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Generalized Quantum Fast Transformations via Femtosecond Laser Writing Technique

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Quantum computers promise to be able to solve tasks beyond the reach of standard computational platforms. Among the others, photonic quantum walks prove to be great candidates for their implementation, since single photon sources, passive linear optics and photo-detectors are sufficient for universal quantum computation. To this aim, a device performing the quantum Fourier transform represents a fundamental building block for quantum algorithms, whose applications are not limited to the field of quantum computation. Recently, an algorithm has been developed to efficiently realize a quantum Fourier transform of an input photonic state by using a quantum walk on elementary linear-optical components. Here we provide a simple operative description of the algorithm, introducing a whole class of quantum transformations achievable through a generalization of this procedure. We finally discuss how femtosecond laser writing technology well represents an efficient and scalable platform for the implementation of this class of photonic quantum walks.

KEYWORDS: quantum information, quantum Fourier transform, integrated photonics

Introduction

Modern research deeply relies on the computational power of state-of-the-art standard technologies. In the last decades, a novel concept of quantum computational platform has been introduced [1], which is strongly believed to go beyond the reach of that currently available [2]. Quantum walk-based computing devices, composed of only linear-optical elements and able to manage adaptive measurements, may indeed become universal quantum computers promising to solve problems that are intractable for classical ones [3–6]. Thus, it becomes necessary to develop tools for efficiently realizing and characterizing any kind of photonic quantum walk, aiming at building up interferometric architectures of higher dimensionality and practicality. Indeed, it was already shown that any unitary evolution can be decomposed in the combined action of a number of beamsplitters and phase shifters that grows polynomially with the size of the interferometer [7, 8]. However, the high versatility of this decomposition comes at the cost of requiring, in general, more elements than those really sufficient for a specific transformation [9]. A new algorithm has been recently developed by Barak and Ben-Aryeh (BB) [10] which, following the famous classical algorithm of Cooley and Tukey [11], minimizes the number of beamsplitters and phase shifters for implementing the 2^n-dimensional quantum Fourier transform (QFT) over the optical modes. Indeed, the BB construction of the m-mode Fast Quantum Fourier Transform (qFFT) reduces the optical elements to only $O(m \log m)$, to be compared with the $O(m^2)$ elements necessary in the more general decomposition. Besides this qFFT implementation, whose importance stands out among quantum processing routines, other remarkable examples of transformations that can be decomposed in a BB-like architecture may find application in future quantum optical experiments [12]. Thus, it is of a broad interest, also from a fundamental point of view, the possibility of exploiting this fast decomposition for more general unitary evolutions on quantum walks.

Fast Quantum Hadamard and Fourier Transforms

Here we introduce a whole class of generalized fast quantum transformations that present a fast decomposition over the optical modes of a 2^n-dimensional interferometer, i.e. transformations that require a number of optical elements

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scaling as $O(m \log m)$. To achieve this task, we reverse the way of thinking of BB, starting from the most general 2^n-dimensional fast architecture to express the dependence of the related unitary transformation $G^{(2^n)}$ on the parameters describing the beamsplitters and phase shifters employed in the construction. The algorithm to design an interferometer of dimension 2^n resembling the fast architecture of the BB decomposition can be pictorially summarized as follows [13]. Each optical mode $k \in [1, 2^n]$ is associated to a vertex of a n-dimensional hypercube in $\mathbb{R}^n$ with coordinates $(b_1, b_2, \ldots, b_n)_k$, where $b_i = -1$ $(+1)$ if the i-th bit of the binary representation of k is 0 (1). The architecture consists of n steps (layers): at the step s, all pairs of modes differing only for the s-th bit are connected. The position of each mode in the cross-section of a fictitious plane can be defined by projecting on it the vertices of the hypercube. Each layer connects, by means of phase shifters and beamsplitters, corresponding to the edges of the hypercube with a given direction, the pairs of modes that have to interact at that step.

Once the overall architecture is arranged, specific unitary evolutions can be tailored for a given application. Firstly, by simply setting all beamsplitters to be 50:50 and zeroing all relative phases within the evolution, the Hadamard matrix is directly recovered. Thus, the simplest transformation achievable with a fast architecture is the Hadamard transformation, whose definition is well-known to be given recursively by defining $H^{(0)} = 1$ and $H^{(m)}$ as

$$H^{(2^{n+1})} = \frac{1}{\sqrt{2}} \begin{pmatrix} H^{(2^n)} & H^{(2^n)} \\ H^{(2^n)} & -H^{(2^n)} \end{pmatrix} \tag{1}$$

Other definitions can be given, for instance, specifying the single matrix elements $H^{(2^n)}_{i,j} = \frac{1}{2^{n/2}} (-1)^{\vec{i}\cdot\vec{j}}$, where the vectors $\vec{i}, \vec{j}$ denote the usual binary representations of the matrix indexes (i, j), or equivalently as a tensor product

$$H^{(1)} = 1 \quad H^{(2)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad H^{(2^n)} = \bigotimes_{s=1}^{n} H^{(2)} \tag{2}$$

A first generalization is obtained by separately setting the transmissivities τ of all beamsplitters and the phases of all phase shifters in the layer s to be equal to a fixed value $\tau_s = \cos\theta_s$ and ϕ_s, respectively. The general transformation in this case has the form

$$G^{(2^n)} = \bigotimes_{s=1}^{n} \begin{pmatrix} \cos\theta_s & e^{i\phi_s} \sin\theta_s \\ \sin\theta_s & e^{i\phi_s} \cos\theta_s \end{pmatrix} \tag{3}$$

It can be shown by directly expanding (3) that the evolution closely resembles the structure of the most general quantum transformation acting on the n-qubit separable state $|x\rangle = |x_0\ x_1\ \ldots\ x_n\rangle$ in the Bloch sphere representation

$$|x\rangle = |x_0\ x_1\ \ldots\ x_n\rangle \rightarrow |\psi\rangle = \bigotimes_{k=1}^{n} \left(\cos\frac{\theta_k(x)}{2} |0\rangle_k + \sin\frac{\theta_k(x)}{2}\ e^{i\phi_k(x)}\ |1\rangle_k \right) \tag{4}$$

which can be recast in a weighted superposition of the computational basis vector $\{|j\rangle\}$

$$|x\rangle \rightarrow \frac{1}{2^{n/2}} \sum_{j=0}^{2^n-1} c_j^{(n,x)} |j\rangle \tag{5}$$

$$c_j^{(n,x)} = \frac{1}{2^{n/2}}\ e^{i(\vec{\phi}(x)-\frac{\pi}{2})\cdot\vec{j}} \prod_{k=1}^{n} (1 + e^{i(\theta_k(x)+\pi \vec{j}_k)}) = e^{i(\vec{\phi}(x)-\frac{\pi}{2})\cdot\vec{j}} \sum_{k=0}^{2^n-1} (e^{i\vec{\theta}\cdot\vec{k}}\, H^{(2^n)}_{k,j}) \tag{6}$$

It is worth noting that, from the definitions above, also the QFT can be recovered [2, 14–16] by directly setting in (4) or (5) all beamsplitters to be 50:50 ($\theta_k = \pi/2$) and all phases $\phi_k(x)$ to

$$\vec{\phi}_k^{(QFT)}(x) = 2\pi \sum_{s=0}^{n-k} x_{n-s}\, 2^{k+s-n-1} \tag{7}$$

Depending on the dimension of the qFFT, a final relabeling of the output modes is necessary to recover the unitary [2, 10].

Generalized Fast Quantum Transform

So far, parameters have been set with a certain degree of symmetry to retrieve the fast implementations of the Hadamard and Fourier transformations over the optical modes. Relaxing the level of symmetry in the architecture, it can be shown that the element $G^{(2^n)}_{i,j}$ of the most general 2^n-dimensional fast quantum transformation, implemented using the BB decomposition, can be parametrized in (τ, ϕ) as

$$G^{(2^n)}_{i,j} = \sum_{k_1=1}^{2^n} \cdots \sum_{k_{n-1}=1}^{2^n} \prod_{s=1}^{n} \mathbf{L}^{(n,s)}_{k_{s-1},k_s} \tag{8}$$

where the factors $\mathbf{L}^{(n,s)}_{k_{s-1},k_s}$ describe the action of the optical components connecting the modes (k_{s-1}, k_s) at the step s of the 2^n-dimensional interferometer. Each matrix $\mathbf{L}^{(n,s)}$ consists of a layer of 2^{n-1} beamsplitters $\mathbf{B}^{(n,s)}_{k_{s-1},k_s}$, placed between the modes (k_{s-1}, k_s), and 2^{n-1} phase shifters $e^{i\phi_{k_s}(n,s)}$ placed on one of the two interacting modes (k_{s-1}, k_s)

$$\mathbf{L}^{(n,s)}_{k_{s-1},k_s} = \mathbf{B}^{(n,s)}_{k_{s-1},k_s}\, e^{i\,\phi_{k_s}(n,s)} \tag{9}$$

having also absorbed the indexes i and j in k_0 and k_n respectively. Here, by denoting with $\tau^{(n)}_{s,k_s}$ the beamsplitters' transmissivities on the mode k_s, the generic matrix associated to the layer of beamsplitters has the form

$$\mathbf{B}^{(n,s)}_{k_{s-1},k_s} \equiv \begin{cases} \tau^{(n)}_{s,k_s} & k_{s-1} = k_s \\ i\sqrt{1-\tau^{(n)2}_{s,k_s}} & (k_{s-1},k_s) \in \{(\alpha,\beta)\}^{(n,s)} \\ 0 & otherwise \end{cases} \tag{10}$$

where, for a given layer s, $\{(\alpha,\beta)\}^{(n,s)}$ is the set of pairs of modes that have to interact for the fast implementation:

$$\{(\alpha,\beta)\}^{(n,s)} = \{(a+2^s\,b, a+2^s\,b+2^{s-1})\} \tag{11}$$

with $a \in \{1,\ldots,2^{s-1}\}$, $b \in \{0,\ldots,2^{n-s}-1\}$.
As an example (Fig. 1), the 8-dimensional qFFT can be obtained within the generalized fast architecture by setting all beamsplitters to be 50:50 ($\tau^{(3)}_{s,k_s} = \sqrt{2^{-1}}\ \forall\ \{k,s\}$) and by setting, for each layer s, the phase shifts $\phi_{k_s}(3,s)$ to be

$$\vec{\phi}(3,1) = \{0,0,0,0,0,0,0,0\}$$
$$\vec{\phi}(3,2) = \left\{0,0,0,0,0,0,\frac{\pi}{2},\frac{\pi}{2}\right\}$$
$$\vec{\phi}(3,3) = \left\{0,0,0,\frac{\pi}{2},0,\frac{\pi}{4},0,\frac{3\pi}{4}\right\}$$

Again, a final relabeling of the output modes is necessary to retrieve the correct form of the qFFT [10, 13].

Experimental Realization of a Fast QFT

Femtosecond laser writing technique for fast architectures. In [13], one first experimental realization of a quantum walk implementing the qFFT has been reported as an effective platform for validating genuine quantum interference with many-photon states. The implementation of the qFFT interferometer has been realized on a photonic integrated platform by exploiting the 3-D capabilities of femtosecond laser writing technology [17, 18], which enables the fabrication of quantum walks arranged in three-dimensional scalable architectures with arbitrary layouts [20–23]. Indeed, the integration of all beamsplitters and phase shifters into a monolithic structure leads to a fundamental increase in the stability and scalability of the whole apparatus. Moreover, while various fabrication techniques exist for realizing integrated photonic devices, femtosecond laser writing offers unique advantages thanks mainly to cost-effective ease of production, low waveguides birefringence, control over the polarization and capability of engineering three-dimensional architectures [19], which prove to be essential tools for the realization of photonic quantum walks.

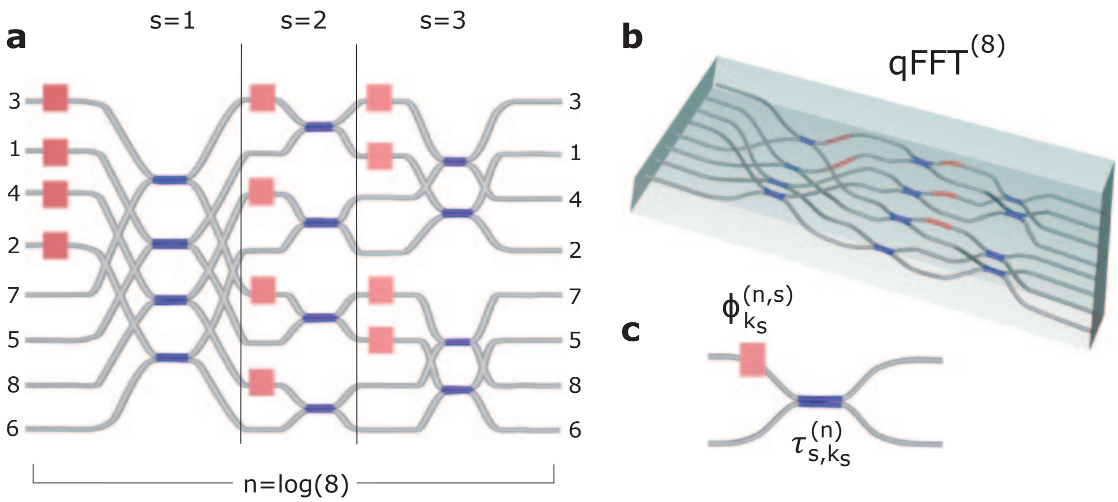

Fig. 1. **a)** Circuital scheme of the integrated interferometer implementing the 8-dimensional quantum fast Fourier transform over the optical modes. Phase shifters are represented by pink squares, while blue segments on the waveguides represent the directional couplers (the integrated equivalent of a beamsplitter). The architecture consists of a sequence of n steps, each including one layer of phase shifters and one of beamsplitters. Additional swaps in the arrangement of the optical modes, highlighted in this scheme, may be required for a more efficient fabrication process or to formally recover the desired transformation. **b)** Three-dimensional integrated interferometer implementing the 8-dimensional qFFT, fabricated via femtosecond laser writing technique [13]. **c)** Parametric description in (τ,ϕ) of the optical components in the circuital representation.

Fabrication of the 3-D fast QFT. The 8-mode integrated interferometer reported in [13] was fabricated in EAGLE2000 (Corning Inc.) boro-aluminosilicate glass chips. The inscription of the waveguides building up the quantum walk was carried out by focusing laser pulses (300 fs duration, 1 MHz repetition rate and 220 nJ energy from an Yb:KYW cavity dumped oscillator at 1030 nm) in the bulk of the glass with a 0.6 NA microscope objective. All waveguides, single mode at 800 nm with about 0.5 dB cm^{-1} propagation losses along the structure, were placed nearly 170 μm under the sample surface. The length of each fan-in and fan-out section, required to bring the waveguides at a distance of 127 μm, was 13.2 mm, while the cross-section of the 3-D interferometer was about 95 μm × 95 μm for a length of 14.7 mm. The Fidelity between the ideal 8-mode quantum Fourier transform and the implemented transformation, characterized via single and two-photon measurements at 785 nm, is $F = 0.9527 \pm 0.0006$, thus proving the high quality of femtosecond laser writing technology for quantum walk applications.

Conclusions

We have presented a new class of fast transformations that can be realized by extending the algorithm of Barak and Ben-Aryeh to more general photonic architectures. Following their algorithm, all fast transformations can be experimentally implemented by properly concatenating linear-optical elements, specifically beamsplitters and phase shifters. The class of generalized quantum walks arises from the parametrization of each optical component, thus spanning the whole space of 2^n-dimensional unitaries achievable with a given fast architecture. An operative recipe for retrieving within this class the Hadamard transform and the quantum Fourier transform is also reported. Lastly, we have discussed how this class of transformations can be efficiently implemented on a photonic platform, by employing the femtosecond laser writing technique to inscribe any quantum walk in an interferometric integrated structure. Further technological advances involving reconfigurable circuits may enable real time modifications in the unitary implemented, thus paving the way for designing efficient transformations whose particular form depends on the heralded photonic input state injected in the interferometer. Together, the approach represents a versatile tool for an efficient realization of photonic quantum walks, for broader applications in quantum computation and quantum information theory.

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