エッセイ: 市場の不一致性から、行為主義の立場からの観点

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ABSTRACT

In this thesis, we try to understand the volatility anomaly and the trading volume anomaly in the financial market by one of the most important behavioristic biases investors are facing, their extrapolation belief.

Firstly, we use the empirical test to investigate if investor’s extrapolation belief can significantly impact the volatility and trading volume. According to the recent groundbreaking work of Greenwood and Shleifer (2014), extrapolation investors’ expected return is “a weighted average of past returns.” Therefore, we construct the Greenwood Shleifer Index (GSI) to quantitatively represent investors’ extrapolative belief. Then we use this index as an explaining variable to test its relation to the volatility and trading volume of different financial markets.

Chapter 1 represents our empirical test results about the relation between GSI and volatility. It is shown in this chapter that in most of the financial markets, GSI can significantly impact volatility even if we include economic factors in our regression, indicating the changing extrapolation belief could be a reason causing volatility to change, especially when individual investors dominate the market. We also find an asymmetric GSI-volatility relationship that volatility is more easily affected by individuals’ extrapolation belief during the declining market.

In the light of previous extrapolative models, Chapter 2 builds a new model to explain the empirical finding of Chapter 1. In our new model, extrapolative investors also pay attention to information innovations, but with confirmation bias when they
evaluate the new arriving information. By questionnaire survey, we give direct evidence that confirmation bias and extrapolation bias could impact individual investors simultaneously. Then by analytical proportions and numerical simulation, we show our model can provide specific links between volatility and investors’ extrapolation belief. Additionally, we find our theory can also help us to explain one of the most stylized facts of volatility, the volatility clustering.

Chapter 3 seeks the relation between GSI and trading volume. Using simple ARIMA structure regression tests, we find individuals’ extrapolation belief can significantly impact the trading volume, but the effect is different according to different market. Specifically, in the emerging stock market where short-sale constraint exists, when GSI<0, trading volume is negatively correlated with |GSI|, the magnitude of individuals’ extrapolation belief, but when GSI>0, trading volume is positively correlated with |GSI|. On the contrary, trading volume is positively correlated with |GSI| for both positive and negative GSI in the future market, where investors are free to sell short. The reason for this wacky relationship is in need of further discussion.

Chapter 4 tries to explain this confusing relation between trading volume and individuals’ extrapolation belief. We find that with a simple modification, our model in Chapter 2 can efficiently explain this intriguing relation. The only modification we make is that extrapolators are heterogeneous with each other in the way that every extrapolator has his idiosyncratic bias when evaluating the information innovations. In this new model, their heterogeneity is amplified by their extrapolation belief. We prove that, in the future market where people can sell short, the trading volume grows as the
magnitude of individuals’ extrapolation belief grows. On the contrary, during the bear market, in the emerging stock market, extrapolators gradually quit the market as $GSI$ increases, the market is left will only fundamentalists who are homogenous with each other, the trading volume reduces accordingly. Moreover, using simulation, we show that our model can efficiently explain the most two important features of financial bubbles: the high trading volume, and the high volatility. Volatility rises because extrapolator’s expectation is becoming more volatile, trading volume increases as extrapolators are getting more heterogeneous with their peers.

To summarize, this thesis makes several contributions. Firstly, this thesis is the first one to empirically investigate the relationship between volatility, trading volume and individuals’ extrapolation bias. Secondly, we confirmed some stylized facts that previous researches are missing, the asymmetric GSI-volatility relationship, for example. Thirdly, we build a new extrapolative model which not only can help us to understand the trading volume and volatility in financial markets but also gives a better explanation for the financial bubbles. We also suggest it may be a more promising way to study irrational individual investors’ behavior from multiple angles, like what we did in building the new model.
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1. The time-varying Volatility and Extrapolation Belief

Abstract: This paper empirically studies the relationship between the time-varying volatility and individual’s extrapolation belief in several financial markets. Based on the groundbreaking work of Greenwood and Shleifer (2014) that extrapolative investors’ expected return is a weighted average of previous returns, we construct the Greenwood Shleifer Index (GSI) to represent investors’ extrapolative belief. Results of our empirical test indicate the changing extrapolation belief could be a reason causing volatility to change, especially when individual investors dominate the market. We also find an asymmetric GSI-volatility relationship that volatility is more easily affected by individuals’ extrapolation belief during the declining market.

1.1 Introduction

Volatility, which is thought to be one of the most essential benchmarks of the financial market, changes over time (Fama (1965), Castanias (1979)). There are a lot of scholars attempting to understand the fluctuation in volatility from the view of economic factors. For example, Flannery and Protopapadakis (2002) try to investigate the relation between the conditional volatility and macroeconomic variables, but just find valid results for only a small part of economic variables they are using. Other researchers, including Cutler, Poterba, and Summers (1990), Fleming and Remolona
(1999), Andersen and Bollerslev (1998), Andersen et al. (2003), try to relate volatility changes in the stock market to micro-economic factors that can affect expected returns. Recent works like Engle and Rangel (2008), Christiansen et al. (2012), Mittnik, Spindler and Robinzonov (2015) try to use more sophisticated models like Spine-Garch to revisit how can stock market volatility be affected by macroeconomic activities. But they only find similar conclusions that the economic factors have limited power in explaining the movements in stock volatility (Mittnik, Spindler, and Robinzonov (2015)).

Instead of paying attention to economic factors, this paper tries to seek the relationship between the time-varying volatility and one of the most important individual investors’ biases, their extrapolation belief. By empirical tests, this paper shows that volatility index is highly correlated with the extent of investors’ extrapolation belief in most financial markets. We believe individual investors’ extrapolation behavior can help us to understand the fluctuation in volatility from a new perspective.

Extrapolation means investors tend to form their expectation of future returns by previous price trend (Barberis et al. (2016)). They believe the stock price will always keep its trend—the price will continue to rise if it has a positive cumulative price change, and vice versa. Both psychology papers and financial literature provide evidence proving that extrapolation bias can deeply impact people’s decision-making process (see, Gilovich et al. (1985); Hirshleifer (2001); Barberis and Thaler, (2003); Fuster et al., (2010), e.g.). Theoretical models have also demonstrated extrapolation can account
for many capital market phenomenon, e.g., the overreaction anomalies (Barberis et al.,
(1998)), the bubble generation (Barberis et al., (2016)), herding investment (Barberis
and Shleifer (2003)) and so on.

Recently, a groundbreaking development of extrapolation theory is given by
Greenwood and Shleifer (2014). They use survey evidence from multiple resources,
empirically prove that “investors’ extrapolative belief is a weighted average of past
price changes, where more recent price changes are weighted more heavily.” Based on
their work, this paper structures Greenwood and Shleifer Index (GSI) to quantitatively
measure the extent of extrapolation belief of investors. We choose daily market data
from several kinds of financial markets, including emerging stock markets (Chinese
stock market), developed stock markets, the Brent crude oil future market, and the
currency markets (JPYUSD and EURUSD), to empirical seek its impact on movements
in volatility.

To characterize the time varying volatility, this paper applies the Realized Volatility
method using 5-min high frequency intraday data. Then we use a simple least squares
regression model to investigate if volatility is related to individuals’ extrapolation belief.
This paper also distinguishes positive GSI from negative GSI with two dummy
variables to explore the possible asymmetric influence in raising market and in
declining market. Besides, to eliminate possible spurious regression, we also introduce
economic factors into our equitation, to see if the regression result will change
significantly.
Strikingly, we find significant regression results that GSI, no matter positive or negative, and no matter with or without economic factors, does statistically affect volatility index for most of the financial markets. Besides, all the regression results are much better when we include GSI as explaining variables than those with only economic factors. These regression results strongly prove that extrapolation is a reason causing volatility to change.

Besides, it can also be seen from our test that when individual investors take a bigger fraction of the whole population, the relationship between GSI and volatility get closer. For example, the Chinese stock market, with its reputation for “individual investors dominated immature market”, has the biggest individual trading volume proportion as well as the best regression result. On the contrary, no significant correlation between GSI and volatility index can be found in the currency market where individual investors only trade a very small proportion of the whole amount. When individual investors’ proportion maintains a moderate size, like Japanese stock market, Nasdaq stock market and the Brent Crude future market, only weak significant regression result can be discovered. Besides, the significance of the estimated parameter of both positive GSI and negative GSI holds even if we include macroeconomic factors in our empirical test. These empirical results indicate that volatility is indeed influenced by individuals’ extrapolation belief.

These diverse empirical test results of different markets can be explained from the different characters of market participators. Individual investors, also called as “retail investors” or “noise traders” (Kyle (1985)), is well documented to hold irrational biases
and make detrimental investment decisions (Barber and Odean (2011), De long et. al. (1990), Shefrin and Thaler (2004), Shiller (2015)). Extrapolation, of course, is one of the most pervasive ones. De Bondt (1998), for example, argues that “Perhaps the best-established stylized fact is extrapolation bias...” Institutional investors, on the contrary, are supposed to have the ability to get all information and correctly evaluate the fundamental price of equity which can help them to arbitrage the mispricing away (Fama (1965), Black (1972), Huberman (2005)). But recent empirical and theoretical findings show that, when the market is filled with enthusiastic extrapolators, institutional investors may not trade against them, or, in some circumstances, they will even try to ride the bubble, buy in asset which they believe it is already over-priced and hoping to sell it to latter arriving irrational investors who will buy the asset in an even higher price (De Long et al. 1990, Abreu and Brunnermeier 2003, Brunnermeier and Nagel 2004). So, a higher proportion of individual trading will indicate a bigger influence of extrapolation belief to the price, therefore a better explanatory power of GSI to volatility index.

To test the robustness of the effect that individual investors’ proportion has on the explanatory power of GSI to volatility index, this paper goes further to compare the regression results between the two indexes in different stages of the same market by dividing the samples of Chinese stock market into core-bubble stage and non-core stage. During bubbles, the extraordinary enthusiasm of individual investors keeps them rushing into the market and becoming more active (Shiller 2015), individual investors will take a bigger proportion as well as a higher influence to the market. Hence,
individual investors’ irrational behavior will have a more notable impact on the market, and volatility should also have a closer relation with GSI. As a result, our test confirms this conjecture. For all three samples of Chinese stock market, we do find better regression result in core-bubble stages than in non-bubble stages — after casting the bubble period, the rest of the sample only show poor fitting effect while the core-bubble stage samples still have highly significant regression results.

According to these comparisons, we may cautiously get the conclusion that changes in extrapolation belief can be one of the reasons causing volatility to change, especially when individual investors take a big proportion of the market.

Besides, our test gives clear evidence of the asymmetric relationship between GSI and volatility. To begin with, our empirical result shows the regression coefficients of the negative GSI are bigger than the coefficient of the positive GSI for all the samples. Also, the coefficient significance level of negative GSI is much higher. When GSI>0, the regression coefficient is only highly significant in Chinese stock market, which turns to weakly significant for other markets. Especially for Nasdaq stock market, where we cannot find significant relation between positive GSI and volatility. Contrarily, coefficient of negative GSI is still highly significant (p<0.01) across all the samples (except currency market). These results demonstrate the asymmetric GSI-volatility relation that volatility is more easily affected by individuals’ extrapolation belief during declining market, but during bull market, this relation is weaker.

This paper’s findings have meaningful implications. Firstly, to the author’s best knowledge, this paper is the first one to empirically study the dynamics of volatility
across time from the perspective of individuals’ extrapolation bias. Although many behavioral papers have documented how individuals’ irrational behaves can exaggerate volatility (DeLong et. al. (1990), Odean (1998), Scheinkman and Xiong (2003), Barberis, Greenwood and Shleifer (2015)), most of these works are focusing on why volatility is “excess” relative to predictions of standard theory model, i.e. the “excess volatility puzzle” of Shiller (Shiller 1980, LeRoy and Porter 1981). Few of them try to study the time-varying nature of volatility from behavioral perspective (for more detail, see Section 1.2). This paper shows that individuals’ irrational bias can not only account for “excess volatility”, but can also drive volatility to change across time.

Secondly, our finding is an important supplement to researches about the changing volatility. As demonstrated above, previous papers mainly try to explain the fluctuation in volatility with economic factors. Pitifully, according to their results, the economic factors can only explain small part of the movements in volatility (Bollerslev, Engle, and Wooldridge (1988), Schwert (1989), Christiansen et. al. (2012), Mittnik, Spindler and Robinsonov (2015)). But according to this paper’s result, volatility index is correlated with extrapolation belief in most of the markets, even for the well-developed stock markets such as Nasdaq stock market and Japanese stock market. Therefore, people’s irrational bias (extrapolation belief in our paper) can be other factors causing volatility to change. That is what previous paper doesn’t take into account. Thus, it is more suitable to understand the volatility fluctuation by combining economic factors and people’s irrational behave.
However, we cannot find satisfying theoretical explanations for this empirical result in previous papers. It is the only thing emphasized in previous papers that Extrapolation, as a bias, can affect individual’s expectation about future returns, which also becomes the starting point for previous papers to explain market anomalies such as the generation of financial bubbles, overreaction (Barberis et al. (2016), Hong and Stein (1999)). But, most of these papers describe extrapolation of investors is a deterministic process—it is only determined by past returns. Therefore, there is no fluctuation in individuals’ extrapolation belief in these models. Nevertheless, according to our empirical results, extrapolation can also influence instantaneous volatility. So, how extrapolation belief can lead changes in volatility still need future theoretical explanation.

The rest of the paper is organized as follows. Section 1.2 briefly reviews related behavioral literatures about volatility, Section 1.3 describes the data and characters of each financial market. Section 1.4 presents the calculation method for volatility and our Greenwood and Shleifer Index (GSI), then illustrates the results of our regression. Further implications of our findings are discussed in Section 1.5.

1.2 Literature Review

The "excess volatility puzzle", which has been demonstrated in many researches, such as Shiller (1980), Campbell and Shiller (1987), West (1988), Gilles and LeRoy (1991), is focusing on the aggregate level of volatility but not about why volatility changes over time. Motived by this anomaly, a group of researchers have started to search for the relation between volatility and investors’ irrational behave. DeLong et al.
(1990) call individual investors “noise traders” and point out noise trader can cause price to depart significantly from its fundamental values, and can also cause the excess volatility of asset. But they don’t specify which bias noise traders are suffering. Starting from prospect theory, Barberis, Huang and Santos (2001) study a model where investors make decisions according to both consumption and the fluctuations of their financial wealth. They prove that their theory can illustrate excess volatility and other anomalies existing in the market. Overconfidence is another irrational bias individual investors are facing which is proved to be capable of increasing volatility (Odean, (1998); Scheinkman and Xiong, (2003)).

Extrapolation bias is also proved to be capable of exaggerating volatility. For example, Barberis et al. (2015) study a model in which investors try to maximize their consumption utility according to their investment performance. In extrapolators’ belief, future return of the asset is determined by past price changes. When a positive cash-flow shock drives the price to rise, extrapolators will try to buy in more asset and hence the stock prices will be pushed even higher. As a consequence, price will be more fluctuate than its fundamental value. Similar results can be found in Cutler, Poterba, and Summers (1990), DeLong, Shleifer, Summers, and Waldmann (1990).

But all of these models are aiming at accommodating the excess volatility puzzle, i.e., why the aggregate level of volatility is higher than the prediction of standard models. Few of them pay attention to volatility fluctuation across time. Take Barberis, Greenwood, Jin and Shleifer (2015) for example, they calculate the volatility over a fixed period predicted by their extrapolative capital asset pricing model, find it is much
higher than the fundamental fluctuations. But, no information about volatility fluctuation in this time horizon can be found in their model.

1.3 Data and Descriptive Statistics

To explore the relation between extrapolative belief and the time-varying volatility, we choose eight different indexes of different financial markets from Choice Database, one of the biggest financial data service enterprises in China. Specifically, the data covers SSEC (Shanghai Security Composite Index) and GEI (Shenzhen Growth Enterprise Index) of Chinese stock market. The SSEC is designed to show the overall performance of Shanghai Security Exchange while GEI represents Shenzhen Stock Exchange. Being developed for decades, the Chinese stock market is now the world's 5th largest stock market by market capitalization at US$3.5 trillion as of February 2016, and 2nd largest in Asia. This paper also chooses indexes from developed countries’ stock markets, N225(Japanese Nihon Keizai Shinbun Index) of Japanese stock market and IXIC (American Nasdaq Composite Index) of American stock market, two of the biggest stock market around the world. For the commodity future market, the data picks Brent Crude Index, for it is the most active commodity future ranked by trading volume (Stoll, Whaley (2010)). EURUSD (Euro to USD dollar exchange rate) Index and JPYUSD (Japanese Yen to USD dollar exchange rate) Index are also included on behalf of the currency market. Choosing EURUSD and USD not only because EURUSD and JPYUSD are the most popular currency pairs in the world but also for the free convertibility and floating exchange rate system of these economic entities. The time
intervals, listed in Table 1.1, are different because of data source restriction. Especially, for SSEC, two separated time series data are available. Also, the data manages to cover two distinguished bubble periods in Chinese stock market the 2005-2008 stock bubble and 2015-2016 stock bubble. Annualized Volatility, calculated as standard deviation of

Table 1.1 Brief report of each market

<table>
<thead>
<tr>
<th>Date range</th>
<th>Average Log Return</th>
<th>Annualized Volatility</th>
<th>Individual trading Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>EURUSD 2015/10/15-2016/12/9</td>
<td>-0.015%</td>
<td>10.21%</td>
<td>&lt;5%</td>
</tr>
<tr>
<td>JPYUSD 2015/10/15-2016/12/9</td>
<td>-0.009%</td>
<td>11.41%</td>
<td>&lt;5%</td>
</tr>
<tr>
<td>Brent Crude Index 2015/9/16-2016/12/9</td>
<td>0.002%</td>
<td>44.53%</td>
<td>25%</td>
</tr>
<tr>
<td>N225 2014/12/19-2016/12/1</td>
<td>0.026%</td>
<td>24.55%</td>
<td>23.5%</td>
</tr>
<tr>
<td>IXIC 2015/4/29-2016/11/16</td>
<td>0.029%</td>
<td>16.59%</td>
<td>&lt;30%</td>
</tr>
<tr>
<td>SSEC 2005/2/1-2008/12/31</td>
<td>0.039%</td>
<td>29.95%</td>
<td>85%</td>
</tr>
<tr>
<td>SSEC 2013/12/23-2016/10/31</td>
<td>0.049%</td>
<td>32.52%</td>
<td>83%</td>
</tr>
<tr>
<td>GEI 2013/12/26-2016/10/31</td>
<td>0.072%</td>
<td>39.19%</td>
<td>85%</td>
</tr>
</tbody>
</table>

Data range, average log return, annualized volatility and individual trading proportion of each market are reported.

Annualized volatility is calculated as standard deviation of all returns, such as

\[
\text{Annualized volatility} = 100 \cdot \sqrt{\frac{252}{n} \sum_{t=1}^{n} (r_t - \bar{r})^2}
\]

where 252 represents the constant representing the approximate number of trading days in a year, \( n \) means the number of samples and \( r_t \) is logarithm return at time \( t \), \( \bar{r} \) is the average return.
returns for the whole interval is also reported in Table 1 as well as the average log return. Because extrapolative belief is usually found in individual investors, Table 1 also gives their trading volume proportion in each market.

As shown in this table, the currency market has the lowest level of individual investors activity and the lowest volatility, it is also considered as one of the most efficient markets (Kristoufek and Vosvrda (2016)). According to Triennial Central Bank Survey (2016), released by the Bank for International Settlements(BIS), daily retail-driven transaction volume (volume traded by individuals) is about 283 billion US dollars, only about 5% compared with the total amount of 5,067 billion by all counterparties. On the contrary, individual investors take more than 80% of the whole population, more than any other market. According to CSDC (China Securities Depository and Clearing Corporation) Report 2016, although only holding 24% of market capitalization, individual investors in China own about 1.2 billion trading accounts (99% percent of all trading accounts) and account for more than 80% of total trading volume. Besides individual dominated markets, Chinese stock market is also depicted as “opaque, chaotic, inefficient, and rather irrational” (Eun and Huang (2007)). Similarly, the Wall Street Journal (August 22, 2001) use casinos to portray Chinese stock market: “In ten years since they were founded, China's stock markets have operated like casinos, driven by fast money flows in and out of stocks with little regard for their underlying value.” The volatility in Chinese stock market is also the highest, especially for GEI, which reaches 39.19% annually.
Compared with Chinese stock market, there are much less individual investors in Japanese and US stock market, volatility of 24.55% and 16.59% also indicate these two markets are more stable. Brent Crude Commodity future has a similar proportion of individual investors like Japanese or US stock market, about 25% of this future is traded by individual investors (Stoll, Whaley 2009). But the volatility of Brent Crude Index dominates with the size of 44.53%. After suffering a big collapse from about 110 USD to the lowest 34 USD in 2014 and 2015, the price of Brute Crude Oil has rebounded to about 50 USD at the end of 2016. Extremely high volatility accompanies this progress which surged to its highest level in seven years (seen in Appendix Figure 1.2).

In a word, the Chineses stock market which has the biggest proportion of individual investors, performs the highest and most volatile volatility compared with other financial markets.

1.4 Calculation Method and Result Discussion

1.4.1 Index calculation method

Greenwood Shleifer Index(GSI). As mentioned above, Greenwood and Shleifer (2014) demonstrate that extrapolators’ expectation about the risky asset’s future return is a “weighted average of past price changes, where more recent price changes weighted more heavily”. Following their research, we build the Greenwood Shleifer Index(GSI) as

$$GSI_t = \sum_{i=1}^{n} r_{t-i} \cdot \lambda^i$$  

(1)
where $r_t$ means the return at time $t$, $\lambda$ governs the weights investors put into each period which is set according to empirical results of Greenwood and Shleifer (2014).

The specification in (3) is commonly used in previous extrapolative models about market anomalies. For example, overreaction and under-reaction of price to information, momentum trading, stylized trading and so on (De Long et al. 1990, Cutler, Poterba, and Summers 1990, Hong and Stein 1999, Barberis and Shleifer 2003, Barberis et al. 2015). It may be necessary to emphasize that this index can also be negative if the price has fallen for some time when individual investors hold pessimistic extrapolation belief about future.

**Volatility Index.** We use Realized Volatility (RV) to measure volatility index. The fast-growing papers on Realized Volatility show that, when high frequency data is available, it is a more efficient and accurate measure than other methods such as ARCH or GARCH model (Andersen and Bollerslev (1998), Barndorff-Nielsen and Shephard (2002), Andersen et al. (2003), McAleer and Medeiros (2008)). Following these literature, this paper measures volatility index with 5-min high frequency intraday data of each market. In high-frequency theory, the realized variance is defined as

$$RV_t = \sum_{i=1}^{n_t} r_{t_i}^2,$$

(2)

where $n_t$ means observation frequency at day $t$. $r_{t_i} = p_{t_i} - p_{t_{i-1}}$ represents the $ith$ intraday sub return, calculated as the difference between logarithm value of price, $p_{t_i}$. Accordingly, we can calculate the annualized realized volatility, such as
\[ V_t = \sqrt{RV_t} \times \sqrt{T} = \sqrt{T} \sum_{i=1}^{n_t} r_{ti}^2, \quad (3) \]

in which T means the total number of trading days in a year.

Using high frequency RV gives many advantages. Firstly, studies have proved RV provides “an unbiased and highly efficient measurement of return volatility” (Andersen et al. (2001)). Further, as stressed by Andersen and Bollerslev (1998) and Andersen et al. (2001), this measurement is independent of the model we use, as well as independent of the sampling frequency. Thirdly and most importantly, this method, unlike other volatility measurements, can ensure volatility index is mathematically independent from our Extrapolative Belief Index. As shown in Fig 1.1, other volatility index calculation methods, like time rolling window standard deviation or the GARCH Model, are using basically the same daily return data as GSI (except for the current return \( r_t \)).

![Figure 1.1 Different volatility calculation methods](image)

Although calculation methods are different, it can be proved that these volatility measure results are mathematically correlated with GSI. On the contrary, as
demonstrated by equation (3) and equation (4), Realized Volatility picks the high-frequency information by using the intraday squared returns \( r_t^2 \) at time \( t \), whereas GSI is calculated by low frequency daily returns \( r_{t-i} \), \( i = 1,2,3 \cdots \), data before time \( t \). Using the independent data resources makes sure no mathematical connections between volatility index and GSI.

The basic statistics of calculated GSI with \( \lambda = 0.8 \) and volatility for different financial markets are reported in Appendix Table 1.2. In Greenwood and Shleifer (2014), for different data sources, their estimated results of \( \lambda \) are different, which ranges from 0.33 to 0.92. Of all the six samples, the empirical result for Gallup survey, which has the second largest sample size and the most significant result, seems to be the most reliable test result. So, we discuss the statistics of GSI with \( \lambda = 0.8 \) which approximates the result of the regression for Gallup survey.

From this table, we can see that, being used to measure how volatile the market is, the volatility index itself is very volatile. The minimum values of the estimated volatility of these indexes are all approximately equal to 4% (except for volatility of Brent Crude Index which reaches to 18.81%). Although the maximum values vary across different markets, they are all far beyond the minimum values. For EURUSD and JPYUSD, the maximum volatility is about 40%, ten times that of its minimum value, but for Brent Crude Index, GEI and SSEC in 20013 and 2016, the maximum volatility hovers to more than 120%, about 30 times bigger than its minimum value. This huge difference indicates that volatility can change very intensely, especially in Brent Crude Oil market and in Chinese stock market. Standard deviation(SD), another signal for
volatility fluctuation, is also much higher in Brent Crude Oil market and Chinese stock market than in other markets. Besides, the mean value of estimated volatility calculated by high frequency approach is smaller than the standard deviation of returns used in Table 1.1. This result is similar to previous papers such as Liu and Tse (2012), Amsköld (2011), as extreme returns are more stressed in standard deviation method (Masset 2011). Although the size is different, the order is same. Brent crude oil future market has the highest level of volatility, Chinese stock market is more volatile than other stock markets, and currency market is the most stable one. These statistics document that, Brent Crude Oil market and Chinese stock market present not only higher but also more unstable volatility than other markets.

Although the standard deviation of Extrapolation belief index, GSI, is not high, its value can become very extreme. For instance, the max value of GSI for Chinese GEI index, is 9.79%, meaning extrapolative investors is so optimistic about future that they believe the price can still raise for about another ten percent. On the contrary, the minimum value of GSI for GEI index drops to -17.52%, indicating extrapolation belief make investors hold extreme pessimistic opinions about future returns. Besides, these severe preconceptions caused by extrapolation bias most occurred during the bubble period when volatility was also extremely high simultaneously.

Appendix Figure 1.3 is a demonstration of the fluctuation of volatility (blue line) and the changing GSI (red line) while the black line means the GEI index. As shown in this picture, volatility starts to raise as the bubble grows since March 2015, and it continues raising even after the bubble busted. Eventually it reaches to its peak--more
than 120% on 9\textsuperscript{th} July 2015, the most panicking time of the whole market. GSI, the extrapolation belief of individuals, also grows as the bubble generates like volatility index, showing the increasing enthusiasm of extrapolative investors. When the bubble begins to fall, GSI turns to negative as investors become pessimistic. It decreases to its minimum when the volatility reaches its peak. It seems the evolution of investors extrapolation belief, positive or negative, is associated with the evolution of volatility.

\textbf{Appendix Figure 1.4}, the scatterplot of volatility versus GSI for Chinese Growing Enterprise stock market (GEI), gives a further demonstration of the contemporaneous GSI--volatility relationship. We can clearly see that, no matter positive or negative, the development of individuals’ extrapolation belief, measured by GSI, is associated with growing volatility.

Similar things happen to other two samples of Chinese stock market where distinguished bubble period can also be easily noticed, as shown in \textbf{Appendix Figure 1.5} and \textbf{Figure 1.6}. Also, high volatility is accompanied by extreme values of GSI can also be found in Japanese stock market, Nasdaq stock market and in Brent Crude oil market (seen \textbf{Appendix Figure 1.7}, \textbf{Figure 1.8} and \textbf{Figure 1.9}).

From these figures, we could assume that volatility can be affected by the magnitude of GSI. Individuals’ extrapolative belief, positive and negative, can both lead volatility to increase. To empirically investigate this, we use the following regression form

\[
Volatility_t = a + \beta_1 GSI_t D_1 (GSI_t > 0) + \beta_2 |GSI_t| D_2 (GSI_t \leq 0) + u_t \tag{4}
\]
where $D_1$ and $D_2$ are two dummy variables aiming to distinguish positive or negative regions of GSI. Results are shown in the following section.

1.4.3 Empirical Test Result and Comparison of different markets

Firstly, as we don’t know exactly the weight that individual investors put into each past period, we calculate GSI with different values of $\lambda$ ranging from 0.3 to 0.9, as suggested by Greenwood and Shleifer (2014). The results for different markets are listed in Appendix from Table 1.3 to Table 1.10.

As we see from these tables, the changing value of $\lambda$ has little impact on the empirical test result. For example, for the currency market, for all the value of $\lambda$, all the empirical results for $\beta_1$ and $\beta_2$ are all insignificant. But for three samples of Chinese stock market, although the size of estimated $\beta_1$ and $\beta_2$ are different with different value of $\lambda$, they are all highly significant. Besides, the R-squared value also changes limitedly. Similar things can also be found in Brent crude oil markets, the Japanese stock market as well as the Nasdaq stock market. Although the estimated $\beta_1$ for Japanese stock market changes from weakly significant to insignificant as $\lambda$ increases from 0.3 to 0.9, and the estimated $\beta_1$ for Nasdaq stock is only insignificant when $\lambda = 0.9$, the estimated results don’t have a major difference with each other, as the significance of $\beta_2$ insists for different $\lambda$, the R-squared value only has small changes. The estimated results don’t have major difference with each other.

Therefore, for the convenience of comparison, the empirical results for different financial markets with $\lambda = 0.8$ are summarized in Appendix Table 1.11. Because
previous papers already show economic factors can partly explain the changes in volatility, we also introduce macro-economic factors into our regression to eliminate possible spurious regression as:

\[
\text{Volatility}_t = a + \beta_1 \cdot \text{GSI}_t \cdot D_1 (\text{GSI}_t > 0) + \beta_2 \cdot |\text{GSI}_t| \cdot D_2 (\text{GSI}_t \leq 0) + \beta_3 \cdot E_t + u_t \quad (5)
\]

where \(E_t\) means macro-economic factors. As we use daily data in our empirical test, the daily macro-economic factors are limited. Specifically, we use the Domestic Interbank Offered Interest Rate for the stock markets, and we use both countries’ Interbank Offered Rate for the currency market (although we only show the regression result with US. Interbank Offered Interest Rate). Because the Crude Oil Future is traded world-widely, we introduce the US Dollar Index as the explaining economic factor. All the empirical test results of regression form (5) for different financial markets are listed in Table 1.11 too.

Some meaningful conclusions can be established. Firstly, according to our empirical test result, GSI can significantly impact volatility in most of the financial markets. Moreover, it has much higher explaining power than economic factors. Particularly, as summarized in Table 1.11, \(\beta_2\) is highly significant (P<0.01) for almost all the samples (except for the currency markets). \(\beta_1\) is also highly significant for Chineses stock market samples. When we only use the economic factors as the explaining variable, the \(R^2\) is quite small, which is consist with previous researcher’s’ finding that economic factors have little ability to explain the time varying volatility. On the contrary, if we use GSI as the explaining variable, \(R^2\) increases significantly, indicating a much better
regression result. More importantly, even if we include the macroeconomic factors in the regression, there is no significant change in the regression results. Taking Chinese GEI market for example, although the coefficient of Shibor is strictly positive, the R-squared value is only 0.12 when we only use Shibor as the explaining variable. But it increases to as much as 0.48 when we introduce GSI into the regression. Furthermore, if we introduce economic factors into equation (4), the significance of $\beta_1$ and $\beta_2$ insists, the R-squared value is similar as before. These significant results verify our assumption that volatility can be caused by individuals' extrapolation belief.

Secondly, the significance level for different markets is different. For instance, Chinese stock market, where individual investors’ trading volume takes the biggest proportion, has the most significant regression results--both $\beta_1$ and $\beta_2$ are significant at the 1% significance level. R-squared values also indicate GSI has the biggest explanatory power for volatility in Chinese stock market. With a size of 0.51, R-squared value for the second Chinese stock market sample (SSEC index from 2013/12/23 to 2016/10/13) implies a well-fitting regression. For other two Chinese stock market samples, R-squared values are both above 0.3, still bigger than other financial markets. As for Japan Stock market, Nasdaq stock market and Brent oil commodity future market, we only find less significant empirical regression results or a weaker explanatory power of GSI for volatility. Although $\beta_2$, the coefficient of the negative GSI, is highly significant (at the 1% significance level) for all these three markets, $\beta_1$, the coefficient for the positive GSI, performs much worse. It is only significant at the 5% significance level for Brent crude oil market and at the 10% significance level for
Japanese stock market while non-significant for Nasdaq stock market. Besides, R-squared values for these three markets are only about 0.1, which also suggest a weaker relation between GSI and volatility. For the currency markets, where individual investors take the minimum trading volume proportion, no significant regression result can be found. In a word, GSI has different ability to explain volatility in different financial markets.

Besides the diverse significant levels among different financial markets, the asymmetric GSI-Volatility relation is also proved. Our empirical results indicate $\beta_2 > \beta_1$ for all samples which have significant regression result. Besides magnitude, significant levels for these two coefficients are also different. For Japan Stock market, Brent oil commodity future market and Nasdaq stock market, $\beta_2$ are all more significant than its counterpart, especially for the Nasdaq stock market, where the positive GSI cannot significantly affect volatility according to our empirical test.

Moreover, to formally test the asymmetric GSI-Volatility relation, we use the following regression form:

$$Volatility_t = a + \beta_3 |GSI_t| + \beta_4 \cdot |GSI_t| \cdot D_2(GSI_t \leq 0) + u_t$$

in which we take $\beta_2 = \beta_1 = \beta_3$ and $\beta_4$ is insignificant as the null hypotheses. But according to our test result of 3.13 (represented in Table 1.12), $\beta_4$ are significant for all the financial markets (except for the currency market). Also, for all the samples with the significant $\beta_4$, we can find $\beta_4 + \beta_3 = \beta_2$, and $\beta_1 = \beta_3$, which officially reject the
null hypotheses—there does exist an asymmetric GSI-Volatility relation in these financial markets.

This inequality reveals that negative GSI has stronger explanatory power to volatility than positive GSI. In other words, volatility is more easily affected by extrapolation belief in a declining market.

1.4.3 Result comparison of different stages

From the regression results of Table 1.11, we can find that when there is a bigger proportion of individual investors, there will be a more significant regression result. So, we could make the assumption that the effect extrapolation belief has on volatility is determined by the proportion of individual investors or the influence of individual investors to pricing. To further verify this assumption, this paper also compares different stages of the same market: bubble stages and other stages.

From Figure 1.3, Figure 1.5 and Figure 1.6 we can easily notice two bubble stages in Chinese stock market, the 2005-2008 stock bubble and 2015-2016 stock bubble. A financial bubble is often portrayed as the asset price rushes abnormally high compared with its fundamental value (Brunnermeier and Oehmke, 2012). As displayed in these figures, from 2013 to 2016, the SSEC index hovered to 5178 point followed by a big collapse of 50% within three months, while GEI index performs similarly by losing more than 56% in the same short time after its rising from around 1300 to a maximum of 4037 point. In both figures, volatility grows and fluctuates with bubble’s generation,
and become even higher and more fluctuant during bubble’s collapse. Similar things happen to SSEC index during 2005 and 2008.

Inexperienced individual investors are continually attracted to “gamble” in the market during the bubble periods. Kindleberger and Aliber (2005, p. 25) suggest that people who are indifferent to such investment are brought into the market. “Even chimney-sweeps and old clothes women dabbled in tulips.” as described by Mackay (1841). “Youth had taken over Wall Street.” happened during the stock market boom of the late 1960s (Brooks’ (1973, p. 211)). The same thing happens to Chineses stock market. Appendix Figure 1.10 shows the development of monthly new individual investors (blue line) and proportion of active account (red line) according to monthly reports of CSDC (China Securities Depository and Clearing Corporation) from 2014 Jan. to 2016 Nov., the second bubble period of Chinese stock market. Because individual investor’ accounts occupy about 99% of total accounts in Chinese stock market, the ups or downs of active account can be regards as the return or leave of individual investors who have already own a stock account. In this picture, we can feel the enthusiasm of new individual investors and the newfound interest of “old” individual investors when the price rises (black line in Figure 1.10), or the depression of individual investors as price falling. In a word, individual investors are much more active during bubbles. Hence, the correlation between volatility index and GSI should be bigger during bubbles.

To compare the different influence of extrapolation belief to volatility during different periods (bubbles period or non-bubble periods), each of the samples drawn from Chinese stock market is divided into two stages based on the price level, the core
bubble stage and the non-core stage. The core-bubble stage is selected according to: (1) it should contain the most distinguished part of the bubble, i.e., the stage when the price reaches a very high level. (2) To avoid reliable test issues, the sample interval should not be too short. The rest time is the non-core stage. Even if we cannot calculate the fundamental value of each index, but based on the price level, we can at least distinguish when is bubble more severe. The core-bubble stage of each sample is represented by the green box in Figure 1.3, Figure 1.5 and Figure 1.6. The comparison result is reported in Appendix Table 1.13.

As represented in Table 1.13, empirical test result varies significantly across different stages. For all three samples, the least squares regression does perform better in core-bubble stages than in non-core stages. Especially for the latter two samples (from 2013 to 2016), after casting the most distinguished bubble stage, $\beta_1$, which is highly significant for the core-bubble stage and for the whole sample (see Table 2), only shows weak significance (at 10% level). R-squared value also drops to less than 0.1, which means the non-core stage of these two samples only have the similar performance with developed stock market and the Brent crude oil future market. Non-core stage of the third sample (SSEC 2005-2008) have a better regression result but it is still incomparable with the core-bubble stage. Besides the different regression significance between core-bubble stage and non-core stage, the asymmetric explanatory power of positive and negative GSI to volatility index is also reflected as $\beta_2 > \beta_1$ still holds for all the stages.
These significant differences verify the conjecture that when the market has a bigger proportion of individual investors, the correlation between volatility index and individuals’ extrapolation belief should be higher. So, we may cautiously get the conclusion that individuals’ trading proportion is critical of explanatory power of GSI to volatility index. Therefore, extrapolation belief can be a reason that drives volatility to change, especially when individual investors have a large influence on pricing, like in Chinese stock market.

1.5 Further Discussion

To the author’s best knowledge, our paper is the first to use empirical test to investigate how individuals’ extrapolation belief can affect volatility across time. According to this paper’s result, extrapolation beliefs can not only cause the "excess volatility puzzle" as show in the existing papers, but also can drive volatility to vary over time. As discussed above, micro and macroeconomic variables can only partly explain why volatility changes across time (Bollerslev, Engle, and Wooldridge (1988), Schwert (1989)), so this approach offers a new method to study the variation of volatility.

Now the challenge is that existing theories about extrapolation are not enough for us to fully understand this relation between GSI and volatility. To further illustrate this, let’s assume the price follows the stochastic process as:

\[ dp_t = \mu_t dt + \sigma_t dW_t \]  

(6)
where $p_t$ is the log value of current price, $\mu_t$ stands for drift component of this process, i.e. the instantaneous conditional mean of return at time $t$, $W$ is a standard Brownian motion while $\sigma_t$ represents a stochastic process independent of $W_t$, it also signifies the instantaneous volatility. Therefore, we can calculate the return as

$$r_{t_i} = p_{t_i} - p_{t_{i-1}} = \int_{t_{i-1}}^{t_i} \mu_t dt + \int_{t_{i-1}}^{t_i} \sigma_t dW_t$$  \hspace{1cm} (7)

and its quadratic variation $QV(t_i, t_{i-1})$ is

$$QV(t_i, t_{i-1}) = \int_{t_{i-1}}^{t_i} \sigma_t^2 dt$$  \hspace{1cm} (8)

Equation (8) shows that according to quadratic variation theory, innovations of the mean component $\mu_t$ cannot change the variation of the return $r_{t_i}$. Further, by semi-martingale theory, when the observation number increases, realized volatility will eventually become equivalent to the return quadratic variation $QV$ (Protter 1990):

$$RV_t = \sum_{i=1}^{n_t} r_{t,i}^2 \rightarrow QV_t = \int_{t_{i-1}}^{t_i} \sigma_t^2 dt = \sigma_t^2 \quad \text{as } n_t \rightarrow \infty$$  \hspace{1cm} (9)

Equation (9) proves the unbiasedness and accuracy of using Realized Volatility to estimate the instantaneous volatility. It also shows that $\mu_t$ won’t affect instantaneous volatility estimated by high frequency Realized Volatility approach. But, the existing extrapolation theories all emphasize it is investor’s expectation of future returns that can be affected by past price changes. In most of these models, the equilibrium price is determined through the interaction between rational inverter’s expectation (which is determined by the stochastic information process), and extrapolative investors’ expectation which is only about past price changes. Therefore, according to previous
studies, we cannot connect the instantaneous volatility $\sigma_t$ with individuals’ extrapolation belief. Hence, why extrapolation belief can affect instantaneous volatility still needs theoretical explanations.

Although extrapolation is one of the most important biases individual are facing, it is not all of them. Overconfidence, for example, is another irrational belief individual may suffer. Overconfident means irrational traders are too “confident” about the accuracy of their private information signals (Gervais and Odean (2001)). Therefore, they usually ignore other investors’ trading behave when they make investment decisions. This bias can also generate several market anomalies, like the “excess volatility” puzzle and high trading volume (Odean 1998, Dumas, Kurshev and Uppal 2009), or even a sustainable bubble (Scheinkman and Wei Xiong 2003). Nevertheless, there is no foundational work as Greenwood and Shleifer (2014) about overconfidence, which can help to calculate how “overconfidence” individuals are.

Therefore, we currently may not be able to empirically study the effect of other individuals’ irrational beliefs on the fluctuation in volatility. We are hoping related works based on survey evidence or experimental evidence can emerge to help us with this dilemma. After all, it is not enough to study the aggregate market phenomenon with only one individual’s irrational bias.
1.6 REFERENCE


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Appendix 1

Figure 1.2 Realized volatility of Brent Crude oil index

Calculation method: 30 days rolling windows of historical realized volatility Source: Yahoo finance

Figure 1.3 GSI and volatility index in Shenzhen Growth Enterprise stock market

We present its absolute value of GSI, which is computed according to Greenwood and Shleifer (2014), with red line (red axis) while red-dot line shows its negative values. The blue line denotes volatility index estimated by high frequency realized volatility approach (blue axis). Black line (black axis) represents IXIC, the index of Nasdaq stock market.
Figure 1.4 Scatterplot of GSI versus volatility index for GEI

Figure 1.5 GSI and volatility index in Shanghai mainboard stock market (2005 -2008)

We present its absolute value of GSI, which is computed according to Greenwood and Shleifer (2014), with red line (red axis) while red-dot line shows its negative values. The blue line denotes volatility index estimated by high frequency realized volatility approach (blue axis). Black line (black axis) represents IXIC, the index of Nasdaq stock market.
Figure 1.6 GSI and volatility index in Shanghai mainboard stock market (2013-2016)

We present its absolute value of GSI, which is computed according to Greenwood and Shleifer (2014), with red line (red axis) while red-dot line shows its negative values. The blue line denotes volatility index estimated by high frequency realized volatility approach (blue axis). Black line (black axis) represents IXIC, the index of Nasdaq stock market.

Figure 1.7 GSI and volatility index in Brent Crude future market

We present its absolute value of GSI, which is computed according to Greenwood and Shleifer (2014), with red line (red axis) while red-dot line shows its negative values. The blue line denotes volatility index estimated by high frequency realized volatility approach (blue axis). Black line (black axis) represents IXIC, the index of Nasdaq stock market.
We present its absolute value of GSI, which is computed according to Greenwood and Shleifer (2014), with red line (red axis) while red-dot line shows its negative values. The blue line denotes volatility index estimated by high frequency realized volatility approach (blue axis). Black line (black axis) represents IXIC, the index of Nasdaq stock market.

**Figure 1.8 GSI and volatility index in Japanese stock market**

We present its absolute value of GSI, which is computed according to Greenwood and Shleifer (2014), with red line (red axis) while red-dot line shows its negative values. The blue line denotes volatility index estimated by high frequency realized volatility approach (blue axis). Black line (black axis) represents IXIC, the index of Nasdaq stock market.

**Figure 1.9 GSI and volatility index in Nasdaq stock market**

We present its absolute value of GSI, which is computed according to Greenwood and Shleifer (2014), with red line (red axis) while red-dot line shows its negative values. The blue line denotes volatility index estimated by high frequency realized volatility approach (blue axis). Black line (black axis) represents IXIC, the index of Nasdaq stock market.
Figure 1.10 New individual investors and active account in Chinese stock market

The red line denotes the monthly increase of new individual investors (thousands). The red line denotes expectations the proportion of active accounts. Black line (black axis) represents the close index of Shanghai Stock market.
<table>
<thead>
<tr>
<th>Index</th>
<th>Date range</th>
<th>Volatility</th>
<th>GSI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>min</td>
<td>Mean</td>
</tr>
<tr>
<td>EURUSD</td>
<td>2015/10/15-2016/12/9</td>
<td>3.99%</td>
<td>8.82%</td>
</tr>
<tr>
<td>JPYUSD</td>
<td>2015/10/15-2016/12/9</td>
<td>3.41%</td>
<td>7.93%</td>
</tr>
<tr>
<td>Brent Crude Index</td>
<td>2015/9/16-2016/12/9</td>
<td>18.81%</td>
<td>44.31%</td>
</tr>
<tr>
<td>N225</td>
<td>2014/12/19-2016/12/1</td>
<td>3.86%</td>
<td>13.56%</td>
</tr>
<tr>
<td>IXIC</td>
<td>2015/4/29-2016/11/16</td>
<td>3.76%</td>
<td>15.57%</td>
</tr>
<tr>
<td>SSEC</td>
<td>2005/2/1-2008/12/31</td>
<td>4.43%</td>
<td>18.25%</td>
</tr>
<tr>
<td>SSEC</td>
<td>2013/12/23-2016/10/31</td>
<td>4.52%</td>
<td>22.06%</td>
</tr>
<tr>
<td>GEI</td>
<td>2013/12/26-2016/10/31</td>
<td>6.57%</td>
<td>25.58%</td>
</tr>
</tbody>
</table>

Mean, standard deviation (SD), extreme values and correlation between Volatility and GSI are reported. Volatility are calculated in the approaches of realized volatility with 5-min intraday data. GSI is calculated by \( X_t = \sum_{i=1}^{n} r_{t-i} \cdot \lambda^i \), where \( r_t \) means the return at time \( t \), \( \lambda \) governs the weights investors put into each period which is set as \( \lambda = 0.8 \) according to Greenwood and Shleifer (2014).
Table 1.3 Summary of empirical results with difference value of $\lambda$ for SSEC (2005-2008)

<table>
<thead>
<tr>
<th>Index</th>
<th>$\lambda = 0.3$</th>
<th>$\lambda = 0.4$</th>
<th>$\lambda = 0.5$</th>
<th>$\lambda = 0.6$</th>
<th>$\lambda = 0.7$</th>
<th>$\lambda = 0.8$</th>
<th>$\lambda = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.10***</td>
<td>0.10***</td>
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<td>[11.0]</td>
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<td>[30.94]</td>
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<tr>
<td>$\beta_1$</td>
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<td>4.31***</td>
<td>3.38***</td>
<td>2.70***</td>
<td>2.12***</td>
<td>1.56***</td>
<td>0.92***</td>
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<td>[10.67]</td>
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<tr>
<td>$\beta_2$</td>
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<td>7.14***</td>
<td>5.59***</td>
<td>4.45***</td>
<td>3.48***</td>
<td>2.49***</td>
<td>1.60***</td>
</tr>
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</tr>
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<td>0.277</td>
<td>0.289</td>
<td>0.300</td>
<td>0.307</td>
<td>0.302</td>
<td>0.281</td>
</tr>
<tr>
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</table>

Data: 2005/01/02~2008/12/31

We estimate time-series regressions of the form

$$\text{Volatility}_t = a + \beta_1 \cdot \text{GSI}_t \cdot D_1(\text{GSI}_t > 0) + \beta_2 \cdot |\text{GSI}_t| \cdot D_2(\text{GSI}_t \leq 0) + u_t$$

where $D_1$ and $D_2$ are two dummy variables. ***, **, * denote statistical significance at the 1%, 5%, 10% level respectively. Newey-West-based $t$-statistics are in brackets.

Table 1.4 Summary of empirical results with difference value of $\lambda$ for SSEC (2014-2016)

<table>
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<th>$\lambda = 0.8$</th>
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<tbody>
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<td>0.24***</td>
<td>0.24***</td>
<td>0.25***</td>
<td>0.25***</td>
<td>0.25***</td>
</tr>
<tr>
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<td>[33.8]</td>
<td>[32.60]</td>
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<td>[31.01]</td>
<td>[33.74]</td>
<td>[31.23]</td>
<td>[30.94]</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>5.9**</td>
<td>4.31***</td>
<td>3.38***</td>
<td>2.70***</td>
<td>2.12***</td>
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<td>0.92***</td>
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<td>[11.16]</td>
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<td>[7.97]</td>
<td>[8.46]</td>
<td>[11.6]</td>
<td>[11.37]</td>
<td>[10.67]</td>
</tr>
<tr>
<td>$\beta_2$</td>
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<td>7.14***</td>
<td>5.59***</td>
<td>4.45***</td>
<td>3.48***</td>
<td>2.54***</td>
<td>1.60***</td>
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<td>[11.53]</td>
<td>[11.889]</td>
<td>[20.3]</td>
<td>[20.12]</td>
<td>[19.17]</td>
</tr>
<tr>
<td>$R^2$</td>
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<td>0.277</td>
<td>0.289</td>
<td>0.300</td>
<td>0.307</td>
<td>0.52</td>
<td>0.281</td>
</tr>
<tr>
<td>N</td>
<td>951</td>
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Data: 2005/01/02~2008/12/31

We estimate time-series regressions of the form

$$\text{Volatility}_t = a + \beta_1 \cdot \text{GSI}_t \cdot D_1(\text{GSI}_t > 0) + \beta_2 \cdot |\text{GSI}_t| \cdot D_2(\text{GSI}_t \leq 0) + u_t$$

where $D_1$ and $D_2$ are two dummy variables. ***, **, * denote statistical significance at the 1%, 5%, 10% level respectively. Newey-West-based $t$-statistics are in brackets.
Table 1.5 Summary of empirical results with difference value of $\lambda$ for GEI

<table>
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<th>$\lambda = 0.8$</th>
<th>$\lambda = 0.9$</th>
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<tbody>
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<td>0.18***</td>
<td>0.17***</td>
<td>0.17***</td>
<td>0.17***</td>
<td>0.17***</td>
<td>0.15***</td>
</tr>
<tr>
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<td>[8.28]</td>
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<td>[21.13]</td>
<td>[21.24]</td>
<td>[20.23]</td>
<td>[16.31]</td>
</tr>
<tr>
<td>$\beta_1$</td>
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<td>4.31***</td>
<td>3.38***</td>
<td>2.70***</td>
<td>2.12***</td>
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<td>0.92***</td>
</tr>
<tr>
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<td>[11.16]</td>
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<td>[7.97]</td>
<td>[8.46]</td>
<td>[11.6]</td>
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<td>[10.67]</td>
</tr>
<tr>
<td>$\beta_2$</td>
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<td>7.14***</td>
<td>5.59***</td>
<td>4.45***</td>
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<td>1.60***</td>
</tr>
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<td>[11.73]</td>
<td>[11.53]</td>
<td>[11.89]</td>
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<td>[19.17]</td>
</tr>
<tr>
<td>$R^2$</td>
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<td>0.277</td>
<td>0.289</td>
<td>0.300</td>
<td>0.307</td>
<td>0.302</td>
<td>0.281</td>
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</tr>
</tbody>
</table>

Data 2005/01/02–2008/12/31

We estimate time-series regressions of the form

$$\text{Volatility}_t = a + \beta_1 \cdot \text{GSI}_t \cdot D_1(\text{GSI}_t > 0) + \beta_2 \cdot |\text{GSI}_t| \cdot D_2(\text{GSI}_t \leq 0) + u_t$$

where $D_1$ and $D_2$ are two dummy variables. ***, **, * denote statistical significance at the 1%, 5%, 10% level respectively. Newey-West-based t-statistics are in brackets.

Table 1.6 Summary of empirical results with difference value of $\lambda$ for Brent Crude Oil

<table>
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<th>$\lambda = 0.7$</th>
<th>$\lambda = 0.8$</th>
<th>$\lambda = 0.9$</th>
</tr>
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<tbody>
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<td>0.34***</td>
<td>0.35***</td>
<td>0.35***</td>
<td>0.35***</td>
<td>0.35***</td>
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<td>0.37***</td>
</tr>
<tr>
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<td>[21.20]</td>
<td>[22.19]</td>
<td>[22.74]</td>
<td>[22.83]</td>
<td>[21.31]</td>
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</tr>
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<td>7.53***</td>
<td>5.4***</td>
<td>3.70***</td>
<td>2.54***</td>
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<td>[3.33]</td>
<td>[2.68]</td>
<td>[1.10]</td>
</tr>
<tr>
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<td>9.70***</td>
<td>7.21***</td>
<td>5.35***</td>
<td>4.01***</td>
<td>2.97***</td>
<td>1.90***</td>
</tr>
<tr>
<td></td>
<td>[4.26]</td>
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<td>[4.33]</td>
<td>[5.61]</td>
<td>[4.37]</td>
<td>[4.33]</td>
<td>[3.77]</td>
</tr>
<tr>
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<td>0.117</td>
<td>0.110</td>
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<tr>
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</tr>
</tbody>
</table>

data 2015/9/16-2016/12/9

We estimate time-series regressions of the form

$$\text{Volatility}_t = a + \beta_1 \cdot \text{GSI}_t \cdot D_1(\text{GSI}_t > 0) + \beta_2 \cdot |\text{GSI}_t| \cdot D_2(\text{GSI}_t \leq 0) + u_t$$

where $D_1$ and $D_2$ are two dummy variables. ***, **, * denote statistical significance at the 1%, 5%, 10% level respectively. Newey-West-based t-statistics are in brackets.
Table 1.7 Summary of empirical results with difference value of $\lambda$ for Nasdaq

<table>
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<tr>
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<th>$\lambda = 0.3$</th>
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<th>$\lambda = 0.6$</th>
<th>$\lambda = 0.7$</th>
<th>$\lambda = 0.8$</th>
<th>$\lambda = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.14*** [22.28]</td>
<td>0.14*** [22.24]</td>
<td>0.14*** [22.36]</td>
<td>0.14*** [21.31]</td>
<td>0.14*** [21.24]</td>
<td>0.15*** [20.23]</td>
<td>0.15*** [22.41]</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>3.3 [1.41]</td>
<td>2.1 [1.20]</td>
<td>1.08 [0.79]</td>
<td>0.30 [0.22]</td>
<td>0.49 [0.45]</td>
<td>0.63 [0.55]</td>
<td>0.65 [1.14]</td>
</tr>
<tr>
<td>$R^2$</td>
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<td>0.073</td>
<td>0.075</td>
<td>0.079</td>
<td>0.082</td>
<td>0.084</td>
<td>0.11</td>
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<tr>
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<td></td>
</tr>
</tbody>
</table>

Data 2015/04/29–2016/1/16

We estimate time-series regressions of the form

$$\text{Volatility}_t = \alpha + \beta_1 \cdot \text{GSI}_t \cdot D_1 (\text{GSI}_t > 0) + \beta_2 \cdot |\text{GSI}_t| \cdot D_2 (\text{GSI}_t \leq 0) + u_t$$

where $D_1$ and $D_2$ are two dummy variables. ***, ***, * denote statistical significance at the 1%, 5%, 10% level respectively. Newey-West-based t-statistics are in brackets.

Table 1.8 Summary of empirical results with difference value of $\lambda$ for N225

<table>
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<th>$\lambda = 0.6$</th>
<th>$\lambda = 0.7$</th>
<th>$\lambda = 0.8$</th>
<th>$\lambda = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.59*** [5.84]</td>
<td>0.59*** [5.60]</td>
<td>0.60*** [5.07]</td>
<td>0.64*** [5.23]</td>
<td>0.69*** [5.58]</td>
<td>0.74*** [5.37]</td>
<td>0.71*** [4.74]</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.57* [1.91]</td>
<td>0.39* [1.89]</td>
<td>0.26* [1.85]</td>
<td>0.14 [1.36]</td>
<td>0.06 [0.06]</td>
<td>0.07 [0.09]</td>
<td>0.06 [0.09]</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>1.96*** [4.37]</td>
<td>1.43*** [4.60]</td>
<td>1.10*** [4.85]</td>
<td>0.86*** [4.95]</td>
<td>0.65*** [4.89]</td>
<td>0.50*** [4.86]</td>
<td>0.38*** [5.51]</td>
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<tr>
<td>$R^2$</td>
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<td>0.11</td>
<td>0.12</td>
<td>0.12</td>
<td>0.13</td>
<td>0.14</td>
<td>0.17</td>
</tr>
<tr>
<td>$N$</td>
<td>951</td>
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<td></td>
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</tbody>
</table>

Data 2014/12/19–2016/11/30

We estimate time-series regressions of the form

$$\text{Volatility}_t = \alpha + \beta_1 \cdot \text{GSI}_t \cdot D_1 (\text{GSI}_t > 0) + \beta_2 \cdot |\text{GSI}_t| \cdot D_2 (\text{GSI}_t \leq 0) + u_t$$

where $D_1$ and $D_2$ are two dummy variables. ***, ***, * denote statistical significance at the 1%, 5%, 10% level respectively. Newey-West-based t-statistics are in brackets.
Table 1.9 Summary of empirical results with difference value of $\lambda$ for EURUSD

<table>
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<th>$\lambda = 0.7$</th>
<th>$\lambda = 0.8$</th>
<th>$\lambda = 0.9$</th>
</tr>
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<tbody>
<tr>
<td>$a$</td>
<td>0.17***</td>
<td>0.18***</td>
<td>0.17***</td>
<td>0.17***</td>
<td>0.17***</td>
<td>0.17***</td>
<td>0.15***</td>
</tr>
<tr>
<td></td>
<td>[8.28]</td>
<td>[22.24]</td>
<td>[22.36]</td>
<td>[22.11]</td>
<td>[21.24]</td>
<td>[20.23]</td>
<td>[16.31]</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.56</td>
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<td>0.21</td>
<td>0.13</td>
<td>0.07</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>[1.08]</td>
<td>[1.30]</td>
<td>[0.79]</td>
<td>[1.26]</td>
<td>[1.13]</td>
<td>[0.82]</td>
<td>[0.53]</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>1.89</td>
<td>0.14</td>
<td>0.10</td>
<td>0.06</td>
<td>0.02</td>
<td>0.04</td>
<td>1.92</td>
</tr>
<tr>
<td></td>
<td>[0.68]</td>
<td>[0.69]</td>
<td>[0.62]</td>
<td>[0.59]</td>
<td>[0.31]</td>
<td>[0.23]</td>
<td>[0.14]</td>
</tr>
<tr>
<td>$R^2$</td>
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<td>0.008</td>
<td>0.007</td>
<td>0.007</td>
<td>0.005</td>
<td>0.003</td>
<td>0.001</td>
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</table>

Data: 2015/10/10–2016/09/12

We estimate time-series regressions of the form

$$\text{Volatility}_t = a + \beta_1 \cdot \text{GSI}_t \cdot D_1(\text{GSI}_t > 0) + \beta_2 \cdot |\text{GSI}_t| \cdot D_2(\text{GSI}_t \leq 0) + u_t$$

where $D_1$ and $D_2$ are two dummy variables. *** , ** , * denote statistical significance at the 1% , 5% , 10% level respectively. Newey-West-based t-statistics are in brackets.

Table 1.10 Summary of empirical results with difference value of $\lambda$ for JPYUSD

<table>
<thead>
<tr>
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<th>$\lambda = 0.3$</th>
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<th>$\lambda = 0.5$</th>
<th>$\lambda = 0.6$</th>
<th>$\lambda = 0.7$</th>
<th>$\lambda = 0.8$</th>
<th>$\lambda = 0.9$</th>
</tr>
</thead>
<tbody>
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<td>0.45***</td>
<td>0.45***</td>
<td>0.45***</td>
<td>0.45***</td>
<td>0.45***</td>
<td>0.45***</td>
</tr>
<tr>
<td></td>
<td>[3.81]</td>
<td>[3.76]</td>
<td>[3.76]</td>
<td>[3.71]</td>
<td>[3.74]</td>
<td>[3.78]</td>
<td>[3.81]</td>
</tr>
<tr>
<td>$\beta_1$</td>
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<td>0.29</td>
<td>0.21</td>
<td>0.13</td>
<td>0.07</td>
<td>0.65</td>
</tr>
<tr>
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<td>[1.08]</td>
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<td>[0.79]</td>
<td>[1.26]</td>
<td>[1.13]</td>
<td>[0.82]</td>
<td>[0.53]</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>1.89</td>
<td>0.14</td>
<td>0.10</td>
<td>0.06</td>
<td>0.02</td>
<td>0.04</td>
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<td>[0.69]</td>
<td>[0.62]</td>
<td>[0.59]</td>
<td>[0.31]</td>
<td>[0.23]</td>
<td>[0.14]</td>
</tr>
<tr>
<td>$R^2$</td>
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<td>0.008</td>
<td>0.007</td>
<td>0.007</td>
<td>0.005</td>
<td>0.003</td>
<td>0.001</td>
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Data: 2015/10/10–2016/09/12

We estimate time-series regressions of the form

$$\text{Volatility}_t = a + \beta_1 \cdot \text{GSI}_t \cdot D_1(\text{GSI}_t > 0) + \beta_2 \cdot |\text{GSI}_t| \cdot D_2(\text{GSI}_t \leq 0) + u_t$$

where $D_1$ and $D_2$ are two dummy variables. *** , ** , * denote statistical significance at the 1% , 5% , 10% level respectively. Newey-West-based t-statistics are in brackets.
<table>
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<th>(\beta_2)</th>
<th>(\beta_3)</th>
<th>(R^2)</th>
</tr>
</thead>
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<tr>
<td>EURUSD</td>
<td>2015/10/15-2016/12/9</td>
<td>302</td>
<td>0.17***</td>
<td>2.64 [0.82]</td>
<td>4.12 [0.03]</td>
<td>-0.29 [-0.75]</td>
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<tr>
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<td></td>
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<td></td>
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<td>4.12 [0.03]</td>
<td>-0.36 [-1.05]</td>
<td>0.003</td>
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<tr>
<td>JPYUSD</td>
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<td>0.45**</td>
<td>14.64 [0.71]</td>
<td>9.12 [0.95]</td>
<td>0.43 [-0.82]</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
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<td>0.44**</td>
<td>[3.92]</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>0.45**</td>
<td>14.64 [0.71]</td>
<td>9.12 [0.95]</td>
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<td>0.008</td>
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<tr>
<td>Brent Crude Index</td>
<td>2015/9/16-2016/12/9</td>
<td>292</td>
<td>0.36***</td>
<td>1.47** [2.68]</td>
<td>2.97*** [4.44]</td>
<td>0.02*** [3.42]</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
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<td>[-2.54]</td>
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</tr>
<tr>
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<td>1.92***</td>
<td>[6.21]</td>
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<td></td>
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<td>0.76***</td>
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<tr>
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<tr>
<td>SSEC</td>
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<td>1.56*** [9.47]</td>
<td>2.49*** [11.89]</td>
<td>0.02*** [-15.29]</td>
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<td></td>
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<td></td>
<td>0.18***</td>
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<td>0.11***</td>
<td>[18.32]</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>SSEC</td>
<td>2013/12/23-2016/10/31</td>
<td>696</td>
<td>0.25***</td>
<td>4.85*** [9.23]</td>
<td>6.95*** [10.49]</td>
<td>0.31*** [11.89]</td>
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</tr>
<tr>
<td></td>
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<td>0.44***</td>
<td>[21.20]</td>
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<td>[14.23]</td>
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<td>0.14</td>
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<tr>
<td>GEI</td>
<td>2013/12/26-2016/10/31</td>
<td>694</td>
<td>0.17***</td>
<td>2.64*** [7.05]</td>
<td>4.12*** [8.90]</td>
<td>-0.10*** [-10.73]</td>
<td>0.36</td>
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<td></td>
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<td>0.54***</td>
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<td></td>
<td></td>
<td></td>
<td>0.39***</td>
<td>[14.78]</td>
<td></td>
<td></td>
<td>0.25</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.48</td>
</tr>
</tbody>
</table>
We estimate time-series regressions of the three forms

\[
\text{Volatility}_t = a + \beta_1 \cdot \text{GSI}_t \cdot D_1 (\text{GSI}_t > 0) + \beta_2 \cdot |\text{GSI}_t| \cdot D_2 (\text{GSI}_t \leq 0) + u_t
\]

where \(D_1\) and \(D_2\) are two dummy variables, \(E_t\) indicates the economic factor that may influence the volatility. For different markets, we use different economic factors. ****, ***, * denote statistical significance at the 1%, 5%, 10% level respectively. Newey-West-based t-statistics are in brackets.

<table>
<thead>
<tr>
<th>Index</th>
<th>Date range</th>
<th>(a)</th>
<th>(\beta_3)</th>
<th>(\beta_4)</th>
<th>N</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EURUSD</td>
<td>2015/10-2016/12/9</td>
<td>0.17**</td>
<td>2.64</td>
<td>4.12</td>
<td>302</td>
<td>0.003</td>
</tr>
<tr>
<td>JPYUSD</td>
<td>2015/10-2016/12/9</td>
<td>0.45**</td>
<td>14.64</td>
<td>9.12</td>
<td>320</td>
<td>0.008</td>
</tr>
<tr>
<td>Brent Crude Index</td>
<td>2015/9-2016/12/9</td>
<td>0.36***</td>
<td>1.47***</td>
<td>1.50***</td>
<td>292</td>
<td>0.10</td>
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<td>1.65*</td>
<td>4.06***</td>
<td>474</td>
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<tr>
<td>IXIC</td>
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<td>0.63</td>
<td>1.64***</td>
<td>394</td>
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<tr>
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<tr>
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<td>4.85***</td>
<td>2.10***</td>
<td>696</td>
<td>0.51</td>
</tr>
<tr>
<td>GEI</td>
<td>2013/12-2016/10/31</td>
<td>0.17***</td>
<td>2.64***</td>
<td>1.48***</td>
<td>694</td>
<td>0.36</td>
</tr>
</tbody>
</table>

We test the volatility-GSI asymmetric relation with the time-series regressions of the form

\[
\text{Volatility}_t = a + \beta_3 |\text{GSI}_t| + \beta_4 |\text{GSI}_t| \cdot D_2 (\text{GSI}_t \leq 0) + u_t
\]

where \(D_2\) are the dummy variable to distinguish negative GSI. ****, ***, * denote statistical significance at the 1%, 5%, 10% level respectively. Newey-West-based t-statistics are in brackets.
### Table 1.13 Summary of empirical results in different stages

<table>
<thead>
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<th>Core bubble Stage</th>
<th>Non-core stage</th>
</tr>
</thead>
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<tr>
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<td>a</td>
<td>( \beta_1 )</td>
</tr>
<tr>
<td>Date range</td>
<td>( a )</td>
<td>( \beta_1 )</td>
</tr>
<tr>
<td>SSEC</td>
<td>2007/03/20-2008/06/11</td>
<td>0.16***</td>
</tr>
<tr>
<td>SSEC</td>
<td>2014/11/03-2016/1/04</td>
<td>0.16***</td>
</tr>
<tr>
<td>GEI</td>
<td>2014/12/29-2016/2/06</td>
<td>0.24***</td>
</tr>
</tbody>
</table>

We estimate time-series regressions of the form

\[
\text{Volatility}_t = a + \beta_1 \cdot \text{GSI}_t \cdot D_1(\text{GSI}_t > 0) + \beta_2 \cdot |\text{GSI}_t| \cdot D_2(\text{GSI}_t \leq 0) + u_t
\]

where \( D_1 \) and \( D_2 \) are two dummy variables. *** , ** , * denote statistical significance at the 1% , 5% , 10% level respectively. Newey-West-based t-statistics are in brackets.

Core bubble stage is marked by green box be in Figure 3, Figure 5 and Figure 6. The rest is classified as non-core stage.
2. Extrapolation, Confirmation Bias and Volatility

Abstract: To explain the empirical findings that volatility is affected by individual investors’ extrapolation belief, we propose a new model in which extrapolative investors also care about information innovations, but with confirmation bias when they evaluate the new arriving information. By questionnaire survey, we give direct evidence that confirmation bias and extrapolation bias could impact individual investors simultaneously. Then by analytical proportions and numerical simulations, we show our model can provide specific links between volatility and investors’ extrapolation belief. Additionally, we find our theory can help to explain one of the most important stylized facts of volatility, the volatility clustering.

Keywords: extrapolation, confirmation bias, volatility clustering

2.1 Introduction

In the first chapter, we empirically demonstrate that volatility index is affected by individuals’ extrapolation belief in most financial markets, and when there is a bigger proportion of individual investors, the connection between them is stronger. Also, it is shown in the first chapter that there is no satisfying theory or model in previous papers that can help us to explain this phenomenon. To address this question, we propose a
new extrapolative model with two significant modifications with previous models. Firstly, in our model, extrapolative individual investors also pay attention to information innovations. Secondly, because of the confirmation bias, their evaluation of information innovations is influenced by their extrapolation belief.

Information need of individuals is what the most extrapolative models don’t take into account. The most common assumption of existing extrapolation models is that individual investors only care about past price changes instead of paying attention to information innovations. Although this concise framework does successfully explain some market anomalies, it is too simple to be realistic. As illustrated by Barker and Haslem (1973), individual investors appear to have “a strong demand for information about product safety and quality, and about the company’s environmental activities”. They also find individual investors will try to use many different methods to get information. Their finds are future supported by lots of following research, such as Chenhall and Juchau (1977), Epstein and Freedman (1994), Lawrence (2013), Xiong et al. (2013). With the fast-growing impact on the internet, information is much easier to access for individual investors. Li and Liu (2017), using in-depth interview results conducted in China, show that the internet has turned to be the principal information channel for individual investors—they will seek different types of information online, such as economic information, individual stock information, economic information, policy information and so on. They also argue the abundant information sources online can help individual investors to improve their investment.
Nevertheless, even if individual investors are eager for seeking information, they exhibit many biases when evaluating information as well as making decisions, for example, *excessive optimism* (Meinert, 1991; Golden, Miliewicz, and Herbig, 1994; Brown and Cliff, 2005), *overconfidence* (Cooper, Folta, and Woo, 1995; Hirshleifer, 2001; Barber and Odean, 2002), *familiarity heuristic* (Ashcraft, 2006; Shefrin, 2007).

*Confirmation bias* is one of the most important biases commonly found among individuals.

*Confirmation bias* is a cognitive bias which makes people be more willing to accept information which coordinates their previous beliefs. But they are reluctant to receive information that is contradictory to their previous views (Shefrin, 2007). Attitude polarization can also be a consequence of this bias which means people with different prior beliefs can renew their belief in entirely different ways for the same information.

Been firstly established by Lord, Ross, and Lepper (1979), it has been widely demonstrated by psychological researchers like Rabin and Schrag (1999) and Bodenhausen (1998). Forsythe, Nelson, Neumann, and Wright (1992) use data from the Iowa Political Stock Market and find that after the third running debate, people become more confident that the candidate they supported before is more likely to win the election.

There is clear evidence indicating *confirmation bias* is common within investment decision process too. Using an experimental market, Forsythe et al. (1992) show that confirmation bias will make investors perform worse than other investors who are not
suffered from this bias. Duong, Pescetto and Santamaria (2014) build a well-defined framework to empirically test the Confirmation bias in UK stock market. Their empirical result shows that pessimistic investors tend to underreact to good financial news while optimistic traders often fail to update their expectation for bad innovations. However, all the investors will even overreact to information which is constant with their prior beliefs—a clear evidence of Confirmation bias. Similarly, using field experimental data of 502 investor responses from South Korea, Park, Konana and Kumar (2010) find investors would like to seek information which can support their prior prediction from the virtual communication, and this propensity of investors would lead investors to make detrimental decisions. Even more sophisticated experts will commit this bias. By analyzing the Syntex Corporation case, Shefrin (2007) demonstrates the managers of big companies will sometime make unreasonable choices because of this bias.

To test more directly whether individual investors are affected by confirmation bias as well as extrapolation bias, we use online questionnaire survey to learn individual investors’ behaviors when facing real investment questions. In our survey, we first investigate what our respondents will take into account when evaluating a stock. After that, respondents are shown with two symmetric stock charts, a price-rising one and a price-declining one. Then, they are asked which news will have a bigger impact on stocks’ future returns, namely “net profit will increase 5% than that in pre-disclosure” or “net profit will decrease 5% than that in pre-disclosure” (for detail, see Appendix
3). The price charts are used to test if respondents are impacted by *extrapolation bias* while the symmetry of two charts ensures their extrapolation belief will have the same magnitude. Additionally, according to the classic Discounted Cash Flow model (DCF model), two news should have the same impact on stock’s future return. If investors can evaluate two news with no bias, they will be neutral with them.

On the contrary, our survey results show that they are holding biases when choosing two options, as only about 30% of the respondents believe two news will have the same impact on the stocks. On the contrary, more than 42% of our respondents choose the positive news for the price-rising stock while more than 46% of them believe the negative news will have a bigger impact on the price-declining stock. As we demonstrated above, *extrapolation bias* makes investors believe the price will continue its trend, and the *confirmation bias* causes them to overreact to information which is consisted of their prior belief. Therefore, this gives us solid evidence that most of the individual investors are affected by *confirmation bias* and *extrapolation bias* at the same time. Additionally, our survey also suggests that despair of their background, respondents tend to be more easily affected by those two bias during the bear market, as more respondents choose negative news for the price-declining stock.

Based on our questionnaire survey findings and in the light of the previous works by De Long et al. (1990), Barberis and Shleifer (2003), Barberis et al. (2015) and Barberis et al. (2016), we build a new extrapolative model in which two types of investors try to allocate their wealth between two assets, a risk-free one and a risky one. Like in
previous models, the first type investors are called fundamentalists, who will try to arbitrage against the mispricing, buying in devalued assets and selling out overpriced ones. The second type is also called extrapolator as in previous models, whose expectations about future returns of the risky asset are affected by their *extrapolation bias*, believing the risky asset’ price will keep its previous trends. The fundamental value of the risky asset is determined by exogenous cash-flow innovations while the actual price is the equilibrium result of two types of investors’ expectations.

The first departure of our model from earlier ones is that, extrapolator’s expectations are also affected by cash-flow innovations. Existing researches like Epstein and Freedman (1994), Lawrence (2013), Xiong et al. (2013), and our survey results all stress the information needs of individual investors. Hence it is more reasonable to formulate individuals’ expectations with the impact of information innovations. The second modification is that extrapolators are also suffered from the *confirmation bias* in the way that extrapolator would be more willing to receive the information which is consistent with their extrapolation beliefs, but will underreact to those which are not. In other words, extrapolative investors will enforce their expectation by new arriving information if the new information is consistent with their prior extrapolation belief. When contradictive information comes, they may also update their expectation but to a much less extent. Nevertheless, in both circumstances, extrapolator’s expectation about the future returns are the combination of extrapolation belief and the biased update caused by information innovations. In our model, it is this update that makes individual
investors’ expectations be a fluctuating process instead of a deterministic one as in most earlier works.

By proposition analyses and high-frequency simulations, we show directly how our framework explains why individuals’ extrapolation belief can affect volatility. As the price is endogenously determined by two types of investors’ decisions, the volatility comes from two resources: the volatility of fundamentalists’ expectations which is determined by the exogenous process of information innovations, and the volatility of extrapolators’ expectations which is determined by both the information innovations as well as their extrapolation belief. Hence, the volatility is affected by extrapolators’ extrapolation belief through their irrational trading behavior. Naturally, if the proportion of extrapolators is bigger, the volatility will be more determined by extrapolators’ extrapolation belief. We confirm this conjecture by proposition analyses and high-frequency simulations. This coordinates with our empirical conclusion in **Chapter 1** that the explanatory power of extrapolation belief to volatility is determined by the proportion of extrapolators.

Besides, as found in our questionnaire survey, respondents are more easily be affected by their *confirmation bias* during the bear market. In **section 1.3**, we show this phenomenon can explain the asymmetric GSI-volatility relation that volatility is more easily affected by individuals’ extrapolation belief during declining market (negative GSI), another important empirical finding of **Chapter 1**.
Our model also shows that the determinant power of extrapolation belief to volatility is affected by the magnitude of extrapolation belief itself. Because when the magnitude of extrapolation belief is bigger, extrapolators will trade more aggressively. Thus, the price will be more affected by their expectations, and the volatility is more determined by their extrapolation belief. To prove this finding, we empirically test it using the same data of Chapter 1 by the regressions of the form

\[ \text{Volatility}_{t} = a + \beta_{1} \cdot |GSI_{t}| \cdot D_{1}(GSI_{t} \geq 0) + \beta_{2} \cdot |GSI_{t}| \cdot D_{2}(GSI_{t} < 0) + \beta_{3} \cdot (GSI_{t})^{2} + u_{t} \quad (1) \]

As a result (see Appendix Table 2.7), the parameter of \(|GSI_{t}|^{2}\) is significant for several markets, which partly proves our prediction.

Additionally, we find our model can help to explain volatility clustering, a well-known distinguished fact of volatility. Volatility clustering means high volatilities tend to appear at the same short period but low volatility is always accompanied by low volatility (Wang et.al. 2016). This phenomenon is extensively discovered in most of the financial markets. It is also wildly recycled in statistical models such as ARCH and GARCH models. Pitifully, little theoretical papers try to explain why volatility clustering occurs. On the contrary, our model offers a natural explanation about volatility clustering. In our model, the volatility is proportional with individuals’ extrapolation belief which decays quite slow. In other words, the clustering Extrapolation Belief drives volatility to cluster. Our model predicts the volatility clustering will be more significant when individual investors dominate the market. By
comparison of the real financial markets and our simulation result, we show our model
can greatly reveal the volatility clustering in the real world.

In summary, our analysis shows that, by combining individual investors’
confirmation bias and extrapolate bias, our model not only can give theoretical
explanations of the empirical conclusion of Chapter 1, but also can lead us to get new
findings. To our best knowledge, our model is the first to combine two individual
investors’ biases together. Earlier models based on just one bias may be able to explain
some market phenomenon, our model seems more fruitful. Therefore, our model
provides a new method to study investors’ behavior.

In section 2.2, we illustrate our questionnaire survey in detail. The new model is
presented in Section 2.3. Section 2.4 introduces the high-frequency simulation method
and our simulated results. Section 2.5 concludes.

2.2 Questionnaire Survey

To learn more directly about how individual investors are affected by extrapolation
bias and confirmation bias, we use questionnaire survey method to investigate investors’
behavior. In our survey, we ask investors to choose which news will have a bigger
impact on the stock which has an obvious trend. The trend will make investors have a
significant extrapolation belief if they are affected by the extrapolation bias. The
positive news and negative news are designed to have the same size of impacts on
stock’s future return according to classical pricing theory. Using this question, we can
test if respondents are influenced by behavioral biases.
To conduct this survey, we post our questionnaire on www.xueqiu.com, one of the most active social platform used by Chinese investors, and ask investors (users of www.xueqiu.com) to answer our questionnaire. Our questionnaire and survey result are also available at https://www.wjx.cn/report/18183893.aspx where just questionnaire in Chinese is provided as we only conduct our survey among Chinese investors. In the end, 233 valid samples are received. For the detail of our questionnaire, please see Appendix 3.

As shown in Appendix 3, our questionnaire consists of two parts, namely Background Survey and Behavior Survey. To verify whether our sample is as good represent of the whole population, we investigate participators’ background and compare the survey result to official data drawn from China Securities Investor Survey Report (2012) by China Securities Investor Protection Fund Corporation (SIPF), an official organization established by China Securities Regulatory Commission. In the first part, we ask respondents’ education level (Question 1), their investment value (Question 2) and investment experience (how long have they invested in the stock market, Question 3). Survey result and official data are summarized in Table 2.2 for comparison. As shown in this table, the distribution of the results for all three questions are in line with official investigation data, showing that our samples are a qualified representation for all Chinese stock market investors. Especially, for the second question, namely investment value, about 42.49% of participators invest less than 100 thousand Chinese Yuan (about 15K USD dollars) in the market, and more than 75%
participators’ investment is under 500 thousand Chinese Yuan (about 77K USD dollars). In other words, most of the investors in Chinese are small investors or retail investors.

For the next part of our survey, we formally collet data to test our assumption. Question 4 asks investors what they will take into account when evaluating a stock. In this question, we use the phrase “technical analysis” which means “to identify trend changes at an early stage and to maintain an investment posture until the weight of the evidence indicates that the trend has reversed” (Pring 2002). As Allen and Taylor (1990) describe “…those traders who employ chart analysis - i.e., those who base their strategies on the analysis and extrapolation of past price movements alone”.

*Extrapolation bias* is thought to be the most important psychological reason for individual investors to seek patterns where there are not (De Bondt 1998). As a result, only 4.7% investors claim they just use technical analysis (price trends) to select stocks and pay no attention to the fundamental value (information releasement). Likewise, about 10% percent of our samples only care about the fundamental value. On the contrary, although with different importance, the majority of the investors care both the trend of price and information innovations. These results suggest that when modeling individuals’ behavior, it is more reasonable to consider the impact of both the information innovations and the price trends analysis (extrapolation belief).

Question 5 investigates how investors seek information. The underlying assumption of this question is that if investors try to estimate the fundamental value of a stock, the correct way should be collecting all the available information instead of just paying
attention to the recent news. Nevertheless, even if more than 85% respondents assert they pay attention to information releasement, only about 32% respondents choose to seek all the available information. Thus, we may get the conclusion that, most of our respondents cannot correctly evaluate the fundamental value of a stock and the most crucial signal for individual investors is the trend signal. This conclusion is consistent with the survey result of China Securities Investor Protection Fund Corporation. In their annual China Securities Investor Survey Report (2011), they find more than 55.35% of investors rely on technical analysis when evaluating stocks, and 44.8% of investors claim they value fundamental analysis more, but still they will take technical analysis as a reference.

Question 6 and Question 7 test if confirmation bias and extrapolation bias can affect investors when they evaluate the news. In Question 6, the chart of a stock which displays an apparent positive (rising) price trend is first showed to respondents, and then we ask them to choose which news will have a bigger impact, or they will have the same impact. Question 7 changes the chart into one with a symmetric but declining price trend and asks respondents to choose with same options. Two news, net profit rising 5% or declining 5%, should have the same impact according to traditional pricing theory. However, according to our survey result which is reported in Appendix Table 2.3, only 32% respondents perform as traditional pricing theories predict. On the contrary, more than 40% respondents will choose the positive news for a price-rising
stock while more than 46% respondents believe the negative news will have a bigger impact on the price-declining stock.

This result gives us clear proof of confirmation bias and extrapolation bias. As demonstrated in Chapter 1 and above, extrapolation bias makes investors believe the price trend will continue. An upward price trend makes extrapolative investors optimistic while a downward trend frustrates them. In Question 6, optimistic investors are more declined on the positive news while in Question 7, pessimistic investors believe the negative news will have a bigger impact. This survey suggests that when answering Question 6 and Question 7, extrapolation bias and confirmation bias affect most of the individual investors at the same time. Therefore, when studying individual investors’ behavior, it is reasonable to combine their extrapolation bias and confirmation bias.

We also compare the performance of different investors with different backgrounds. As a result, we find unsophisticated investors are more likely to be affected by their bias. Like in Table 2.3, respondents are divided according to their investment experience. For the respondents with only one-year investment experience, more than 60% of investors will choose the news which coordinates with the price trend, but only about 20% of these inexperienced investors can correctly estimate two options. Similarly, for the respondents who only started investing stock three years ago, about 24% investors using the right method while 42% people choose positive news for Question 5 and 46% people choose negative news for Question 6, still bigger for the
whole population. This situation gets better for next two group of more experienced investors, as more than 36% of them believe the two news have the same impact while much less proportion of them will choose the news consisting with the price trend.

Similarly, Table 2.4 compares investors with different education levels. As shown in this table, although the proportion of respondents who choose the third option is almost the same for different groups, the investors without undergraduate education are more likely to choose the news consisting with the price trend while the post-graduate respondents are unlikeliest affected by confirmation bias. These comparisons indicate that extrapolation bias and confirmation bias are more significant for unsophisticated investors.

Another noticeable phenomenon is that our respondents appear to be more affected by confirmation bias when the stock price declines. As shown in Table 2.3 and Table 2.4, about 40% of the whole samples choose the positive news for a price-rising stock, but more than 45% of them believe the negative news will have a bigger impact on a price-declining stock. And, this phenomenon holds true for almost all the groups of respondents. Additionally, more people choose the third option (the right option according to classical pricing theory) when facing an upward valued stock, despite their different background. The reason lying behind may be that investors are more easily panicked during the bear market and hence be affected by irrational beliefs.

As a conclusion of our questionnaire survey, at least several findings can be established:
1) Like price trend (technical analysis), information (news) is also important for individual investors to make investment decisions.

2) Price trend impacts individual investors when they try to evaluate the information, which means

3) Individual investors are affected by extrapolation bias and confirmation bias at the same time, especially for unsophisticated individual investors.

4) It seems that, in the bear market, individual investors are more likely to be affected by this two bias.

By these findings and in light of previous works, we build a new extrapolative model which is presented in next section.

2.3 The model

Based on our survey findings and in the light of earlier works (Hong and Stein 1999, Barberis and Shleifer 2003, Barberis et al. 2016), we build a new extrapolative discrete time model with the modification that extrapolators are also suffered from confirmation bias. Then we use analytical proportions to show how our model can explain the empirical findings of our first chapter.

2.3.1 Model Introduction

There are two assets in our new model: one risk-free asset which is in perfectly elastic and one risky asset which has a fixed amount of $Q$. The risk-free asset earns a constant return which is normalized to zero while the dividend of the risky asset is announced at
each period but will only be paid at the last period. The logarithm evolution of $D_T$ is given by

$$D_T = D_0 + \varepsilon_1 + \cdots + \varepsilon_t + \cdots + \varepsilon_T \quad \text{and} \quad \varepsilon_t \sim N(0, \sigma^2_\varepsilon) \quad \text{i.i.d.} \quad (2)$$

where $\varepsilon_t$ is the information released to all investors at period $t$. Actual price of the risky asset in our model is determined by the interaction of two kinds of investors, fundamental traders (fundamentalists) and extrapolators. All of the investors exhibit the same CARA (constant absolute risk aversion) risk $\gamma$, and choose to maximize his utility determined by his investment performance in next period. Assume at time $t$, he owns a risk-free wealth $W_t$, when he chooses to invest the risky asset which will pay back at the next time $t+1$. His wealth will follow

$$W_t \to W_t + N_t \cdot \overline{R_{t+1}} = W_t + N_t (\overline{P_{t+1}} - P_t) \quad (3)$$

where $N_t$ is his share demand at time $t$, $\overline{R_{t+1}}$ is his expected return of the asset, $P_t$ denotes the current logarithm price when $\overline{P_{t+1}}$ means his expected logarithm price at time $t+1$. Then his utility objective will be

$$U_t = \max_{N_t} E_t (-e^{-\gamma(W_t + N_t \overline{R_{t+1}})}) \quad \text{or} \quad \max_{N_t} E_t (-e^{-\gamma(W_t + N_t (\overline{P_{t+1}} - P_t)}) \quad (4)$$

Assume $\overline{R_{t+1}}$ is norm-distributed which indicates his utility objective $U_t$ is logarithm distributed. Then

$$E(U_t) = -e^{-\gamma(W_t + N_t \overline{R_{t+1}})} + \frac{1}{2} \text{var}(-\gamma(W_t + N_t \overline{R_{t+1}})) \quad (5)$$

denote
\[ \Omega = -\gamma (W_t + N_t \overline{R_{t+1}}) + \frac{1}{2} Var(-\gamma (W_t + N_t \overline{R_{t+1}})) \] (6)

To maximize \( \exp(U_t) \) equals to minimize \( \Omega \). From the first-order condition that

\[ \frac{\partial \Omega}{\partial N_t} = -\gamma \overline{R_{t+1}} + \gamma^2 N_t Var(\overline{R_{t+1}}) \] (7)

then, we can get the time \( t \) per-capita optimal demand as

\[ N_t = \frac{E_t(\overline{R_{t+1}})}{\gamma Var(\overline{R_{t+1}})} = \frac{E_t(\overline{P_{t+1}} - P_t)}{\gamma Var(\overline{P_{t+1}} - P_t)} \] (8)

Equation (6) means the demand of every trader is determined by the gap between his Acceptable Price in time \( t+1 \) and the current price. When he expects the price at time \( t+1 \) will surpass the current price, he will decide to buy in shares and increase his demand of the risky asset. On the contrary, if he has a pessimistic expectation of the price changing, he will reduce his demand and sell his shareholdings to other counterparts.

2.3.1.1 Optimal Demand of Fundamentalists

Fundamentalists, which make up a fraction of \( \mu^F \), are capable of correctly processing all available information and observing the fundamental value. Their demand is determined by the differences between the current price and the fundamental value. Assume the fundamentalists are bounded rational investors in the way that even though they understand the gap between the two prices is caused by other investors whose belief are different with themselves, they do not trade on the likeliness of the price gap may grow bigger but even and rather choose to believe the mispricing will disappear in the next period.
In the light of Barberis et al. (2016), we use a backward induction method to get their time $t$ optimal demand as follows. At the final period, date $T$, the price of the risky asset $P_T$ must equal the cash flow realized on that date, so that $P_T = D_T$. At time $T-1$, the fundamental trader’s first-order condition implies that his share demand is

$$N_{T-1}^F = \frac{E_{T-1}(\hat{p}_T^F) - P_{T-1}}{\gamma \cdot \text{Var}(\hat{p}_T^F - P_{T-1})} = \frac{E_{T-1}(D_T) - P_{T-1}}{\gamma \cdot \text{Var}(E_0(D_T) - P_{T-1})} = \frac{D_{T-1} - P_{T-1}}{\gamma \cdot \sigma_e^2}$$  \hspace{1cm} (10)

where we have used the fact that $E_{T-1}(D_T) = D_{T-1}$, and have also assumed, for simplicity, that the fundamentalists set the conditional variance of price changes equal to the variance of cash-flow shocks. When there are only fundamentalists in the market, market clearing implies

$$N_{T-1}^F = Q$$  \hspace{1cm} (11)

Which means

$$P_{T-1}^F = D_{T-1} - \gamma \cdot \sigma_e^2 \cdot Q$$  \hspace{1cm} (12)

The superscript $F$ in (9) means the price is solely determined by fundamentalists, i.e., the fundamental value of the risky asset. As to the time $T-2$ demand of fundamentalists, it also follows the equation

$$N_{T-2}^F = \frac{E_{T-2}(\hat{p}_T^F) - P_{T-2}}{\gamma \cdot \text{Var}(\hat{p}_T^F - P_{T-2})}$$  \hspace{1cm} (13)

As we have demonstrated that fundamentalists hold bounded rationality belief that they ignore other traders’ impact to the market and simply believe the price will convert
to the fundamental value at the next time. In other words, in their mind, \( E_{T-2}(\hat{P}_{T-1}^F) = P_{T-1}^F \), then

\[
N_{T-2}^F = \frac{E_{T-2}(\hat{P}_{T-1}^F) - P_{T-2}}{\gamma \text{var}(\hat{P}_{T-1}^F - P_{T-2})} = \frac{E_{T-2}(P_{T-1}^F - \gamma \sigma^2 \varepsilon^2) - P_{T-2}}{\gamma \text{var}(\varepsilon_i(\hat{P}_{T-1}^F - \gamma \sigma^2 \varepsilon^2) - P_{T-2})} = \frac{D_{T-2} - \gamma \sigma^2 \varepsilon^2 - P_{T-2}}{\gamma \sigma^2 \varepsilon^2} \tag{14}
\]

In equation (13), we also suppose he will take conditional variance of price changes as \( \sigma^2 \varepsilon \) in the same pattern. Similarly, when market is full of fundamentalists, the equilibrium price will be

\[
P_{T-2}^F = D_{T-2} - 2 \cdot \gamma \cdot \sigma^2 \varepsilon \cdot Q \tag{15}
\]

With this process continues, we can easily get the low frequency optimal demand of fundamentalists at time \( t \) as

\[
N_t^F = \frac{D_{T-(T-t-1)} - \gamma \sigma^2 \varepsilon^2 - p_t}{\gamma \sigma^2 \varepsilon} \tag{16}
\]

then, the risky asset’s fundamental value will be determined when only fundamentalist exist in the market \( (N_t^F = Q) \), then

\[
P_t^F = D_t - (T - t) \cdot \gamma \sigma^2 \varepsilon Q \tag{17}
\]

So, in this situation, the expected return at time \( t \) will be

\[
R_t^F = P_{t+1}^F - P_t^F = \varepsilon_t + \gamma \cdot \sigma^2 \varepsilon \cdot Q \tag{18}
\]

and

\[
E(R_t^F) = \gamma \sigma^2 \varepsilon Q \quad \text{and} \quad \text{var}(R_t^F) = \sigma^2 \varepsilon^2 \tag{19}
\]
Equation (14) means, if the risky asset price follows the fundamental value, investors (fundamentalists) will earn an expected return of \( \gamma \sigma^2 Q \) for his risk tolerance. And the return volatility of this risky asset just equals the variance of cash-flow shocks. For the common situation when extrapolators are also participating, Equation 11 can be rewritten as

\[
N_t^E = \frac{P_t^F + \gamma \sigma^2 Q - P_t}{\gamma \sigma^2} \tag{20}
\]

which describes the fact that fundamentals’ demand is determined by the gap between the fundamental value and the current price, as well as the risk compensating \( \gamma \sigma^2 Q \).

2.3.1.2 Optimal demand for extrapolators

Like fundamental investors, the optimal demand of extrapolators is also determined by

\[
N_t^E = \frac{E_t \left( R_{t+1}^E \right)}{\gamma \text{Var}(E_t(R_{t+1}^E))} = \frac{E_t \left( P_{t+1}^E \right) - P_t}{\gamma \text{Var}(E_t(R_{t+1}^E))} \tag{21}
\]

where \( E_t \left( R_{t+1}^E \right) \) means extrapolators’ expected return for the next trading period. As we have demonstrated above, there are two sources where extrapolators’ expectation return comes from. The first one is their extrapolation belief. Following previous researches, we formulate this extrapolation belief as

\[
X_t = \sum_{k=1}^{t-1} \lambda^k \beta (P_{t-k} - P_{t-k-1}) \tag{22}
\]

where \( 0 < \lambda < 1, \beta \) means extrapolation coefficient, \( \lambda \) is the memory effect which is well documented by Greenwood and Shleifer (2014) that extrapolative investors’
expected return is “a weighted average of past price changes with more recent price changes weighted more heavily”. This specification in (17) is widely used to reveal how individuals’ extrapolation belief can cause many market anomalies such as overreaction and under-reaction of price to information, momentum trading, stylized trading and other market anomalies (De Long et al. 1990, Hong and Stein 1999, Barberis and Shleifer 2003, Barberis et al. 2015, Barberis et al. 2016). Also, it is similar to that used in Chapter 1. But because in our model setting, there is a constant expected return $\exp(R^E_t) = \gamma \sigma^2 \xi$ for the risk tolerance, and we assume extrapolators understand this. Accordingly, we modify (17) as

$$X_t = \sum_{k=1}^{n} \lambda^k \beta (P_{t-k} - P_{t-k-1} - \eta) = \lambda X_{t-1} + \lambda R^E_t$$

(23)

$\zeta$ denotes extrapolators’ initial enthusiasm which is set as $\zeta = 2\gamma \sigma^2 Q$, where $\eta = \gamma \sigma^2 Q$ means extrapolators only care about the extra price changes $R^E_t$. For the necessity of this modification, see Appendix 2.1.

The second factor impacting extrapolators’ expectation is the information innovations. As we have already demonstrated, information innovations are essential to individual investors when they make decisions. Extrapolators will update their expectations by observing the information process. Also, as we have proved, because of the confirmation bias, their expectation evaluation is a biased process in the way that extrapolative investor will enforce their expectation by new arriving information when it is consistent with their prior extrapolation belief. Nevertheless, when contradictive
information comes, they also update their expectation but to a much less extent.

Therefore, the expected price change of extrapolators is modified as

\[
X_t = \begin{cases} 
X_t \cdot e^{e_t} & X_t \geq 0 \\
X_t \cdot e^{-e_t} & X_t < 0
\end{cases}
\]

(24)

This equation illustrates the two recourses of extrapolators expectation, the extrapolation belief \(X_t\), and the impact \(X_t \cdot (e^{e_t} - 1)\) coming from the information innovations and their confirmation bias.

As shown in Figure 2.3, at time \(t - 1\), with an existing price changing process \(P_0 \rightarrow P_{t-1}\), extrapolators will believe the price will continue to change by \(X_t\). Because of the information innovation, they will adapt this price change into \(X_t\). For simplicity, we assume they believe this price change can be realized in one single trading period. Therefore, in their expectation

\[
E_t(\hat{r}^E_{t+1}) = X_t \quad N_t^E = \frac{X_t}{\gamma \sigma^2_x}
\]

(25)

Hence, their optimal demand will be \(N_t^E = \frac{X_t}{\gamma \sigma^2_x}\), if they only care about their extrapolation belief (where we suppose that extrapolators set the conditional variance of \(E_t(\hat{r}^E_{t+1})\) equals to the information innovation variance \(\sigma^2_x\)). But, as we show in our survey, a great proportion of individual investors claim they also care about the fundamental value, although it seems they use incorrect methods. So, we assume extrapolators in our model pay limited attention to the fundamental value, which causes their demand become...
where we keep the weight that they put in fundamental value be a minute value (0.1 in this paper). With a large size of $\phi$, this equation ensures extrapolators’ demand is mainly determined by their extrapolation belief.

2.3.2 Equilibrium Price and implications

In a market described above, the equilibrium price emerges when the sum of all participators’ demand equals the fixed supply of the risky asset, such that

$$\mu^F \cdot N_t^F + \mu^E \cdot N_t^E = Q$$  \hspace{1cm} (27)$$

where $\mu^F$ and $\mu^E$ represent the proportion of fundamentalists and extrapolators respectively, $\mu^F + \mu^E = 1$. Thus, the equilibrium price would be

$$P_t = P_t^F + \frac{\phi^E}{\mu^F + \phi^E} \overline{X}_t$$  \hspace{1cm} (28)$$

The very concise Equation (28) indicates that, in our economy, the actual price will depart from the fundamental value just because extrapolators’ irrational bias. Accordingly, the actual return would be

$$R_t = P_t - P_{t-1} = \varepsilon_t + \gamma \sigma^2 \cdot Q + \frac{\phi^E}{\mu^F + \phi^E} (\overline{X}_t - \overline{X}_{t-1})$$  \hspace{1cm} (29)$$

$$R_t = \varepsilon_t + \gamma \sigma^2 \cdot Q + \frac{\phi^E}{\mu^F + \phi^E} X_t \left( e^{e_t - \frac{1}{\lambda} e^{e_{t-1}}} \right) + \frac{\phi^E}{\mu^F + \phi^E} R_{t-1} e^{e_{t-1}}$$  \hspace{1cm} (30)$$

The proposition below lays out why our model can explain the empirical findings of the first chapter.
Proposition 1. In the economy described above, volatility is partially determined by individuals' extrapolation belief.

At the beginning of time interval \( t - 1 \rightarrow t \), with an unknown \( \varepsilon_t \) and all other variables value known in (31), the expected volatility of \( R_t \) will be

\[
E[Var(R_t|\Pi_{t-1})] = E[Var(\varepsilon_t|\Pi_{t-1})] + E[Var\left(\frac{\phi \mu E}{\mu_F + \varphi \mu E} \cdot \overline{X_t}|\Pi_{t-1}\right)] \\
+ E[Var\left(\frac{\phi \mu E}{\mu_F + \varphi \mu E} \cdot X_{t-1}|\Pi_{t-1}\right)]
\]

(31)

where \( \Pi_{t-1} \) means all the known information set at time \( t-1 \). At this circumstance, \( \overline{X_{t-1}} \) is a just known value, instead of a changing variable. Hence, when \( X_t \geq 0 \),

\[
E[Var(R_t|\Pi_{t-1})] = \sigma_e^2 + \left(\frac{\phi \mu E}{\mu_F + \varphi \mu E}\right)^2 E[Var(X_t \cdot e^{\varepsilon_t}|\Pi_{t-1})] \\
+ 2E[Cov\left(\varepsilon_t, \frac{\phi \mu E}{\mu_F + \varphi \mu E} X_t e^{\varepsilon_t}|\Pi_{t-1}\right)] \\
= \sigma_e^2 + \left(\frac{\phi \mu E}{\mu_F + \varphi \mu E}\right)^2 X_t^2 \sigma_e^2 + 2 \mu^F \cdot \frac{\phi \mu E}{\mu_F + \varphi \mu E} X_t Cov(\varepsilon_t, e^{\varepsilon_t})
\]

(32)

and when \( X_t < 0 \)

\[
E[Var(R_t|\Pi_{t-1})] = \sigma_e^2 + \left(\frac{\phi \mu E}{\mu_F + \varphi \mu E}\right)^2 E[Var(X_t \cdot e^{-\varepsilon_t}|\Pi_{t-1})] \\
+ 2E[Cov\left(\varepsilon_t, \frac{\phi \mu E}{\mu_F + \varphi \mu E} X_t e^{-\varepsilon_t}|\Pi_{t-1}\right)] \\
= \sigma_e^2 + \left(\frac{\phi \mu E}{\mu_F + \varphi \mu E}\right)^2 X_t^2 \sigma_e^2 - 2 \mu^E \cdot \frac{\phi \mu E}{\mu_F + \varphi \mu E} X_t Cov(\varepsilon_t, e^{\varepsilon_t})
\]

(33)

Equation (32) and (33) can be integrated as

\[
E[Var(R_t|\Pi_{t-1})] = \sigma_e^2 + \left(\frac{\mu E}{\mu_F + \varphi \mu E}\right)^2 |X_t|^2 \sigma_e^2 + 2 \mu^F \left(\frac{\phi \mu E}{\mu_F + \varphi \mu E}\right)^2 |X_t| Cov(\varepsilon_t, e^{\varepsilon_t})
\]

(34)
Based on equation (32), (33) and (34), the relation between the expected volatility \( Var(R_t) \) and \( X_t \) can be shown by the following figure.

**Figure 2.1  Relation between expected volatility and Extrapolation belief**

The solid line reveals what happens between volatility and extrapolation belief. The blue line denotes \( X_t \geq 0 \) while red line means \( X_t < 0 \).

Thus, volatility in our economy is just a quadratic function of the magnitude of individual’s extrapolation belief. When \( |X_t| \) grows, the expected value of volatility will increase accordingly. Also, we can see that this determinant power of extrapolation belief to volatility is also affected by the magnitude of \( X_t \) itself, for

\[
\frac{\partial E[Var(R_t)|\mathcal{F}_{t-1}]}{\partial |X_t|} = 2|X_t| \left( \frac{\phi \mu^E}{\mu^E + \phi \mu^E} \right)^2 \sigma^2_e + 2 \mu^E \left( \frac{\phi \mu^E}{\mu^E + \phi \mu^E} \right)^2 \text{Cov}(\epsilon_t, e_{t+1}) \quad (35)
\]

Then we can get our second proposition that

**Proposition 2.** The determinant power of extrapolation belief to volatility is affected by the magnitude of extrapolation belief itself.
The reason behind Proposition 2 is very intuitively. As $X_t$ or $GSI$ of the first chapter measures extrapolators’ expectation of future returns, so a large magnitude of $X_t$ (or $GSI$ ) will indicate extrapolators are extraordinary optimistic (or extraordinary pessimistic when $X_t < 0$) about the future when they will trade more aggressively. Thus, the price will be more affected by extrapolators’ irrational belief, and therefore, the volatility will be more determined by $X_t$. To test this proposition, we modify the regression equation used in 3.1, Chapter 1 as

$$Volatility_t = a + \beta_1 \cdot |GSI_t| \cdot D_1(GSI_t \geq 0) + \beta_2 \cdot |GSI_t| \cdot D_2(GSI_t < 0) + \beta_3 \cdot (GSI_t)^2 + u_t$$ (36)

Table 2.6 summarizes the new empirical test result which gives us evidence of Proposition 2. As shown in this table, for three market samples, the Brent Crude Oil market, the Japanese stock market, and the SSEC index (2005-2008) of Chinese stock, the parameter of $GSI_t^2$ are all significant, which indicates that the magnitude of extrapolation belief significantly affects its connection with volatility index for these samples. The insignificant regression result for GEI index and SSEC index (2014-2016) may be caused by fact that individuals’ trading has already largely impact the market since their trading proportion is more than 80%. Then, the increasing GSI may not help to continue increasing their impact to the market. Naturally, for the currency market, where individual investors only have minimum impact to the market, no significant test result can be found.

Proposition 1 shows why our model can theoretically explain the empirical finding of our first chapter that the changing extrapolation belief could be a reason causing
volatility to change. But **Proposition 2** extend this conclusion that the connection between extrapolation belief and volatility may be also affected by its magnitude.

Like the magnitude of $X_t$, other parameters can also affect extrapolators’ impact to the market, $\mu^E$, for example. From equation (29), we can easily get

$$\frac{\partial E[Var(R_t|\Pi_{t-1})]}{\partial |X_t|\partial \mu^E} > 0$$

(37)

which means when there is a bigger proportion of individual investors, there will be a closer correlation between the volatility and extrapolation belief, also an empirical finding in **Chapter 1**.

**Proposition 3.** *In the economy described above, where there is bigger proportion of individual investors, there will be a bigger correlation between $X_t$ and the volatility index.*

Besides, it can be easily noticed that

$$\frac{\partial E[Var(R_t|\Pi_{t-1})]}{\partial |X_t|\partial \lambda} > 0$$

(38)

which is strictly positive. Therefore, we can get the following proposition that

**Proposition 4.** *Besides the proportion of individual investors $\mu^E$, individuals’ memory effect coefficient $\lambda$ can also impact the correlation between volatility and $X_t$.*

This proposition is quite straightforward. Since $\lambda$ measures the extent individuals are affected by past price changes. For the fixed price changes, when $\lambda$ is bigger, individuals will be more impressed and have a more noticeable extrapolation belief. Accordingly, they will have a higher impact on the price and volatility.
In a word, our model provides explicit links between individuals’ extrapolation belief and the instantaneous volatility. Also, as individuals’ impact on price is determined both by the size of extrapolation belief and by the proportion of individual investors, we can also get the conclusion that the relation between volatility and \( X_t \) is affected both by individuals’ proportion \( \mu^E \) and individuals’ enthusiasm about past price changes \( \lambda \).

### 2.3.3 Volatility Clustering

**Proposition 5.** Volatility in our economic tend to “cluster”, and, the bigger the extrapolators’ proportion is, the more significant volatility clustering will be.

Proof: Denote \( \frac{\phi \mu^E}{\mu^E + \rho \mu^E} = A \), then from equation (29), we can get

\[
\begin{align*}
Var(R_t|\Pi_{t-1}) &= Var(\varepsilon_t|\Pi_{t-1}) + A^2Var(\overline{X_t}|\Pi_{t-1}) + A^2Var(\overline{X_t-1}|\Pi_{t-1}) - \\
&\quad 2A^2Cov(\overline{X_t},\overline{X_{t-1}}|\Pi_{t-1}) + 2ACov(\varepsilon_t,\overline{X_t}|\Pi_{t-1}) - 2ACov(\varepsilon_t,\overline{X_{t-1}}|\Pi_{t-1})
\end{align*}
\]

(39)

since \( \varepsilon_t \) is independent identically distributed, then

\[
Cov(\overline{X_t},\overline{X_{t-1}}|\Pi_{t-1}) = 0 \text{ and } Cov(\varepsilon_t,\overline{X_{t-1}}|\Pi_{t-1}) = 0
\]

(40)

and

\[
Var(R_t|\Pi_{t-1}) = Var(\varepsilon_t|\Pi_{t-1}) + A^2Var(\overline{X_t}|\Pi_{t-1}) + A^2Var(\overline{X_{t-1}}|\Pi_{t-1}) - 2ACov(\varepsilon_t,\overline{X_t}|\Pi_{t-1}) - 2ACov(\varepsilon_t,\overline{X_{t-1}}|\Pi_{t-1})
\]

(41)

It can easily get that
\[ R_{t+1} = P_{t+1} - P_t = \epsilon_{t+1} + \gamma \sigma_{\hat{\chi}}^2 Q + \frac{\phi \mu^g}{\mu^{E+\phi \mu^g}} (\overline{X_{t+1}} - \overline{X_t}) \] 

(42)

So,

\[ \text{Var}(R_{t+1} | \Pi_{t-1}) = \text{Var}(\epsilon_{t+1} | \Pi_{t-1}) + A^2 \text{Var}(\overline{X_{t+1}} | \Pi_{t-1}) + A^2 \text{Var}(\overline{X_t} | \Pi_{t-1}) \]

\[ 2A \text{Cov}(\epsilon_{t+1}, \overline{X_{t+1}} | \Pi_{t-1}) \]

(43)

Apparently,

\[ \text{Cov}(\text{Var}(R_{t+1} | \Pi_{t-1}), \text{Var}(R_t | \Pi_{t-1})) = A^4 \text{Var}(\overline{X_t} | \Pi_{t-1}) \]

(44)

and

\[ \text{Cor}(\text{Var}(R_{t+1} | \Pi_{t-1}), \text{Var}(R_t | \Pi_{t-1})) = \frac{\text{Cov}(\text{Var}(\overline{X_t} | \Pi_{t-1}))}{\text{Var}(R_t | \Pi_{t-1}) \text{Var}(R_{t+1} | \Pi_{t-1})} \]

\[ = A^4 \frac{\text{Var}(\overline{X_t} | \Pi_{t-1})}{\text{Var}(R_t | \Pi_{t-1}) \text{Var}(R_{t+1} | \Pi_{t-1})} \]

(45)

which proves Proposition 5.

The reason of volatility clustering in our model is very intuitive. As we demonstrate, volatility in our economy has two sources the exogenous information innovation and the endogenous product of information and Extrapolation Belief. Extrapolation belief is persistent as it is the weighted average of past returns—a large size extrapolation belief is likely followed by another large one. So, when volatility is more determined by extrapolation belief (\(|X_t|\) is large), volatility is more likely to cluster. In the simulation part, we show our model can efficiently reveal the real financial market.
2.3.4 Asymmetric Relation between Extrapolation Belief and Volatility

In (19), we assume extrapolators will be symmetrically affected by confirmation bias, however, according to our questionnaire survey result, they would be more easily influenced during the bear market. Therefore, we can modify (24) as

\[ X_t = \begin{cases} X_t \cdot e^{\epsilon_t} & X_t \geq 0 \\ X_t \cdot \rho e^{-\epsilon_t} & X_t < 0 \end{cases} \quad (46) \]

where \( \rho > 1 \) which indicates, the declining price makes individual investors be more affected by their confirmation bias. Thus (26) will become

\[ E[Var(R_{t+1}|I_{t-1})] = \mu^2 \cdot \sigma_t^2 + \rho^2 \left( \frac{\phi \mu^2}{\sigma^2 + \rho^2 \sigma^2} \right)^2 X_t^2 \sigma_t^2 + 2 \rho^2 \mu \left( \frac{\phi \mu^2}{\sigma^2 + \rho^2 \sigma^2} \right) X_tCov(\epsilon_t, \epsilon_t) \quad (47) \]

Accordingly, Figure 2.1 should be changed into

![Figure 2.2 Relation between expected volatility and Extrapolation belief](image)

The solid line reveals what happens between volatility and extrapolation belief. The blue line denotes \( X_t \geq 0 \) while red line means \( X_t < 0 \).
**Figure 2.2** indicates if individual investors are more influenced by confirmation bias in the bear market, the slope of the curve in the fourth dimension should be bigger than in the first dimension. In other words, volatility will be more affected by extrapolation belief when \( x_t < 0 \). This asymmetric relation between volatility and extrapolation belief is coordinate with the symmetric GSI-volatility relation that empirically demonstrated in the first chapter.

### 2.4 High-frequency simulation

**Figure 2.3 High-frequency Price Process**

Using the new model built in **Section 2.3**, we continue to simulate it with high-frequency method to coordinate the volatility calculating method in **Chapter 1**. In the first chapter, volatility is estimated by RV (Realized Volatility) which requires high-frequency data, but our model introduced in section 3 is designed as a low-frequency model in which investors only trade once for each period. Therefore, we discretize the basic model into a high-frequency discrete time version.
2.4.1 Simulation method

Firstly, we divide each trading period into \( n \) sub-periods. For consistency, we specify as follows: as shown in Figure 2.3, for each time interval, the beginning time \( t-1 \) is also named as \( (t-1)_0 \), and the ending point of this time \( t \) can be seen as \( (t-1)_n \). Then the low frequency price \( P_{t-1} \) will become the starting high frequency price \( p_{t-1_0} \) of time interval \( t-1 \rightarrow t \), or the ending high frequency price \( p_{t-2_n} \) for previous time interval \( t-1 \rightarrow t \). Such that

\[
P_t = p_{t_0} = p_{t-1_n}
\]

(48)

Then, we discretize the information \( \varepsilon_t \) of each period into an information set which consists of a number of temporally even distributed innovations \( \varepsilon_{t_i} \), such as

\[
\varepsilon_t = \{\varepsilon_{t_1}, \varepsilon_{t_2}, \varepsilon_{t_3} \cdots \varepsilon_{t_n}\} \quad \text{where } \varepsilon_{t_i} \sim N(0, \sigma^2_{\varepsilon}), \text{ i.i.d. over time}
\]

(49)

Then

\[
\varepsilon_t \sim N(0, \sigma^2_{\varepsilon}), \text{ i.i.d. over time where } \sigma^2_{\varepsilon} = n \cdot \sigma^2_{\varepsilon}
\]

(50)

Based on this modification, we can simply deduct the fundamentalists’ demand of every high-frequency time \( t_i \) using the same backward induction method. As in our basic model, fundamentalists also choose to maximize their utility objective

\[
\max_{N_{t_i}} E_{t_i} \left( -e^{-\gamma(W_{t_i} + N_{t_i}(p_{t_i} - p_i))} \right)
\]

(51)

In this high-frequency process, we use the lowercase \( p_{t_i} \) to distinguish it with the low-frequency price \( P_t \). Accordingly, using backward induction, fundamentalists’ optimal demand is also determined by
\[ N_t^F = \frac{E_{t_i}(p_{t_i+1}^F) - p_{t_i}}{\gamma \cdot \text{Var}\{E_{t_i}(p_{t_i+1}^F) - p_{t_i}\}} \] (52)

Similarly, we can find the fundamental value of risky asset at \( t_i \) would be

\[ p_{t_i}^F = D_{t_i} - (T - t) \cdot \gamma \sigma_e^2 Q + i \cdot \gamma \sigma_e^2 Q \] (53)

(38) can simply be rewritten as

\[ N_{t_i}^F = \frac{p_{t_i}^F + \gamma \sigma_e^2 Q - p_{t_i}}{\gamma \sigma_e^2} \] (54)

by simple calculation, we can get

\[ r_{t_i}^F = p_{t_i+1}^F - p_{t_i} = \epsilon_{t_i} + \gamma \sigma_e^2 Q \]

\[ E(r_{t_i}^F) = \gamma \sigma_e^2 Q = \frac{1}{n} R_t^F \quad \text{var}(r_{t_i}^F) = \sigma_e^2 = \frac{1}{n} \text{var}(R_t^F) \]

when \( i = 0 \) \( p_{t_0}^F = P_t^F = D_t - (T - t) \cdot \gamma \sigma_e^2 Q \)

\( i = n \) \( p_{t_n}^F = p_{t+1}^F = D_{t+1} - (T - t - 1) \cdot \gamma \sigma_e^2 Q \) (55)

As shown above, when all the investors were fundamentalists, expected return of the fundamental value for every sub-period \( t_i \) becomes \( \frac{1}{n} R_t^F \), just because the risk premium of every sub-period is just \( \text{one-nth} \) of the whole period. Return volatility of the sub-period also changes accordingly. In brief, after dividing the discrete low frequency cash flow shock \( \epsilon_t \) into a set of high frequency innovations, the fundamentalists will also adapt their demand according to the high-frequency innovation process. In this way, our model manages to derivative an inner-consistent fundamental value evolution process (both in high frequency and low frequency) by characterizing investors’ optimal demand.
With respect to high-frequency evolution process of optimal demand for extrapolators, it can also be denoted as

$$N^E_{t_1} = \frac{E_{t_1}(p^E_{t+1}) - p_{t_1}}{\gamma \text{Var}(E_{t_1}(r^E_{t,n}))} = \frac{x_{t_1}}{\gamma \sigma^2_\varepsilon}$$  \hspace{1cm} (56)

which also most the same with equation (28). Lowercase $x_{t_1}$ is also used to distinguish with low frequency expected changes. Like that in the basic model, it also has two recourses, the extrapolation belief $x_{t_1}$, and his adaptation for the sub-period information innovation, such that

$$\begin{cases}
\tilde{x}_{t_1} = e^{e_{t_1}} \cdot x_{t_1} & \text{if } x_{t_1} > 0 \\
\bar{x}_{t_1} = e^{-e_{t_1}} \cdot x_{t_1} & \text{if } x_{t_1} < 0
\end{cases}$$  \hspace{1cm} (57)

When calculating $x_{t_1}$, it should be noticed that the already happened price change $P_t(p_{t_0}) \rightarrow p_{t-1}$ would also cause investors to extrapolate just as other past price changes (say $P_{t-1} \rightarrow P_t$) do. Therefore, the price change $P_t(p_{t_0}) \rightarrow p_{t-1}$ should be considered. Because investors are more used to judge the trend according to low frequency returns (daily price changes, mostly). Besides, it is hard for investors to acquire high-frequency price process data sometimes. We also assume extrapolators only care about the whole price change of the current period, instead of the high-frequency price changing process, such that

$$x_{t_1} = \sum_{k=1}^{n} \lambda^{k+1} \beta (P_{t-k} - P_{t-k-1} - \eta) + \lambda [p_{t-1} - P_t - (i-1)\eta'] + \zeta'$$  \hspace{1cm} (58)

where $\eta' = \gamma \sigma^2_\varepsilon$ means the due return of every sub-period, and $\zeta' = 2\gamma \sigma^2_\varepsilon$. Equation (54) means, because trade happens multiple times in each trading period, extrapolators need to adapt their judgement according to the already happened price

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change in the current period $p_{t-1} - p_t$ and in previous periods. Then we can get another concise price equation of the high-frequency equilibrium process

$$p_t = p_t^F + \frac{\phi\mu^e}{\mu + \phi\mu^e} \bar{x}_t$$

(59)

2.4.2 Numerical Simulation result

Figure 2.5 of Appendix 2 plots the simulation result using the parameter value listed in Table 2.1. Especially, $\lambda$, the parameter of memory effect is set to be 0.8, just the same value we used to calculate individuals’ extrapolative belief for the empirical test in Chapter 1. It is also one of the empirical test results of Greenwood and Shleifer (2014). The proportion of extrapolative investors is set to be 80% to mimic the Chinese stock market, where the most significant empirical test result in Chapter 1 can be found.

Table 2.1 Numerical benchmark parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.8</td>
<td>Memory effect</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.2</td>
<td>Extrapolation coefficient</td>
</tr>
<tr>
<td>$\mu^F$</td>
<td>20%</td>
<td>Proportion of Fundamentalists</td>
</tr>
<tr>
<td>$\mu^E$</td>
<td>80%</td>
<td>Proportion of Extrapolators</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.1</td>
<td>Coefficient of absolute risk aversion</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>0.2</td>
<td>Volatility of high frequency information innovations</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>$\sqrt{2}$</td>
<td>Volatility of low frequency information innovations</td>
</tr>
<tr>
<td>$n$</td>
<td>50</td>
<td>Number of sub-period of one time interval</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.1</td>
<td>Extrapolators’ expectation Weight on fundamental value</td>
</tr>
</tbody>
</table>
The first picture in Figure 2.5 gives the high-frequency price process while the corresponding low-frequency price is plotted in second picture. Apparently, the high frequency picture contains more price information than the other one, which makes us be able to calculate the volatility using RV method. By the same method used in Chapter 1, Volatility and GSI (Greenwood and Shleifer Index, the indexes of individuals’ extrapolation belief) are calculated and represented in Figure 2.6.

Interesting implications can be found from our simulation. Mispricing, for instance. The actual price, plotted by the red line in Figure 2.5, departs from the fundamental value (black line) for most of the time. This departure, which reflects the influence of extrapolators’ expectation to the price, can grow enormously. In the beginning, with an initial price decline, extrapolators expect the price will continue to fall and begin to sell their shares, which makes the price decline even lower than the fundamental value. This devaluation grows severe as their extrapolation belief increases. Also, the equity can be overvalued when the extrapolators are optimistic about future price. Like that happens from \( T = 60 \), the increasing price makes extrapolator believe the price will continue rising and buy in more shares, the actual price surpasses the fundamental value accordingly. In a word, what happens in our economy is similar to the old analogy about the likeness between the stock market and a man walking his dog. The man’s course (the fundamental value) is steady but the dog (the actual price) is easily distracted and excited. Although they may reach the same goal together as the man holding the leash,
the dog’ course is hard to predict as it may “lunge at a pigeon or scurry backward in fear of a speeding bicycle” (Jorgensen, 1994).

The following several figures illustrate how our simulation can explain several stylized facts of volatility.

2.4.2.1 excess volatility

As we have analyzed, if all investors were fundamentalists, the expected value of volatility should just be the volatility of information $\sigma_e$, displayed by the black line in Figure 2.6. Nonetheless, things change when extrapolators emerge in the market. In this figure, the green line represents the actual volatility index, calculated by Realized Volatility mothed used in Chapter 1. For most of the time, the actual volatility is larger than the information volatility $\sigma_e$, showing that extrapolators’ irrational behavior will cause volatility to increase. Therefore, our model, like many previous models based on individuals’ extrapolation belief (Cutler, Poterba, and Summers, 1990, DeLong, Shleifer, Summers, and Waldmann, 1990, Barberis et.al. 2015), can explain the famous “excess volatility puzzle” which is firstly argued by Shiller that the volatility in actual price is much higher than the volatility of the cash flow innovations.

2.4.2.2 Volatility and Extrapolation Belief

More importantly, our model can explain why extrapolation belief can affect the instantaneous volatility. For simplicity, we use $\text{cor}(|GSI|, \text{Volatility})$ as an indicator of the explanatory power of extrapolation belief to the instantaneous volatility. The absolute value of Greenwood and Shleifer Index (the index of extrapolation belief) is
also plotted by the red line in Figure 2.6. Apparently, it is positively correlated with the volatility index (green line). By our simulation, if the parameters are set as Table 2.1, $\text{cor}(|GSI|, \text{Volatility}) = 0.535$, a similar result as that in Chinese stock market.

As demonstrated by Proposition 3 and 4, our model also predicts the proportion of extrapolators $\mu^E$ as well as the memory effect $\lambda$ can affect the explanatory power of extrapolation belief to volatility. We now use simulation to prove this two propositions. Figure 2.7 shows the impact of changing $\lambda$ which follows the prediction of Proposition 3, that with a bigger $\lambda$, the correlation between volatility and absolute GSI should be higher. Figure 2.8 shows the effect of changing $\mu^E$ when other parameters hold. As shown in these two figures, there is a monotone increasing relationship between the proportion of individual investors with the correlation between volatility and absolute GSI.

To show our simulation can reveal what happens in the actual market, we also list the correlation test result between volatility index and $|GSI|$ for all the real markets as well as our simulation results in Appendix 2, Table 2.6. For example, when the extrapolators proportion is about 80%, the correlation between $|GSI|$ and volatility is 0.53, almost the same as its value in Chinese stock market where about 85% of investors are individual invests. When extrapolators’ proportion drops to 25%, the similar value with the proportion of individual investors in the developed stock markets, $\text{cor}(|GSI|, \text{Volatility})$ drops to 0.27, also similar to its value in the developed stock market. When extrapolators only take just 5% of the population,
\[ \text{cor}(GSI, \text{Volatility}) \] only equals 0.067, just like what happens in the currency market.

Additionally, by Proposition 2, we suggest, the relation between volatility and individuals’ extrapolation belief is a quadratic function, and the size of extrapolation belief itself can influence its

2.4.2.3 Volatility clustering

Figure 2.9 plots the Volatility Clustering (characterized by autocorrelation of volatility index) of our simulation result and in the real markets. Figure 2.9.a compares different financial markets. For the IXIC (Index of Nasdaq Stock Market) and the N225 (index of Japanese stock market), represented by gray line and black line respectively, or for the market where individual investors have limited impact, volatility displays certain degree of clustering—the correlation decays from about 0.26 at lag 1 to about 0.1 at lag 20. As a contrast, three indexes (represented by the red, pink and purple line) of Chinese stock market, exhibit a much higher level of clustering. As can be seen in this picture, the volatility correlation at lag 1 is as high as 0.8 which reduces much slower to 0.5 at lag 20. Apparently, when the proportion of individual investors is higher, the more significant clustering volatility exhibit.

Figure 2.9.b displays our simulation result which shows how well our model can fit the real market. The red line plots the volatility autocorrelation when extrapolators take a proportion of 80%, while the black line shows when only 20% investors are extrapolators. As what happens in Chinese stock market, the correlation also decays
from around 0.8 at lag 1 to about 0.4 at lag 20. And, just like in the developed market, the simulated volatility autocorrelation also decreases from 0.25 to about 0.1. This well fitted simulation result pictorially explains our models’ ability to explain the volatility clustering.

To summarize, our simulation results prove our propositions in Section 2.3. Combining these propositions and our high-frequency simulation results, our model shows a great ability to explain the empirical findings of Chapter 1. Additionally, our model can also help to understand volatility clustering, another most important stylized fact of volatility.

2.5 Conclusion

Using the questionnaire survey, we emphasize the information needs of individual investors. We also prove individual investors tend to be influenced by confirmation bias and extrapolative bias at the same time. By these findings and in the light of earlier extrapolative models, we build a new extrapolative model in which extrapolators are also affected by confirmation bias when they evaluate new arriving information. In our model, extrapolators’ expectation of future returns is a production of their extrapolation belief and the biased update for the information innovations. Using proposition analysis and high-frequency simulations, we show that our model can efficiently explain our findings in Chapter 1 as well as the real world. Furthermore, we show that the slowing decay Extrapolation Belief can also drive volatility to cluster. By simulation we show
how well our simulation fits the real financial markets, which makes our model be more convincing.

2.6 REFERENCE:


Appendix 2.1

To explain the necessity of this modification, assume the finance market is in an absolute stable situation (by “absolute stable situation”, we mean all the information innovation equals zero, $\varepsilon_t = 0, D_T = D_0 = D_t$). Then according to (12), the fundamental price evaluation process will become a straight line like Figure 1. Although no innovation is released, fundamental investors won’t confirm this situation in advance until the last period $T$, so they will also require a risk compensation $\exp(R^F_t) = \gamma \sigma_t^2 Q$ for every investment period.
Figure 2.4 Fundamental Price process in “absolute stable” situation.

In this picture, fundamental price is denoted by the black line. “Absolute stable” situation means $\epsilon_i = 0, D_i = D = 100, \gamma = 0.01, \sigma^2 = 2, Q = 1, \gamma \sigma^2 Q = 0.2$.

Assume extrapolators are aware of this risk compensation return and observe this price change process, they will not speculate in the equity because they understand this constant price increase $\gamma \sigma^2 Q$ doesn’t mean the price gains “extra” increase, but just means a “due” rise. Accordingly, to describe their only care about the “extra increment”, we modify the equation (27) into (28).

Appendix 2.2

---

a. High-frequency price of Numerical simulation result
b. Low-frequency price of Numerical simulation result

**Figure 2.5 Numerical simulation result**

This figure plots the price process when the parameter value is set by Table 1. The black line plots the fundamental value of the asset for the same cash-flow sequence while the red line reveals the actual price.
Figure 2.6 Volatility and GSI of Numerical simulation result

Volatility is measured by RV method, the same method used in Chapter 1. Extrapolation belief is also calculated by the same method in Chapter 1.

Figure 2.7 Correlation and Changing Memory effect

This picture shows the change of $\text{corr}(\text{Volatility}_{t}, |GSI_{t}|)$ when the memory effect $\lambda$ is gradually changed from 0 while other parameters are set according Table 1.

Figure 2.8 Correlation and Changing proportion of extrapolators

This picture shows the change of $\text{corr}(\text{Volatility}_{t}, |GSI_{t}|)$ when the proportion of extrapolators $\mu^{E}$ is gradually changed from 0 while other parameters are set according Table 1.
Figure 2.9 volatility clustering in real financial markets and by our simulation result

Autocorrelation of volatility is shown in this figure. The first picture shows autocorrelation of volatility in Chinese stock market, Japanese stock market as well as Nasdaq stock market, while the second picture displays our simulation result.

Table 2.2 Background survey statistics

<table>
<thead>
<tr>
<th>Question 1: Education level</th>
<th>Survey Sample</th>
<th>Official Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Junior High School</td>
<td>2.58% (6)</td>
<td>4.51%</td>
</tr>
<tr>
<td>High School</td>
<td>12.02% (28)</td>
<td>17.52%</td>
</tr>
<tr>
<td>Undergraduate</td>
<td>67.38% (157)</td>
<td>72.56%</td>
</tr>
<tr>
<td>Post graduated and above</td>
<td>18.03% (42)</td>
<td>5.40%</td>
</tr>
</tbody>
</table>

| Question 2: Investment value                                   |               |               |
| Smaller than 100 thousand                                      | 42.49% (99)   | 52.27%        |
| (100 thousand, 500 thousand)                                  | 33.48% (78)   | 32.93%        |
| (500 thousand, 1 million)                                     | 15.02% (35)   | 8.61%         |
The second column reports statistics of our survey result about participators’ background with distributions as well as the actual number (reported in brackets). Official Data comes from China Securities Investor Survey Report (2012) by China Securities Investor Protection Fund Corporation.

### Question 3: Years of investment

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Less than 1 year</td>
<td>(1 year, 3 years)</td>
</tr>
<tr>
<td></td>
<td>10.3% (24)</td>
<td>26.61% (62)</td>
</tr>
<tr>
<td></td>
<td>4.31%</td>
<td>24.37%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100% (233)</strong></td>
<td><strong>100%</strong></td>
</tr>
</tbody>
</table>

### Table 2.3 Investors’ behavior survey statistics

<table>
<thead>
<tr>
<th></th>
<th>Survey Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Question 4: Technical Analysis VS. Fundamental Value Analysis</strong></td>
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<tr>
<td>Only rely on Technical Analysis</td>
<td>4.72% (11)</td>
</tr>
<tr>
<td>Only rely on Fundamental Value Analysis</td>
<td>10.3% (24)</td>
</tr>
<tr>
<td>Rely on both, but on Technical Analysis more</td>
<td>25.75% (60)</td>
</tr>
<tr>
<td>Rely on both, but on Fundamental Value Analysis more</td>
<td>21.46% (50)</td>
</tr>
<tr>
<td>Rely on both with the same importance</td>
<td>37.77% (88)</td>
</tr>
</tbody>
</table>
Question 5: Horizon of information seeking

<table>
<thead>
<tr>
<th></th>
<th>Percentage</th>
<th>Actual Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Several days</td>
<td>12.4%</td>
<td>(29)</td>
</tr>
<tr>
<td>Several Months</td>
<td>54.94%</td>
<td>(128)</td>
</tr>
<tr>
<td>All the times</td>
<td>32.62%</td>
<td>(76)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100%</strong></td>
<td><strong>(233)</strong></td>
</tr>
</tbody>
</table>

Statistics of our survey result about participators’ investment behavior. The actual number is reported in brackets.
### Table 2.4 Investors’ behavior survey statistics

<table>
<thead>
<tr>
<th>Years of investment</th>
<th>Whole Sample</th>
<th>Less than 1 year</th>
<th>(1year, 3years)</th>
<th>(3 years, 5years)</th>
<th>More than 5 years</th>
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<tbody>
<tr>
<td><strong>Question 6: Rising trend and information innovation</strong></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Increase 5%</td>
<td>42.92% (95)</td>
<td>60% (15)</td>
<td>43.55% (27)</td>
<td>36.73% (17)</td>
<td>35.42% (34)</td>
</tr>
<tr>
<td>Decrease 5%</td>
<td>27.04% (63)</td>
<td>16% (4)</td>
<td>33.87% (22)</td>
<td>26.53% (13)</td>
<td>28.13% (27)</td>
</tr>
<tr>
<td>Same impact</td>
<td>32.76% (75)</td>
<td>24% (6)</td>
<td>23.44% (15)</td>
<td>36.73% (18)</td>
<td>36.46% (35)</td>
</tr>
<tr>
<td><strong>Question 7: Declining trend and information innovation</strong></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Increase 5%</td>
<td>23.61% (54)</td>
<td>16% (4)</td>
<td>33.87% (22)</td>
<td>20.41% (12)</td>
<td>25% (24)</td>
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<tr>
<td>Decrease 5%</td>
<td>47.64% (109)</td>
<td>64% (16)</td>
<td>44.61% (28)</td>
<td>42.86% (19)</td>
<td>40.63% (39)</td>
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<tr>
<td>Same impact</td>
<td>30.04% (67)</td>
<td>20% (5)</td>
<td>22.58% (14)</td>
<td>36.73% (17)</td>
<td>34.38% (33)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>233</td>
<td>25</td>
<td>64</td>
<td>49</td>
<td>99</td>
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Table 2.5 Investors’ behavior survey statistics

<table>
<thead>
<tr>
<th>Education Level</th>
<th>Whole Sample</th>
<th>Middle school</th>
<th>Under-graduated</th>
<th>Postgraduate</th>
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<tbody>
<tr>
<td>Question 5: Rising trend and information innovation</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Increase 5%</td>
<td>42.92% (95)</td>
<td>44.11% (15)</td>
<td>40.51%(64)</td>
<td>39.02%(16)</td>
</tr>
<tr>
<td>Decrease 5%</td>
<td>27.04% (63)</td>
<td>23.52% (8)</td>
<td>29.75%(47)</td>
<td>29.27%(12)</td>
</tr>
<tr>
<td>Same impact</td>
<td>32.76% (75)</td>
<td>32.35% (11)</td>
<td>29.75%(47)</td>
<td>31.71%(13)</td>
</tr>
<tr>
<td>Question 3: Declining trend and information innovation</td>
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</tr>
<tr>
<td>Increase 5%</td>
<td>23.61% (54)</td>
<td>17.64% (6)</td>
<td>25.95% (41)</td>
<td>29.27% (12)</td>
</tr>
<tr>
<td>Decrease 5%</td>
<td>47.64% (109)</td>
<td>52.94% (18)</td>
<td>45.57% (72)</td>
<td>36.59% (15)</td>
</tr>
<tr>
<td>Same impact</td>
<td>30.04% (67)</td>
<td>29.41% (10)</td>
<td>28.48% (45)</td>
<td>34.15% (14)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>233</strong></td>
<td><strong>34</strong></td>
<td><strong>158</strong></td>
<td><strong>41</strong></td>
</tr>
</tbody>
</table>
Table 2.6 Summary of empirical results in different financial market

<table>
<thead>
<tr>
<th>Index</th>
<th>Date range</th>
<th>a</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>N</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EURUSD</td>
<td>2015/10/15-2016/12/9</td>
<td>0.39**</td>
<td>-0.89</td>
<td>-15.44</td>
<td>967.9</td>
<td>302</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>2015/10/15-2016/12/9</td>
<td>[5.75]</td>
<td>[-0.40]</td>
<td>[0.46]</td>
<td>[0.43]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>JPYUSD</td>
<td>2015/10/15-2016/12/9</td>
<td>0.36***</td>
<td>52.05</td>
<td>37.8</td>
<td>-1244.5</td>
<td>320</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>2015/10/15-2016/12/9</td>
<td>[2.26]</td>
<td>[1.43]</td>
<td>[1.02]</td>
<td>[-0.83]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brent Crude</td>
<td>2015/9/16-2016/12/9</td>
<td>0.41***</td>
<td>-2.61*</td>
<td>-1.39</td>
<td>57.0***</td>
<td>292</td>
<td>0.13</td>
</tr>
<tr>
<td>Index</td>
<td>2015/9/16-2016/12/9</td>
<td>[14.61]</td>
<td>[-1.62]</td>
<td>[-0.86]</td>
<td>[2.95]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N225</td>
<td>2014/12/19-2016/12/1</td>
<td>0.83***</td>
<td>-1.79</td>
<td>0.91</td>
<td>8.63***</td>
<td>474</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>2014/12/19-2016/12/1</td>
<td>[5.76]</td>
<td>[-1.27]</td>
<td>[0.58]</td>
<td>[3.33]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IXIC</td>
<td>2015/4/29-2016/11/16</td>
<td>0.15***</td>
<td>-0.8</td>
<td>2.46***</td>
<td>3.08</td>
<td>394</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>2015/4/29-2016/11/16</td>
<td>[20.69]</td>
<td>[-0.86]</td>
<td>[2.15]</td>
<td>[0.13]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSEC</td>
<td>2005/2/1-2008/12/31</td>
<td>0.11***</td>
<td>1.06***</td>
<td>1.93***</td>
<td>7.94***</td>
<td>951</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>2005/2/1-2008/12/31</td>
<td>[25.19]</td>
<td>[3.46]</td>
<td>[5.82]</td>
<td>[1.82]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSEC</td>
<td>2013/12/23-2016/10/31</td>
<td>0.11***</td>
<td>5.29***</td>
<td>7.63***</td>
<td>-7.06</td>
<td>696</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>2013/12/23-2016/10/31</td>
<td>[14.63]</td>
<td>[6.09]</td>
<td>[5.38]</td>
<td>[0.38]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEI</td>
<td>2013/11/26-2016/10/31</td>
<td>0.18***</td>
<td>2.42***</td>
<td>3.79***</td>
<td>2.74</td>
<td>694</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>2013/11/26-2016/10/31</td>
<td>[15.16]</td>
<td>[4.58]</td>
<td>[4.98]</td>
<td>[0.43]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 summarize estimate results of the regressions form

$$Volatility_t = a + \beta_1 \cdot |GSI_t| \cdot D_1(GSI_t \geq 0) + \beta_2 \cdot |GSI_t| \cdot D_2(GSI_t < 0) + \beta_3 \cdot (GSI_t)^2 + u_t$$

***, **, * denote statistical significance at the 1%, 5%, 10% level respectively. Newey-West-based t-statistics are in brackets.
### Table 2.7 Comparison of Correlations between |GSI| and Volatility

<table>
<thead>
<tr>
<th>Index</th>
<th>Date range</th>
<th>Correlation Between</th>
<th>Individual</th>
<th>Correlation Between</th>
<th>Proportion of extrapolators</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>GSI and Volatility</td>
<td>trading</td>
<td>GSI and Volatility</td>
</tr>
<tr>
<td>EURUSD</td>
<td>2015/10/15-2016/12/9</td>
<td>0.09</td>
<td>&lt;5%</td>
<td></td>
<td>0.068</td>
</tr>
<tr>
<td>JPYUSD</td>
<td>2015/10/15-2016/12/9</td>
<td>0.06</td>
<td>&lt;5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brent Crude Index</td>
<td>2015/9/16-2016/12/9</td>
<td>0.26</td>
<td>25%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N225</td>
<td>2014/12/19-2016/12/1</td>
<td>0.27</td>
<td>23.5%</td>
<td></td>
<td>0.27</td>
</tr>
<tr>
<td>IXIC</td>
<td>2015/4/29-2016/11/16</td>
<td>0.20</td>
<td>&lt;30%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSEC</td>
<td>2005/2/1-2008/12/31</td>
<td>0.52</td>
<td>85%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSEC</td>
<td>2013/12/23-2016/10/31</td>
<td>0.65</td>
<td>83%</td>
<td></td>
<td>0.53</td>
</tr>
<tr>
<td>GEI</td>
<td>2013/11/26-2016/10/31</td>
<td>0.56</td>
<td>85%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We use the same method to calculate volatility and GSI index for all real markets and our simulated economy. Volatility is calculated in the approaches of realized volatility with 5-min intraday data. GSI is calculated by $|GSI_t| = \left| \sum_{t=1}^{\lambda} R_t \cdot \lambda \right|$, where $R_t$ means the return at time $t$, $\lambda$ governs the weights investors put into each period which is set as $\lambda = 0.8$ according to Greenwood and Shleifer (2014).
Appendix 2.3 Questionnaire

Background survey

1. Your graduation

A: Below undergraduate  B: Undergraduate  C: Postgraduate

2. The total value of your investment in stock market.

A: <10K  B: 10k< <100K.  C:100k< <500K.  D:500K<<1000K.  E:>1000K

3. How long since you first invested in the stock market

A: Less than 2 years  B: more than 2 years but less than 5  C: >5

Behavior survey

1. Which one will you choose to evaluation a stock

A: I only care about the trend (Technical Analysis)

B: I only care about the information (Fundamental Value).

C: I pay attention to both trend and information, but I think trend (Technical Analysis) is more important

D: I pay attention to both trend and information, but I think information (Fundamental Value) is more important.

E: I pay attention to both trend and information, and they have the same importance.

3. When you seek information, what will you do

A. I only pay attention to the latest information (only in resent several days).

B. I pay attention to the information in resent several months.

C. I will try my best to search for all the information for all the time even since IPO.
4. A stock has the following trend.

For this stock, which news will have the bigger impact?

A. Net profit will be 5% more than pre-disclosure

B. Net profit will be 5% less than pre-disclosure

C. A and B will have same impact

5. The same with Question 3, say a stock has following trend,

For this stock, which news will have the bigger impact?

A. Net profit will be 5% more than pre-disclosure

B. Net profit will be 5% less than pre-disclosure

C. A and B will have same impact
3. Extrapolation Belief and the Trading Volume

**Abstract:** Using empirical regression tests, we find individuals’ extrapolation belief can significantly impact the trading volume, but the effect is different according to different markets. Specifically, in the emerging stock market where short-sale constraint exists, when GSI<0, trading volume is negatively correlated with |GSI|, the magnitude of individuals’ extrapolation belief. But when GSI>0, trading volume is positively correlated with |GSI|. On the contrary, in the market where short-sale is available for investors, trading volume is positively correlated with |GSI| for both positive and negative GSI.

3.1 Introduction

Like volatility, the high trading volume is also an anomaly to prove market inefficient, for there should be minimal trading as rational models of investing predict. Nevertheless, real equity markets perform much higher trading volume that can hardly be explained by traditional theories (Thaler and Barberis 2003). Figure 3.1 plots the annual turnover ratio of domestic shares of several stock markets which directly illustrates the unreasonable high trading volume. As shown in this figure, the average ratio of all stock markets in the world (red line) is higher than 100% for the whole time, while the turnover ratio of Chinese stock market, as a representative of the emerging market, is more than 200% at the same time.
Besides enormous amount, trading volume also changes across time. As shown in Figure 3.1, the turnover rate of United States market doubles its size, reaching about 400% in 2008 when the World Financial Crisis happens. Figure 3.2 describes the daily changing hands of Shanghai Stock Market in detail. The trading volume is relatively moderate during the bear market, about 100M hands in 2010~2014. But it hovers to more than 800M hands in the 2015 bubble. Besides, the high trading volume is a distinguished feature of financial bubbles. Like Cochrane (2011) demonstrated: “Every asset price “bubble” . . . has coincided with a similar trading frenzy, from Dutch tulips in 1620 to Miami condos in 2006…”

However, claimed by many researchers, it is harmful for investors to trade in such a high amount (Odean 1988, Biais et al., 2005). For example, Barber and Odean (2000) investigate a large group of investors’ investment records obtained from a national brokerage company and confirm that, because of the transaction costs, the more an investor trade, the lower return he may earn. So, as Cochrane (2011) asks, “why do investors trade such enormous quantities?” And, what makes them trade more during certain periods, financial bubbles for example, than during other periods?

To find the reason for investors’ irrational trading behavior, we try to understand the trading volume by investors extrapolation bias. By empirical test, we find individual’s extrapolation belief can significantly impact the trading volume in most of the markets.

Firstly, to test if individuals’ extrapolation belief and trading volume are related, we collect data from several types of financial markets, including the developed stock
markets, the future markets, the emerging stock markets and the Bitcoin market. The key difference to be noticed between these markets is the short-sale constraint.

Short-sale constraint, as a sign of market incompleteness, can impact stock market enormously. The influence of short-sale constraint on the market is well demonstrated by many scholars. Miller (1977) demonstrates short-sale constraints can prevent pessimistic investors to trade against the optimistic opponents that lead the stocks to be overpriced. And this conclusion is proved by Lamont and Thaler (2003), who find some technology IPOs, which require a higher cost for people to short, are easily to be overvalued. Although short-sale is crucial to build an efficient market as it ensures arbitragers be able to get profit when they believe the equity is overvalued (Miller 1977), in many emerging stock markets (and in the Bitcoin market), the ability to short sell is nonexistent (Kraus and Rubin 2003). But unlike the emerging stock market, in the future market, investors are free to choose long-position (buying in the contract) or short-position (selling out contract). “Free to sell short” is always a distinguished feature of the future market compared with other financial markets. Short-sale is also not difficult for investors in the developed stock market. As in these markets, most of the investors are institutional investors who have more financial tools and opportunities than individual investors when they want to sell short. The better market environment can also provide hedging tools for them to cover potential losses. Therefore, we can divide financial markets into two groups, markets with short-sale constraint (like the emerging stock markets) and markets without the short-sale constraint (developed stock markets and future markets).
Then, using the same method of Chapter 1, we construct the GSI (Greenwood and Shleifer Index) to quantitatively represent investors’ extrapolation belief and test its relationship with trading volume. Strikingly, several interesting findings appear from our simple ARIMA regression.

First of all, trading volume is significantly affected by GSI in all of the financial markets. According to our empirical test, in the emerging stock market and the future market, both positive and negative GSI can significantly affect trading volume. Specially, for the developed stock market, we can only find significant regression result for positive GSI in the Hong Kong stock market. But the trading volume are all significantly affected by negative GSI in the developed stock markets. These different significances of positive GSI and negative GSI are consistent with our finding in previous chapters that, investors tend to be more easily affected by their extrapolation belief in the bear market.

The more interesting finding is the different relation between the negative GSI and trading volume in different financial markets. As we show in Appendix Table 3.4, in the emerging stock market or in the Bitcoin market, where short-sale is difficult for investors, trading volume is positively related to |GSI| when GSI>0, but negatively related to |GSI| when GSI<0, which indicates, in these markets, individuals’ positive extrapolation belief increases trading volume while their negative extrapolation belief decreases the trading volume. This finding is consistent with some previous researches that trading volume is positively correlated with past returns (Statman, Tholey, and Vorikink (2006), Glaser and Weber (2007), Zaiane and Abaoub (2009)). Although they
employ lagged returns instead of the cumulative weighted average of past returns this paper use.

But our finding about the relation between GSI and trading volume in the future market and in the developed stock market distinguishes our research with previous ones. According to our regression results, when GSI<0, the relation between |GSI| and trading volume is always positive in these markets. In other words, in the market where selling short is available to investors, individuals’ negative extrapolation belief can also increase the trading volume, which is just the opposite to the emerging stock markets and the Bitcoin market.

These empirical findings raise at least two questions. Firstly, why trading volume can be impacted by individual investors’ extrapolation belief. Secondly, how can short-sale constraint plays such an important role in determining the relation between trading volume and individual’s extrapolation belief?

To the best of my knowledge, there are only two papers directly discuss the relationship between trading volume and individuals’ extrapolation belief, Barberies et al. (2016) and Defusco, Nathanson, and Zwick (2017). Barberies and his coauthors build a model about financial bubbles in which extrapolators “waver” between two conflictive signals, the fundamental value and their extrapolation belief. When the extrapolation belief grows, the trading volume introduced by extrapolators’ wavering increases too. Their theory can explain the positive relation between Extrapolation belief and the trading volume during the bubble period. Also focusing on bubbles, Defusco, Nathanson, and Zwick (2017) present a model with two type of investors:
short-term investors who are also extrapolators and long-term investors. The trading volume is amplified by short-term investors who trade more aggressively as they disturb the price by their extrapolative self-contribute trading. However, neither of this two papers pays attention to the situations when GSI is negative, or what happens to the bear markets. Nor do they care about the future market, where the relation between negative GSI and trading volume is totally different. In a word, the relation between GSI and trading volume still need further discussion.

In the following section, we briefly review related researches about trading volume and behavioristic explanations. Empirical test is represented in Section 3.3. Section 3.4 concludes.

3.2 Literature Review

The first attempt to understand changes of trading volume is through the simultaneous positive return-volume relation. This is a well-established empirical finding supported by many studies (Karpoff 1987). Crouch (1970) suggests that positive correlations between the variance of daily price changes and volumes not only can be found for individual stocks, but can also be found for aggregate market indexes. Using daily and monthly data, Morgan (1976) also proved that the positive relationship between absolute price changes and trading volume is pervasive in the market. These studies are followed by the finding of the asymmetric volume–price relationship that volume is more closely connected with positive price changes than negative ones. This asymmetric relation is firstly found by Ying (1966), and supported by other researchers,
such as Epps (1975, 1977), Smirlock and Starks (1985), and Al-Deehani (2007). Several authors who attempt to explain this simultaneous volume-price relation have offered some theoretical explanations, like the “different attitude to risk hypothesis” by Epps (1975), the “mixture of distribution hypothesis” by Harris (1983), and the “asymmetric information theory” which can introduce trading among investors (Wang, 1994; Campbell and Kyle, 1993; Heaton and Lucas, 1993).

But these theories have little ability to explain the following finding that trading volume is also influenced by past price changes. This is also well supported by many researchers. The positive cross-autocorrelation relationship between trading volume and past returns is supported by Chordia and Swaminathan (2000) by empirical test. Using vector auto-regressions and associated impulse response functions, Statman, Thorley, and Vorkink (2004) find a bi-directional positive impact of past trading volume on the returns and past returns on the trading volume. By the same method, Zaiane and Abaoub (2009) find similar evidence indicating a positive trading volume-past return relationship in Tunisian stock market.

Behavior theories, unlike traditional ones, can offer great help to explain this volume-past return correlation. Firstly, the “disposition effect” theory, which means traders tend to realize the paper gains of their successful investment. On the contrary, they would avoid selling out the stocks that are defective (Odean (1998)). Hence, when past returns are good, trading volume will increase as people try to realize their gains, and vice versa (Shefrin and Statman (1985)). Many researchers verified this theory by empirical tests,
such as Lakonishok and Smidt (1986), Ferris, Haugen and Makhija (1988), and Heath, Huddart and Lang (1999).

Another common behavioristic explanation is overconfidence, which means “people believe that they have information strong enough to justify a trade, whereas in fact the information is too weak to warrant any action” (Barberis 2003). It is theoretically proved by Daniel, Hirshleifer, and Subrahmanyam (1998), and Odean (1998) that if investors are “overconfidence” about their own trading signals, they would trade with others more frequently. Trading volume rises accordingly. Following development is given by Gervais and Odean (2001) who suggest investors will become more “overconfidence” if they experienced positive investment profits although those profits may also be experienced by most of other investors. Therefore, the positive past returns will increase the trading volume.

This overconfidence theory is empirically tested by Statman, Tholey, and Vorikink (2006), who use monthly trading volume data of both individual stacks and the aggregate stock market to investigate the relationship between trading volume and past price changes. According to their empirical result, the positive volume-past return relationship can be found both in the aggregate index and in individual stocks. They also prove when more stock shares are held by individuals, this volume-past return relationship become more significant. Their research and following similar works, such as Glaser and Weber (2007), Zaiane and Abaoub (2009), Chiang and Zheng (2010), make overconfidence theory be a convicting theory to explain the relationship between the volume-past return relation.
Being similar to researches like Statman, Tholey, and Vorikink (2006), this paper also tries to seek if trading volume is influenced by past returns. But this paper is distinguished from previous works by several profound aspects. Firstly, in our paper, to coordinate with the Extrapolation Belief, we calculate GSI as a weighted average of lagged returns, instead of just the original lagged returns. Secondly, we distinct the positive and the negative GSI, to seek the possible asymmetric relation between trading volume and GSI. Thirdly and most importantly, we extend this empirical test to future markets, where overconfidence theory doesn’t fit. As in the future markets, the past positive returns cannot indicate individuals are becoming more overconfidence—they are free to the choose short position or the long position just as institutional investors do. Besides, we get a new finding which previous papers are missing that in the future markets, the negative GSI, will increase the trading volume, instead of reducing it as in the emerging stock market.

In short, previous theories cannot help us to fully understand this GSI-trading volume relationship, especially for the GSI-trading volume relationship in the market where people can freely sell short. Further research is needed.

3.3 Empirical test

3.2.1 Data description and Empirical methodology

As has been done in the first chapter, we first conduct the empirical test for several types of different financial markets. Our data includes the daily changing hands (as the estimation of trading volume) and indexes of these markets which are drawn from the
Choice Database. Specifically, these markets include the Brent Oil Future Index, Chinese Gold Future Index, Chinese Silver Future Index, and the Chinese Copper Future Index from the future market. We also cover Dow Jones 500 index, S&P index, IXIC (Nasdaq stock market index), N225, CAC 40 (France stock market index), HSI index (Hong Kong stock market index) on behalf of the developed stock market. For the emerging stock market, this paper chooses three Chinese stock market indexes, SSEC, SZI and GEI, as well as TWII, the index of Taiwan stock market. Bitcoin index is also included as another representative of individual investor dominated, irrational market.

As we have demonstrated in Chapter 1, individual investors take much higher proportion in Chinese stock market than in other markets. Similar things happen to Chinese Future market. As summarized by Wang et al. (2015), individuals trading proportion in Copper Future market, Gold Future market and the Silver Future market is about 86%, 87%, 92% respectively, compared with about 30% individual investor in the Brent Crude Oil future market. Bitcoin Market is another market which is dominated by individual investors, as the institutional investors are reported to enter this market since 2017 (ESTEVES, 2018).

Trading volume patterns are also different between these markets. Figure 3.1 and Figure 3.2 not only show the turnover rate in Chinese stock market is much higher than in other markets, but also show a positive correlation between the trading volume and the price level. But, trading volume of Chinese Gold Future market, as shown in Figure
doesn’t seem related to the price level. So, to answer what drives trading volume to change, we empirically test its relation with individuals’ extrapolation belief.

To do that, taking the same vein like Chapter 1, we contrast Greenwood Shleifer Index (GSI) to measure individual’s extrapolation belief, as

\[ GSI_t = \sum_{i=1}^{n} r_{t-i} \cdot \lambda^i \]  

where \( r_t \) means the return at time \( t \), \( \lambda \) is the memory effect that governs the weights investors put into each period. \( n \) is set to be 20. For the trading volume (changing hands), we take its logarithm value to eliminate the influence of scales. Then following Ajinkya and Jain (1989), Girard and Biswas (2007), we estimate the relation between log trading volume and GSI using ARIMA \((p, d, q)\) models.

As ARIMA model requires the time series to be stationary, we use ADF-GLS test as well as KPSS test to check the stationary of the logarithm trading volume for every market. ADF-GLS test is developed by Elliot, Rothenberg and Stock (1996), which is a modification of ADF test aiming at employing the detrending transformation to improve the test power. KPSS test which is proposed by Kwiatkowski, Phillips, Schmidt and Shin (1992), whose null-hypothesis is that the time series is stationary. It is suggested that KPSS unit root test eliminates a possible low power against stationary unit root that occurs in the ADF (Katircioglu, Feridun, & Kilinc, 2014; Jafari, Othman, & Nor, 2012). Therefore, KPSS unit root test results yield more robust results. Test results are shown in Table 3.1.
According to our test result, the log trading volume data from the developed stock markets, the Brent crude oil future market and the Shanghai Copper future market all pass the DF-GLS test and KPSS test, indicating stationary time series. On the contrary, the log value of trading volume from TWII, SZI and GEI are all failed. Although the log trading volume from SSEC, Shanghai Silver Future market and Shanghai Gold Future Market pass the DF-GLS test, it failed in the KPSS test. Therefore, we also treat trading volume from these markets as non-stationary.

Following Chase (2013), we apply the first order of differencing to these markets with non-stationary logarithm trading volume. Then, by AIC and BIC criterion, we choose different ARIMA structures for different financial markets in our empirical tests. Specifically, for the market with stationary logarithm trading volume, we use

$$\log (Vol_t) = a + \beta_1 \cdot |GSI_t| \cdot D_1(GSI_t > 0) + \beta_2 \cdot |GSI_t| \cdot D_2(GSI_t \leq 0)$$

$$+ \sum_{i=1}^{p} \varphi_i \log (Vol_{t-i}) + \sum_{i=1}^{q} \theta_i u_{t-1} + u_t$$  \hspace{1cm} (2)

where $D_1$ and $D_2$ are two dummy variables aiming to distinguish positive or negative regions of GSI. $Vol_t$ represent the trading volume at time $t$. And when log $(Vol_t)$ is not stationary, we use

$$Dv_t = a + \beta_1 \cdot GSI_t \cdot D_1(GSI_t > 0) + \beta_2 \cdot |GSI_t| \cdot D_2(GSI_t \leq 0) +$$

$$\sum_{i=1}^{p} \varphi_i Dv_{t-i} + \sum_{i=1}^{q} \theta_i u_{t-i} + u_t$$  \hspace{1cm} (3)

where $$Dv_t = \log (Volume_t) - \log (Volume_{t-1})$$  \hspace{1cm} (4)

Detailed empirical test result is demonstrated in the following section.
3.2.2 Empirical test result

From Table 3.2 to Table 3.7, we show the specific regression equation and the empirical test result for different value of $\lambda$ for several markets (Due to space limitations, we only list empirical test result of several markets). As we illustrate above, we use different ARIMA structure for different markets. For example, for IXIC, we apply ARIMA (1,0,1) model while for the SSEC, we use ARIMA (1,1,1). Although the regression equation is different, the meaning of $\beta_1$ and $\beta_2$ is similar. A positive value always indicates $|\text{GSI}|$ can increase the trading volume and vice versa.

From Table 3.2 to Table 3.7, we can see, like the empirical test between GSI and volatility, changes in the value of $\lambda$ cannot make fundamental differences to the empirical test results. When $\lambda$ changes, the empirical test results are almost the same. For example, for the Nasdaq stock market, when $\lambda$ increases from 0.3 to 0.9, $t$ value of $\beta_2$, the coefficient of the negative GSI, varies from 2.2 to 3.5 which ensure $\beta_2$ be most significant across all the regressions. Similarly, $\beta_1$, the coefficient of the positive GSI maintains insignificant for all regression results. Besides, the $R^2$ values are very similar in all these regressions. Similar things happen to other markets that not only $\beta_1$ and $\beta_2$ are all significant for all regression results, but also the $R^2$ changes very slightly. In a word, the relation between trading volume and individuals’ extrapolation belief is robust no matter what value we choose for $\lambda$ to calculate GSI.

For the convenience of comparison, Table 3.8 summarizes the ARIMA structure we use for different financial markets as well as the empirical test results for $\beta_1$ and $\beta_2$.
when $\lambda = 0.8$. Several profound conclusions can be established from our empirical tests and the comparison of all these results.

Firstly, *Extrapolation belief*, as the most common bias that individual investors are suffering, does impact the trading volume. As shown in Table 3.4, $\beta_2$, the coefficient of the negative GSI, is significant for all the markets, whiles $\beta_1$, the coefficient of the positive GSI, is significant for most of the emerging markets. In some developed stock markets, positive GSI can still significantly affect trading volume. This suggests that we should pay attention to investors’ extrapolation belief when researching trading volume.

What is more interesting is the asymmetric relation between GSI and trading volume. As shown in this table, the coefficient of the positive GSI, $\beta_1$, is always positive for all the markets when it is significant. This positive correlation indicates that when individual investors are optimistic, their extrapolation belief will increase the trading volume. On the contrary, the coefficient of the negative GSI, $\beta_1$, is only positive for the future market and the developed stock market, but it is significantly negative for all the emerging stock markets and the Bitcoin market. That means, for the future markets and the developed stock markets, during the bear market, when investors are pessimistic because of cumulative negative price changes, extrapolation belief can also increase the trading volume just as in the bull market. On the contrary, negative extrapolation belief of individual investors actually reduces trading volume in the emerging stock market.

As illustrated above, the key that distinguishes the emerging stock markets (and Bitcoin market) with other financial markets is the short-sale constraint. It seems short-
sale constraint is crucial for the relationship between GSI and trading volume. More specifically, when selling short-sale is totally available to every investor, an increasing magnitude of the negative GSI indicates a rise in the trading volume. Nevertheless, in the emerging stock market or in the Bitcoin market, where selling short is quite difficult for investors, trading volume reduces as extrapolators are more suffered from their negative extrapolation belief during the bear market.

At first sight, it may be straightforward to explain the relationship between trading volume and individuals’ extrapolation bias in the future market, but, it is very confusing for the stock market where short-sale constraint exists. A bigger magnitude of |GSI| may indicate a bigger difference between extrapolators’ opinion and fundamentalists’ belief. In the future market, with no short-sale constraint, individuals’ extrapolation bias will lead them to trade with the fundamentalists who try to arbitrage the mispricing away. Then when |GSI| rises, trading between extrapolators and fundamentalists will also increase. But, why, in the stock markets or in the Bitcoin market, trading volume reduces as the magnitude of negative GSI increases. If the reason lies in the fact that short sale constraint will keep the extrapolators who are pessimistic about future return out of the market, which also makes the market with only identical fundamentalists and hence little trading volume, then, why, on the contrary, in the bull market, positive GSI makes trading volume increase? As during the bull market, especially during the bubble period, fundamentalists gradually quit the market because of the short-sale constrain, if extrapolators are all optimistic about future returns, no trade will happen among them.
In a word, the relation between trading volume and individuals’ extrapolation belief still needs theoretical explanations.

3.4 Conclusion and Discussion

To conclude, using daily data from several kinds of markets, this paper empirically demonstrates individuals’ extrapolation belief can significantly impact the trading volume. More importantly, we also find the short-sale constraint is crucial in determining this relationship. In the future market and developed stock market, where selling short-sale is available, both the negative GSI and the positive GSI can raise the trading volume. On the contrary, in the emerging stock market or in the Bitcoin market, where investors are facing short-sale constraint, the negative GSI reduces the trading volume, just being the opposite to what happens in the future market. Positive GSI can also increase trading volume in the emerging market.

As we illustrated above, existing theories cannot explain our findings. For instance, previous theories about extrapolation belief only discuss the bull market situations (or financial bubbles). “Overconfidence” theory and “disposition effect” theory cannot deal with what happens in the future market. So, new theories about extrapolation belief and trading volume is needed.
3.5 REFERENCE


GERVAIS, S. & ODEAN, T. 2001. Learning to be overconfident. the Review of


**Appendix 3**

**Figure 3.1 Stock traded, turnover ratio of domestic shares**

Turnover ratio is the value of domestic shares traded divided by their market capitalization. The value is annualized by multiplying the monthly average by 12. Source: World Federation of Exchanges database.

**Figure 3.2 Changing Hands of Shanghai Stock Market**

This figure displays the daily Changing Hands of Shanghai Stock Market with brow line. The index (SSEC) is represented by the blue line. Source: Choice Database.
Figure 3.3 Changing Hands of Chinese Gold Future Market

This figure displays the daily Changing Hands of Chinese Gold Future Market with brown line. The index is represented by the blue line. Source: Choice Database.

Table 3.1 Stationary test for logarithm value of trading volume (log $(Volume_t)$)

<table>
<thead>
<tr>
<th>Market</th>
<th>ADF-DLS test</th>
<th>KPSS test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t-Statistic</td>
<td>LM-Statistic</td>
</tr>
<tr>
<td>Dow Jones 500</td>
<td>-8.79***</td>
<td>0.61**</td>
</tr>
<tr>
<td>S&amp;P</td>
<td>-10.53***</td>
<td>0.53**</td>
</tr>
<tr>
<td>IXIC</td>
<td>-9.75***</td>
<td>0.12***</td>
</tr>
<tr>
<td>N225</td>
<td>-3.52***</td>
<td>0.25***</td>
</tr>
<tr>
<td>France</td>
<td>-6.57***</td>
<td>0.34***</td>
</tr>
<tr>
<td>HSI</td>
<td>-3.93**</td>
<td>0.19***</td>
</tr>
<tr>
<td>TWII</td>
<td>-1.79</td>
<td>1.02</td>
</tr>
<tr>
<td>SSEC</td>
<td>-2.18**</td>
<td>2.18</td>
</tr>
<tr>
<td>SZI</td>
<td>-1.50</td>
<td>4.08</td>
</tr>
<tr>
<td>GEI</td>
<td>-0.46</td>
<td>1.67</td>
</tr>
<tr>
<td>Brent Oil</td>
<td>-7.60***</td>
<td>0.53*</td>
</tr>
<tr>
<td>Bit Coin</td>
<td>0.19</td>
<td>3.89</td>
</tr>
<tr>
<td>Shanghai Silver Future</td>
<td>-2.24*</td>
<td>0.87</td>
</tr>
<tr>
<td>Shanghai Gold Future</td>
<td>-2.17*</td>
<td>1.19</td>
</tr>
<tr>
<td>Shanghai Copper Future</td>
<td>-6.64***</td>
<td>0.36**</td>
</tr>
</tbody>
</table>

This table reports the unit test results for the logarithm value from different financial markets by ADF-GLS test and KPSS test. ***, **, * means the non-hypothesis is rejected at the 1%, 5% and 10% confidence level for ADF-GLS test. For KPSS test, ***, **, * means the time series is stationary at the 1%, 5% and 10% confidence level.
Table 3.2 Empirical results for IXIC with ARIMA (1,0,1) structure for different $\lambda$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\text{ar}(1)$</th>
<th>$\text{ma}(1)$</th>
<th>$R^2$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>2.19</td>
<td>-8.69**</td>
<td>0.71***</td>
<td>-0.33***</td>
<td>0.244</td>
<td>987</td>
</tr>
<tr>
<td>0.4</td>
<td>0.95</td>
<td>-6.15**</td>
<td>0.71***</td>
<td>-0.33***</td>
<td>0.243</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>-0.17</td>
<td>-4.36**</td>
<td>0.71***</td>
<td>-0.32***</td>
<td>0.243</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>-0.85</td>
<td>-3.37***</td>
<td>0.70***</td>
<td>-0.33***</td>
<td>0.243</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>-1.20</td>
<td>-2.85**</td>
<td>0.69***</td>
<td>-0.32***</td>
<td>0.245</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>-0.95</td>
<td>-2.79**</td>
<td>0.66***</td>
<td>-0.31***</td>
<td>0.246</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>-0.44</td>
<td>-2.72****</td>
<td>0.65***</td>
<td>-0.29***</td>
<td>0.249</td>
<td></td>
</tr>
</tbody>
</table>

This table reports empirical results of the form

$$\log(Volume_t) = a + \beta_1 \cdot GSI_t \cdot D_1(GSI_t > 0) + \beta_2 \cdot |GSI_t| \cdot D_2(GSI_t \leq 0) + \varphi_1 \log(Volume_{t-1}) + \theta_1 u_{t-1} + u_t$$

where GSI is calculated with different values of $\lambda$. ***, **, * denote statistical significance at the 1%, 5%, 10% level respectively. Newey-West-based t-statistics are in brace.
Table 3.3 Empirical results for GEI with ARIMA (1,1,1) structure for different λ

<table>
<thead>
<tr>
<th>λ</th>
<th>a</th>
<th>β₁</th>
<th>β₂</th>
<th>ar(1)</th>
<th>ma(1)</th>
<th>R²</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>-0.01*</td>
<td>6.09***</td>
<td>-3.60***</td>
<td>0.40***</td>
<td>-0.80***</td>
<td>0.195</td>
<td>1619</td>
</tr>
<tr>
<td>0.4</td>
<td>-0.01*</td>
<td>3.91***</td>
<td>-2.18***</td>
<td>0.41***</td>
<td>-0.80***</td>
<td>0.186</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>-0.01*</td>
<td>2.60***</td>
<td>-1.32***</td>
<td>0.42***</td>
<td>-0.81***</td>
<td>0.177</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>-0.01*</td>
<td>1.72***</td>
<td>-0.76***</td>
<td>0.44***</td>
<td>-0.82***</td>
<td>0.167</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>-0.01</td>
<td>1.03***</td>
<td>-0.41***</td>
<td>0.46***</td>
<td>-0.82***</td>
<td>0.155</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>-0.00</td>
<td>0.51***</td>
<td>-0.19***</td>
<td>0.48***</td>
<td>-0.83***</td>
<td>0.143</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>-0.00</td>
<td>0.17*</td>
<td>-0.04**</td>
<td>0.49***</td>
<td>-0.83***</td>
<td>0.132</td>
<td></td>
</tr>
</tbody>
</table>

This table reports empirical results of the form

\[ Dv_t = a + \beta_1 \cdot GSI_t \cdot D_1(GSI_t > 0) + \beta_2 \cdot [GSI_t] \cdot D_2(GSI_t \leq 0) + \phi_1 Dv_{t-1} + \theta u_{t-1} + w_t \]

where

\[ Dv_t = \log(\text{Volume}_t) - \log(\text{Volume}_{t-1}) \]

where GSI is calculated with different value of λ. ***, **, * denote statistical significance at the 1%, 5%, 10% level respectively. Newey-West-based t-statistics are in brace.
Table 3.4 Empirical results for SSEC with ARIMA (3,1,1) structure for different $\lambda$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\lambda = 0.3$</th>
<th>$\lambda = 0.4$</th>
<th>$\lambda = 0.5$</th>
<th>$\lambda = 0.6$</th>
<th>$\lambda = 0.7$</th>
<th>$\lambda = 0.8$</th>
<th>$\lambda = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>[-0.36]</td>
<td>[-0.26]</td>
<td>[-0.18]</td>
<td>[-0.16]</td>
<td>[0.16]</td>
<td>[0.84]</td>
<td>[0.16]</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>8.07***</td>
<td>5.08***</td>
<td>3.28***</td>
<td>2.07***</td>
<td>1.25***</td>
<td>0.62***</td>
<td>0.26***</td>
</tr>
<tr>
<td></td>
<td>[6.97]</td>
<td>[6.39]</td>
<td>[5.77]</td>
<td>[5.05]</td>
<td>[4.27]</td>
<td>[3.85]</td>
<td>[3.11]</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-7.28**</td>
<td>-4.63**</td>
<td>-3.04**</td>
<td>-1.99**</td>
<td>-1.25**</td>
<td>-0.76**</td>
<td>-0.39**</td>
</tr>
<tr>
<td>ar(1)</td>
<td>0.46***</td>
<td>0.46***</td>
<td>0.49***</td>
<td>0.51***</td>
<td>0.53***</td>
<td>0.59***</td>
<td>0.62***</td>
</tr>
<tr>
<td></td>
<td>[14.01]</td>
<td>[14.56]</td>
<td>[15.24]</td>
<td>[16.12]</td>
<td>[17.40]</td>
<td>[25.84]</td>
<td>[29.36]</td>
</tr>
<tr>
<td>ar(2)</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.04</td>
<td>0.06**</td>
<td>0.04***</td>
</tr>
<tr>
<td></td>
<td>[1.01]</td>
<td>[0.98]</td>
<td>[1.04]</td>
<td>[1.19]</td>
<td>[1.47]</td>
<td>[2.35]</td>
<td>[2.70]</td>
</tr>
<tr>
<td>ar(3)</td>
<td>0.06*</td>
<td>0.06*</td>
<td>0.06**</td>
<td>0.07**</td>
<td>0.07**</td>
<td>0.11***</td>
<td>0.12**</td>
</tr>
<tr>
<td></td>
<td>[1.93]</td>
<td>[1.95]</td>
<td>[1.99]</td>
<td>[2.11]</td>
<td>[2.37]</td>
<td>[3.20]</td>
<td>[4.50]</td>
</tr>
<tr>
<td>ma(1)</td>
<td>-0.88***</td>
<td>-0.89***</td>
<td>-0.89***</td>
<td>-0.89***</td>
<td>-0.89***</td>
<td>-0.89***</td>
<td>-0.88***</td>
</tr>
<tr>
<td></td>
<td>[-34.28]</td>
<td>[-35.28]</td>
<td>[-36.30]</td>
<td>[-37.80]</td>
<td>[-40.46]</td>
<td>[-81.41]</td>
<td>[-105.64]</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.19</td>
<td>0.18</td>
<td>0.17</td>
<td>0.16</td>
<td>0.16</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>$N$</td>
<td>1619</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Data: 2010/12/23 -- 2016/11/31

This table reports empirical results of the form

$$DV_t = a + \beta_1 \cdot GSI_t \cdot D_t(GSI_t > 0) + \beta_2 \cdot |GSI_t| \cdot D_t(GSI_t \leq 0) + \sum_{i=1}^{3} \phi_i DV_{t-i} + \theta u_{t-i} + u_t$$

where

$$DV_t = \log(Volume_t) - \log(Volume_{t-1})$$

where GSI is calculated with different value of $\lambda$. ***, **, * denote statistical significance at the 1%, 5%, 10% level respectively. Newey-West-based t-statistics are in brace

$$\sum_{i=1}^{3} \theta_i u_{t-i} \sum_{j=1}^{3} \phi_j DV_{t-j}$$
### Table 3.5 Empirical results for Shanghai Silver Future with ARIMA (1,1,1) structure for different $\lambda$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$a$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\text{ar}(1)$</th>
<th>$\text{ma}(1)$</th>
<th>$R^2$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>-0.01**</td>
<td>5.12*</td>
<td>5.00***</td>
<td>0.29***</td>
<td>-0.78***</td>
<td>0.202</td>
<td>2278</td>
</tr>
<tr>
<td>0.4</td>
<td>-0.02***</td>
<td>4.25**</td>
<td>4.02***</td>
<td>0.29***</td>
<td>-0.79***</td>
<td>0.203</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>-0.02***</td>
<td>3.52***</td>
<td>3.17***</td>
<td>0.30***</td>
<td>-0.80***</td>
<td>0.204</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>-0.02***</td>
<td>2.72***</td>
<td>2.41***</td>
<td>0.30***</td>
<td>-0.80***</td>
<td>0.205</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>-0.02***</td>
<td>1.89***</td>
<td>1.70***</td>
<td>0.31***</td>
<td>-0.80***</td>
<td>0.204</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>-0.02***</td>
<td>1.13***</td>
<td>1.15***</td>
<td>0.31***</td>
<td>-0.80***</td>
<td>0.204</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>-0.01***</td>
<td>0.55**</td>
<td>0.74***</td>
<td>0.32***</td>
<td>-0.80***</td>
<td>0.205</td>
<td></td>
</tr>
</tbody>
</table>

where $GSI$ is calculated with different value of $\lambda$. ***, **, * denote statistical significance at the 1%, 5%, 10% level respectively. Newey-West-based t-statistics are in brace.

This table reports empirical results of the form

$$DV_t = a + \beta_1 \cdot GSI_t \cdot D_1(GSI_t > 0) + \beta_2 \cdot D_2(GSI_t \leq 0) + \psi_t DV_{t-1} + \theta_t u_t + u_t$$

where

$$DV_t = \log(Volume_t) - \log(Volume_{t-1})$$

where GSI is calculated with different value of $\lambda$. ***, **, * denote statistical significance at the 1%, 5%, 10% level respectively. Newey-West-based t-statistics are in brace.
Table 3.6 Empirical results for Shanghai Gold Future with ARIMA (1,1,1) structure for different $\lambda$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$a$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\text{ar}(1)$</th>
<th>$\text{ma}(1)$</th>
<th>$R^2$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>-0.02***</td>
<td>-1.09***</td>
<td>1.66***</td>
<td>0.79***</td>
<td>-0.98***</td>
<td>0.277</td>
<td>2278</td>
</tr>
<tr>
<td></td>
<td>[-2.79]</td>
<td>[-6.54]</td>
<td>[8.98]</td>
<td>[25.82]</td>
<td>[-118.65]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>-0.03***</td>
<td>-0.86***</td>
<td>1.45***</td>
<td>0.84***</td>
<td>-1.00</td>
<td>0.288</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[-4.92]</td>
<td>[-7.81]</td>
<td>[11.42]</td>
<td>[35.91]</td>
<td>[-0.17]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>-0.03***</td>
<td>-0.74***</td>
<td>1.24***</td>
<td>0.87***</td>
<td>-1.00</td>
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<td></td>
<td>[-4.83]</td>
<td>[-8.48]</td>
<td>[12.32]</td>
<td>[40.40]</td>
<td>[-0.15]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>-0.03***</td>
<td>-0.66***</td>
<td>1.10***</td>
<td>0.89***</td>
<td>-1.00</td>
<td>0.297</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[-4.64]</td>
<td>[-8.91]</td>
<td>[13.31]</td>
<td>[46.56]</td>
<td>[-0.14]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>-0.04***</td>
<td>-0.58***</td>
<td>1.00***</td>
<td>0.92***</td>
<td>-1.00</td>
<td>0.299</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[-6.44]</td>
<td>[-8.69]</td>
<td>[14.29]</td>
<td>[55.48]</td>
<td>[-0.13]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>-0.02</td>
<td>-1.07***</td>
<td>1.28***</td>
<td>0.28**</td>
<td>-0.07</td>
<td>0.279</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[-0.95]</td>
<td>[-3.19]</td>
<td>[3.91]</td>
<td>[2.36]</td>
<td>[0.20]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>-0.02*</td>
<td>-0.08*</td>
<td>0.25***</td>
<td>0.11</td>
<td>-0.56***</td>
<td>0.274</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[-1.82]</td>
<td>[-1.84]</td>
<td>[2.83]</td>
<td>[1.14]</td>
<td>[-3.80]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This table reports empirical results of the form

$$Dv_t = a + \beta_1 \cdot GSI_t \cdot D_1(GSI_t > 0) + \beta_2 \cdot |GSI_t| \cdot D_2(GSI_t \leq 0) + \varphi_1 Dv_{t-1} + \theta_1 u_{t-1} + u_t$$

where

$$Dv_t = \log(\text{Volume}_t) - \log(\text{Volume}_{t-1})$$

where GSI is calculated with different value of $\lambda$. ***, **, * denote statistical significance at the 1%, 5%, 10% level respectively. Newey-West-based t-statistics are in brace.
Table 3.7 Empirical results for Brent Crude Oil Future with ARIMA (1,0,2) structure for different \( \lambda \)

<table>
<thead>
<tr>
<th>( \lambda = 0.3 )</th>
<th>( \lambda = 0.4 )</th>
<th>( \lambda = 0.5 )</th>
<th>( \lambda = 0.6 )</th>
<th>( \lambda = 0.7 )</th>
<th>( \lambda = 0.8 )</th>
<th>( \lambda = 0.9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>12.42**</td>
<td>12.42***</td>
<td>12.42***</td>
<td>12.42***</td>
<td>12.42***</td>
<td>12.43***</td>
</tr>
<tr>
<td></td>
<td>[253.41]</td>
<td>[251.81]</td>
<td>[249.11]</td>
<td>[245.73]</td>
<td>[241.15]</td>
<td>[234.60]</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>-0.19</td>
<td>-0.57</td>
<td>-0.76</td>
<td>-0.91</td>
<td>-1.09</td>
<td>-1.20</td>
</tr>
<tr>
<td></td>
<td>[-0.07]</td>
<td>[-0.30]</td>
<td>[-0.49]</td>
<td>[-0.70]</td>
<td>[-0.92]</td>
<td>[-1.10]</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>7.17**</td>
<td>5.61**</td>
<td>4.67**</td>
<td>3.91**</td>
<td>3.11**</td>
<td>2.24**</td>
</tr>
<tr>
<td></td>
<td>[2.35]</td>
<td>[2.39]</td>
<td>[2.45]</td>
<td>[2.46]</td>
<td>[2.31]</td>
<td>[2.05]</td>
</tr>
<tr>
<td>( ar(1) )</td>
<td>0.88***</td>
<td>0.88***</td>
<td>0.88***</td>
<td>0.89***</td>
<td>0.89***</td>
<td>0.89***</td>
</tr>
<tr>
<td></td>
<td>[28.81]</td>
<td>[29.08]</td>
<td>[29.48]</td>
<td>[29.95]</td>
<td>[30.33]</td>
<td>[30.55]</td>
</tr>
<tr>
<td>( ma(1) )</td>
<td>-0.33***</td>
<td>-0.34***</td>
<td>-0.34***</td>
<td>-0.34***</td>
<td>-0.35***</td>
<td>-0.34***</td>
</tr>
<tr>
<td>( ma(2) )</td>
<td>-0.18***</td>
<td>-0.18***</td>
<td>-0.19***</td>
<td>-0.19***</td>
<td>-0.19***</td>
<td>-0.09***</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.415</td>
<td>0.416</td>
<td>0.416</td>
<td>0.417</td>
<td>0.417</td>
<td>0.416</td>
</tr>
</tbody>
</table>

\( N = 2278 \)

Data 2009/01/05 -- 2018/05/23

This table reports empirical results of the form

\[
\log(Volume_t) = a + \beta_1 \cdot GSI_t \cdot D_1(GSI_t > 0) + \beta_2 \cdot |GSI_t| \cdot D_2(GSI_t \leq 0) + \phi_1 \log(Volume_{t-1}) + \sum_{i=2}^\infty \theta_i u_{t-i} + u_t
\]

where GSI is calculated with different value of \( \lambda \). ***, **, * denote statistical significance at the 1%, 5%, 10% level respectively. Newey-West-based t-statistics are in brace.
<table>
<thead>
<tr>
<th>Index</th>
<th>Date range</th>
<th>a</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>ARIMA Structure</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dow Jones 500</td>
<td>2014/01/30 - 2016/11/14</td>
<td>18.8***</td>
<td>-3.68</td>
<td>7.27***</td>
<td>ARIMA (1,0,1)</td>
<td>0.27</td>
</tr>
<tr>
<td>S&amp;P</td>
<td>2014/01/04 - 2017/12/29</td>
<td>21.99***</td>
<td>-2.02</td>
<td>2.17*</td>
<td>ARIMA (1,0,1)</td>
<td>0.41</td>
</tr>
<tr>
<td>IXIC</td>
<td>2015/10/15 - 2016/12/9</td>
<td>21.34***</td>
<td>-1.01</td>
<td>1.67**</td>
<td>ARIMA (1,0,1)</td>
<td>0.20</td>
</tr>
<tr>
<td>Developed stock market</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N225</td>
<td>2016/12/19 - 2016/12/21</td>
<td>11.5***</td>
<td>-0.78</td>
<td>1.22**</td>
<td>ARIMA (1,0,2)</td>
<td>0.41</td>
</tr>
<tr>
<td>France</td>
<td>2014/1/29 - 2017/12/29</td>
<td>18.47***</td>
<td>-1.97</td>
<td>3.78***</td>
<td>ARIMA (2,0,2)</td>
<td>0.39</td>
</tr>
<tr>
<td>HSI</td>
<td>2014/1/29 - 2016/12/15</td>
<td>21.25***</td>
<td>3.18***</td>
<td>3.39***</td>
<td>ARIMA (1,0,1)</td>
<td>0.33</td>
</tr>
<tr>
<td>Emerging Stock market and Bitcoin market</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TWII</td>
<td>2013/11/26 - 2016/10/31</td>
<td>0.01</td>
<td>0.20</td>
<td>-1.32***</td>
<td>ARIMA (2,1,2)</td>
<td>0.19</td>
</tr>
<tr>
<td>Bit Coin</td>
<td>2014/3/30 - 2018/01/31</td>
<td>-0.01</td>
<td>0.16</td>
<td>-0.86***</td>
<td>ARIMA (2,1,0)</td>
<td>0.11</td>
</tr>
<tr>
<td>SSEC</td>
<td>2010/1/01 - 2016/11/03</td>
<td>0.001</td>
<td>0.62***</td>
<td>-0.76***</td>
<td>ARIMA (3,1,1)</td>
<td>0.14</td>
</tr>
<tr>
<td>SZI</td>
<td>2010/12/23 - 2016/11/31</td>
<td>0.02</td>
<td>0.47***</td>
<td>-0.19***</td>
<td>ARIMA (3,1,1)</td>
<td>0.14</td>
</tr>
<tr>
<td>GEI</td>
<td>2010/12/23 - 2016/11/31</td>
<td>0.001</td>
<td>0.51***</td>
<td>-0.32***</td>
<td>ARIMA (1,1,1)</td>
<td>0.13</td>
</tr>
<tr>
<td>Brent Oil</td>
<td>2012/1/23 - 2016/1/13</td>
<td>12.43***</td>
<td>-1.30</td>
<td>2.13**</td>
<td>ARIMA (1,0,2)</td>
<td>0.41</td>
</tr>
<tr>
<td>Shanghai Silver Future</td>
<td>2008/1/14 - 2018/2/22</td>
<td>-0.01***</td>
<td>1.68***</td>
<td>1.21***</td>
<td>ARIMA (1,1,1)</td>
<td>0.20</td>
</tr>
</tbody>
</table>
This table compares the empirical test results of different financial markets with the regression form

\[ \ln(Volume_t) = a + \beta_1 \cdot GSI_t \cdot D_1(GSI_t > 0) + \beta_2 \cdot |GSI_t| \cdot D_2(GSI_t \leq 0) + ar(1) + ma(1) + u_t \]

***, **, * denote statistical significance at the 1%, 5%, 10% level respectively. Newey-West-based t-statistics are in brace.
4. Extrapolation Belief, Trading Volume and Bubbles

Abstract: To explain the confusing relationship between trading volume and individuals’ extrapolation belief that we confirmed in Chapter 3, we modify our model in Chapter 2 by assuming extrapolators are heterogeneous in the way that every extrapolator has his idiosyncratic bias when evaluating the information innovations. We prove that our new model not only can efficiently explain the relation between extrapolation belief and trading volume in different financial markets, but also can explain the two most distinguished features of financial bubbles: high trading volume and high volatility which other models are struggling to explain.

4.1 Introduction

In Chapter 3, we empirically prove that individual investors’ extrapolation belief can significantly impact the trading volume, especially for the market with a high proportion of individual investors. Specifically, during the bear market, individuals’ negative extrapolation belief increases the trading volume in the future markets but decreases the trading volume in the stock market. Nevertheless, during the bull market, their positive extrapolation always increases the trading volume for all the markets. We also show that previous behavioristic theories, like “overconfidence theory”,
“disposition effect” theory, or existing extrapolative models cannot fully explain our findings.

Fortunately, this paper finds, with a simple modification on our model in Chapter 2, we can effectively address these rubs. And we prove our new model built on heterogeneous extrapolators not only can explain why trading volume is related with extrapolation belief in these financial markets, but also can help to understand the two most distinguished features of bubbles that other theories are struggling with: the coexistent of high trading volume and the high volatility.

The only modification we made, with other model settings being the same, is that extrapolators are heterogeneous with each other. Their heterogeneity lies in the sense that, at every time, each extrapolator has his own idiosyncratic bias when evaluating the news about the risky asset’s cash flow. This modification is well evidenced by many facts that can generate differences among investors, like “Gradual Information Flow” (Hong and Stein 2007), “Limited attention” (Hirshleifer and Teoh 2003, Peng and Xiong 2006, Corwin and Coughenour 2008), “Heterogeneous Prior” (Harris and Raviv 1993, Kandel and Pearson 1995), and so on.

This modification is crucial to our new model, because this makes extrapolators not only trade with fundamentalists but also trade with other extrapolators. This is because, as discussed in Chapter 2, news innovations can affect extrapolators by enhancing their expectation when the news is consistent with their prior extrapolation belief or by weakening it when the news is contradictive. Since extrapolator has his idiosyncratic bias about the news innovation, his expectation will also be heterogeneous with other
extrapolators’ expectations. In other words, some extrapolators will be more optimistic than others even if they observe the same price trend and hold the same extrapolation bias, just because the news they are receiving is better than others. Or, they may be pessimistic compared to others because their “biased” news is not as good as others. Therefore, optimistic extrapolators will buy shares from those who are pessimistic.

Additionally, the difference between extrapolators’ expectation grows as their extrapolation belief increases, as in our model, extrapolators’ difference about the news innovation is amplified by their previous extrapolation belief. The stronger extrapolation belief they are holding, the more diffuse their expectation will be.

We discuss our model’s prediction in two situations, “no short-sale constraint” and “short-sale constraint”. When investors can sell short, like in the future market, the positive correlation between |GSI| and high trading volume is a natural prediction of our model. As in our model, when individuals’ extrapolation belief (|X_i|) increases, the difference between extrapolators and fundamentalists, and the differences between different extrapolators all become more significant. Their demand changes more significantly too. Consequently, trading volume of the whole market increases as the magnitude of individuals’ extrapolation belief grows, like what happens in the future market. Trading volume is accounted for all investors.

In the second situation, when investors are facing short sale constraint, things are different. During the bull market, the cumulative positive price changes strike extrapolator’ enthusiasm. In the beginning, they try to increase their demand, buying from the rational fundamental traders. Their enthusiasm pushes the price even higher
and maybe gradually drive fundamental traders away from the market. But trading volume wouldn’t reduce even when fundamentalists quit the market. As we demonstrated above, when the price continues rising, individuals’ extrapolation belief will become stronger, and accordingly, the difference between them will grow bigger. Therefore, the same bias which introduces only little volume in the beginning, can eventual bring in enormous volume. Because the difference caused by the news adjustment on their expectation is now so large in magnitude. In this situation, the trading volume mainly happens among heterogeneous extrapolators.

Nevertheless, during the bear market, where extrapolators are too pessimistic than they should be, they will first try to sell their own shares out. But because of the short-sale constraint, extrapolators are forced to leave the market just as the fundamentalists do when the asset is too overvalued. Market are only left with fundamentalists. Trading volume will now be caused by the difference among fundamentalists instead of by the difference between extrapolators. As fundamentalists are much more homogenous compared with extrapolators, the trading volume would become smaller when individual’s extrapolation bias grows stronger, which just explains the negative relation between |GSI| (GSI<0) and trading volume as our empirical test finds.

Further, by simulation, we show our model can help us to get a better understanding of financial bubbles.

Financial bubbles seem to be a chronic disease of the financial market when the price of equities surges extremely high compared with its fundamental value (Xiong 2013). Although these bubbles are distributed in different financial markets and in different
periods, high trading volume and high volatility can always be found at the same time in these torrential stages (Xiong and Scheinkman 2003). Nevertheless, despite all the efforts scholars have devoted to financial bubbles, we still cannot fully understand them. As Appendix Table 4.1 shows, traditional models of financial bubbles could address why bubble occurs, but not the fact of the co-occurrence of high trading volume and the high volatility. The same holds true for the first generation of behavioral theories. Although, more recent models like Scheinkman and Xiong (2003) and Barberis et al. (2016), can partly explain the two features, they all have their own shortages. Unlike previous theories, we believe our model can efficiently explain the origination of financial bubbles, as well as their most important features.

By our theory, the bubble occurs as follow: in the initial stage, some positive news innovation push price up, causing individuals to extrapolate, leading price to rise even higher, which will make the individual extrapolate more in return. At certain circumstance, as this process continues, a bubble will be generated by their extrapolation belief. As demonstrated in our second chapter, individual investors are affected both by their confirmation bias as well as their extrapolation bias. The volatility of their expectations, and consequently the volatility of the asset price, will grow endogenously with their extrapolation belief as a result of the adjustment of confirmation bias about the news innovation. And by the heterogeneous extrapolators modification, we prove that, in the bull market, the trading volume also increases as extrapolation belief grows. Therefore, volatility and trading volume will grow endogenously as they are positively correlated with individual investors’ extrapolation
belief. With simulation, we show how powerful of our model to explain the financial bubbles.

To summarize, this paper makes at least two contributions. Firstly, we develop a new model by a simple modification of our model in Chapter 2 which can help us to understand why irrational investors’ extrapolation belief can affect trading volume. Secondly, by simulation this paper proves our new model is especially useful to understand two distinguished features of financial bubbles, the high trading volume and the high volatility.

In the next section, we review related researches about financial bubbles. Section 4.3 presents our model and discusses its implications. Sections 4.4 describe the simulation of an experimental bubble which reveals the two features of financial bubbles. Section 4.5 concludes. All proofs are in the Appendix.

4.2 Literature Review

High trading volume and high volatility are believed to be the most two distinguished features of bubbles (Ofek and Richardson 2001, Cochrane 2002).

The high trading volume, which is the positive correlated with the high price, is thought to be the first characteristic feature of the financial bubbles (Galbraith 1954, Carlos, Neal, and Wandschneider 2006). Ofek and Richardson (2001) demonstrate that “between early 1998 and February 2000, pure internet firms represented as much as 20% of the dollar volume in the public equity market, even though their market capitalization never exceeded 6%.” But after the internet bubble busted, the turnover rate of Nasdaq
stock market declined quickly to average size. Barberis et al. (2016) confirm that the high trading volume is also positively correlated with the price level during the bubble periods, such as the stock market boom of 1928-1929 and the 2004-2005 real estate market.

High volatility is also an obvious character of bubble periods (Scheinkman and Wei Xiong 2003). Cochrane (2002) refers to the much-discussed Palm case: “Palm stock was tremendously volatile during this period, with 15.4% standard deviation of 5-day returns, which is about the same as the volatility of the S&P 500 index over an entire year”. Using different calculation methods, Lowry and Schwert (2002) estimate the volatility of internet stocks, finds that during the tech-bubble, the volatilities of technical stocks are extremely high according to other periods and according to non-technic stocks, also he points out that this phenomenon deserves more study.

To my best knowledge, there are two models directly illustrate the characters of financial bubbles, Scheinkman and Xiong (2003) and Barberis et al. (2016). Scheinkman and Xiong (2003) build a model in which investors are “overconfidence” about their private signals which lead them to hold different opinions about the equity’s price with each other. They assume that, when investors are facing short-sale constraints, investors can choose to buy in assets in order to resale more optimistic ones in the future. The “resale option” can generate a sustainable bubble as people value this option too much. High volume and high price volatility can also be created by investors irrational trading behavior as their enthusiasm on the “resale” option.
Barberis et al. (2016) present a multiple-generation heterogeneous model where many investors are extrapolative investors. Extrapolative investors form their expectation of future return according to their extrapolative signals—they would purchase assets just because of the past positive cumulative return. But in their model, extrapolative investors are also affected by another signal—the asset’s fundamental value. They assume extrapolators also “waver” independently between these two conflictive signals. Heterogeneity between extrapolators generates because of their independent wavering. By a consumption-based equilibrium model, they show that, bubble will be generated by extrapolators’ irrational behavior. Also in this process, the heterogeneity also increases as the gap between two signals grows. Then the trading volume will be endogenously exaggerated by the price level.

But these models still cannot help us fully understand why bubbles are companied with both high trading volume and volatilities. In Scheinkman and Xiong (2003), the disagreement between investors which governs the trading volume is exogenous. As a consequence, the trading volume is exogenous to the price level. This is confirmed by Barberis et al. (2016) who use simulations to test the trading volume pattern in the model of Scheinkman and Xiong (2003). Therefore, Scheinkman and Xiong (2003) cannot explain why during bubbles, trading volume is positively correlated with the price level.

In Barberis et al. (2016), the positive relation between trading volume and the price level is a natural prediction of their model. But, because they build the model with the irrational bias, extrapolation, which requires the future price changes be smaller than
past price changes, their model cannot explain high volatilities during bubbles. Even, their model may predict contradict result: the absolute return scales of each generation will become smaller when bubble grows, hence the volatility, if there is any, will be smaller as bubble growing.

In a word, given the characters of financial bubbles, previous models don’t give us a satisfactory explanation especially why volatility is always extremely high during bubbles.

4.3 A New Model with Heterogeneous Extrapolators

To address why trading volume is correlated with investors’ extrapolation, this paper introduces heterogeneous extrapolators into our model while other model settings are similar.

In this new economy, there are also two assets: a risk-free asset and a risky asset. Risk-free is in perfectly elastic supply and earns a constant return which is normalized to zero, while the risky asset is in fixed supply of amount $Q$. The logarithm evolution of its dividend $D_T$, which is paid in the last term, is given by

$$D_T = D_0 + \varepsilon_1 + \cdots + \varepsilon_t + \cdots + \varepsilon_T$$

$$\varepsilon_t \sim N(0, \sigma^2) \text{ i.i.d. over time.}$$  \hspace{1cm} (1)

where $\varepsilon_t$ is the information released to all investors at period $t$. As in original model, only two kinds of investors trade in the market: fundamentalists and extrapolators. Fundamentalists, who just try to arbitrage the miss pricing, is identical with each other
and make up a fraction of $\mu^F$ of all the investors. As shown in Chapter 2, the fundamental value will be

$$P_t^F = D_t - (T - t) \cdot \gamma \sigma^2_t Q$$  \hspace{1cm} (2)

The other type investor is extrapolators, whose expectation is affected by past price changes. Following Chapter 2, their extrapolation belief can be expressed as

$$X_t = \sum_{k=1}^{t-1} \lambda^k \beta (P_{t-k} - P_{t-k-1} - \eta)$$  \hspace{1cm} (3)

where $0 < \lambda < 1$, $\beta$ means extrapolation coefficient, $\lambda$ is the memory effect, $\eta$ is the risk compensation. And as demonstrated in Chapter 2, they are also suffered with confirmation bias in the way that extrapolative investor will enforce their expectation by new arriving information if it is consistent with their prior extrapolation belief. Nevertheless, when contradictory information comes, they also update their expectation but to a much less extent.

But, unlike previous model, we assume extrapolators are heterogeneous when evaluating the new arriving news. Every time news is announced to the public, unlike fundamentalists who can receive immediately and correctly, extrapolator hold his idiosyncratic bias in the process of evaluating the arrival news.

The idiosyncratic bias of individual investors on information processing has solid proofs. For example, the “Gradual Information Flow” mechanism which refers to the fact that some investors will receive certain important news earlier than other investors (Hong and Stein, 1999). This fact draws many researchers’ attention such as Huberman and Regev (2001), Menzly and Ozbas (2006). Cohen and Frazzini (2006) gives a
specific paradigm that changes in a company performance will be easily detected by its customers or suppliers who will take this advantage to trade with uninformed investors.

“Limited Attention” which means investors can only focus on a subsection of all the overwhelming publicly information (Hirshleifer and Teoh 2003, Peng and Xiong, 2006). This shortage of individual investors is more important in current financial market for the high-complexity of these market and the accelerating speed of world changing. Besides, Harris and Raviv (1993) and Kandel and Pearson (1995) suggest that “Heterogeneous Priors” can also generate difference among investors even if they receive all the valuable information at the same time, because the priority of different news is heterogeneous to different investors. Besides, investors heterogeneousness of information processing can also be generated by other mechanisms, such as Distorted Transmission (Hong, Scheinkman and Xiong, 2008, Malmendier and Shantikumar, 2007), representativeness and conservatism (Hirshleifer (2001), Barberis and Thaler (2003)), and so on.

Therefore, this paper introduces heterogeneous investors into our model in the way that extrapolative investors have their own bias when evaluating the information innovation $\varepsilon_t$. Assume their bias are norm distributed, such that

$$\varepsilon_t^i = \varepsilon_t + \omega_t^i \text{ where } \omega_t^i \sim \mathcal{N}(0, \sigma^2_{\omega_i}) \text{ i.i.d.} \quad (4)$$

where $\omega_t^i$ means extrapolator $i$’s bias about the information innovation $\varepsilon_t$. For simplicity, we assume it is independent and identically distributed. Then

$$\varepsilon_t^i \sim \mathcal{N}(\varepsilon_t, \sigma^2_{\omega_i}) \quad (5)$$
Assume there are \( n \) types of extrapolators, indexed by \( i \in \{1, 2, 3 \ldots, n\} \), and each type takes up the fraction \( \mu^{E,i} \), such that:

\[
\mu^F + \sum \mu^{E,i} = 1
\]  

(6)

More simply, we assume each type of extrapolator take the same fraction of the whole population. Then,

\[
\mu = \mu^{E,i} = \mu^{E,i} = \frac{1-\mu^F}{l}
\]  

(7)

Following our model setting in Chapter 2, extrapolator i’s expected price changes of next period should be,

\[
\hat{X}_t^i = \begin{cases} 
X_t \cdot e^{\varepsilon_t^i} & X_t \geq 0 \\
X_t \cdot e^{-\varepsilon_t^i} & X_t < 0 
\end{cases}
\]  

(8)

Therefore, extrapolators’ irrational belief \( \hat{X}_t^i \) is a log-normal distributed, such that

\[
\mathbb{E}(\hat{X}_t^i) = \begin{cases} 
X_t \cdot e^{\varepsilon_t^i + \frac{\sigma_t^2}{2}} & X_t \geq 0 \\
X_t \cdot e^{-\varepsilon_t^i + \frac{\sigma_t^2}{2}} & X_t < 0 
\end{cases}
\]  

(9)

On basis on above model settings, we discuss the prediction of our model in two situations, “no short-sale constraint” and “short-sale constraint”.

4.3.1 No short-sale constraint

When investors are free to sale short, like in the future market, they can borrow shares form others to arbitrage as long as they think the asset is overpriced, even if they don’t currently have any in their hands. In this situation, investors can freely adjust their
demand by choosing long position (positive demand) or short position (negative demand). Therefore, similar with Chapter 2, demand of fundamentalists should be

$$N_t^F = \frac{D_t - (\gamma - 1)^2 - Q - P_t}{\gamma \sigma^2} = \frac{P_t^F + \gamma \sigma^2 Q - P_t}{\gamma \sigma^2}$$

(10)

This means the demand of fundamentalist will become negative as long as the actual price surpasses the fundamental value by $\gamma \sigma^2 Q$. Similarly, we assume extrapolators also pay attention to fundamental value, then, the demand of the $i$th extrapolator should be

$$N_t^{E,i} = \theta \frac{D_t - (\gamma - 1)^2 - Q - P_t}{\gamma \sigma^2} + \phi \frac{R_i^f + \gamma \sigma^2}{\gamma \sigma^2}$$

(11)

or

$$N_t^{E,i} = \theta N_t^F + \phi \frac{R_i^f + \gamma \sigma^2}{\gamma \sigma^2}$$

(12)

$\theta$ and $\omega$ denote the weight extrapolator puts in the two signals, fundamental value signal and irrational signal, respectively. Then the equilibrium price will be determined when the sum of all investors’ demand equals the fixed supply $Q$, as

$$\mu^F N_t^F + \sum \mu_i^{E,i} N_i^{E,i} = Q$$

(13)

Therefore, the equilibrium price would be

$$P_t = \begin{cases} 
  P_t^F + \phi X_t e^{\varepsilon_t + \frac{\sigma^2}{2}} & X_t \geq 0 \\
  P_t^F + \phi X_t e^{-\varepsilon_t + \frac{\sigma^2}{2}} & X_t < 0
\end{cases}$$

(14)

Then, when $X_t \geq 0$, from period $t - 1$ to $t$, the demand change of fundamentalists,
\[ O_t^F = N_t^F - N_{t-1}^F = \frac{1}{\gamma \sigma_{\varepsilon}^2} [(P_t - P_t^F) + (P_t - P_{t-1})] \]

\[ = \frac{1}{\gamma \sigma_{\varepsilon}^2} (X_{t-1}e^{\varepsilon_{t-1} + \frac{\sigma_0^2}{2}} - X_t e^{\varepsilon_t + \frac{\sigma_0^2}{2}}) \tag{15} \]

From equation (3), we know that

\[ X_t = \lambda X_{t-1} + \lambda R_t \quad \text{or} \quad X_{t-1} = \frac{1}{\lambda} X_t - R_t \tag{16} \]

Then equation (15) can be rewritten as

\[ O_t^F = \frac{1}{\gamma \sigma_{\varepsilon}^2} \left[ \left( \frac{1}{\lambda} X_t - R_t \right) e^{\varepsilon_{t-1} + \frac{\sigma_0^2}{2}} - X_t e^{\varepsilon_t + \frac{\sigma_0^2}{2}} \right] \]

\[ = \frac{1}{\gamma \sigma_{\varepsilon}^2} X_t e^{\frac{\sigma_0^2}{2}} \left( \frac{1}{\lambda} e^{\varepsilon_{t-1} - \varepsilon_t} \right) - \frac{1}{\gamma \sigma_{\varepsilon}^2} R_t e^{\varepsilon_t + \frac{\sigma_0^2}{2}} \tag{17} \]

For simplicity, assume \( R_t \) is a martingale process with mean value of zero. Then,

\[ E(\Omega_t^F) = \frac{1}{\gamma \sigma_{\varepsilon}^2} X_t \Delta e^{\frac{\sigma_0^2}{2}} \quad \text{where} \quad E \left( \frac{1}{\lambda} e^{\varepsilon_{t-1} - \varepsilon_t} \right) = \Delta \tag{18} \]

Similarly, for the \( i \)th extrapolator, his demand change can be expressed as

\[ O_{t}^{E,i} = N_{t}^{E,i} - N_{t-1}^{E,i} = \varphi O_{t}^F + \phi \left( \frac{X_{t+1}^{i} + \gamma Q \sigma_{\varepsilon}^2}{\gamma \sigma_{\varepsilon}^2} - X_t^{i} + \gamma Q \sigma_{\varepsilon}^2 \right) \]

\[ = \varphi \frac{1}{\gamma \sigma_{\varepsilon}^2} (X_{t-1} e^{\varepsilon_{t-1} + \frac{\sigma_0^2}{2}} - X_t e^{\varepsilon_t + \frac{\sigma_0^2}{2}}) + \phi \frac{X_{t+1} e^{\varepsilon_{t+1} + \omega i - X_t e^{\varepsilon_{t-1} + \omega i - 1}}}{\gamma \sigma_{\varepsilon}^2} \tag{19} \]

Then its conditional expectation will be

\[ E(\Omega_{t}^{E,i}) = \varphi \frac{1}{\gamma \cdot \sigma_{\varepsilon}^2} \Delta X_t e^{\frac{\sigma_0^2}{2}} - \phi \frac{1}{\gamma \cdot \sigma_{\varepsilon}^2} \Delta X_t \]

\[ = \frac{1}{\gamma \sigma_{\varepsilon}^2} \Delta X_t (\varphi e^{\frac{\sigma_0^2}{2}} - \phi) \tag{20} \]
For the whole market, trading volume equals the half of the sum of all investors’ absolute demand changes, such as

\[ V_t = \frac{1}{2} (\mu^F |O_t^F| + \sum_{i=1}^I \mu^E_i |O_t^{E,i}|) \]  

then

\[ E(V_t) = \frac{1}{2} [\mu^F E(|O_t^F|) + \mu E(|O_t^{E,i}|)] \]  

From equation (18) and (19), we know

\[ \frac{\partial E(V_t)}{\partial |X_t|} > 0 \quad \text{and} \quad \frac{\partial E(|O_t^{E,i}|)}{\partial |X_t|} > 0 \]  

so,

\[ \frac{\partial V_t}{\partial |X_t|} > 0 \]  

It is easy to prove, when \( X_t < 0 \), the trading volume also satisfy inequality (24).

Therefore, the trading volume of the whole market increases as the magnitude of \( |X_t| \) increases, just like what happens in the future market. In this situation, both the demand of extrapolators and fundamentalists change more greatly as \( |X_t| \) increases. But it should be noticed that, extrapolators not only trade with fundamentalist but may also trade with other extrapolators.

4.3.2 Short-sale constraint

In this situation, investors are facing short sale constraint. Even though in their belief, the risky asset is overpriced to a large extent which can ensure good profit from short-
selling, they cannot short to arbitrage. Hence, the price is only determined by more
optimistic investors. Therefore, every type of investors’ demand is determined by

\[ N_t^\mathcal{F} = \max \left[ \frac{p_t^\mathcal{F} + \gamma \alpha^z Q - p_t}{\gamma \sigma^z}, 0 \right] \]  

\[ N_t^{E,i} = \max \left[ \varphi \frac{p_t^\mathcal{F} + \gamma \alpha^z Q - p_t}{\gamma \sigma^z} + \varphi \frac{\bar{x}^i + \gamma \bar{\sigma}^\mathcal{F} Q}{\gamma \bar{\sigma}^\mathcal{F}}, 0 \right] \]

To show how investors behave in this situation, we firstly introduce the definition of
investors’ Acceptable Price (\( \widehat{p}_{t}^{\mathcal{F}} \)): for the \( i \)th extrapolator, at time \( t \), his Acceptable Price is the price at which he will not change his demand if he can freely buy in or sell short. In other words, his Acceptable Price is the price when other investors hold the same belief with him, then the market is determined by his will. If the actual price is higher than his Acceptable Price, he will try to reduce his demand, and vice-versa.

Then, for the fundamentalist, their Acceptable Price is just the fundamental value, \( \widehat{p}_{t}^{\mathcal{F}} = \mathcal{P}_t^{\mathcal{F}} \). But for extrapolator, his Acceptable Price will be determined by:

\[ \varphi \frac{p_t^\mathcal{F} + \gamma \alpha^z Q - p_t}{\gamma \sigma^z} + \varphi \frac{\bar{x}^i + \gamma \bar{\sigma}^\mathcal{F} Q}{\gamma \bar{\sigma}^\mathcal{F}} = Q \]  

then:

\[ \widehat{p}_{t}^{E,i} = \varphi P_t^\mathcal{F} + \varphi \bar{x}_t^i \]

Equation (28) means, extrapolator’s Acceptable Price is just the weighted average of two signals he cares, the fundamental value and his irrational bias. Accordingly, his Acceptable Price is also log-normal distributed, and the mean value should be
Also from (25) and (26), we can figure when will investors quit the market. For the fundamentalists, as long as $N_t^F \leq 0$, they will keep their demand be 0 and stay away, at this time,

$$P_t \geq D_t - (T - t - 1)\gamma \sigma^2 \gamma Q \quad \text{or} \quad P_t \geq P_t^F + \gamma \sigma^2 \gamma = \overline{P}_t^F + \gamma \sigma^2 \gamma Q \quad (30)$$

Similarly, for extrapolators, if $N_{t,i}^E \leq 0$,

$$P_t \geq \varphi P_t^F + \phi X_t^i + \gamma \sigma^2 \gamma Q \quad \text{or} \quad P_t \geq \overline{P}_t^E,i + \gamma \sigma^2 \gamma Q \quad (31)$$

From equation (30) and (31), we can see if the actual price $P_t$ surpasses investor’s Acceptable Price $\overline{P}_t^E,i$ by $\gamma \sigma^2 \gamma Q$, or, if investor’s Acceptable Price $\overline{P}_t^E,i$ is smaller than $P_t - \gamma \sigma^2 \gamma$, he will quit the market. So, we can define the Threshold Price as

$$P_t^* = P_t - \gamma \sigma^2 \gamma Q \quad (32)$$

Whenever $\overline{P}_t^E,i \leq P_t^*$, investor $i$ will quit the market, leaving the market with more optimistic ones. But it should be noticed here that, for the same extrapolator who is more optimistic than other extrapolators in some period, may become pessimistic than his peers in the next period. This is because his random idiosyncratic bias will make him overestimate the news sometime, but underestimate it at other time. Isaac Newton’s failed speculation in the South Sea bubble is a good certification of extrapolators who are going in and out of the market. (Barberis et, al. 2016).

Accordingly, the equilibrium price is determined by those investors who are more optimistic than others. To unify, we denote $\mu^F = \mu_0$, $\mu^E,i = \mu_i$, $N_t^F = N_t^0$, $N_t^{E,i} =$
\( N_i^* \). \( i^* \) is the subset of \( i \in \{ 0,1,2,3 \ldots , n \} \), such that every trader in this set has positive demand for the risky asset. Likewise, define \( I \in \{ 1,2,3 \ldots , n \} \) to specify extrapolators who have positive demand. Then the equilibrium price will be resulted by

\[
\sum_{i \in I} \mu_i \cdot N_i = Q
\]

(33)

where \( I^* \) is the subset of \( i \in \{ 0,1,2,3 \ldots , n \} \), such that any trader in this set has positive demand for the risky asset. Then by simple calculation we can get, when fundamentalists are still in the market \((P_t^F > P_t^*)\),

\[
P_t = P_t^F + \sum \frac{\mu_i}{\mu^F + \Sigma_{i \in I} \mu_i} \cdot X_i^t - \frac{1 - (\mu^F + \Sigma_{i \in I} \mu_i)}{\mu^F + \Sigma_{i \in I} \mu_i} \gamma \sigma_x^2 Q
\]

(34)

Likewise, when fundamentalists left the market,

\[
P_t = P_t^F + \sum \frac{\mu_i}{\Sigma_{i \in I^*} \mu_i} \cdot X_i^t - \frac{1 - \Sigma_{i \in I^*} \mu_i}{\Sigma_{i \in I^*} \mu_i} \gamma \sigma_x^2 Q
\]

(35)

Combining (34) and (35), we can get,

\[
P_t = \sum \frac{\mu_i}{\Sigma_{i \in I^*} \mu_i} \cdot P_t^E - \frac{1 - \Sigma_{i \in I^*} \mu_i}{\Sigma_{i \in I^*} \mu_i} \gamma \sigma_x^2 Q
\]

(36)

Equation (36) shows the actual price is a little smaller than the weighted average value of the existing investors’ Acceptable Price. This devaluation can be regarded as the risk compensation asked by the existing investors because they have to absorb other investors’ shares who have already left the market.

On basis of this, we can understand now how the trading volume changes during the bull market or during the bear market. We start with the bull market.
4.3.2.1 Trading volume in the bull market

Figure 4.1 illustrates how price evolves in this circumstance. Assume at time $t = 0$, $X_0 = 0$, $P_0 = P_0^F = \hat{P}_0^E$. At time $t = 1$, when a positive information innovation is released, the price rises because fundamentalists understand the positive news and try to buy in more. Subsequently, the extrapolators who observe this positive price change will become optimistic about future returns. Then their enthusiasm pushes the price even higher ($P_1 > P_1^F$). In this progress, fundamentalists reduce their demand while extrapolators increase their demand. But because the price is not too high ($P_1^F > P_1^*$), fundamentalists still exist in the market.

At the next time, $t = 2$, with $X_t$ continues growing, extrapolators become more exited which eventually lead actual price be much bigger than the fundamental value. As shown in the second picture of Figure 4.1, in this situation, as $P_2 > P_2^F + \gamma \sigma_\varepsilon^2 Q$, fundamentalists have quit the market, leaving the market with only extrapolators. Then, the actual price is determined by the existing extrapolators’ expectations. But, because some extrapolators underestimate the new-innovated information ($\varepsilon_1^i < \varepsilon_1^*$), they are pessimistic because their adjusted Acceptable Price is smaller than other extrapolators, and even smaller than the threshold value. As shown in this picture, for any extrapolator whose Acceptable Price is smaller than the Threshold Price $P_2^* = P_2 - \gamma \sigma_\varepsilon^2$ (dash line), he will quit the market. The dash area denotes his possible Acceptable Price which is smaller than $P_2^*$. Therefore, the actual price should be higher than the mean value of all extrapolator’s Acceptable Price (the dash blue line), or $P_2 > E_1 \left( \hat{P}_t^E \right)$. 

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Figure 4.1 Demonstration of price evolution in the bull market

Horizontal axis in this figure indicates the price level while the curve shows Probability density function of extrapolator’s Acceptable Price $f(x)$.

Things are similar when the price continues to rise. Like shown in the third picture, $X_t$ has grown much higher because of past cumulative price changes, the actual price surpasses the fundamental value to a larger extent, fundamentalists stay away from the
market. But as extrapolators are different with each other about the news, their Acceptable Prices are diffused as before. The actual price will be determined solely by those more optimistic extrapolators. As we have discussed above, the \( i \)th extrapolator may be the optimist at one period (in the blank area of Figure 4.1.b), but become pessimist in next period (drops into the shadow area of Figure 4.1.c), and vice versa. So, for the \( i \)th extrapolator, his trading induced by the difference between \( N_{t}^{E,i} \) and \( N_{t-1}^{E,i} \) can be categorized into three classes:

1. He exists in the market in both periods. In this situation, his Acceptable Price is in the blank area for both period. His demand change just follows equation (19) and equation (20), as

\[
m_{t}^{i} = O_{t}^{E,i} = N_{t}^{E,i} - N_{t-1}^{E,i} = \varphi O_{t}^{E,i} + \phi \left( \frac{x_{t+1}^{i} + y \sigma_{x}^{2}}{\gamma \sigma_{t}^{2}} - \frac{x_{t}^{i} + y \sigma_{x}^{2}}{\gamma \sigma_{t}^{2}} \right)
\]

\[
= \varphi \frac{1}{\gamma \sigma_{t}^{2}} (X_{t-1} e^{e_{t-1}^{i} + \frac{\sigma_{b}^{2}}{2}} - X_{t} e^{e_{t}^{i} + \frac{\sigma_{b}^{2}}{2}}) + \phi \frac{x_{t+1}^{i} e^{e_{t}^{i} + \omega t} - x_{t}^{i} e^{e_{t-1}^{i} + \omega t-1}}{\gamma \sigma_{t}^{2}}
\]

then its expected value will be

\[
E(m_{t}^{i}) = E(O_{t}^{E,i}) = \varphi \frac{1}{\gamma \sigma_{t}^{2}} \Delta X_{t} e^{\frac{\sigma_{b}^{2}}{2}} - \phi \frac{1}{\gamma \sigma_{t}^{2}} \Delta X_{t}
\]

\[
= \frac{1}{\gamma \sigma_{t}^{2}} \Delta X_{t} (\varphi e^{\frac{\sigma_{b}^{2}}{2}} - \phi)
\]

Therefore, trading volume accounted by this extrapolator in situation is the absolute value of \( m_{t}^{i} \). Simply

\[
\frac{\partial E(|m_{t}^{i}|)}{\partial |x_{t}|} > 0
\]
(2) He exists in the market for only one period, or his position is in the blank area for one period but is in the shadow area in the other.

\[ n_t^i = \mathcal{N}_t^{E,i} = (\varphi \frac{p_t^i + \gamma \sigma_z^2 Q - p_t}{\gamma \sigma_z^2} + \varphi \frac{\hat{X}_t^i + \gamma \sigma_z^2 Q}{\gamma \sigma_z^2}) \]  

(40)

As the fundamentalists have already left the market, then the equilibrium price now will follow equation (35). Substitute it into (40), we can get,

\[ n_t^i = \frac{1}{\gamma \sigma_z^2} (\varphi \hat{X}_t^i - \sum_{i=1}^{\mu_i} \cdot \hat{X}_t^i) + \frac{1}{\sum_{i=1}^{\mu_i}} Q \]

\[ = \frac{1}{\gamma \sigma_z^2} X_t (\varphi \hat{e}_t^i - \sum_{i=1}^{\mu_i} \cdot \hat{e}_t^i) + \frac{1}{\sum_{i=1}^{\mu_i}} Q \]  

(41)

it can easily prove,

\[ \frac{\partial E(|n_t|)}{\partial |X_t|} > 0 \]  

(42)

(3) He doesn’t exist in the market for both periods (being in the shadow area for both periods)

\[ \sigma_t^i = 0 \]  

(43)

Therefore, the expected value of trading volume induced by this extrapolator will be

\[ \bar{\nu}_t^i = |m_t^i| (\int_{p_t}^{\phi \sigma_z^2} f(x))^2 dp + 2|n_t^i| \cdot \int_{p_t}^{\phi \sigma_z^2} f(x) \int_{p_t}^{\phi \sigma_z^2} f(x) dp \]

\[ + \sigma_t^i \int_{p_t}^{\phi \sigma_z^2} f(x) \int_{p_t}^{\phi \sigma_z^2} f(x) dp \]  

(44)

where \( f(x) \) is the probability density function of extrapolator’s Acceptable Price. As \( \omega_t^i \) is independent identical distributed, we can assume \( f(x) \) being the same for different periods. Denote:
\[ \emptyset_1 = (\int_\rho^\infty f(x))^2 \]
\[ \emptyset_2 = 2 \cdot \int_\rho^\infty f(x) \cdot \int_0^{\rho_r} f(x) \]
\[ \emptyset_3 = 1 - \emptyset_1 - \emptyset_3 = \int_0^{\rho_r} f(x) \cdot \int_0^{\rho_r} f(x) \]

Then
\[ \overline{v}_t = \emptyset_1 |m_t^i| + \emptyset_2 |n_t^i| \]  
(46)

When fundamentalists leave the market, like what happens in Figure 4.1.b and Figure 4.1.c, the price will be solely determined by extrapolators, then \( \emptyset_1, \emptyset_2 \) and \( \emptyset_3 \) are stable as their heterogeneous bias on news evolution is normal distributed by the same variance \( \sigma^2 \) across time. Therefore,

\[ \frac{\partial \nu_t}{\partial |x_{el}|} > 0 \]  
(47)

Every extrapolator fits in one of above three types. Other extrapolator’s expected value of trading volume will be the same with \( \overline{v}_t \), such that

\[ \overline{v}_t = \nu_t^j \quad i \neq j \]  
(49)

Thus, when fundamentalists stay away from the market, trading volume of whole market will be

\[ V_t = \frac{1}{2} \sum_{i \in I} \mu^i * \nu_t^i = \frac{1}{2} \overline{v}_t \sum_{i \in I} \mu^i \]  
(50)

\[ \frac{\partial \nu_t}{\partial |x_{el}|} > 0 \]  
(51)
In a word, when $X_t > 0$ and fundamentalists quit the market, $\emptyset_1, \emptyset_2, \emptyset_3$ are stable. Then the trading volume will be positively correlated with $X_t$.

4.3.2.2 Volatility during the bull market

As we demonstrated in equation (36) that, during the bull market, the equilibrium price approximates the weighted average of existing extrapolators’ Acceptable Price, or those extrapolators’ expectation whose Acceptable Price lies in the blank area in Figure 4.1. Therefore, it is a little higher than the mean value of all extrapolators’ Acceptable Price (the blue dash line in Figure 4.1). More officially, it should be

$$P_t = \int_p^\infty f(x)P \, dp - \frac{1 - \sum_{i \in I} \mu_i}{\sum_{i \in I} \mu_i} \gamma \sigma_e^2 Q$$  \hspace{1cm} (52)

It can be proved that the weighted average of existing extrapolators’ Acceptable Price is a constant ratio compared with $\exp \left( \frac{\sum_{i \in I} \mu_i}{\sum_{i \in I} \mu_i} \right)$, assume

$$\int_p^\infty f(x)P \, dp / \int_0^\infty f(x)P \, dp = A$$  \hspace{1cm} (53)

For the detail deduction, see Appendix 4.2. As we illustrated that, $f(x)$, the probability density function of extrapolator’s Acceptable Price, is stable across the process. Substitute equation (29) and (53) into (52), we will get

$$P_t = A(\varphi P_t^F + \phi X_t \cdot e^{\sigma_e^2/2}) - \frac{1 - \sum_{i \in I} \mu_i}{\sum_{i \in I} \mu_i} \gamma \sigma_e^2 Q$$  \hspace{1cm} (54)

Then the return of this process at time $t$ should be:

$$R_t = P_t - P_{t-1} = A \varphi \varepsilon_t + A \phi e^{\sigma_e^2/2} (X_t \cdot e^{\varepsilon_t} - X_{t-1} \cdot e^{\varepsilon_{t-1}})$$  \hspace{1cm} (55)
Simply, as in the second chapter, volatility at time $t$ can be represented as:

$$
E[\text{Var}(R_t|\Pi_{t-1})] = A^2 \varphi^2 E[\text{Var}(\varepsilon_t|\Pi_{t-1})] + (A \phi e^{\varepsilon_t+\frac{\sigma^2}{2}X_t})^2 E[\text{Var}(e^{\varepsilon_t}|\Pi_{t-1})] \\
+ A^2 \phi \Phi e^{\varepsilon_t+\frac{\sigma^2}{2}X_t} E[\text{cov}(\varepsilon_t, \varepsilon_t|\Pi_{t-1})]
$$

Equation (56) means, in the bull market, not only the trading volume increases with $X_t$, but also the volatility increases too. This conclusion coordinates with our finding in Chapter 1 and Chapter 2. Specially, according to our theory, when the price has continually raised to a significant extent and the bubble occurs, the trading volume and volatility will be both unusually high than other periods. Therefore, our model is particularly helpful to understand the trading volume anomaly and the volatility anomaly of financial bubbles.

### 4.3.2.3 Trading volume in the bear market

During the bear market, things are different when $X_t$ decreases from zero to negative. Similarly, we demonstrate this process with Figure 2. As shown in this figure, at the beginning $t = 0$, $X_0 = 0$, $P_0 = \tilde{P}_0 = \tilde{P}^{F}_{0}$. Then, a negative information innovation shocks the market, the price drops immediately for the fundamentalists can correctly observe the fundamental value. The negative price change makes extrapolators be pessimistic about future returns, hence, their expected-price drops for their negative extrapolation belief. Like in Figure 2.a, the actual price drops from $P_0$ to $P_1$ while the fundamental price becomes $P^{F}_1$ from $P^{F}_0$. The overreaction is just caused from the pessimistic extrapolators.
Figure 4.2 Demonstration of price evolution in the bear market

Horizontal axis in this figure indicates the price level while the curve shows Probability density function of extrapolator’s Acceptable Price $f(x)$.

As in the bull market, extrapolators hold heterogeneous belief on the risky asset’s price, because their bias on the information innovation. Then, at time $t = 1$, there are already some pessimists whose Acceptable Price is lower than the threshold price. For
those who are too pessimistic ($P_i^{E,t} < P_1^*$), they have already quit the market. Then, the actual price is also determined by more optimistic ones.

At the next time, as prices drops, extrapolators become more pessimistic. Much more of them quit the market as their Acceptable Price $\tilde{P}_{i}^*$ is under the threshold price $P^*$. The shadow area in Figure 1.b become bigger than in Figure 1.a, indicating more extrapolators stay out of the market. As this process continues, $X_t$ decreases and the shadow area grow bigger and bigger, more and more extrapolators quit the market. Eventually, there are only few extrapolators stay in the market, as in Figure 4.c.

Like in the bull market, for one extrapolator, his trading activity can also be categorized into three classes, stays in both period, stay only in one period and stay away in both period. The trading volume in three situations can also be expressed as:

$$
m_t^i = O_{t}^{E,i} = N_t^{E,i} - N_{t-1}^{E,i} = \varphi O_t^E + \phi \left( \frac{X_{t+1}^i + \gamma Q \sigma_2^2}{\gamma \sigma_2^2} - \frac{X_t^i + \gamma Q \sigma_2^2}{\gamma \sigma_2^2} \right)
$$

$$
= \varphi \frac{1}{\gamma \sigma_2^2} \left( X_{t-1} e^{-\varepsilon_{t-1} + \frac{\sigma_2^2}{2}} - X_t e^{-\varepsilon_t + \frac{\sigma_2^2}{2}} \right) + \phi \frac{X_{t+1} e^{-\varepsilon_{t+1} + \omega_1} - X_t e^{-\varepsilon_t + \omega_1}}{\gamma \sigma_2^2}
$$

Assume $E \left( \frac{1}{\lambda} e^{-\varepsilon_{t-1}} - e^{-\varepsilon_t} \right) = \nabla$, then the expectation of (51) will be

$$
E \ m_t^i = E \ O_t^{E,i} = \varphi \frac{1}{\gamma \sigma_2^2} \nabla X_t e^\frac{\sigma_2^2}{2} - \phi \frac{1}{\gamma \sigma_2^2} \nabla X_t
$$

$$
= \frac{1}{\gamma \sigma_2^2} \nabla X_t (\varphi e^\frac{\sigma_2^2}{2} - \phi)
$$

Simply,

$$
\frac{\partial E \ m_t^i}{\partial |X_t^i|} > 0
$$
Likewise, for the second situation,

\[ n^1_t = \frac{1}{\gamma \sigma^2_t} X_t (\phi e^{-\epsilon^1_t} - \sum_{i \in I} \frac{\mu_i}{\Sigma_{i \in I} \mu_i} \cdot e^{-\epsilon^1_t}) + \frac{1}{\Sigma_{i \in I} \mu_i} Q \]  

and

\[ \frac{\partial E[|n^1_t|]}{\partial |X_t|} > 0 \]  

For the third class, when he quit the market for both period,

\[ \sigma^1_t = 0 \]  

Therefore, the expected value of trading volume by extrapolator in the bear market can also be expressed as

\[ \bar{v}_t^1 = |m^1_t| (\int_{p_1}^{p_2} f(x) \, dx)^2 dp + 2 |n^1_t| \cdot \int_{p_1}^{p_2} f(x) \, dx \int_{0}^{p_2} f(x) \, dp \]

\[ + \sigma^1_t \int_{0}^{p_2} f(x) \, dp \]

\[ = \phi_1 |m^1_t| + \phi_2 |n^1_t| \]  

But as we shown in Figure 4, the probability of investors stay in the market (the blank area) reduces as \(|X_t|\) increases. Or, both \(\phi_1\) and \(\phi_2\) reduces as \(|X_t|\) increases. Therefore,

\[ \frac{\partial \bar{v}_t^1}{\partial |X_t|} < 0 \text{ when } \frac{\partial |m^1_t|}{\partial |X_t|} < - \frac{\phi_1}{\phi_2} \text{ and } \frac{\partial |n^1_t|}{\partial |X_t|} < - \frac{\phi_2}{\phi_2} \]  

Which indicates, as long as the increasing speed of extrapolators’ demand change is smaller than the speed of they leaving the market, the trading volume will reduce \(|X_t|\) increases.
The most significant difference between the bull market and the bear market is the probability of extrapolators who trade in the market. In the bull market, most of the extrapolators insist in the market for the whole process. As the difference between extrapolators grows rapidly when their extrapolation belief increases, the trading volume of the whole market rises accordingly. But in the bear market, when extrapolators are more pessimistic than fundamentalists, more and more of they will quit the market. When the market is full of more identical fundamentalists, little trading volume happens. On the contrary, as we illustrated above, in the future, no extrapolator quits the market even in the bear market. The trading volume will consequently increase as their difference grows with a growing $|X_e|$.

In a word, our model which emphasize the heterogeneity among extrapolators not only can explain the relation between individuals’ extrapolation belief in the future market, but can also explain their relation in the markets with strict short-sale constraint.

4.4 Simulation of the bubble

As been demonstrated in sector 4.3, our model can help to understand the two most distinguish feature of financial bubbles, high trading volume and the high volatility. We now use simulation to illustrate this.

In section 4.3.2, we prove that, when short-sale constraint exists, the price evolution should follow equation (36). Therefore, we firstly simulate this equation to see what will happen to the price, volatility and trading volume according to our model. Then, we observe the demand and the Acceptable Price evolution process of every investor to
see if our analysis method in section 4.3.2.1 can also explain the trading volume of our simulation.

4.4.1 Simulation of the basic model

| Table 4.2: Numerical benchmark parameters for bubble simulation |
|------------------|------------------|------------------|
| Parameter        | Value            | Description                  |
| $I$              | 100              | Number of Extrapolators     |
| $\lambda$       | 0.8              | Memory effect               |
| $\beta$         | 0.2              | Extrapolation coefficient   |
| $\mu^F$         | 30%              | Proportion of Fundamentalists|
| $\mu^{E,l}$     | 0.7%             | Proportion of Fundamentalists|
| $\gamma$        | 0.1              | Coefficient of absolute risk aversion |
| $\sigma_\epsilon$ | 1.5              | Volatility of high frequency information innovations |
| $\sigma_\omega$ | 0.2              | Volatility of Extrapolators’ idiosyncratic bias |
| $D$             | 50               | Initial expected dividend   |
| $N$             | 60               | Number of Period            |

Figure 4.3 shows one possible simulated result of equation (36) with parameters set according to Table 4.2. In this simulation, there are 100 different types of extrapolators who are different from each other in evaluating information innovations. While the volatility of the information is 1.5, we assume the variation of their bias on the innovation to a very small value (only 0.2), to show that even a slight difference between extrapolators can generate interesting results. Simulated price, trading volume, volatility as well as extrapolator’s extrapolation belief $|X_t|$ are displayed in Figure 4.3,
from which we can see the simulation based on our model can realistically mimic the bubble’s generation, as well the high volume and high volatility during this process.

As shown in Figure 4.3.a, during the first 10 periods, there is no news announced to the public \((\epsilon_t = 0)\), price rises by a fixed value every time. This is the compensation risk as we demonstrated in Chapter 2. Since \(t = 11\), the market starts to be affected by cash-flow news (information innovations). Initially, the fundamental value (red line in Figure 4.3.a) is driven to fall. The actual price (black line in Figure 4.3.a) drops as well. Fortunately, this trend doesn’t persist as most following news become positive which makes the fundamental value to rise. The actual price rises too. Then, after a short period, the actual price starts to transcend the fundamental value (the black line begins to suppose the red one) until a very distinguished bubble arises. As shown in the price picture, the peak of the actual price is about 180 while the fundamental value is only about 100, indicating the asset is about 80% overvalued. This bubble doesn’t last long until it bursts. Eventually, the actual price converges to the fundamental value by a short time. Moreover, we also plot the price with the green line when extrapolators are homogenous with each other in Figure 4.3. a. Our simulation result clearly shows that the bubble will grow much bigger when extrapolators are heterogeneous. This is consistent with other papers’ result that with short constraint and heteronogous investors, the asset is easily overpriced (Scheinkman and Xiong, 2004, Miller 1977, Harrison and Kreps 1978).

During this process, the trading volume and volatility also increase intensively. For simplicity, following Grange and Sin (1999), Masset (2011), Frank and Westerhoff
(2016) and other researchers, we use the absolute daily returns $|R_t|$ as the measurement of volatility which we plot in Figure 4.3.b. Combining Figure 4.3.c which displays extrapolators’ extrapolation belief, we can clearly see the volatility is positively correlated with $|X_t|$. As we demonstrated in the second chapter, our model based on extrapolation bias and confirmation bias can explain the positive relation between extrapolation belief and volatility as well. Volatility, in our model rises as the expectation of extrapolators is not only determined by their extrapolation belief, but also is affected by the information innovations. The same volatility in the information innovation will be fundamentally amplified by extrapolator’s extrapolation belief in the bubble period.

Besides volatility, the volume also grows rapidly as the bubble grows. As shown by Figure 4.3.d, the turnover rate is promoted to about 90% at the top of the bubble, which means almost all the shares are handed over from one group of people to another just in one trading period. To explain why trading volume increases into such amount, we plot every trader’s changing demand in Figure 4.4.

Firstly, from Figure 4.4 we can understand why does the bubble generate. As shown in this figure, during the initial stage ($t = 1 \ldots,10$), extrapolators and fundamentalists just hold their shares, and no trading happens. Then, when the negative news arrives which leads the price to fall, extrapolators start to sell their shares to the fundamentalist as their extrapolation belief makes them be pessimistic about future. During this stage, fundamentalist’s demand (the dotted line) increases, and most of the extrapolators choose to reduce their demand (the colored solid line). But when the actual price starts
to exceed the fundamental value, extrapolators gradually increase their demand as they become enthusiasm about future returns while the demand of fundamentalists in this process drop quickly to zero. With more positive news comes and extrapolators become more excited about future, the overvaluation grows so big that a distinguished bubble generates.

Figure 4.4 also explains why the trading volume increases even after the fundamentalist quit the market. As can be seen in this picture, the fluctuation of share demands (solid lines) of extrapolators grows much stronger during the bubble period. One extrapolator’s demand being zero in one period may become very large in the next period, or it can decrease from a large value to zero just in one period. This rapidly changing just reveals the fact that some extrapolators who quit the market in some periods of the bubble (his Acceptable Price is smaller than the threshold value), will come back and bet with others again (his Acceptable Price is now higher than the threshold value), as Newton did in the South Sea bubble. On the other hand, solid colored lines in Figure 4.3.c, which portray extrapolator’s demand, disperse with each other as the bubble grows. Considering every extrapolator’s demand is set to be the same value initially, this proves the fact that, as the bubble grows, the difference among extrapolators, induced by the same level of bias at the beginning, can grow so huge now. And this huge difference makes them to changes their demand rapidly, and lead the volume to grow bigger and bigger. Therefore, this picture also illustrates why the trading volume is positively correlated with investors extrapolation belief $|X_t|$. 
In a word, this simulation build on our new model can generate a more realistic bubble as during this process trading volume and volatility both increase enormously, which makes our new model more convincing.

4.4.2 Simulated Acceptable Price of different investors

We now use the simulated Acceptable Price and its histogram during this process to show our trading volume analysis method of section 4.3.2 also fits our simulation.

Figure 4.5 compares ten extrapolators’ Acceptable Prices (solid colored lines) with the Threshold value (blue dotted line) during the bubble period (from \( t = 15 \) to \( t = 45 \)). From this picture we can see, for one particular extrapolator, (take the pink line for example), his Acceptable Price fluctuates around the Threshold value. As we illustrated above, he only stays in the market when his Acceptable Price is bigger than the Threshold value. So, these fluctuating lines reveal the fact that, when some extrapolators return the market after they left before, other extrapolators who was in the market may choose to escape. This also proves our classification of extrapolator’s trading behavior, that during two adjacent periods, he may stay in the market for both periods, stay in the market for only one period or stay out the market for both periods. Meanwhile, this picture shows, as the bubble grows, the Acceptable Prices of extrapolators diffuse because of their idiosyncratic bias on information, just as revealed in Figure 4.4.

Figure 4.6 continues to compare histogram of extrapolators’ Acceptable Price in different periods. At the early stages (\( t=14 \)), fundamentalist exists in the market since
their Acceptable Price (the fundamental value, green line in this picture) is bigger than the threshold value (blue line). Meanwhile, the Acceptable Prices of extrapolators diffuse because of their idiosyncratic bias on information. At this time, some extrapolators have already left the market as their Acceptable Price is smaller than the threshold value. In the middle stages (t=27 or t=34), fundamentalist has already quit the market. But things are similar for extrapolative investors—there are always some extrapolators who are more pessimistic than their peers and staying away from the market. By rough comparison, we can see, the proportion of extrapolator who is staying in the market are similar in these periods, just as we assumed in Section 4.3.

Therefore, Figure 4.5 and Figure 4.6 demonstrate what happens to extrapolators in our simulation is very similar to what we show in Figure 4.1, indicating our theory in Section 4.3 is a realistic and persuasive method to explain trading volumes in the bubble period.

In short, by numerical simulation, we prove our new model can generate a very realistic bubble which displays high trading volume and the high volatility simultaneously. Therefore, our new model can improve our understanding of financial bubbles. We also show our method in section 4.3.2 is a proper way to analyze the changing of trading volume and its relation with extrapolation belief.

4.6 Conclusion

To answer our empirical test finding in Chapter 3 that trading volume can be impacted by individuals’ extrapolation belief, we modify our extrapolative model in Chapter 2
with heterogeneous extrapolators. In this new model extrapolators are facing idiosyncratic bias when evaluating the information innovations. Consequently, their heterogeneity grows as their idiosyncratic bias about the information is amplified by their extrapolation belief. We prove that, in the future market and in the rising stock market, the trading volume grows as the magnitude of individuals’ extrapolation belief grows. On the contrary, in the declining stock market, extrapolators gradually quit the market as $|X_t|$ increases, the market is left will only fundamentalists who are homogenous with each other, the trading volume reduces accordingly.

Moreover, using simulation, we show that our model can effectively explain the most two important features of financial bubbles: the high trading volume, and the high volatility. The bubble occurs in response to positive news and extrapolators’ irrational behave, a phenomenon Kindleberger (1978) called displacement. Volatility rises because extrapolator’s expectation is becoming more volatile, trading volume increases because extrapolators become more heterogeneous with their peers.

Although our model is built on individuals’ extrapolation bias and their confirmation bias, we are not suggesting other behavioristic biases, overconfidence bias, for example, are not important. Actually, extrapolators in our model are also “overconfidence”. They are too “confident” about their extrapolation belief and their biased information, which leads them to commit unwise trading decisions. Otherwise, they would pay more attention to the market performance and check the credibility of their “private information”. We do suggest that, individual investors, as an irrational group, is too
complex to be simply understood by one or two behavioristic bias. It is more reasonable to understand them from multiple aspects.

REFERENCE


COHEN, L. & FRAZZINI, A. 2008. Economic links and predictable returns. The


### Appendix 4.1

Table 4.1: Selected models of bubbles

<table>
<thead>
<tr>
<th></th>
<th>Generation of bubbles</th>
<th>Excess Volume</th>
<th>Positive Relation between price and Volume</th>
<th>Excess Volatility</th>
<th>Positive Relation between extrapolation belief and Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rational model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>asymmetrically information</td>
<td>Allen, Morris, and Postlewaite (1993)</td>
<td>○</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Limited liability</td>
<td>Allen and Gale (2000)</td>
<td>○</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td><strong>Behavior model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Biased self-attrition</td>
<td>Barberis, Shleifer, and Vishny (1998)</td>
<td>○</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Extrapolation</td>
<td>Hong and Stein (1999)</td>
<td>○</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Extrapolation</td>
<td>DeLong and others (1990)</td>
<td>○</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Overconfidence</td>
<td>Scheinkman and Xiong (2003)</td>
<td>○</td>
<td>○</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Extrapolation</td>
<td>Barberis, Shleifer (2016)</td>
<td>○</td>
<td>○</td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>

This table summarizes several models which try to explain bubbles. “○” means the fact which the model can explain while “x” means the aspect the model failed to cover.
a: Simulated Price Process

The black overlaid line displays the actual price while the red line plots the fundamental value. The green line shows the actual price when extrapolators are homogenous.

b: Simulated volatility

Volatility is measured by absolute return.

c: The changing $|X_t|$

Extrapolation Belief Index ($X_t$) is displayed in this picture with green line, while its absolute value ($|X_t|$) is shown by black line.
**Simulated trading volume**

Trading volume is displayed with black line in this picture.

**Figure 4.3 Simulation result of a financial bubble**

We present a simulated financial bubble according to equation (36). Parameter values are set according to Table 4.2.

**Figure 4.4 The changing demand of investors**

This picture illustrates every investor’s changing demand during this process with the dot line representing fundamental investor’s demand and colored solid lines showing every extrapolators’ demand.
Figure 4.5 Evolution of extrapolators’ Acceptable Price

10 extrapolators’ Acceptable Prices, represented by colored dotted line, are plotted with the threshold value which is plotted by the blue dotted line.
Figure 4.6 Histogram of extrapolators’ Acceptable Price

This picture illustrates histograms of extrapolators’ Acceptable Price when t equals 14, 27 and 34 respectively. The red line shows the actual price while the blue dotted line represents the threshold value. The fundamental value is plotted by green dotted line.
Appendix 4.2

Recall

\[ \mathbb{P}_t^{E,i} = \phi P_t^E + \phi \tilde{X}_t^i \]  \hspace{1cm} (a)

and during the bull market

\[ \tilde{X}_t^i = X_t \cdot e^{\varepsilon_t^i} \quad \text{when} \quad X_t \geq 0 \]  \hspace{1cm} (b)

Then we can get

\[ \mathbb{P}_t^{E,i} = \phi P_t^E + \phi X_t \cdot e^{\varepsilon_t + \omega_t^i} \]  \hspace{1cm} (d)

As extrapolators only put minimal weight on the fundamental signal, we assume \( \phi = 0 \).

\[ d \left( \mathbb{P}_t^{E,i} \right) = \phi X_t \cdot e^{\varepsilon_t} d\left( \omega_t^i \right) \]  \hspace{1cm} (e)

Substitute it into (52)

\[ \int_{p_t}^{\infty} f(x) P \, dp = \int_{p_t}^{\infty} f(x) \mathbb{P}_t^{E,i} \, d\left( \mathbb{P}_t^{E,i} \right) = \phi X_t e^{\varepsilon_t} \int_{\omega_t^i}^{\infty} f'(x) e^{\omega_t^i} \, d\omega_t^i \]  \hspace{1cm} (f)

where \( e^{\omega_t^i} = \frac{P_t-P_F}{\phi X_t e^{\varepsilon_t}} \), and \( f'(x) \) is the probability density function for logarithm distribution \( e^{\omega_t^i} \). Similarly,

\[ \int_{0}^{\infty} f(x) P \, dp = \phi X_t e^{\varepsilon_t} \int_{0}^{\infty} f'(x) e^{\omega_t^i} \, d\omega_t^i \]  \hspace{1cm} (g)

then we can get:

\[ \frac{\int_{p_t}^{\infty} f(x) P \, dp}{\int_{0}^{\infty} f(x) P \, dp} = \frac{\int_{\omega_t^i}^{\infty} f'(x) e^{\omega_t^i} \, d\omega_t^i}{\int_{0}^{\infty} f'(x) e^{\omega_t^i} \, d\omega_t^i} \]  \hspace{1cm} (h)
which means the ratio of the actual price (calculated by the mean value of existing extrapolators’ Acceptable Price) to the mean value of all extrapolators’ Acceptable Price is not correlated with the price level \((X_t \text{ or } P_t)\), and just correlated with the proportion of existing extrapolators. And as we assume this proportion is stable during the bull market, we can get

\[
\frac{\int_{p^*}^{\infty} f(x) P dp}{\int_{0}^{\infty} f(x) P dp} = \frac{\int_{\omega_t^*}^{\infty} f'(x)e^{\omega_t} d\omega_t}{\int_{0}^{\infty} f'(x)e^{\omega_t} d\omega_t} = A
\]

which is fixed across the whole process.