouin zone and is located at \( \Gamma \) point (umklapp-process mode). The frequencies of such phase oscillation modes, which are located above \( \omega_{pl} \) in the strong magnetic field region, are inversely proportional to the lattice constant of the vortex lattice \( a \). Hence, one understands that the JPR in a single Josephson junction system should be observed in the frequency region above \( \omega_{pl} \) and its resonant frequency is an increasing function of the external magnetic field.\(^11\)

The phase dynamics of a Josephson vortex lattice in a stack of intrinsic Josephson junctions was first investigated by Bulaevskii et al.\(^11\),\(^12\) They claimed, on the basis of an asymptotic analysis in the region of \( a/\lambda_J \ll 1 \), \( \lambda_J \) being the Josephson length, that the JPR in a periodic Josephson vortex lattice is similar to that in a single Josephson-junction system, that is, the lowest resonant frequency is larger than

![FIG. 1. Magnetic field dependence of the resonant frequencies observed in Bi-2212 (Ref. 7).](image-url)
\[ \omega_{pl} \] is proportional to \( 2 \pi a^{-1} H \) in the strong field limit. Hence, the JPR at a frequency lower than \( \omega_{pl} \) observed in Bi-2212 has not been derived in their calculations.

In this paper we calculate numerically the small phase oscillation modes of a Josephson vortex lattice in the moderate field region of \( a/\lambda_j > 1 \) to elucidate the nature of the JPR lying below the zero-field plasma frequency, using a model of inductively and also capacitively coupled intrinsic Josephson junctions. The intrinsic Josephson junctions have two spatial scales, i.e., the Josephson length \( \lambda_j \) and the \( c \)-axis penetration depth \( \lambda_c \). The Josephson length is dominated by the inductive coupling between the in-plane currents and gives a measure of the size of Josephson vortices. On the other hand, the \( c \)-axis penetration depth is related to the Josephson coupling between superconducting layers and \( \lambda_c/a \) yields the time scale of the Josephson plasma. The phase oscillation modes in the Josephson vortex lattice depend on both the spatial scales. As is well known, the \( c \)-axis penetration depth is much longer than the Josephson length, i.e., \( \lambda_c/\lambda_j = \mathcal{O}(10^{-3}) \gg 1 \), in the high-\( T_c \) superconductors. This fact makes it difficult to perform accurate numerical calculations for the phase oscillation modes. To overcome this difficulty we derive carefully the dynamical equation suitable for numerical calculations for the oscillatory parts of the gauge-invariant phase differences. Solving numerically this equation, we obtain the eigenfrequencies and the eigenfunctions as a function of the magnetic field. Our numerical results for the phase oscillation modes well reproduce the experimentally observed field dependence of the resonant frequencies. In this paper, for simplicity, we restrict ourselves to the case where Josephson vortices enter all the junctions and form a periodic triangular lattice in the region of \( a/\lambda_j \gg 1 \).

II. FORMULATION

First we summarize the derivation of the dynamical equations describing small phase oscillations under an in-plane magnetic field in a stack of intrinsic Josephson junctions. In the following calculations we adopt the coordinate system in which the \( x \) and \( y \) axes are in the in-plane direction and the \( c \) axis is perpendicular to the junctions. Suppose that an external magnetic field is applied in the \( y \) axis. Then, the system is assumed to be uniform in the \( y \) direction. The Josephson effects in this system can be described in terms of the dynamics of the gauge-invariant phase difference defined on each junction site. We denote the gauge-invariant phase difference at \( x \) on \( l \)th junction at time \( t \) by \( \Theta_l(x,t) \). It has been established that the capacitive and inductive couplings between junctions dominate the dynamics of the phase differences in the intrinsic Josephson junction systems. In the presence of these interactions the first and the second Josephson relations are generalized as

\[
\frac{\hbar c}{e*} \frac{\partial}{\partial x} \Theta_l(x,t) = \Phi_l(x,t) - \eta \Phi_{l+1}(x,t) - 2 \Phi_l(x,t) + \Phi_{l-1}(x,t),
\]  

where \( \Phi_l(x,t) \) and \( V_l(x,t) \) denote, respectively, the line density of the magnetic flux and the voltage difference at \( x,t \) on \( l \)th junction.\(^{13-18}\) The dimensionless parameters \( \eta \) and \( \alpha \) in Eqs. (1) and (2) are, respectively, the inductive and capacitive coupling constants, which are related to the in-plane London penetration depth \( \lambda_{ab} \) and the charge screening length of the superconducting layers \( \mu \) as follows:

\[
\eta = \frac{\lambda_{ab}^2}{s d}, \quad \alpha = \frac{\varepsilon \mu^2}{s d},
\]

where \( \varepsilon \) is the dielectric constant of the insulating layers and \( s \) (\( d \)) is the thickness of the superconducting (insulating) layers. In high-\( T_c \) cuprates \( \alpha \) is estimated to be \( \mathcal{O}(1) \,^{16-18} \). On the other hand, \( \eta \) takes a value of \( \mathcal{O}(10^3) \), that is, the inductive coupling is very large in the intrinsic Josephson junction systems. However, one notices that the inductive coupling term, i.e., the second term on the right-hand side of Eq. (1) vanishes for the in-phase configuration, i.e., \( \Phi_{l+1} = \Phi_l = \Phi_{l-1} \), that is, the first term on the right-hand side of Eq. (1) cannot be neglected, though \( \eta \) is very large. To complete the dynamical equations for \( \Theta_l(x,t) \), \( \Phi_l(x,t) \), and \( V_l(x,t) \) one can use the Maxwell equation in addition to Eqs. (1) and (2),

\[
\frac{\partial}{\partial x} \Phi_l(x,t) = \frac{\hbar c}{e*} \sin \Theta_l(x,t) + \frac{\varepsilon}{c} \frac{\partial}{\partial t} V_l(x,t),
\]

with \( \lambda_c \) being the \( c \)-axis penetration depth.

Let us now study small phase oscillations in the Josephson vortex lattice state on the basis of Eqs. (1), (2), and (4). Let \( \Theta_l^{(0)}(x) \) and \( \Phi_l^{(0)}(x) \) be a set of static solutions of the above coupled equations for \( \Theta_l(x,t) \) and \( \Phi_l(x,t) \). Solutions corresponding to small phase oscillations around the static solutions can be expressed as

\[
\Theta_l(x,t) = \Theta_l^{(0)}(x) + \theta_l(x,t),
\]

\[
\Phi_l(x,t) = \Phi_l^{(0)}(x) + \frac{\hbar c}{e*} \phi_l(x,t),
\]

\[
V_l(x,t) = \frac{\hbar}{e*} v_l(x,t),
\]

where \( \theta_l(x,t), \phi_l(x,t), \) and \( v_l(x,t) \) represent the small time-dependent parts. In this paper we confine ourselves to the case in which Josephson vortices form a regular triangular lattice (see Fig. 2). In this case the static solution \( \Theta_l^{(0)}(x) \) satisfies the coupled equations

\[
\frac{d^2}{dx^2} \Theta_l^{(0)}(x) - \frac{d^2}{dx^2} \Theta_2^{(0)}(x) = \frac{4}{\lambda_j^2} \left[ \sin \Theta_1^{(0)}(x) - \sin \Theta_2^{(0)}(x) \right],
\]
\[
\frac{d^2}{dx^2} \Theta_1^{(0)}(x) + \frac{d^2}{dx^2} \Theta_2^{(0)}(x) = \frac{1}{\lambda_c^2} [\sin \Theta_1^{(0)}(x) + \sin \Theta_2^{(0)}(x)],
\]

(9)

where \( \Theta_1^{(0)}(x) [\Theta_2^{(0)}(x) \) is the phase difference at \( x \) on the junctions with odd (even) indices and \( \lambda_f \) is the Josephson length defined as

\[
\lambda_f = \frac{\lambda_c}{\sqrt{\eta + \frac{1}{2}}}. \quad (10)
\]

Then, the small oscillation parts \( \theta(x,t), \phi_f(x,t), \) and \( u_f(x,t) \), obey the following linearized equations:

\[
\frac{d}{dx} \theta_i(x,t) = \phi_i(x,t) - \eta [\phi_{i+1}(x,t) - 2 \phi_i(x,t) + \phi_{i-1}(x,t)],
\]

(11)

\[
\frac{d}{dt} \phi_i(x,t) = u_i(x,t) - \alpha [u_{i+1}(x,t) - 2 u_i(x,t) + u_{i-1}(x,t)],
\]

(12)

\[
\frac{d}{dx} \phi_i(x,t) = \frac{1}{\lambda_c^2} \cos \Theta_1^{(0)}(x) \phi_i(x,t) + \frac{\epsilon}{c^2} \frac{d}{dt} u_i(x,t).
\]

(13)

In the present triangular-lattice case one can introduce the Fourier series expansion along the \( c \) axis for the oscillatory part of the phase differences as

\[
\theta_i(x,t) = \sum_q \theta^{(1)}(x;\omega,q)e^{imq2d-iot}, \quad (14)
\]

for \( l = 2m-1 \) and

\[
\theta_i(x,t) = \sum_q \theta^{(2)}(x;\omega,q)e^{imq2d-iot}, \quad (15)
\]

for \( l = 2m \) with \( m \) being an integer. In this paper we consider only the \( q = 0 \) case. Then, from Eqs. (11)–(15) it follows the coupled equations for \( \theta^{(1)}(x;\omega,q=0) = \theta^{(1,2)}(x;\omega), \)

\[
\begin{align*}
&\left[ \frac{1 + 2\alpha}{1 + 4\alpha} \frac{\epsilon}{c^2} \omega^2 + \frac{2\eta}{1 + 4\eta} \frac{\partial^2}{\partial x^2} \right] \theta^{(1)}(x;\omega) \\
&+ \left[ \frac{2\alpha}{1 + 4\alpha} \frac{\epsilon}{c^2} \omega^2 + \frac{2\eta}{1 + 4\eta} \frac{\partial^2}{\partial x^2} \right] \theta^{(2)}(x;\omega)
\end{align*}
\]

(16)

\[
= \frac{1}{\lambda_c^2} \cos \Theta_1^{(0)}(x) \theta^{(1)}(x;\omega),
\]

The small phase oscillation modes in the triangular Josephson vortex lattice can be obtained by solving these coupled equations as an eigenvalue problem. However, since \( \eta \) is very large, i.e., \( \eta \sim \mathcal{O}(10^5) \), a careful treatment is required for solving numerically Eqs. (16) and (17). For this we introduce the quantities

\[
\theta^{(\pm)}(x;\omega) = \theta^{(1)}(x;\omega) \pm \theta^{(2)}(x;\omega), \quad (18)
\]

which satisfy the equations

\[
\begin{align*}
&\left[ \frac{\epsilon}{c^2} \omega^2 + \frac{2\eta}{1 + 4\alpha} \frac{\partial^2}{\partial x^2} - \frac{\cos \Theta_1^{(0)}(x) + \cos \Theta_2^{(0)}(x)}{2\lambda_c^2} \right] \theta^{(\pm)}(x;\omega) \\
&= \cos \Theta_1^{(0)}(x) - \cos \Theta_2^{(0)}(x) \theta^{(\mp)}(x;\omega),
\end{align*}
\]

(19)

\[
\begin{align*}
&\left[ \frac{1}{1 + 4\alpha} \frac{\epsilon}{c^2} \omega^2 + \frac{2\eta}{1 + 4\eta} \frac{\partial^2}{\partial x^2} - \frac{\cos \Theta_1^{(0)}(x) + \cos \Theta_2^{(0)}(x)}{2\lambda_c^2} \right] \theta^{(-)}(x;\omega) \\
&= \cos \Theta_1^{(0)}(x) - \cos \Theta_2^{(0)}(x) \theta^{(+)}(x;\omega).
\end{align*}
\]

(20)

Note that Eqs. (19) and (20) have a zero-frequency \( (\omega = 0) \) solution, which is given by

\[
\theta^{(\pm)}(x;0) = \frac{d}{dx} \Theta_1^{(0)}(x) \pm \frac{d}{dx} \Theta_2^{(0)}(x). \quad (21)
\]

This solution corresponds to the sliding mode of the triangular Josephson vortex lattice.

Let us now calculate the eigen-frequencies of the small oscillation modes. First, we have to solve Eqs. (8) and (9). Suppose that the Josephson vortex lattice has a period \( a \) along the \( x \) direction (see Fig. 2). In this case Eqs. (8) and (9) have solutions of the form

\[
\Theta_1^{(0)}(x) = \frac{2\pi}{a} x + g(x) + f(x),
\]

\[
\Theta_2^{(0)}(x) = \frac{2\pi}{a} x + g(x) - f(x),
\]

(22)
Then, from Eqs. (52) these equations indicate numerically these equations

\[ \frac{d^2}{dx^2} f(x) = \frac{4}{\lambda_j} \sin \left( \frac{2\pi x}{a} + g(x) \right) \cos f(x), \quad (23) \]

\[ \frac{d^2}{dx^2} g(x) = \frac{1}{\lambda_j^2} \cos \left( \frac{2\pi x}{a} + g(x) \right) \sin f(x). \quad (24) \]

These equations indicate $g(x) \sim O(\eta^{-1})$. Hence, neglecting $g(x)$, we utilize solutions of the approximate equation

\[ \frac{d^2}{dx^2} f(x) = \frac{4}{\lambda_j} \sin \left( \frac{2\pi x}{a} \right) \cos f(x), \quad (25) \]

for $f(x)$ in Eq. (22). In Fig. 3 we present numerical solutions of Eq. (25) under the boundary condition $f(0) = f(a) = 0$ for several values of $a$ in the region of $\lambda_j < a < \lambda_c$. In this approximation Eqs. (19) and (20) are reduced to

\[ \frac{\omega^2}{\omega_{pl}^2} \left( \frac{\pi}{a} \right)^2 \sin \left( \frac{2\pi x}{a} \right) \sin f(x) \theta^+(x;\omega) = \cos \left( \frac{2\pi x}{a} \right) \cos f(x) \theta^-(x;\omega), \quad (26) \]

\[ \frac{1}{1+4\alpha} \frac{\omega^2}{\omega_{pl}^2} \frac{\lambda_j^2}{4} \frac{\partial^2}{\partial x^2} + \sin \left( \frac{2\pi x}{a} \right) \sin f(x) \theta^+(x;\omega) = \cos \left( \frac{2\pi x}{a} \right) \cos f(x) \theta^-(x;\omega), \quad (27) \]

where $\omega_{pl}$ is the Josephson plasma frequency defined as $\omega_{pl} = c/\sqrt{\varepsilon \lambda_c}$. Note that the potential terms in Eqs. (26) and (27) are periodic. Hence, one can utilize the plane-wave expansion based on the Bloch theorem for $\theta^{\pm}(x;\omega)$ to solve numerically these equations

\[ \theta^{\pm}(x;\omega) = e^{ikx} \sum_{m=-\infty}^{\infty} \theta^{\pm}_m(\omega) e^{i(2\pi m/a) x}. \quad (28) \]

Then, from Eqs. (26) and (27) it follows for $k = 0$...
Calculations indicate that the frequency of the lowest excitation mode $\omega_1$ does not show such behavior, that is, $\omega_1$ is located below $\omega_{pl}$ and is a decreasing function of $\lambda_j/a$, though $\omega_2$ has the character predicted by Bulaevskii et al. Let us now briefly discuss the origin of the field dependence of the $\omega_1$ mode. If the umklapp processes are neglected in Eqs. (~29) and (~30), that is, only $n=0$ components are retained, Eq. (~30) gives the approximate eigenfrequency $\omega_2^2/\omega_{pl}^2 = -(1 + 4\alpha)M_0 < 1$, for the $\theta_1(\omega)$ component. From this fact one may infer that there exists an eigenfrequency in which the Fourier component $M_0$ dominates the field dependence. Since the Fourier components $M_n$’s, defined in Eq. (~31) originate from the periodic modulation of the phase differences $f(x)$, $M_0$ decreases with increasing $H$ since $f(x) \rightarrow 0$ for $H \rightarrow \infty$. Thus, we have an eigenfrequency which decreases as the magnetic field increases in the $u(2)$ channel.

In the microwave absorption experiments for Bi$_2$Sr$_2$CaCu$_2$O$_8$ two resonant modes have been observed in the presence of an in-plane magnetic field. In the case of $a/l_J = 4$ the observed field dependence of the lower (higher) resonant mode is qualitatively very similar to that of $\omega_1$ ($\omega_2$). In our calculations $\omega_3$ is located close to $\omega_2$, that is, three resonant modes may be expected to appear around $\omega_{pl}$ from our theory in a rigorous sense. However, the modes of $n=2$ and $n=3$ are nearly degenerate when the capacitive coupling is weak, i.e., $\alpha \ll 1$ as seen in Figs. 4 and 5. In the analysis of the $c$ axis $I$-$V$ characteristics the capacitive coupling constant $\alpha$ is estimated to be $\alpha = 0.1 - 0.2$ in Bi$_2$Sr$_2$CaCu$_2$O$_8$. As a result, the nearly degenerate $n=2$ and $n=3$ modes are expected to form a single resonant mode in the microwave absorption experiments. Thus, our results for the magnetic field dependence of the phase oscillation modes are consistent with the microwave absorption experiments in Bi$_2$Sr$_2$CaCu$_2$O$_8$. It is also noted that the eigenfrequencies in the case of $\alpha \neq 0$ are larger than those in the case of $\alpha = 0$. This increase originates from the charging energy existing in the case of $\alpha \neq 0$.

To clarify the character of the phase oscillation modes we study their eigenfunctions. In Figs. 6–10 we plot the spatial variation of the eigenfunctions $\theta_1(x, \omega_n)$ and $\theta_2(x, \omega_n)$ in the case of $a/l_J = 4$ and $\alpha = 0.2$, which are obtained via Eqs. (18) and (28) from the numerical results for $\theta_m^{(n)}(\omega_n)$. Note that the $0$th-order functions $\theta_1^{(0)}(x)$ and $\theta_2^{(0)}(x)$, in Eq. (22) represent the phase differences in the triangular vortex lattice state in which Josephson vortices are situated at $x/l_J = 4m$ on the junction 1 and at $x/l_J = 4m + 2$ on the junction 2 with.
Let us next study the \( n = 1 \) mode. As seen in Fig. 7, its eigenfunctions are nodeless and satisfy the relation \( \theta_1(x, \omega_1) = -\theta_2(x, \omega_1) \), that is, the motion of the phase differences on the junctions 1 and 2 takes place mostly in the opposite direction. Then, the \( n = 1 \) mode may be considered as an out-of-phase oscillation mode of the Josephson vortices. This is a new mode that does not exist in single Josephson-junction systems and whose eigenfrequency decreases with increasing the flux density.

In Fig. 8 we present the eigenfunctions of the \( n = 2 \) mode. It is seen that \( \theta_1(x, \omega_2) \) and \( \theta_2(x, \omega_2) \) are periodic functions with nodes and their period is the same as that of the 0th-order function \( f(x) \), though their phases of the spatial oscillations are shifted. Note that the Josephson vortices are moved by this phase-shift. Then, the \( n = 2 \) mode may be considered primarily as an oscillation mode of the Josephson vortex lattice, too. However, this mode is also affected by the background plasma modes, since the shape of the Josephson vortices are modulated in this oscillation mode. It is also seen that the eigenfunctions are shifted downward, i.e., \( \int_0^\pi dx \theta_{1,2}(x, \omega_2) < 0 \), that is, the \( n = 2 \) mode has a small in-phase component \( (\theta_1 + \theta_2 \neq 0) \).

Figure 9 shows the eigenfunctions of the \( n = 3 \) mode, \( \theta_1(x, \omega_3) \) and \( \theta_2(x, \omega_3) \). These eigenfunctions have two nodes in the period of the Josephson vortex lattice and their phases of the spatial variation coincide with the phase of \( f(x) \). Consequently, the centers of the Josephson vortices do not move in this oscillation mode, i.e., \( \theta_1(x, \omega_3) = 0 \) and \( \theta_2(x, \omega_3) = 0 \) at \( x/\lambda_J = 4m \) and \( 4m + 2 \). Thus, one may conclude that the \( n = 3 \) mode has only the character of the plasma mode.

Finally, we present the eigenfunctions of the \( n = 4 \) mode in Fig. 10. The eigenfunctions have four nodes within the period of the vortex lattice. It is also noted that the nodes of the eigenfunctions always appear on the centers of the Josephson vortices, that is, the \( n = 4 \) mode should be considered as the plasma mode with a large wave number which is folded inside the reduced Brillouin zone and is located at the \( \Gamma \) point. From these results one may conclude that the modes of \( n = 0, 1, 2 \) have the character of vibration modes of the Josephson vortex lattice and the other higher-frequency modes have their origin in the background plasma modes in the umklapp process.

**IV. SUMMARY**

In this paper we have calculated small phase oscillation modes in a triangular Josephson vortex lattice in a stack of intrinsic Josephson junctions and clarified the field dependence of the eigenmodes. In the triangular lattice case the period of the gauge-invariant phase difference doubles in the \( c \) direction. As a result, we have two phase oscillation modes at \( \Gamma \) point in the normal process, i.e., the out-of-phase motion on consecutive two junctions, i.e., \( \theta_1 - \theta_{2,1} \neq 0 \), and the in-phase motion corresponding to the sliding mode in the \( c \) direction. We have shown that the low frequency excitation modes in the triangular Josephson vortex lattice are primarily composed of the out-of-phase oscillation modes. This result originates from the strong inductive coupling between junctions. It has been also shown that the frequency of the lowest excitation mode in the triangular Josephson vortex lattice \( \omega_4 \) is less than \( \omega_{pl} \) and is a decreasing function of the applied magnetic field. This low frequency mode was missing in the previous works based on the asymptotic analysis in the strong field region \( \lambda_J/\theta \gg 1 \). As shown in this paper, to get this mode an elaborate numerical calculation is required in the moderate field region of \( \lambda_J/\theta < 1 \) where an analytical calculation is difficult because the nonlinearity of the phase differences is strong. The excitation modes with higher...
eigenfrequencies $\omega_n$ for $n \geq 2$ have the strong character of the plasma oscillation. The eigenfrequencies of these excitation modes increases as the external field increases. The two resonant frequencies observed in the microwave absorption experiments for Bi-2212 can be identified with $\omega_1$ and $\omega_{2,3}$ obtained in our calculations.

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5 See, for example, Y. Matsuda and M.B. Gaifullin, Physica C 362, 64 (2001), and references therein.