Analysis of financial network and the role of asymmetry in stock and fund returns

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Analysis of financial network and the role of asymmetry in stock and fund returns

Doctoral Thesis

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Chapter 1

Introduction

Research on equity markets have always been one of the central topics in finance. Development of diverse set of models and financial data abundance led a new stream of research. This thesis investigates stocks, stock indexes and portfolios, respectively, based on several statistical models that have not been applied into financial research.

First, thesis analyzes the significance of financial networks on equity markets based on a diverse portfolio investments. Actually, the geographical importance of the markets have been thoroughly analyzed by previous studies. Researchers found that investors prefer to invest in domestic markets rather than investing into markets which are in a far distance. Home bias and uncertain foreign markets led investors to focus on domestic markets. However, due to the interconnectedness of economies in today, the high priority of domestic investment decreased and international portfolio investments increased. Our study investigates the significance of the economy’s position in financial network and compares it with geographical location. Study employed spatial panel model with distinct formation of weight matrices. We found that spatial effect is, indeed, significant to explain stock market performance. In addition, we also found that economy’s centrality in the network of debt flows is significant as well.

Secondly, thesis investigates Jensen’s alpha measurement by skew-symmetric model for error term distribution. Due to its simplicity, initially developed asset pricing model in finance - Capital Asset Pricing Model (CAPM) by Sharpe (1964) and Litner (1965) has been used extensively both in research field and industry, until its replacement by multi factor models (Ross (1976) - APT, Fama and French (1993), Hou et al, (2015)). CAPM has been thoroughly studied by a number of papers based on a various international datasets yet controversies still go on about the validity of this one factor model. A study by Jensen, M., (1968) showed that CAPM is not able to explain abnormal returns and α - intercept term from a simple regression is used to account for this unobserved drivers.

Since then, Jensen’s α has been the main research focus in empirical tests of CAPM. CAPM states that expected excess rate of return of stocks and portfolio is equal to β times expected excess return of market portfolio. But Jensen’s α is included to account for unobserved drivers,

\[ E[R_p - r_f] = \alpha + \beta E[R_m - r_f] \] \hspace{1cm} (1.1)

here, \( R_p \) is the rate of return for portfolio, \( R_m \) is the rate of return for market portfolio and \( r_f \) is interest rate.

It has been popular to estimate α by OLS (Ordinary Least Square) method where α is the intercept and β is the regression coefficient in a simple linear model. However, α has not been
analyzed further in the statistical model. Our approach analyzes $\alpha$ in a new statistical model (Generalized Lehmann’s Alternative Model, (GLAM)) that describes $\alpha$ and error terms together in a regression model. GLAM is a semi-parametric and assumes the unknown distribution $F$ to be asymmetric around zero with asymmetry parameter indicating the degree of distortion. OLS method estimates $\alpha$ and $\beta$ simultaneously. But rank (R) statistics method estimates $\beta$ itself and it is known to be robust against outliers in the data or distribution with heavy tail. Moreover, we employ observed residuals to estimate the location and asymmetry parameters by GLAM and parameters are also estimated based on R statistics, which allows $F$ to be unknown.

Actually, estimate of $\beta$ based on rank statistics was introduced by Jureckova (1971) and well described by Jaeckel (1972) in a linear regression model. Estimate of location and asymmetry parameter in GLAM based on rank statistics was introduced by Miura and Tsukahara (1993).

We used daily data from Tokyo, New York and London stock exchanges, which are Nikkei 225, S&P 500 and FTSE 100 indices’ constituent stock prices (almost 830 individual stocks). Time period of data is from 1998 to 2017. The relations of the estimated parameters: $\beta$, location, asymmetry parameters and $\alpha$ are also studied cross-sectionally and chronologically in a quarter-wise manner.

We found that LS and R estimate of $\beta$ are noticeably distinct. Especially, in top (bottom) of $\beta$ distribution ranked based on LS estimate, LS consistently overestimates (underestimates) the parameter than R estimate of $\beta$. As a result, we obtained quite different residuals from both approaches.

We found that residuals are, indeed, skewed and GLAM model shows a significant deviation of residuals from the state of being symmetrical across all stocks, especially during the financially stressful periods. Furthermore, we decomposed $\alpha$ into location and asymmetry part. By looking into the cross sectional quarterly regressions, we found that $\theta$ has statistically significant relation to $\alpha$. Our main results is that Jensen’s $\alpha$ is a sum of location (of error terms) and asymmetry effect.

Lastly, thesis focuses on manager performance evaluation based on Unit Trust data. Due to the increase in a number of investment funds, evaluation of manager performance is another highly visited research area in finance. The very first study of fund managers is Jensen (1969). Paper investigated fund performance based on cross sectional and time series analysis. Besides, the development of multi-factor models led researchers to have a control over risk factors other than the market when analyzing the performance. However, literature lack of studies that applied robust methods but the Ordinary Least Squares (OLS). OLS is found to be imprecise when data has outliers and can lead to different results as our previous study found for the case of stock returns.

First contribution of this research is an application of robust R statistics to evaluate the performance. As mentioned earlier, R statistics is insensitive to outliers which occurs frequently in mutual fund data. Study uses 1097 close-end mutual fund data in monthly frequency for 50-years time span and employs Fama and French 5 factor model (2015) model to estimate $\alpha$ and obtain observed residuals.

$$r_{i,t} = \alpha_i + \beta_iRMRF_t + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + \epsilon_{i,t}$$ (1.2)

for $t=1,...,T$ period and $i=1,...,n$ funds.

Here, $r$ - fund return, $RMRF$ - market excess return, $SMB$ - small minus big, $HML$ - high minus low, $RMW$ - robust minus weak and $CMA$ - conservative minus aggressive risk portfolios.

Furthermore, estimated $\alpha$ are separated into two groups due to manager skill and luck based...
on cross sectional bootstrapping of residuals. Bootstrapping method is used to study the performance of mutual and hedge fund manager (Kosowski et al. 2006, 2007). Persistence of skill is studied by cross sectional regressions of compounded return on estimated $\alpha$ together with skewness and location parameters from skew-normal fitting (Fama and Macbeth (1973), Christopherson et al. (2006)).

$$\eta_{i,t} = \alpha_i + \epsilon_{i,t}$$

Moreover, $\eta_{i,t}$ is modeled by the semi-parametric GLAM method and also fitted skew-normal distribution (Azzalini, 1986) to estimate the asymmetry and location parameters.

Study found that skewness and location of residual distribution for the case of skillful manager persists for 5 year period but not in shorter horizons. In other words, $\alpha$ is significant to explain the longer horizon compounded return together with location and asymmetry parameters. This findings is in line with previous studies on mutual and pension funds that focused just on $\alpha$ assuming symmetrical residual distribution.

In addition, asymmetry in $\eta_i$ varies depending on the mutual fund’s strategy for investments (bond, equity or mixed assets). Funds that invested heavily into bonds or other fixed income securities have symmetrical $\eta_i$ distribution and funds invested larger proportion of assets into equity markets display stronger asymmetry. One possible explanation could be that equity markets are the primary source of asymmetry and it becomes obvious for the case of skillful manager case.

The rest of thesis is constructed as following. Chapter 2 presents the financial network analysis and its main findings. Chapter 3 is about Jensen’s alpha measurement by skew-symmetric model for error term distribution. Chapter 4 presents the research of manager performance evaluation based on close-end funds Unit Trusts data. Conclusion revisits our main findings from three studies and concludes this thesis.
Chapter 2

A Network Analysis of International Financial Flows

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Abstract

Study investigates the effect of international financial network on S&P Global Equity Index’s performance for 21 developed markets and for the period between 2001 and 2015. The Coordinated Portfolio Investment Survey (CPIS) is used to construct the financial network and three distinct centralities, eigenvector, degree and betweenness centralities are estimated.

Based on empirical results from spatial panel model, we found that spatial effect is statistically significant to explain index returns together with economy’s centrality on debt based financial network.
2.1 Introduction

A great number of previous studies have focused on the Great Recession and the European sovereign debt crisis in 2011. Nonetheless, researches have not investigated the market integration and its progress during crisis period. A brief summary of previous papers in finance reveals that only financial institutions’ (e.g., banks, funds, unions, companies and other financial intermediaries) performance are explained by either its role and influences in the global investment pattern, i.e., network, or its location – spatial effect.

However, finance literature lacks of evidence to show a relation between the stock market performance and country’s geographical or network locations in aggregate level. Even though recent papers analyzed micro-level, for instance, a brokerage network. Hence, we expect the stock market performance to be influenced by countries’ centrality in financial network, or alternatively by its geographical location. In this research, we analyze financial interconnectedness based on a bilateral investments and investigate its effect to explain stock market performance, in comparison with geographical location, specifically – a spatial effect.

Our research studied 21 developed economies’ stock markets for 15 years period of time starting from 2001 till 2015. The annual change (in US dollar) in Standard and Poor’s (S&P) Global Equity Indices are used as an indicator of stock market performance. In addition, the Coordinated Portfolio Investment Survey (CPIS) is employed to construct and analyze financial network of economies based on a bilateral investments. International Monetary Fund (IMF) semiannually and annually conducts the CPIS survey to investigate portfolio investment patterns among different economies in the world. A number of covered countries based on the reporting countries’ investment destinations varied between 186 and 215. It enabled us to thoroughly analyze centrality of 21 different markets under the focus of this research.

Using the financial network of economies three types of centralities, eigenvector, degree and betweenness are estimated. Centralities represent every investment patterns distinctly and focus on different characteristics of nodes in a given network. Relying on the constructed network’s centralities, we analyze the spatial neighbor effect on stock market performance by employing the spatial panel model. We expect the stock market performance to be affected not by neighboring markets but the importance of the economy in the global financial network which is represented by three various centralities.

Section 2.2 presents related literature and Section 2.3 thoroughly explains data that is used by our study. Section 2.4 introduces our methodology and Section 2.5 meticulously presents our main findings. Section 2.6 concludes this study by summarizing primary findings and reviewing shortcomings.

2.2 Review of previous studies

Analyzing equity markets is of great interest in finance. In empirical finance literature, stock market performance is studied by inclusion of influential and diverse factors of national and global economy, in order to model and predict its future path. The process of high integration of economies based on a various links (e.g., geographical, cultural, etc.) modified the study of market performance by focusing additionally on international financial networks. A great number of researches covered network of financial companies (e.g., venture capital, hedge funds) or banks (e.g., investment). For instance, Hochberg et al. (2007) found a positive effect of networking by analyzing venture capital (VC) firm’s network effect on its fund performance.
Grullon et al. (2014) conducted a research of investment banking network. They found that by changing underwriters from initial public offering (IPO) to seasonal equity offering (SEO), firm’s stocks co-move less with their previous investment bank, which they used for IPO, and more with the new one – SEO. Nevertheless, based on a traditional finance theory stock markets should be complete and changing underwriters should not affect co-movements of stock prices. Indeed markets are not complete, and there is asymmetric information flow because of several unknown boundaries contradicting traditional finance theory. Information asymmetry obviation is studied by Bailey et al. (2006). Researchers studied the effect of cross listings of non-US firms in US stock markets. Research found that increased level of disclosure decreases information asymmetry between investors and managers, and enhances access for additional capital and liquidity. In addition, greater protection for a small shareholders and improved corporate prestige are the key benefits of cross listing. Technological improvement led companies to cross list their shares in national, as well as in international stock markets. In 1998, the USA and the UK attracted 30% and 7%, respectively, of all of the cross listings in the world (Doige et al. (2009)).

This elimination of asymmetries comes by increasing level of networking through cross listings, cross border mergers and acquisitions (M&A) by foreign direct investment (FDI) or portfolio investment. Research also specifically focused on equity returns and financial network relations. Such as Tesar and Werner (1995), and Brennan and Cao (1997) found that financial inflows have a positive relation with returns. Moreover, Froot, O’Connell, and Seasholes (2001) analyzed daily bank data and found that international financial networking can forecast equity returns in emerging markets.

Financial network assumptions are based on a perfect connection of economies (e.g., highly developed technology, no cross border restrictions, etc.) which puts empirical results in accordance with the benchmark theory. Thus, benchmark financial theory states that investors should have similar portfolios regardless of their geographical location if markets are complete. Nonetheless, imperfect connections due to lack of highly developed technology, other cross border related issues and equity market controls by government do not enable to construct financial investment network based on a perfect and unbiased connections. This incomplete aspect of network derives alternative investment patterns which is far from the benchmark theory. Therefore, research in international investment patterns is of great interest in economics which enables us to analyze thoroughly the current globalization and to identify flaws in financial network of economies. Recently, Lane and Milesi-Ferretti (2008) studied variations in international investment patterns and found strong effect of bilateral trade, close cultural and language connections on current investment arrangements. Therefore, geographical location can matter for investments among countries. Obviously, most of the economies are affected by a stronger neighbor markets. This spatial dependency of subjects is extensively studied in other fields, especially in urban economics, whereas in finance less research have been conducted. Researches in spatial finance found a mixed results, contradicting as well as supporting evidence for the existence of the spatial effect. Grinblatt and Keloharju (2001) found that distance is less prominent for investors with a highly diversified portfolios. On the contrary, Degryse and Ongena (2005) studied 15000 bank loans to firms of variety sizes and locations, and found supporting evidence on the existence of spatial effect on bank loan rates. Fernandez (2011) modifies capital asset pricing model (CAPM) and interpolates spatial dependency features. By applying constructed S-CAPM they analyzed 126 firms and concluded that spatial dependency does exist.

In brief, stock market analysis go beyond the explanations of traditional finance theory to reflect high globalization of equity markets. Most of the researchers considered to analyze finan-
cial network based on some existent linkages (e.g., loan, investment, etc.) while others primarily focused on international investment patterns due to geographical locations and other country specific linkages (e.g., cultural, language, etc.). Thus, in this study we focus on both, spatial and financial network effect of economy on stock market performance.

2.3 Data

Main data of this research, the CPIS is obtained from the IMF database\(^1\) for the period between 2001 and 2015. The CPIS is a global survey that is based on a participation of central banks of national economies around the world. The number of reporting countries and investment destinations, in other words – covered countries in their reports varied slightly each year. In addition to actively investing countries (e.g., USA, UK, Germany, Japan, etc.), the CPIS data included developing economies (e.g., India, Brazil, Russia, etc.) and offshore countries (e.g., Panama, Bermuda, Cayman Islands, etc.). Thus, information rich data set enabled us to construct financial network and enhanced the precision of estimated country centralities. The CPIS data is widely employed in finance literature (Yildirim, 2003), and recently, Lane and Milesi-Ferretti (2008) studied a major driving forces of bilateral investment patterns using the CPIS data for the period between 2001 and 2006.

The CPIS survey attributes cross border portfolio investments made by reporting economies. Portfolio investment consisted of equity (e.g., stock), long and short term debt (e.g., bond) securities. Data set occasionally included negative, which embody short positions in investments, not specified and confidential entries. Clearly, not specified entries are used for unidentified issuers of securities and confidential entries employed for issuers who kept confidentiality of their investment reports\(^2\).

Nonetheless, data is not flawless and has drawbacks (Lane and Milesi-Ferretti (2008)). The CPIS data set has insufficient number of reporting countries to cover the whole world, even though year by year the number of reporting and covered countries increased. Most of the emerging economies do not report their portfolio investments and keep it confidential (e.g., China). The next flaw arises from underreporting of portfolio investments by reporting countries. In the CPIS survey total portfolio investment is consisted of equity and investment fund shares, i.e., long and short term debt securities. In case of being underreported of any subparts of total leads to inaccuracy of total investments made by economies. However, the CPIS data set provides us an opportunity to analyze bilateral investments thoroughly, and evaluate how influential and central are country stock markets.

Annual price change (in US dollar) in S&P Global Equity Indices\(^3\) for 21 countries are used as an indicator of stock market performance. S&P Global Equity Index represent annual price change (in US dollar) in stock markets that is covered by the S&P Frontier Broad Market Index (Frontier BMI), the S&P Global Market Index (Global BMI) and the S&P International Finance Corporation Investible (IFCI). The S&P Frontier BMI and Global BMI includes developed and emerging country’s markets, whereas the S&P IFCI includes only developing markets. S&P index includes foreign investable and most liquid assets in stock markets of all levels of economy (e.g., sector, sub-sector, etc.). In addition, S&P indices are used as a benchmark to evaluate and compare stock and bond performance in all types of markets (e.g., emerging and developed).

\( ^{1}\)The data are available at [http://data.imf.org/?sk=B981B4E3-4E58-467E-9B90-9DE0C3367363](http://data.imf.org/?sk=B981B4E3-4E58-467E-9B90-9DE0C3367363)

\( ^{2}\)Notes and definitions are available at [http://data.imf.org/?sk=B981B4E3-4E58-467E-9B90-9DE0C3367363&s=1410469433565](http://data.imf.org/?sk=B981B4E3-4E58-467E-9B90-9DE0C3367363&s=1410469433565)

\( ^{3}\)The data are available at [http://data.worldbank.org/indicator/CM.MKT.INDX.ZG](http://data.worldbank.org/indicator/CM.MKT.INDX.ZG)
2.4 Methodology

2.4.1 Network

We construct financial network and estimates centralities based on a bilateral investment ties. Empirical studies about networks are widely observed in other fields of research (e.g., sociology, biology, engineering, economics, climatology, information and network sciences, etc.). Network theory is a part of graph theory (Tutte, 1984) which studies directed or undirected bilateral linkages between various objects, i.e., nodes. Thus, network characteristics (e.g., connecting tie, node centrality, etc.) is based on a constructed network and its graphical representation. Edges can represent various types of linkages (e.g., investment, bank loan, remittance, trade, etc.) and directness shows the direction of a connecting tie. Node centralities represent how influential is the node in the given network based on its ties, neighbors and position among other nodes.

Using the CPIS data set a network of investments is constructed for each type of investments (equity, long-term and short-term debt), respectively. By employing reporting country data, a bilateral adjacency matrix of investments is constructed as presented in Eq. (3.40).

\[ I_t = \begin{pmatrix} 0 & i_{13} & \ldots & i_{1k} \\ i_{21} & 0 & \ldots & i_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ i_{k1} & i_{k2} & \ldots & 0 \end{pmatrix} \]  

(2.1)

Adjacency matrix of investments consisted of \( k \) rows and columns for \( k \) different countries, and

\[ i_{jk} = \begin{cases} Investment, & \text{if } j \text{ invests into } k \\ 0, & \text{if } j \text{ does not invest into } k \end{cases} \]  

(2.2)

The number of covered countries by network is increased following the construction of the adjacency matrix of investments. Since, some countries reported their bilateral portfolio investment destinations entirely, while other countries reported partially. Hence, the adjacency matrix of investments covered nearly all of the portfolio investments invested in stock markets every year. This network is based on a directed investments and thus in adjacency matrix column names indicated investing countries and raw names are destinations of investments.

Financial network of bilateral investments in equities are depicted in Fig. 2.1 - 2.3. Network is visualized in a world map to illustrate major investment linkages between countries. Connecting ties represent investments from one country to another weighted by the investment amount (in millions of US dollars). Thus, thick investment linkages among countries imply high volume of investments.

As Fig. 2.1 displays, in 2001 leading investor countries and destinations are European countries, the USA and Japan. In comparison with Fig. 2.1, in 2015 offshore countries (e.g., Bermuda, Panama, Cayman Islands, etc.) and Asian-Pacific markets (e.g., Hong Kong, Thailand, Singapore, etc.) become primary destinations, as well as connecting ties of the USA, European countries and Japan become thicker than in 2001. This significant change implies a high amount of investment circulation and integration of equity markets.

Similarly, Fig. 2.4 - 2.6 illustrate financial networks based on long-term debt investments and Fig. 2.7 - 2.9 presents networks for the case of short-term debt investments. Clearly, the debt investments are in one direction the US markets and the level of investments are much lower.
then equity scenario.

Based on financial networks three distinct centrality measures (eigenvector, degree and betweenness) are estimated for countries, respectively. Network centralities identify central and important vertex in a network based on its connections with other vertices and position among vertices. Analysis of central node is an essential part of financial network study to investigate the role of influential nodes in information diffusion, systemic failure and preserving network stability.

2.4.2 Centrality

Eigenvector centrality is mostly applied form of centrality in empirical finance literature (Colla and Melle (2010), Chuang (2016)) along with degree and betweenness centralities. Eigenvector centrality measures how influential is the given node based on the number of ordinary and extraordinary neighbors ((Bonacich, 1972).

Eigenvector centrality’s is presented in Eq. (2.31).

\[
C_{eigenvector,j} = \frac{1}{\lambda} \sum_{e \in G} a_{j,e} c_{i,j,e} 
\]

Here,

\[
a_{je} = \begin{cases} 
1, & \text{if } j \text{ invests into } e \\
0, & \text{if } j \text{ does not invest into } e 
\end{cases}
\]

\[
c - \text{ is eigenvector centrality of node } j, \ i - \text{ is investment made by country } j \text{ to } e, \ G - \text{ is a network representation } k \text{ economies and } \lambda - \text{ is a constant value. As a } \lambda \text{ the highest eigenvalue of the adjacency matrix is used in order to derive positive eigenvector values (Perron-Frobenius Theorem)}. \text{Investment value is added to the formula to weight centrality based on the investment amount.}

Degree centrality measures how central is the given node based on the number of ties with other vertices. There are two types of degree centralities – in-degree and out-degree centralities. As it is clear from the name, in-degree and out-degree distinctly measure the number of incoming and outgoing edges (Freeman (1978)).

To evaluate degree centrality, adjacency matrix of edges is constructed and the number of connecting ties for each node is enumerated. In Eq. (2.23) \( A \) – is adjacency matrix of edges (in and out degree), \( i \) – is investment made by country \( j \) to \( e \).

\[
C_{degree,j} = \sum_{j:j \neq e} A_{je}i_{je} 
\]

Betweenness centrality gives more credit to the location of nodes in a given network. Vertices’s lying on a higher proportion of the shortest paths connecting other nodes will be assigned higher values, in comparison with other vertices which do not lie. Hence based on betweenness centrality, the most influential nodes locate at the intersection and operate as a bridge or hub for other nodes’ connections (Friedkin, 1991).

Betweenness centrality’s estimation method is presented in Eq. (2.25). Here \( g_{en}(j) \) - is the number of shortest paths connecting \( e \) and \( n \) nodes passing through \( j \) node. \( g_{en} \) - is the total number of shortest paths connecting \( e \) and \( n, \ i - \text{ is investment amount made by } j \text{ to other countries and by other countries to } j \).
$$C_{betweenness,j} = \sum_{j \neq e}^{k} \frac{g_{en}(j) i_{e,j,n}}{g_{en}} \quad (2.6)$$

### 2.4.3 Spatial panel analysis

Following the centrality analysis, a spatial autoregressive model is employed to investigate the neighbor market effect on country’s stock markets together with centrality estimates. Centralities, annual price change in S&P Global Equity Indices and additional variables that are believed to have a relation to market performance are employed as explanatory variables.

First, a list of neighbors for each country is designed based on a contiguity and based on $k$ nearest neighbors by employing longitude, latitude and a distance geographical metrics. Next, using the neighbor list, a spatial weight matrix is formulated to conduct spatial analysis (Eq. (2.7)).

$$W \equiv \begin{cases} 0 & w_{i1} & \ldots & w_{ik} \\ w_{i2} & 0 & \ldots & w_{ik} \\ \vdots & \vdots & \ddots & \vdots \\ w_{ik} & w_{ik} & \ldots & 0 \end{cases} \quad (2.7)$$

Spatial weight matrix consisted of $k$ rows and columns for $k$ different countries, and

$$w_{jk} = \begin{cases} 1, & \text{if } j \text{ and } k \text{ are neighbors} \\ 0, & \text{if } j \text{ and } k \text{ are not neighbors} \end{cases} \quad (2.8)$$

Matrix is row standardized and thus a fraction of unit is assigned for each neighbors in the row for a given country. We construct three types of weight matrices for different types of neighboring schemes as illustrated by Fig. (2.10) - (2.12). Contiguity, $k = 1$ and $k = 2$ based formations of neighbors enables us to investigate the spatial effect in more detail.

Our main model is depicted in Eq. (2.9) (Fernandez (2011), Paramati et al. (2016)).

$$y_{i,t} = \lambda \sum_{j=1}^{k} w_{i,j} y_{j,t} + x_{i,t} \beta + \alpha + u_{i,t} \quad (2.9)$$

Here, $y$ - is annual price change (in US dollar) in S&P Global Equity Indices, $\rho$ - is an estimator for a spatial effect, $w$ - is a spatial weights constructed using neighborhood list, $x$ - are additional variables (countries’ network centrality (e.g., eigenvector, degree, betweenness), earnings to price ratio, book to market ratio, log of GDP, log of house price index and unemployment rate).

Moreover, the disturbance have a spatial dependency as in Cliff and Ord (1973) (Kapoor et al. 2007).

$$u_{i,t} = \rho \sum_{j=1}^{k} w_{i,j} u_{j,t} + \epsilon_{i,t} \quad (2.10)$$

$$\epsilon_{i,t} = \mu_{i} + v_{i,t} \quad (2.11)$$

Here, $\epsilon_{i,t}$ are innovations in period $t$ and $\mu_{i}$ is individual effect. $v_{i,t}$ are independent innovations that vary over both the cross-sectional units and time periods.
2.5 Results

Results are presented in Tables 2.1 - 2.3 when centrality estimates are obtained from the three different networks such as equity, long-term debt and short-term debt, respectively. Thus, Table 2.1 presents results when centralities are obtained from the network of equity investments and employed in a model (2.9) as explanatory variable. Result shows that spatial effect ($\lambda$) is statistically significant together with other essential macroeconomic variables. Interestingly, only in the case of $k$ is 1 and 2, centrality is significant enough to explain stock market index return.

Similarly, Tables 2.2 - 2.3 display results when centralities are obtained from long-term debt and short-term debt investments network, respectively. As expected, spatial effect is significant together with other macroeconomic variables. In comparison with previous results, centralities are statistically significant for all three types of neighboring scheme.

The principal reason of distinct results is that all three types of centralities variously portray financial network. As it is explained in the methodology part, eigenvector centrality puts additional weights for the variety of node neighbors to estimate centrality, thus, not only simple connections but linkages with crucial neighbors are awarded by high centrality values. Betweenness centrality weights more harshly the position of nodes in the network. Hence, node serving as a hub for connections of its neighbors gets higher betweenness values. The most uncomplicated centrality - degree centrality gives more value to nodes based on a number of incoming and outgoing edges.
Table 2.1: Spatial panel results. Centralities are obtained from the network of equity investments.

Table presents results of Eq. 2.9 for three different spatial weights, contiguity, $k$ is 1 and 2, respectively. Models from 1 to 3 show the empirical results when distinct centralities are employed. Here, centralities are obtained from the network of equity investments.

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<td></td>
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<td></td>
<td>$E/P$</td>
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<tr>
<td></td>
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<td>0.11 ** 0.045</td>
<td>0.116 *** 0.036</td>
<td>0.167 *** 0.036</td>
<td>0.165 *** 0.036</td>
<td>0.175 *** 0.036</td>
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<td>0.192 *** 0.042</td>
<td>0.196 *** 0.042</td>
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<td>(0.165)</td>
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<td>(0.191)</td>
<td>(0.192)</td>
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<td>$Mkt$</td>
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<td>$E/P$</td>
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<td>$B/M$</td>
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<tr>
<td></td>
<td>0.93 *** 0.077</td>
<td>0.926 *** 0.077</td>
<td>0.91 *** 0.075</td>
<td>0.79 *** 0.082</td>
<td>0.782 *** 0.082</td>
<td>0.765 *** 0.077</td>
<td>0.818 *** 0.077</td>
<td>0.816 *** 0.077</td>
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<td></td>
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<td>-0.005 0.072</td>
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<td></td>
<td>$log(HPI)$</td>
<td></td>
<td></td>
<td>$Unemployment$</td>
<td></td>
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</tr>
<tr>
<td></td>
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<td>-0.18 0.788</td>
<td>-0.033 0.793</td>
<td>-0.166 0.796</td>
<td>-0.166 0.793</td>
<td>-0.177 0.793</td>
<td>-0.082 0.754</td>
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</tr>
<tr>
<td></td>
<td>$log(HPI)$</td>
<td></td>
<td></td>
<td>$Unemployment$</td>
<td></td>
<td></td>
<td>$Eigenvector$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.841)</td>
<td>(3.848)</td>
<td>(3.755)</td>
<td>(3.816)</td>
<td>(3.838)</td>
<td>(3.711)</td>
<td>(3.566)</td>
<td>(3.609)</td>
<td>(3.534)</td>
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<tr>
<td></td>
<td>$Unemployment$</td>
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<td></td>
<td>$Eigenvector$</td>
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<tr>
<td></td>
<td>-0.918 0.303</td>
<td>-0.96 0.303</td>
<td>-0.941 0.302</td>
<td>-0.985 0.302</td>
<td>-1.021 0.305</td>
<td>-1.033 0.297</td>
<td>-0.872 0.292</td>
<td>-0.898 0.288</td>
<td>-0.907 0.287</td>
</tr>
<tr>
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<td>(0.918)</td>
<td>(0.945)</td>
<td>(0.945)</td>
<td>(0.945)</td>
<td>(0.945)</td>
<td>(0.945)</td>
<td>(0.945)</td>
<td>(0.945)</td>
<td>(0.945)</td>
</tr>
<tr>
<td></td>
<td>$Degree$</td>
<td></td>
<td></td>
<td>$Betweenness$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.031 0.9129</td>
<td>0.004 0.904</td>
<td>-0.007 0.904</td>
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<td>0.004 0.904</td>
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</tr>
<tr>
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<td>(0.038)</td>
<td>(0.038)</td>
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</tr>
</tbody>
</table>

The standard errors are given in parentheses. *, ** and *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.
Table 2.2: Spatial panel results. Centralities are obtained from the network of long-term debt investments.

Table presents results of Eq. 2.9 for three different spatial weights, contiguity, \( k \) is 1 and 2, respectively. Models from 1 to 3 show the empirical results when distinct centralities are employed. Here, centralities are obtained from the network of long-term debt investments.

<table>
<thead>
<tr>
<th></th>
<th>Contiguity based</th>
<th>K 1</th>
<th>K 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model 1</td>
<td>Model 2</td>
<td>Model 3</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.113 **</td>
<td>0.111 ***</td>
<td>0.116 ***</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td>(0.045)</td>
<td>(0.045)</td>
</tr>
<tr>
<td>Mkt</td>
<td>0.925 ***</td>
<td>0.922 ***</td>
<td>0.914 ***</td>
</tr>
<tr>
<td></td>
<td>(0.074)</td>
<td>(0.077)</td>
<td>(0.076)</td>
</tr>
<tr>
<td>E/P</td>
<td>0.015</td>
<td>0.004</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(0.071)</td>
<td>(0.073)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>B/M</td>
<td>0.089</td>
<td>0.092</td>
<td>0.092</td>
</tr>
<tr>
<td></td>
<td>(0.064)</td>
<td>(0.066)</td>
<td>(0.065)</td>
</tr>
<tr>
<td>log(GDP)</td>
<td>0.098</td>
<td>-0.17</td>
<td>-0.132</td>
</tr>
<tr>
<td></td>
<td>(0.779)</td>
<td>(0.794)</td>
<td>(0.792)</td>
</tr>
<tr>
<td></td>
<td>(3.707)</td>
<td>(3.868)</td>
<td>(3.775)</td>
</tr>
<tr>
<td>Unemployment</td>
<td>-0.893 ***</td>
<td>-0.982 ***</td>
<td>-0.939 ***</td>
</tr>
<tr>
<td></td>
<td>(0.298)</td>
<td>(0.307)</td>
<td>(0.303)</td>
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<tr>
<td>Eigenvector</td>
<td>96.726 ***</td>
<td>57.185 *</td>
<td>84.615 ***</td>
</tr>
<tr>
<td>Degree</td>
<td>-0.022</td>
<td>-0.019</td>
<td>-0.019</td>
</tr>
<tr>
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<td>(0.032)</td>
<td>(0.032)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>Betweenness</td>
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<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
</tbody>
</table>

The standard errors are given in parentheses. *, ** and *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.
Table 2.3: Spatial panel results. Centralities are obtained from the network of short-term debt investments.

Table presents results of Eq. 2.9 for three different spatial weights, contiguity, k = 1 and 2, respectively. Models from 1 to 3 show the empirical results when distinct centralities are employed. Here, centralities are obtained from the network of short-term debt investments.

<table>
<thead>
<tr>
<th></th>
<th>Contiguity based</th>
<th>K1</th>
<th>K2</th>
<th>K3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model 1</td>
<td>Model 2</td>
<td>Model 3</td>
<td>Model 1</td>
</tr>
<tr>
<td>λ</td>
<td>0.107 **</td>
<td>0.111 **</td>
<td>0.113 **</td>
<td>0.165 ***</td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td>(0.045)</td>
<td>(0.045)</td>
<td>(0.036)</td>
</tr>
<tr>
<td>Mkt</td>
<td>0.93 ***</td>
<td>0.923 ***</td>
<td>0.925 ***</td>
<td>0.793 ***</td>
</tr>
<tr>
<td></td>
<td>(0.075)</td>
<td>(0.076)</td>
<td>(0.076)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>E/P</td>
<td>-0.001</td>
<td>0.003</td>
<td>0.003</td>
<td>-0.021</td>
</tr>
<tr>
<td></td>
<td>(0.072)</td>
<td>(0.072)</td>
<td>(0.072)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>B/M</td>
<td>0.092</td>
<td>0.089</td>
<td>0.089</td>
<td>0.167 **</td>
</tr>
<tr>
<td></td>
<td>(0.065)</td>
<td>(0.065)</td>
<td>(0.065)</td>
<td>(0.066)</td>
</tr>
<tr>
<td>log(GDP)</td>
<td>-0.221</td>
<td>-0.148</td>
<td>-0.221</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>(0.785)</td>
<td>(0.787)</td>
<td>(0.788)</td>
<td>(0.791)</td>
</tr>
<tr>
<td></td>
<td>(3.761)</td>
<td>(3.832)</td>
<td>(3.77)</td>
<td>(3.731)</td>
</tr>
<tr>
<td>Unemployment</td>
<td>-0.938 ***</td>
<td>-0.953 ***</td>
<td>-0.954 ***</td>
<td>-1.004 ***</td>
</tr>
<tr>
<td></td>
<td>(0.301)</td>
<td>(0.302)</td>
<td>(0.302)</td>
<td>(0.298)</td>
</tr>
<tr>
<td>Eigenvector</td>
<td>66.685 *</td>
<td>59.108 *</td>
<td>57.281 *</td>
<td>57.281 *</td>
</tr>
<tr>
<td>Degree</td>
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<td>-0.025</td>
<td>-0.049</td>
<td>-0.025</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>(0.045)</td>
<td>(0.048)</td>
<td>(0.045)</td>
</tr>
<tr>
<td>Betweenness</td>
<td>-0.003</td>
<td>-0.002</td>
<td>-0.002</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.02)</td>
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</table>

The standard errors are given in parentheses. *, ** and *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.
2.6 Conclusion

Research investigated the importance of market centralities in three distinct financial networks on market performance. The CPIS data is employed to construct the financial network based on country’s bilateral equity, long-term and short-term debt portfolio investments, respectively. Moreover, three different types of centralities are estimated, such as eigenvector, degree and betweenness centralities to capture the distinct characteristics of financial interconnectedness.

Based on spatial panel regression, we found that market performance is significantly affected by the neighboring markets. In addition, macroeconomic variables also found to be statistically significant as expected. Especially, housing price index and unemployment rate show a clear relation between the market performance and slowdown in economic activity.

Financial network also found to be a significant to explain the stock markets. However, only the network based on debt flows are found to have a positive association with market performance. This is possibly due to the reforms and openings of developing markets to foreign investors to have an access for additional funds. Thus, attractive debt markets will have a spillover effect on equity markets.

Our research studied the financial network and stock market performance relation based on official statistics of equity markets published by international organizations (e.g., IMF, World Bank). However, research has some drawbacks and shortcomings. Firstly, data is presented in annualized form by the IMF which normalizes significant changes in investment flows among countries throughout a year even though bilateral portfolio investment data covered 15 years period of time. Secondly, it is a survey data which is based on a voluntary participation of central banks of countries. The similar flaws can be observed in the S&P Global Equity Indices which are represented as a stock market performance indicators in our study. Aggregating annual price change in index led to smoothing real trend by ignoring significant jump or plummet in index that happened during the crisis period in this 15 years.
2.7 Reference


Tutte, W.T., 1984. Graph theory. Addison-Wesley California.

Figure 2.1: Equity based network 2001

Figure 2.2: Equity based network 2008

Figure 2.3: Equity based network 2015
Figure 2.4: Long term debt based network 2001

Figure 2.5: Long term debt based network 2008

Figure 2.6: Long term debt based network 2015
Figure 2.7: Short term debt based network 2001

Figure 2.8: Short term debt based network 2008

Figure 2.9: Short term debt based network 2015
Figure 2.10: Contiguity based

Figure 2.11: $k = 1$

Figure 2.12: $k = 2$
Chapter 3

Jensen’s alpha measured under skew symmetric semi-parametric model for error terms

Navruzbek Karamatov\textsuperscript{1}, Ryozo Miura\textsuperscript{2}

\textsuperscript{1} Graduate School of Economics and Management, Tohoku University
\textsuperscript{2} Graduate School of Business, Hitotsubashi University


Abstract

Due to its simplicity and restrictive assumptions, initially developed asset pricing model - Capital Asset Pricing Model has been used extensively in academics and in financial industry. One of the modesty of this market model is relying on a single market index to explain variations in stock return. Moreover, a simple estimation method, namely the Ordinary Least Squares (LS) is applied for empirical analysis to estimate $\beta$. However, study of M. Jensen., (1968) cleared that CAPM is not able to explain abnormal returns and $\alpha$ is used to account for this unobserved factors. More importantly, Jensen’s Alpha is obtained as a mean value of residuals from a regression. However, LS is sensitive to outliers and it could make estimators to be vulnerable. In reality, observed residuals have exceptions and not symmetrically distributed.

Can asymmetry in error term distribution affect Jensen’s Alpha? This research tries to find the answer by applying robust Rank statistics in comparison with Least Squares to fit a simple linear regression into Nikkei 225, FTSE 100 and S&P 500 stocks. Furthermore, the Generalized Lehmann’s Alternative Model (GLAM) and Skew-t distributions are applied onto observed residuals to analyze the scale of deviation from an assumed normality and symmetricalness. We expect Jensen’s Alpha to have a relationship with skewness that is not taken into account by LS.

We found that residuals are, indeed, not normally distributed. GLAM model shows a significant deviation of residuals from the state of being symmetrical across all stocks, especially during the financially stressful periods. In addition, $\theta$ possesses a statistically significant relation to $\alpha$ as well as to skew effect which is defined as a difference between $\alpha$ and $\mu$. Next, we found that median $\theta$ for each quarter is significant to explain index rate of return and this finding is consistent across all three markets.


3.1 Introduction

Asset pricing is one of the primary drivers for majority of the researches in finance. Despite its simplicity and numerous restrictive assumptions, initially developed asset pricing model in finance Capital Asset Pricing Model (CAPM) by William Sharpe (1964) and John Lintner (1965) has been used extensively both in research field and industry, until the replacement by the multi factor models (Ross, S. A., (1976), E. Fama., K. French (1993), K. Hou et al. (2015)). CAPM has been studied in deep by a number of papers based on a various international datasets yet controversies still go on about the validity of this one factor model. Since assumptions are far from being a reasonable about the financial market and model is based on only one factor to capture all variations in stock rate of return (Perold, 2004). Even though multi factor models replaced it for asset pricing long ago, Da et al., (2012) found CAPM is still in use by financial institutions for cost of equity analysis. Moreover, 73% chief financial officers prefer CAPM (Graham and Harvey (2001)) and 75% of Professors in recommend CAPM for asset pricing studies (Welch (2008)).

A study by Jensen, M., (1968) showed that CAPM is not able to explain abnormal returns and $\alpha$ - intercept term from a simple regression is used to account for this unobserved drivers. More importantly, LS observes Jensen’s Alpha as a mean value of residuals from a regression. Nonetheless, LS is sensitive to outliers and this would make estimators to be vulnerable. Contrary to the assumption of symmetry, in reality, observed residual distribution is not bell shaped and has a significant deformations.

Hence, a simple question arises, does asymmetry affect Jensen's Alpha? We go back to CAPM to estimate $\beta$ by LS and compare it with other robust method - rank statistics (R). Following the review of literature we found that LS is applied in stock return ignoring the notion of sensitiveness of the method to outliers. We expect LS to lack of decisiveness to estimate model parameters precisely when normality assumptions of $\epsilon$ are not met and especially in the presence of outliers. Thus, analysis carried on employing R statistics as well which is accepted in statistics as a robust method when sample error terms are not from a family of normal distributions (Jaeckel, 1972).

In addition, we assume that Jensen’s Alpha could be have relation with asymmetry which is not taken into consideration by other studies. The Generalized Lehmann’s Alternative Model (GLAM) (Miura and Tsukahara, 1993) and Skew-t distributions (Skew-t) (Azzalini, A., 1985) are applied onto observed residuals to analyze the scale of deviation from an assumed normality and symmetricalness.

Relying on GLAM and Skew-t distribution models, paper meticulously estimates the deviation of residuals from normality assumptions by the parameter $\theta$ and $\gamma$. This enables us to thoroughly analyze the underlying residual variation which is left by the market model. We found that residuals are indeed not normally distributed and parameters ($\theta$, $\gamma$) indicate a significant deviation of residuals from normality and symmetricalness across all stocks in Nikkei 225, FTSE 100 and S&P 500 indexes.

Our study contributes finance literature in three ways. First is an application of robust nonparametric R statistics to estimate stock $\beta$. By being insensitive to outliers, R methods outperforms LS to precisely estimate $\beta$. Both of the methods provide us with different error terms which are further used to estimate location and asymmetry parameters. Secondly, as in our knowledge it is the first paper to model error term from a simple linear regression applied in stock return. Specifically, we meticulously derive location and asymmetry parameters through seminonparametric method (GLAM) and compared with parametric counterpart. Third contribution is to analyze skewness effect on Jensen’s Alpha. Our study show that this cause is due to
asymmetry in error terms and its magnitude differs depending on the period.

This paper is organized as follows. The Section 3.2 and 3.3 review previous related literature and models. Section 3.4 presents data and its descriptive summary. In addition, this section meticulously introduces LS and R methods as well as other models applied in this study. Estimated $\beta$ based on two approaches and residual analysis are given in Section 3.5. Besides, section also includes cross sectional study of GLAM and skew-t distribution parameters. This section also presents Jensen’s Alpha decomposition and its relation to asymmetry parameter. Section 3.6 presents relative efficiency of two estimates and the last Section 3.7 sums up our main findings.

3.2 Review of previous studies

Back in 1950s finance was steps away from having its first asset pricing model, even though investors traded stocks in Amsterdam markets since 1602. Portfolio diversification for risk sharing was already in practice in order to minimize the risk and increase expected return (Perold, 2004). Later, based on Markowitz’s (1959) portfolio theory CAPM was developed by William Sharpe (1964) and John Littner (1965). Even later Mossin (1966) and Black (1972) extended CAPM by eliminating some of the restrictive assumptions for applicability in the real world case. However, in the last two decades multi factor models (E. Fama., K. French (1993), K. Hou et al. (2015)) replaced CAPM after its failure to explain abnormal stock returns. CAPM has been tested by numerous studies employing variety of data samples. Yet debate still goes on whether CAPM is still applicable or not (Perold. A. F., (2004), Fama. E. F., French. K. R., (2004)).

Among others, study by Jensen (1968) made clear that CAPM is not able to explain abnormal stock or portfolio returns and $\alpha$, intercept of the linear regression, is added as an additional variable to account for extra variability that is left unexplained by market return. Empirical researches proved $\alpha$ has a non-constant nature and fluctuates during the time period (Arnott., et al, (2018)). It is known as a Jensen’s Alpha and applied as one of the portfolio strategies that exist out in the market today.

Majority of empirical studies of Jensen’s Alpha applied a simple linear regression with a common estimation method - LS. The simplicity of a linear regression comes with a number of restrictive assumptions. From a statistical point of view a normal linear regression has a multiple assumptions and one of them is normally distributed error terms. Application of LS in this case could produce inefficient estimators and reduce method functionality if proper robust estimation techniques are not taken. Since LS is quite sensitive method if the data contains outliers.

This awareness of LS failure in data with outliers made researchers to apply other estimation methods by relaxing some of the assumptions. Such as the Least Absolute Deviations (LAD) (Edgeworth (1887)), the Bounded Influence Estimator (Krasker and Welsch (1982)), the Least Trimmed Squares and the Least Median Squares (Rousseeuw (1984)) are commonly applied when normality tests such as the Jarque-Bera and Kolmogorov-Smirnov tests rejected normality assumptions.

Nonetheless, LS alternatives and modifications of it are based on a number of assumptions and sensitive to outliers clustering found in Onder and Zaman (2003, 2005). Moreover, Hettmansperger and Sheather (1992) showed that the Least Median Squares is instable when centrally located data changes. Recently, Denhere and Bindele (2016) compared Rank based estimation with LS and LAD estimators, and found that R estimators are robust compared to parametric methods when data has outlying observations and fat-tailed error distribution. Besides, we found that finance literature also lacks of study for an application of robust estimation
technique for CAPM β and Jensen’s Alpha estimation, such as a distribution free Rank based methods.

Nonparametric methods gained popularity due to several advantages than traditional approaches and rank statistics is one of the widely used approach. Rank method has been developed extensively by a number of studies such as Jureckova (1971) and Jaeckel (1972). Later developments come from Lehmann (1975) and Gibbons (1997). In specific, Jureckova (1971) mathematically establishes the asymptotic linearity of rank statistics and infers its asymptotic normality for a multiple linear regression case. Besides, Jackel (1972) introduces dispersion measures and minimization procedure in order to derive regression parameters. Asymptotic normality is also shown to be the same as in Jureckova (1971) case. Especially, in the case of a simple linear regression, estimator is a weighted mean of pairwise slopes \( \frac{(Y_j - Y_i)}{(c_j - c_i)} \) \( \{j \neq i\} \).

Rank method does not require the underlying observations to follow normal distributions and it is distribution free estimation technique - which is the main reason for its popularity. Moreover, being insensitive to outliers and efficiency properties are the key reasons for applying these methods in the analysis rather than LS (Hettmansperger and McKean (1977), Hollander and Sethuraman (1978)).

Miura (1985a,b) computed estimates of beta based on monthly data for the period from 1952 January to 1981 December and showed the difference of the two estimates of beta based on LS and nonparametric estimate based on R statistics. Also he fitted Log-Normal distribution to the residuals and showed the relations between the estimated scale parameter of Log-Normal distribution and the estimate of asymptotic variance of the two estimators. Zhou (2001) followed the same scheme as Miura (1985a,b) to compute beta based on daily data. In this paper we use Generalized Lehmann’s Alternative model which can take good care of location. This corrects an ad-hoc treatment of location in the Log-Normal fitting in Miura (1985a,b) and Zhou (2001).

### 3.3 Review of statistical properties

Study employs a simple linear regression in Eq. (3.1) where \( i = 1, ..., n \). Error terms (\( \epsilon_i \)) are expected to be i.i.d and have a distribution \( G(x) \).

\[
Y_i = \alpha + \beta X_i + \epsilon_i \quad \text{(3.1)}
\]

\[
\eta_i = Y_i - \beta X_i = \alpha + \epsilon_i \quad \text{(3.2)}
\]

#### 3.3.1 Optimality of Least Squares

Least Squares estimate \( \hat{\beta} \) is considered to be the best linear unbiased estimate (BLUE) based on Gauss-Markov theorem. It states that \( \hat{\beta} \) is minimum variance and linear unbiased estimator of \( \beta \), as long as the assumptions of classical linear regression model are hold (Greene, 2012). For instance, \( Y \) is assumed to be a linear function of \( X \), strict exogeneity (\( E(\epsilon|X) = 0 \)), full column rank of \( X \) and homoscedastic variance (\( Var(\epsilon_i) = \sigma^2 \)).

#### 3.3.2 Asymptotic normality of estimates

\[
\eta \sim G(x - \alpha) \quad \text{(3.3)}
\]

Asymptotic normality of LS and R estimates are presented below in Eq. (3.4) and (3.5), respectively. When \( n \) is large enough, both estimates will reach to the true parameter \( \beta \). In
addition, variances of both estimates are presented in Eq. (3.6) and (3.7), respectively (Miura, 1985b).

\[ \sqrt{n}(\hat{\beta}_{LS} - \beta) \rightarrow N(0, \sigma_{\beta}^2) \]  
\[ \sqrt{n}(\hat{\beta}_{R} - \beta) \rightarrow N(0, \sigma_{\beta}^2) \]  
\[ \sigma_{\beta,LS}^2 = \frac{1}{c^2} \int_{-\infty}^{\infty} x^2 g(x) dx \]  
\[ \sigma_{\beta,R}^2 = \frac{1}{12c^2 [\int_{-\infty}^{\infty} g^2(x) dx]^2} \]

Here, \( g(x) = G'(x) = h'(F(x) : \theta)f(x) \)

\[ c^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 \]

\[ \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \]

We further focus on error terms by applying Generalized Lehmann’s Alternative Model.

### 3.3.3 The Generalized Lehmann’s Alternative Model

The GLAM method is semi-parametric and based on rank statistics. The following definitions and assumptions of GLAM is from Miura and Tsukahara (1993) and we keep notations unchanged for simplicity.

Let \( \Theta \) be interval in real line. A function \( h(t; \theta) \) for \( t \in (0, 1) \) and \( \theta \in \Theta \) which satisfies the following (1) and (2) is called the Generalized Lehmann’s Alternative model:

1. \( h = (0; \theta) = 0 \) and \( h(1; \theta) = 1 \) for any \( \theta \in \Theta \). \( h(t; \theta) \) is strictly monotone function of \( t \).
2. There exists \( \theta^* \in \Theta \) such that \( h(t; \theta^*) = t \) for \( t \in (0, 1) \). And for \( \theta > \theta^* \), \( h(t; \theta) = t < h(t; \theta^*) \) for all \( t \).

\( X \) observations are assumed to be i.i.d and have an empirical distribution function given by \( G(x : \mu, \theta) \). Deformation in \( G(x : \mu, \theta) \) is captured by the parameter \( \theta \).

\[ h(t; \theta) = 1 - (1 - t)^\theta \]

\[ G(x : \mu, \theta) = h(F(x - \mu); \theta) = 1 - (1 - F(x - \mu))^\theta \]

To obtain \( \mu \) and \( \theta \) parameters we followed estimation procedure presented Miura and Tsukahara (1993). First, the empirical distribution function for observation \( X_i \) is estimated as in Eq. (3.12).

\[ G_n(x) = n^{-1} \sum_{i=1}^{n} I[X_i < x] \]

Next, the estimated empirical distribution function is linearized.

\( X_{(1)} < X_{(2)} \ldots < X_{(n)} \) are ordered values of \( X_i \)'s for \( i = 1, \ldots, n \). \( X_{(0)} = X_{(1)} - 1/n \) and \( X_{(n+1)} = X_{(n)} + 1/n \) are set, respectively.

\[ \tilde{G}_n(x) = \frac{x + iX_{(i+1)} - (i + 1)X_{(i)}}{(n + 1)(X_{(i+1)} - X_{(i)})} \]

where, \( x \in (X_{(i)}, X_{(i+1)}) \).

Following the linearization, \( Z_i \) values are obtained by the inverse of \( \tilde{G}_n(x) \).

\[ Z_i(r) = \tilde{G}_n^{-1}(h(\frac{i}{n + 1}; r)) \]

Then, \( R_i^+(r, q) \) are estimated for a given tentative location parameter \( q \).

\[ R_i^+(r, q) = \text{(the number of } \{j : |Z_j(r) - q| \leq |Z_i(r) - q|\}) \]
\[ S_{\theta,n}(r,q) = \frac{1}{n} \sum_{i:Z_i(r)>q} J_\theta((1 + \frac{R_i^+(r,q)}{n+1})/2) + \frac{1}{n} \sum_{i:Z_i(r)\leq q} J_\theta((1 - \frac{R_i^+(r,q)}{n+1})/2) \] (3.16)

\[ S_{\mu,n}(r,q) = \frac{1}{n} \sum_{i:Z_i(r)>q} J_\mu((1 + \frac{R_i^+(r,q)}{n+1})/2) + \frac{1}{n} \sum_{i:Z_i(r)\leq q} J_\mu((1 - \frac{R_i^+(r,q)}{n+1})/2) \] (3.17)

Score functions given by Eq. (3.33) and (3.34) are used for Eq. (3.16) and (3.17) to estimate \( \theta \) and \( \mu \) parameters simultaneously. Statistics are simultaneously minimized as in Eq. (3.18) to obtain optimal parameters, \( \hat{\mu} \) and \( \hat{\theta} \).

\[ S_{\theta,n} \approx 0 \]
\[ S_{\mu,n} \approx 0 \] (3.18)

\[ D_n \triangleq \{ (r,q) : \sum_{k=1}^{2} |S_k,n(r,q)| = \text{min} \} \]

Asymptotic normality of \( \beta \) proves to be essential for estimation of \( \mu \) and \( \theta \) as shown in Miura (2016). Both parameters could be obtained from residuals if error terms are asymptotically normally distributed and \( \beta \) is asymptotically normal as well.

### 3.4 Data and estimation procedure

#### 3.4.1 Data

Paper relies on three stock market index constituents, Nikkei 225 (N225), FTSE 100 and S&P500 for this study. N225 data is obtained from Quick Financial Data Provider and it is a set of stock prices of all Nikkei 225 stocks in a daily frequency. Similarly FTSE 100 and S&P 500 are in daily frequency as well but obtained through Thomson Reuters Database. Time period covered by datasets is from 1998 January until 2017 October. As a risk free rate - overnight call money rate of the Bank of Japan is employed\(^1\) for N225 stocks, London Interbank Offered Rate (LIBOR) for FTSE 100 stocks and 1-month US Treasury Bill rate for S&P500 stocks. The chosen risk free rate is in line with previous researches for Japanese market (Kubota and Takehara (2010)). Descriptive statistics for index and risk free rates are presented in Table (3.1).

<table>
<thead>
<tr>
<th>Statistic</th>
<th>N</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>N225</td>
<td>4,765</td>
<td>0.0001</td>
<td>0.015</td>
<td>−0.114</td>
<td>0.142</td>
</tr>
<tr>
<td>Call money rate</td>
<td>4,872</td>
<td>0.001</td>
<td>0.002</td>
<td>−0.001</td>
<td>0.007</td>
</tr>
<tr>
<td>FTSE 100</td>
<td>5,152</td>
<td>0.0001</td>
<td>0.012</td>
<td>−0.088</td>
<td>0.098</td>
</tr>
<tr>
<td>1 month LIBOR</td>
<td>4,986</td>
<td>0.0031</td>
<td>0.024</td>
<td>0.002</td>
<td>0.078</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>4,969</td>
<td>0.0003</td>
<td>0.022</td>
<td>−0.090</td>
<td>0.116</td>
</tr>
<tr>
<td>1 month Treasury bill</td>
<td>4,942</td>
<td>0.0020</td>
<td>0.021</td>
<td>0.000</td>
<td>0.064</td>
</tr>
</tbody>
</table>

\(^1\)https://www.boj.or.jp/en/statistics/market/short/mutan/index.htm/
3.4.2 $\beta$ estimation

LS Method

Traditional one factor model is given in Eq. (3.40). LS method relies on minimizing the sum of squared residuals (3.20). Estimation window consisted of moving and non overlapping 3 month. For each stock all available rate of returns are divided into quarters with a given month and year information. Number of returns are not the same for each quarter due to trading and non trading day differences for every month. However, the available number of observations for stock returns per quarter are found to be in the range of 59 and 63. This approach of analysis ensures our estimates to be conducted for every single quarter of the year and makes it possible to gain extra insight of a given stock behavior during the financial market and business cycles. Hence, more than 100 $\beta$ values are estimated for each stock names depending on the availability of stock returns for all sample period.

\[ R_{i,q,t} - R_{f,q,t} = \alpha_{i,q} + \beta_{i,q}^{LS}(R_{m,q,t} - R_{f,q,t}) + \epsilon_{i,q,t} \]  

(3.19)

\[ SSR_{i,q}(\epsilon_{i,q}) = \sum_{t=1}^{T} ((R_{i,q,t} - R_{f,q,t}) - \alpha_{i,q} - (R_{m,q,t} - R_{f,q,t})\beta_{i,q}^{LS})^2 \]  

(3.20)

\[ u_{i,q,t} = (R_{i,q,t} - R_{f,q,t}) - \alpha_{i,q} - \hat{\beta}_{i,q}^{LS}(R_{m,q,t} - R_{f,q,t}) \]  

(3.21)

\[ i = \{1, \ldots, 225\}, q = \{1, \ldots, N\}, t = \{1, \ldots, T\} \]  

(3.22)

Here, $R_i$ - stock rate of return, $R_f$ - risk free rate, $R_m$ - market rate of return, $\epsilon_i$ - LS error term, $u_i$ - LS residual. $i$ is the available stocks in our data set and varies depending on a stock market, $N$ is the a maximum number of quarters available for a given stock and $T$ is the maximum number of stock returns available for a given quarter.

R Method

Eq. (3.23) presents R approach. Similar to LS method, estimation window consisted of moving and non overlapping 3 month. For each stock all available rate of returns are divided into quarters with a given month and year information. However, in the case of rank statistics not sum of squared residuals but the sum of dispersions are minimized (3.25). We employed the simplest and commonly applied score function - Wilcoxon scores (Jaeckel (1972)) as in (3.24).

\[ R_{i,q,t} - R_{f,q,t} = \beta_i^{R}(R_{m,q,t} - R_{f,q,t}) + \eta_{i,q,t} \]  

(3.23)

\[ W_T(R_\eta) = \frac{R_\eta}{T+1} - \frac{1}{2}(\Leftrightarrow J_\beta(t) = t - \frac{1}{2}) \]  

(3.24)

\[ D_i(\eta,q) = \sum_{t=1}^{T} \frac{R_{\eta,i,q,t}}{T+1} - \frac{1}{2}((R_{i,q,t} - R_{f,q,t}) - (R_{m,q,t} - R_{f,q,t})\beta_{i,q}^{R}) \]  

(3.25)

Here, $D_i(\eta,q)$ - sum of dispersion, $R_{\eta,i}$ - rank of $\eta_i$, $W_T(R_\eta)$ - Wilcoxon scores, $v_i$ is residual obtained by R approach.

\[ v_{i,q,t} = (R_{i,q,t} - R_{f,q,t}) - \hat{\beta}_{i,q}^{R}(R_{m,q,t} - R_{f,q,t}) \]  

(3.26)
\[ i = \{1, \ldots, 225\}, q = \{1, \ldots, N\}, t = \{1, \ldots, T\} \]  

Here, \( i \) is the available stocks in our data set, \( N \) is the maximum number of quarters available for a given stock and \( T \) is the maximum number of stock returns available for a given quarter.

### 3.4.3 GLAM

Following the estimation of \( \hat{\beta} \), residuals \((u_i, v_i)\) are observed for every stock and quarterly period. Here, we present the procedure to obtain \( \hat{\theta} \) and \( \hat{\mu} \).

Here \( J_1 \) and \( J_2 \) are score functions for \( \theta \) and \( \mu \) respectively. Estimation of score functions are derived as following:

\[ g(x : \mu, \theta) = \frac{dG(x : \mu, \theta)}{dx} \]  
\[ g_\theta(x : \mu, \theta) = \frac{dg(x : \mu, \theta)}{d\theta} \]  
\[ g_\mu(x : \mu, \theta) = \frac{dg(x : \mu, \theta)}{d\mu} \]  
\[ J_\theta(t) = \frac{g_\theta(G_{\mu, \theta}^{-1}(t) : \mu, \theta)}{g(G_{\mu, \theta}^{-1}(t) : \mu, \theta)} \]  
\[ J_\mu(t) = -\frac{g_\mu(G_{\mu, \theta}^{-1}(t) : \mu, \theta)}{g(G_{\mu, \theta}^{-1}(t) : \mu, \theta)} \]

However, these optimal scores are not available since the fundamental form of \( F \) is unknown. Here, the logistic distribution is applied to derive score functions and complete mathematical derivation of score functions are given in Appendix.

\[ J_\theta(t) = \frac{1}{\theta} + \ln(1 - [1 - (1 - t)^{1/\theta}]) = \frac{1}{\theta} + \ln(1 - t)^{1/\theta} \]  
\[ J_\mu(t) = -\frac{1}{\theta} \left[ (\theta - 1)(-1)(1 - (1 - t)^{1/\theta}) + 1 - 2(1 - t)^{1/\theta} \right] \]

This is because we used \( J_\beta(t) = t - \frac{1}{2} \) in Eq. (3.24) which is an optimal score function for the case \( G_{\mu, \theta} \equiv F(x - \mu) \) and \( F \) is logistic. This makes us keep a consistency of our view on \( F \).

Score functions given by Eq. (3.33) and (3.34) are used for Eq. (3.16) and (3.17) to estimate \( \theta \) and \( \mu \) parameters simultaneously. Statistics are simultaneously minimized as in Eq. (3.35) to obtain optimal parameters \( \mu \) and \( \theta \).

\[ S_{\theta,n} \approx 0 \]  
\[ S_{\mu,n} \approx 0 \]  
\[ D_n \triangleq \{(r, q) : \sum_{k=1}^{2} |S_{h,n}(r, q)| = \min\} \]

\( \hat{\theta} \) and \( \hat{\mu} \) are obtained by minimizing Eq. (3.35).
3.4.4 Skew-t distribution

Random values from a normal distribution have no skewness on either side of the distribution and displays a bell-shape form. However, this behavior is not observed in residuals ($\epsilon$) from a simple linear regression (3.23) fitted into stock return. Hence, we applied a semi-parametric approach - GLAM to captured asymmetry by $\theta$.

To estimate a skewness a widely used skew-t distribution (Azzalini, A., 1985) is used as well which is a parametric approach in order to compare with $\theta$. To make a fair ground for comparison we choose degrees of freedom 8 which makes $t$ distribution close to logistic distribution.

In Eq. (3.36) is presented a linear transformation of random variable $Y$ which follows skew-t distribution$^2$. Here, $\xi$ is location, $w$ scale parameters and $\gamma$ skew parameters. And again we keep notations unchanged as in the original study.

\[ Y \sim St(\xi, \omega^2, \gamma) \quad (3.36) \]

\[ Y = \xi + wX \quad (3.37) \]

Probability distribution function of $X$ is shown in Eq. (3.38) where $\nu$ is degrees of freedom, $\Gamma$ is a gamma function and $\Phi$ is a cumulative $t$-distribution function.

\[ f(x) = 2\phi(x)\Phi(x) \quad (3.38) \]

\[ \phi(x) = \frac{\Gamma(\nu/2)}{\sqrt{\nu\pi}\Gamma(\nu/2)}(1 + \frac{t^2}{\nu})^{-\nu/2} \quad (3.39) \]

We fitted skew-t distribution into observed residuals ($v_{i,t}$, $u_{i,t}$) from simple linear regression and estimated all three parameters by Maximum Likelihood method. Our objective is to use $\gamma$ and $\xi$ to compare with $\theta$ and $\mu$ from GLAM.

However, due to a singularity problem (Azzalini, A., 2013) of information matrix, we used centralized parameters rather than direct parameters and estimated location $\xi$ and skewness $\gamma$. Comparison of different parameters is beyond the scope of this research.

3.5 Empirical results

3.5.1 $\beta$

Relying on Nikkei 225 stock returns a simple linear regression (3.40) is fitted by LS and R methods. $\beta$s are obtained for non-overlapping quarterly windows. Average $\beta$s across Nikkei 225 stocks presented in Table (3.2). Thus, R and LS produce distinct $\beta$s as well as standard deviations, minimum and maximum values.

Tables (3.5) and (3.6) present descriptive statistics of estimated $\beta$ by R and LS methods for a sample 6 different stock names from various industries. Two approaches estimated comparable $\beta$s, nonetheless, discrepancy is clear and supports previous result. Especially standard deviation of $\beta$s from R approach are smaller than its counterpart for most the cases. Depending on terms, estimated $\beta$ is as low as -0.001 or as high as 2.2. This behavior is different according to the

\[ http://azzalini.stat.unipd.it/SN/Intro/intro.html \]
names of stocks. A possible explanation is that this variation in $\beta$s is due to the nature of the industry where companies operate and macroeconomic situation in Japan itself.

### Table 3.2: Quarterly average $\beta$ of N225

<table>
<thead>
<tr>
<th>Statistic</th>
<th>N</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>79</td>
<td>0.939</td>
<td>0.094</td>
<td>0.663</td>
<td>1.150</td>
</tr>
<tr>
<td>LS</td>
<td>79</td>
<td>0.946</td>
<td>0.092</td>
<td>0.700</td>
<td>1.138</td>
</tr>
</tbody>
</table>

### Table 3.3: FTSE100

<table>
<thead>
<tr>
<th>Statistic</th>
<th>N</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>79</td>
<td>0.860</td>
<td>0.166</td>
<td>0.417</td>
<td>1.149</td>
</tr>
<tr>
<td>LS</td>
<td>79</td>
<td>0.875</td>
<td>0.166</td>
<td>0.433</td>
<td>1.183</td>
</tr>
</tbody>
</table>

### Table 3.4: S&P500

<table>
<thead>
<tr>
<th>Statistic</th>
<th>N</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>79</td>
<td>0.993</td>
<td>0.137</td>
<td>0.597</td>
<td>1.269</td>
</tr>
<tr>
<td>LS</td>
<td>79</td>
<td>0.999</td>
<td>0.135</td>
<td>0.579</td>
<td>1.271</td>
</tr>
</tbody>
</table>

### Table 3.5: Descriptive statistics of $\beta$, R method

<table>
<thead>
<tr>
<th>Statistic</th>
<th>N</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toyota Motor Corp</td>
<td>79</td>
<td>0.913</td>
<td>0.234</td>
<td>0.303</td>
<td>1.519</td>
</tr>
<tr>
<td>Taisei Corp</td>
<td>79</td>
<td>0.894</td>
<td>0.324</td>
<td>0.127</td>
<td>2.006</td>
</tr>
<tr>
<td>Takashimaya Co</td>
<td>79</td>
<td>0.911</td>
<td>0.295</td>
<td>0.238</td>
<td>1.726</td>
</tr>
<tr>
<td>Nippon Express Co Ltd</td>
<td>79</td>
<td>0.814</td>
<td>0.247</td>
<td>0.141</td>
<td>1.334</td>
</tr>
<tr>
<td>Canon Inc</td>
<td>79</td>
<td>0.937</td>
<td>0.353</td>
<td>−0.001</td>
<td>2.222</td>
</tr>
<tr>
<td>Mitsubishi Corp</td>
<td>79</td>
<td>1.182</td>
<td>0.243</td>
<td>0.399</td>
<td>1.713</td>
</tr>
</tbody>
</table>

### Table 3.6: Descriptive statistics of $\beta$, LS method

<table>
<thead>
<tr>
<th>Statistic</th>
<th>N</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toyota Motor Corp</td>
<td>79</td>
<td>0.917</td>
<td>0.232</td>
<td>0.366</td>
<td>1.489</td>
</tr>
<tr>
<td>Taisei Corp</td>
<td>79</td>
<td>0.907</td>
<td>0.345</td>
<td>0.126</td>
<td>2.070</td>
</tr>
<tr>
<td>Takashimaya Co</td>
<td>79</td>
<td>0.909</td>
<td>0.296</td>
<td>0.258</td>
<td>1.808</td>
</tr>
<tr>
<td>Nippon Express Co Ltd</td>
<td>79</td>
<td>0.822</td>
<td>0.247</td>
<td>0.141</td>
<td>1.334</td>
</tr>
<tr>
<td>Canon Inc</td>
<td>79</td>
<td>0.934</td>
<td>0.353</td>
<td>−0.012</td>
<td>2.215</td>
</tr>
<tr>
<td>Mitsubishi Corp</td>
<td>79</td>
<td>1.192</td>
<td>0.242</td>
<td>0.420</td>
<td>1.667</td>
</tr>
</tbody>
</table>

### Figure 3.1

Rolling window beta Canon Inc
We can observe this nature of $\beta$ by looking at the stocks one by one for each time period, but lack of a statistical method to capture an overall image will not allow us except conditioning or restricting analysis by industry-wise or size-wise. So we randomly choose a widely known company stock and present results. Results for other stocks are available upon request.

Quarterly estimated $\beta$ for Canon stocks illustrated in Fig. (3.1). In 1999 Canon stocks differently behaved than the rest of the financial market and it is clear from very low $\beta$. Especially, the end of 2000 Canon $\beta$ is still unstable and have the highest peak for the last 20 years period of time. From 2005 until 2009 $\beta$ has increasing trend in a small range, nonetheless, European Sovereign Debt crisis in 2011 possibly caused it to plummet sharply in 2011 - 2012. Afterwards, starting from 2013 Canon experienced less volatile and smaller $\beta$ until the end of data period. Non constant $\beta$ contrary to the static CAPM assumptions is in line with previous studies (Jagannathan and Wang (1996), Lewellen and Nagel (2006)). LS and R estimates are comparable and slightly diverges. Notably, for 2002 and 2011 LS estimated quite distinct $\beta$ than R method.

Cross sectional analysis of $\beta$

In order to get a deeper intuition regarding our estimates we look into cross sectional distribution of $\beta$s for a chosen quarter. Fig. (3.2) - (3.3) present histograms for 2008 Q2 and 2017 Q3, for R and LS cases, respectively. Obviously, estimates are different during crisis and relatively peaceful periods in market. LS histograms have fat tails and more width. In contrast, R $\beta$ histograms display slightly centralized distribution and it is stronger for Q3 in 2017.

Fig. (3.4) displays $\beta$ difference between LS and R estimates. Histogram clearly illustrates the persistent discrepancy between $\beta$s across all N225 stocks in 2Q 2008. Some of the stocks have a significantly distinct $\beta$ estimates. Nonetheless, difference is even stronger in 3Q 2017 in Fig. (3.5). Maximum and minimum of $\beta$ difference is significantly bigger than estimates in crisis period. A possible explanation for this is the intrinsic difference among LS and R methods. During the volatile market, LS and R $\beta$s are similar, contrary to less volatile period estimates.
3.5.2 Shape of residual distribution

Here, we turn to investigate the residuals from a simple linear regression. Fig. (3.53), (3.54), (3.55) and (3.56) in Appendix illustrate histogram of residuals from LS and R methods for Canon stocks as an example. Similar behavior of residuals could be observed for other company stocks as well. As it is illustrated, histograms have a noticeable skewness on both sides, have heavy tails and violates assumptions of a simple normal linear regression.

In addition, Table (3.7) presents average skewness and kurtosis of residual distribution for 225 stocks. Normal distribution has 0.03 skewness and 2.96 kurtosis which verifies a symmetricalness of distribution. However, average skewness and kurtosis among N225 stocks are far from being close to normal distribution.

Table 3.7: Descriptive statistics of quarterly average skewness and kurtosis in n225

<table>
<thead>
<tr>
<th>Statistic</th>
<th>N</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skewness LS</td>
<td>79</td>
<td>0.262</td>
<td>0.157</td>
<td>-0.054</td>
<td>0.626</td>
</tr>
<tr>
<td>Skewness R</td>
<td>79</td>
<td>0.267</td>
<td>0.161</td>
<td>-0.058</td>
<td>0.646</td>
</tr>
<tr>
<td>Kurtosis LS</td>
<td>79</td>
<td>1.693</td>
<td>0.804</td>
<td>0.310</td>
<td>4.221</td>
</tr>
<tr>
<td>Kurtosis R</td>
<td>79</td>
<td>1.819</td>
<td>0.880</td>
<td>0.353</td>
<td>4.791</td>
</tr>
</tbody>
</table>

3.5.3 \( \mu \) and \( \theta \)

Relying on derived score functions for logistic distribution and statistics for parameter inference, GLAM parameters \( \theta \) and \( \mu \) are estimated. The following Fig. (3.6) and (3.7) illustrate estimates for \( \theta \) and \( \mu \) for Canon stocks.

\( \theta \) for Canon stocks has a significant fluctuation during the sample period. Values are higher than one for most of the observation and fluctuation becomes wider from 2006 until 2009. This exceptional variation could be a possible reaction of Canon stock prices to financial market distress around 2008. Interestingly, \( \theta \) behavior changed after 2011, however, from 2017 it revives noticeable fluctuations.

\( \mu \) shows similar pattern. Fluctuations in a narrow corridor is followed by a wide movements during 2006 and 2009. Especially, in 2009 \( \mu \) plummets to the lowest points twice in a year and decline is obviously the effect of stagnation and downfall in financial markets occurred in 2009.

Figure 3.6

Figure 3.7

Fig. (3.8) and (3.9) depict estimated \( \theta \) and \( \mu \) for Mitsubishi Corp., respectively. \( \theta \) for Mitsubishi stocks also has a significant fluctuation throughout the sample period. However, before 2007 variation usually exhibits low frequency. Some years have low \( \theta \) in the range of
1.7 and 1.1. Extreme fluctuation is persistent and periodic, especially after 2007 and a similar behavior could be observed until the end of observation.

\( \mu \) shows similar pattern with the case of Canon. High variation could be observed only from 1998 until 2003. On the contrary, estimated parameter exhibits a clear increasing trend prior to the crisis in 2008 - 2009. Afterwards, \( \mu \) only has a fluctuation in a narrow range.

Figure 3.8

![Theta Mitsubishi Corp](image)

```
0.98 1.02 1.06 1.10
Theta
```

Table 3.9: Descriptive statistics of estimated \( \theta \) for 6 different names of stocks based on R and LS approaches, respectively. Mean values of \( \theta \) clearly indicate that on average residuals have unsymmetrical distribution and \( \theta \) is higher than 1 throughout our sample period.

In addition, Tables (3.10) and (3.11) present descriptive statistics for \( \mu \) parameter for 6 names of stock based on R and LS, respectively. Results support our expectations that a simple linear regression residuals are unsymmetrical and mean values are not equal to zero. Tables (3.12) and (3.15) present descriptive statistics of \( \mu \) and \( \theta \) across N225 stocks. Results are not different from the case of 12 stock names. \( \mu \) parameter is -0.001 and \( \theta \) is 1.039.

Cross sectional analysis of \( \mu \) and \( \theta \)

Distribution of estimated \( \mu \) across 225 stocks are presented below in Fig. (3.10) - (3.11) for LS and R residuals, respectively. Starting with Fig. (3.10), in 2008 \( \mu \) has a noticeable left skewed shape for both approaches, nonetheless, this nature is weak in 2015. In addition, the range of estimated \( \mu \) is slightly larger for R case.
Table 3.10: Descriptive statistics of $\mu$, R

<table>
<thead>
<tr>
<th>Statistic</th>
<th>N</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toyota Motor Corp</td>
<td>79</td>
<td>-0.001</td>
<td>0.001</td>
<td>-0.004</td>
<td>0.002</td>
</tr>
<tr>
<td>Taisei Corp</td>
<td>79</td>
<td>-0.001</td>
<td>0.002</td>
<td>-0.006</td>
<td>0.004</td>
</tr>
<tr>
<td>Takashimaya Co</td>
<td>79</td>
<td>-0.001</td>
<td>0.002</td>
<td>-0.011</td>
<td>0.003</td>
</tr>
<tr>
<td>Nippon Express Co Ltd</td>
<td>79</td>
<td>-0.001</td>
<td>0.002</td>
<td>-0.005</td>
<td>0.002</td>
</tr>
<tr>
<td>Canon Inc</td>
<td>79</td>
<td>-0.001</td>
<td>0.002</td>
<td>-0.004</td>
<td>0.003</td>
</tr>
<tr>
<td>Mitsubishi Corp</td>
<td>79</td>
<td>-0.001</td>
<td>0.002</td>
<td>-0.006</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Table 3.11: Descriptive statistics of $\mu$, LS

<table>
<thead>
<tr>
<th>Statistic</th>
<th>N</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toyota Motor Corp</td>
<td>79</td>
<td>-0.001</td>
<td>0.001</td>
<td>-0.004</td>
<td>0.002</td>
</tr>
<tr>
<td>Taisei Corp</td>
<td>79</td>
<td>-0.001</td>
<td>0.002</td>
<td>-0.006</td>
<td>0.007</td>
</tr>
<tr>
<td>Takashimaya Co</td>
<td>79</td>
<td>-0.001</td>
<td>0.002</td>
<td>-0.006</td>
<td>0.003</td>
</tr>
<tr>
<td>Nippon Express Co Ltd</td>
<td>79</td>
<td>-0.001</td>
<td>0.002</td>
<td>-0.005</td>
<td>0.004</td>
</tr>
<tr>
<td>Canon Inc</td>
<td>79</td>
<td>-0.001</td>
<td>0.002</td>
<td>-0.005</td>
<td>0.003</td>
</tr>
<tr>
<td>Mitsubishi Corp</td>
<td>79</td>
<td>-0.001</td>
<td>0.002</td>
<td>-0.005</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Table 3.12: Quarterly average $\mu$ of N225 stocks

<table>
<thead>
<tr>
<th>Statistic</th>
<th>N</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>79</td>
<td>-0.001</td>
<td>0.001</td>
<td>-0.004</td>
<td>0.001</td>
</tr>
<tr>
<td>LS</td>
<td>79</td>
<td>-0.001</td>
<td>0.001</td>
<td>-0.004</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 3.13: FTSE100 stocks

<table>
<thead>
<tr>
<th>Statistic</th>
<th>N</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>79</td>
<td>-0.007</td>
<td>0.010</td>
<td>-0.035</td>
<td>0.002</td>
</tr>
<tr>
<td>LS</td>
<td>79</td>
<td>-0.006</td>
<td>0.010</td>
<td>-0.034</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Table 3.14: S&P500 stocks

<table>
<thead>
<tr>
<th>Statistic</th>
<th>N</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>79</td>
<td>-0.002</td>
<td>0.006</td>
<td>-0.019</td>
<td>0.011</td>
</tr>
<tr>
<td>LS</td>
<td>79</td>
<td>-0.002</td>
<td>0.006</td>
<td>-0.020</td>
<td>0.012</td>
</tr>
</tbody>
</table>

Table 3.15: Quarterly average $\theta$ of N225 stocks

<table>
<thead>
<tr>
<th>Statistic</th>
<th>N</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>79</td>
<td>1.039</td>
<td>0.006</td>
<td>1.026</td>
<td>1.058</td>
</tr>
<tr>
<td>LS</td>
<td>79</td>
<td>1.039</td>
<td>0.006</td>
<td>1.026</td>
<td>1.060</td>
</tr>
</tbody>
</table>

Table 3.16: FTSE100 stocks

<table>
<thead>
<tr>
<th>Statistic</th>
<th>N</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>79</td>
<td>1.026</td>
<td>0.005</td>
<td>1.016</td>
<td>1.040</td>
</tr>
<tr>
<td>LS</td>
<td>79</td>
<td>1.025</td>
<td>0.004</td>
<td>1.017</td>
<td>1.039</td>
</tr>
</tbody>
</table>

Table 3.17: S&P500 stocks

<table>
<thead>
<tr>
<th>Statistic</th>
<th>N</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>79</td>
<td>1.031</td>
<td>0.005</td>
<td>1.022</td>
<td>1.045</td>
</tr>
<tr>
<td>LS</td>
<td>79</td>
<td>1.031</td>
<td>0.005</td>
<td>1.022</td>
<td>1.046</td>
</tr>
</tbody>
</table>
Similar skewed distribution is observed for $\theta$s across 225 stocks as well, but to the right side as illustrated in Fig. (3.12) - (3.13). In addition, $\theta$ values noticeably form two clusterings around 1 and 1.05 in 2Q 2008. This behavior is still weakly persistent in 1Q 2015, especially in $\theta$s from R residuals.

During financially stressful periods in markets a distinct behavior in stock price could be observed. As our findings for $\theta$ and $\mu$ depicted, this nature of stocks is viable in residuals and it was not explained by market excess return in a simple linear regression. Thus, asymmetry in residual distribution caused by irregularity in stock return could leave traditional results in doubt. Moreover, company specific and industry related factors are possible drivers of unsymmetrical and non-normal shape of error term distribution, and GLAM accurately captures those factors in stock returns.

**Figure 3.10**

![Cross sectional mu, N225](image1)

**Figure 3.11**

![Cross sectional mu, N225](image2)

Fig. (3.14) and (3.15) display a difference between $\theta$ and $\mu$ estimated from LS and R residuals. Histogram clearly supports the notion that both approaches deliver distinct residuals and this discrepancy is consistent across 225 stocks in 2Q 2008. Some of the stocks have a significantly
Figure 3.12

Cross sectional theta, N225

Figure 3.13

Cross sectional theta, N225
diverse $\theta$ and $\mu$ estimates, e.g., -0.08 (far left side of Fig. (3.14)).

**Figure 3.14**

Cross sectional difference theta, N225

**Figure 3.15**

Cross sectional difference mu, N225

**$\beta$ and $\theta$**

Fig. (3.16) illustrate difference of estimated $\beta$ for Canon stocks. Plot has no pronounced time trend, however, the magnitude of contrast is quite significant. Especially, in 2011 and 2013 the divergence of two $\beta$ is noticeable. Similar plot for $\theta$ parameter is presented in Fig. (3.17). $\theta$ difference also has obscure trend by time but the range of fluctuation decays gradually.

**Figure 3.16**

Beta difference Canon

**Figure 3.17**

Theta difference Canon

Moreover, $\beta$ and $\theta$ do not show any sign of correlation as illustrated by Fig. (3.18) and (3.19). Observed $\theta$ values form two distinct clusters with mean being lower and higher than one. However, this behavior of $\theta$ is not related to $\beta$.

**Figure 3.18**

Beta Theta Canon

**Figure 3.19**

Beta Theta Canon
### 3.5.4 Skew-t distribution results

Following the estimation of $\theta$ and $\mu$, we fitted skew-t distribution into observed residuals $(\gamma_{i,t}, \mu_{i,t})$ and estimated $\gamma$, $\gamma$ and $\xi$. As an example Fig. (3.20) and (3.21) illustrate estimated parameters after fitting into residuals for Canon stocks.

Skew-t distribution’s $\xi$ parameter illustrates the location of the residual distribution. We expect $\xi$ to be comparable with $\mu$ from GLAM. $\mu$ and $\xi$ share a similar path in the beginning of the period with high fluctuations. However, $\xi$ plummets significantly in 2009, while $\mu$ shows high fluctuations only (Fig. (3.7)). In addition, $\mu$ varies in the range of -0.002 and 0.002, but $\xi$ has a range of -0.004 and 0.004 which is almost two times wider. This obviously illustrates the fundamental distinction of two different approaches to model error terms from a simple linear regression.

Shape parameter in Fig. (3.21) has a distinct behavior. Initially, $\gamma$ fluctuates in a small range but later reaches the highest point in 2009 and the lowest in 2016. Interestingly, for some periods $\gamma$ is zero which means that residual distribution has skewness on neither side and has a symmetrical form. However, $\theta$ from GLAM in Fig. (3.6) fluctuates quite noticeably during the time period with no sign of symmetricalness. Once again this could be due to a fundamental difference inherited into two approaches.

**Figure 3.20**  
Location for Canon

**Figure 3.21**  
Skew for Canon

**Figure 3.22**  
Cross sectional skew

**Figure 3.23**  
Cross sectional skew

Cross sectional distribution of estimated 225 $\gamma$s for 2Q in 2008 are displayed in Fig. (3.22) and (3.23), for R and LS residuals, respectively. Both histograms illustrate a similar distribution of skewness parameters among 225 stocks. Moreover, $\gamma$ forms two clusterings, one is more negative and the other on a positive side, and it is stronger in case of LS residuals. This is a possible indication that for some stocks residuals are left skewed and for others residuals are right skewed, and it supports our previous $\theta$ results. Similar behavior is observed for other periods as well.
such as in the 1st quarter of 2015 in Fig. (3.24) and (3.25). We choose to present findings for γ
only for crisis and relatively peaceful periods, nonetheless, result for the rest of the time period
is available upon request.

3.5.5 Jensen’s Alpha decomposition

Skew effect

β in a linear regression captures the sensitiveness of excess return to excess market return and
in line with CAPM assumption that the market is the only risk factor. Nonetheless, a growing
number of papers analyze the rate of return with inclusion of intercept term in a regression
known as “Jensen’s Alpha” and introduced by M. Jensen (1968).

\[
\alpha_i = E[R_i - R_f - \beta_i(R_m - R_f)]
\]

(3.40)

Difference of α and location parameter μ gives a skew effect as shown below.

\[
\alpha_i = E[\eta] = \mu_i + E[\epsilon] = \mu_i + \int_{-\infty}^{\infty} xdh(F(x) : \theta)
\]

(3.41)

\[
\alpha_i - \mu_i = \int_{-\infty}^{\infty} xdh(F(x) : \theta)
\]

(3.42)

Fig. (3.26) - (3.29) illustrate the cross sectional distribution of skew-effect for different quar-
ters. In 2005 2Q, histograms are centered between 0 and 0.001, and has a fat tails on the right
side. Skew effect from R and LS do not differ significantly and has a very similar shape of
distribution. However, in 2008 skew effects are quite distinct and R case has a noticeable right
tail.

In addition, skew effect is derived based on skew-t distribution’s location parameter as well
(3.45). Fig. (3.30) - (3.33) illustrate cross sectional skew effect obtained by subtracting skew-t
location parameter μ from α. In 2005 2Q, skew effects from LS and R are centered around 0
as well as have a similar shape. In comparison, skew effect based on skew-t location parameter
has a smaller magnitude than GLAM counterpart but still it has a fat right tail. In 2008 4Q
skew effect has more balanced distribution than Fig. (3.28) and (3.29). Possible explanation is
the intrinsic difference of GLAM and skew-t to capture the location parameter. GLAM seems
to capture the location more accurately and has asymmetrical skew effect during the crisis time.

To analyze this incongruence between GLAM and skew-t, skew effects regressed onto asym-
metry parameters as presented in Eq. (3.44) and (3.46), respectively.
Here, $i = \{1...225\}$ stocks and $q = \{1..79\}$ quarters.

\[
\text{skew effect}^{\text{glam}}_{i,q} = \alpha_{i,q} - \mu^{\text{glam}}_{i,q} 
\]

(3.43)

\[
\text{skew effect}^{\text{skew-t}}_{i,q} = \alpha_{i,q} - \mu^{\text{skew-t}}_{i,q}
\]

(3.45)

Here, $\kappa_1$ and $\kappa^*_1$ is the sensitiveness of skew-effect on asymmetry parameter $\theta$ and $\gamma$. Fig. (3.37) and (3.39) illustrate obtained estimated $\kappa_1$ and $\kappa^*_1$ for R and LS cases, respectively. Time period presented in a quarterly form and ranges form 1998 1st until 2017 3rd quarter. Noticeably, $\kappa$s have completely different path and magnitude, due to the fact that skew-effect is distinct in Eq. (3.43) and (3.45).

As an example, regression result for Q1 2009 presented below in Eq. (3.47) ($t$ - stats are given in parenthesis). Clearly, when $\theta$ is equal to 1 in case of symmetry, skew effect is almost zero for GLAM case ($\kappa_0$ and $\kappa_1 \theta_i$ sum up to zero). Similar relation between skew effect and $\theta$ could be observed for other quarters as well.

\[
\text{skew effect}^{\text{glam}}_{i} = -0.0335 + 0.0333 \cdot \theta_i + \epsilon_i 
\]

\[(-13.90)\] \[14.47\]

(3.47)

In comparison, below in Eq. (3.48) ($t$ - stats are given in parenthesis) is presented regression result for Q1 2009 for skew-t case. Assuming relatively symmetrical error term distribution, skew effect should be equal to $\kappa^*_0$ plus $\kappa^*_1 \gamma_i$ which does not sum up to zero. In comparison with previous findings, $\kappa^*_0$ and $\kappa^*_1$ do not portray a similar behavior as found for the case of GLAM. More importantly, skew-t’s $\gamma$ parameter does not explain skew effect as shown in Eq. (3.48), $\kappa^*_1$ is insignificant, in comparison to Eq. (3.47) where $\kappa_1$ is statistically significant.

\[
\text{skew effect}^{\text{skew-t}}_{i} = -0.0000123 - 0.0000139 \cdot \gamma_i + \epsilon_i
\]

\[(-0.76)\] \[(-0.54)\]

(3.48)

Besides, comparison of both approaches (GLAM and skew-t parameters) based on a $p$ -values from quarterly regressions’ results reveals that $\theta$ explains skew-effect in all quarters across our data time span Fig. (3.34). Skewness parameter of skew-t fails to explain skew-effect in most of the quarters and could not reject the null hypothesis that $\kappa^*_1$ is zero Fig. (3.35).

\[
\alpha \text{ decomposition}
\]

Following the GLAM and Skew-t distribution fit, we obtained location and skew parameters for each of 225 stocks by quarter. Jensen’s Alpha obtained as in Eq. (3.49) and (3.50) (Jensen, 1968).

\[
\alpha^{L,S}_i = E[R_i - \beta^{L,S}_i (R_m - R_f)]
\]

(3.49)
Figure 3.34

p-values from regression

Figure 3.35

p-values from regression

Figure 3.36

Intercept from regression

Figure 3.37

Coefficient from regression

Figure 3.38

Intercept from regression

Figure 3.39

Coefficient from regression
\[
\alpha_i^R = E[R_i - \beta_i^R(R_m - R_f)]
\] (3.50)

Next, we regressed \(\alpha\) from a simple linear regression on \(\mu, \theta, \xi\) and \(\gamma\) for every quarter, respectively, to analyze if \(\alpha\) is explained by location and asymmetry of residual distribution as in Eq. (3.51) and (3.52).

\[
\alpha_{i,q} = \kappa_0 + \kappa_1 \mu_{i,q} + \kappa_2 \theta_{i,q} + \epsilon_{i,q}
\] (3.51)

\[
\alpha_{i,q} = \kappa_0^* + \kappa_1^* \xi_{i,q} + \kappa_2^* \gamma_{i,q} + \epsilon_{i,q}^*
\] (3.52)

Here, \(i = \{1, \ldots, 225\}\) stocks and \(q = \{1, \ldots, 79\}\) quarters.

Fig. (3.40), (3.41) and (3.42) illustrate \(\kappa_0, \kappa_1\) and \(\kappa_2\), respectively. Starting with \(\kappa_1\) in (3.41), estimated coefficient for location parameter \(\mu\) fluctuates noticeably around one and this result is in line with the study of Jensen (1968).

In Fig. (3.40) and (3.42), \(\kappa_0\) and \(\kappa_1\) have a similar path in but different sign. Clearly, during the crisis period in 2008, \(\alpha\) was quite sensitive to \(\theta\) than other periods. As an example, regression results for Q1 2009 presented in (3.53). Assuming no asymmetry in error term distribution from a simple linear regression, \(\kappa_0\) and \(\kappa_2\) cancel each other and Jensen’s Alpha is only equal to \(\mu\).

Similar relation between \(\alpha\) and \(\theta\) could be observed for other quarters as well.

\[
\alpha_i = -0.0325 + 1.0229 \times \mu_i + 0.0325 \times \theta_i + \epsilon_i
\] (3.53)

\[
(-11.91) \quad (32.27) \quad (12.49)
\]

Fig. (3.43), (3.44) and (3.45) illustrate \(\kappa_0^*, \kappa_1^*\) and \(\kappa_2^*\), respectively. In comparison with previous findings, \(\kappa_0^*\) and \(\kappa_2^*\) do not portray a similar behavior which is found earlier. Below in Eq. (3.54) is presented regression result for Q1 2009 for the case of skew-t. Assuming relatively symmetrical error term distribution, \(\alpha\) is equal to \(\kappa_0^*\) and \(\kappa_1^*\) which does not sum up to zero.

\[
\alpha_i = -0.000012 + 1.0093 \times \xi_i - 0.000036 \times \gamma_i + \epsilon_i^*
\] (3.54)

\[
(-0.73) \quad (215.49) \quad (-1.30)
\]

Following the regression result we can conclude that GLAM parameters \(\mu\) and \(\theta\) are superior to skew-t distribution parameters to decompose the \(\alpha\) into location and asymmetry.
3.5.6 VIX and $\theta$

To further explore the relation of estimated $\hat{\theta}$ with uncertainty indicator of stock markets - volatility index is used in Eq (3.55) and Eq (3.56). Here $t$ is quarters.

$$VIX_t = \alpha + \beta \hat{\theta}_{med}^t + \epsilon_t$$  \hspace{1cm} (3.55)

$$\hat{\theta}_{med}^t = \alpha + \beta VIX_t + \epsilon_t$$  \hspace{1cm} (3.56)

Tables (3.18) and (3.19) present results for three markets, respectively. Clearly, increase in median $\theta$ contributes into VIX and it could be observed across markets, however, the magnitude is different. Hence, increase in the number of stocks with asymmetrical error distribution possibly indicates the grow in market uncertainty.

Table 3.18

<table>
<thead>
<tr>
<th>Dependent variable: VIX</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{med}^{jp}$</td>
<td>9.823***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.572)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_{med}^{usa}$</td>
<td>7.644***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.460)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_{med}^{uk}$</td>
<td>6.115***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.612)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-9.765***</td>
<td>-7.605***</td>
<td>-6.050***</td>
</tr>
<tr>
<td></td>
<td>(1.604)</td>
<td>(1.492)</td>
<td>(1.646)</td>
</tr>
<tr>
<td>Observations</td>
<td>79</td>
<td>79</td>
<td>75</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.328</td>
<td>0.253</td>
<td>0.153</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.328</td>
<td>0.253</td>
<td>0.153</td>
</tr>
</tbody>
</table>

Note: *$p<0.1$; **$p<0.05$; ***$p<0.01$

Table 3.19

<table>
<thead>
<tr>
<th>Dependent variable: $\theta_{med}$</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{med}^{jp}$</td>
<td>0.034***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_{med}^{usa}$</td>
<td>0.034***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_{med}^{uk}$</td>
<td>0.027***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1.011***</td>
<td>1.015***</td>
<td>1.016***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Observations</td>
<td>79</td>
<td>79</td>
<td>75</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.336</td>
<td>0.262</td>
<td>0.165</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.328</td>
<td>0.253</td>
<td>0.153</td>
</tr>
</tbody>
</table>

Note: *$p<0.1$; **$p<0.05$; ***$p<0.01$

3.5.7 Stock market index and $\theta$

To further analyze observed $\hat{\theta}$ we investigated the relation between market index and $\hat{\theta}$ as in Eq. (3.57).
\[ R_{m,t} = \gamma_0 + \gamma_1 \hat{\theta}_t^{med} + \epsilon_t \]  

Here, \( i \) are quarters, \( R_m \) is quarterly index rate of return and \( \hat{\theta}^{med} \) is observed median of \( \hat{\theta} \) distribution for the given quarter. We repeated this analysis for all three indexes, FTSE100, N225 and S&P500, respectively, and results are given in Table (3.20).

### Table 3.20

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>n225q</th>
<th>sp500q</th>
<th>ftse100q</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_{jp}^{med} )</td>
<td>-6.937***</td>
<td>(2.410)</td>
<td></td>
</tr>
<tr>
<td>( \theta_{usa}^{med} )</td>
<td>-4.010***</td>
<td>(1.163)</td>
<td></td>
</tr>
<tr>
<td>( \theta_{uk}^{med} )</td>
<td>-5.285***</td>
<td>(1.217)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>7.084***</td>
<td>(2.458)</td>
<td>4.128***</td>
</tr>
<tr>
<td></td>
<td>5.403***</td>
<td>(1.241)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>79</td>
<td>99</td>
<td>131</td>
</tr>
<tr>
<td>R²</td>
<td>0.097</td>
<td>0.109</td>
<td>0.128</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.085</td>
<td>0.100</td>
<td>0.121</td>
</tr>
</tbody>
</table>

Note: *p < 0.1; **p < 0.05; ***p < 0.01

Obviously, median \( \hat{\theta} \) is statistically significant to explain stock index rate of return. In addition, the magnitude of the effect is distinct and negative across all three markets, the largest effect is observed for N225 and the smallest effect is observed for S&P 500.

Results supported by our previous findings regarding VIX and \( \hat{\theta} \). Assuming median \( \hat{\theta} \) is an indication of market uncertainty, negative relation between median \( \hat{\theta} \) and market index rate of return is expected.

### 3.6 F distribution and relative efficiency

#### 3.6.1 Estimation of F distribution

In this section we estimate \( F \) which was assumed to be a symmetric around zero but to have unknown form of distribution. \( Z_i \) values are obtained based on optimal \( \hat{\theta} \) estimated in section 3.4. The proof applies the convergence arguments for estimated empirical distribution functions in section 5 of chapter 5 of Shorack and Wellner (1986).

The advantage of GLAM is that we can estimate \( \mu \) and \( \theta \) without knowing the functional form of \( F \). However, in order to see the accuracy of estimators of \( \beta \) and a comparison of asymptotic variance of estimation error of these two methodology: LS and R-estimator, the functional form of distribution \( F \) and the density \( f \) are required. The role of \( F \) in GLAM model includes, as well as its symmetry, a representation of dispersion of the underlying distribution in the tails of \( F \), while the transformation function \( h(F; \theta) \) only represents how and how much the underlying distribution \( F \) is skewed/asymmetric to fit to the distribution of observed residuals. Thus we choose t-distribution which degree of freedom parametrize the tail-heaviness and unimodal symmetric shape of distribution ranging from Normal to almost close to Cauchy.

For each quarter and each stock, we fitted t-distribution to a set of \( Z_i \) (estimated \( \theta \)). Then we found that the estimated degree of freedom varies cross-sectionally very much from as small
as 3 or 4 to as large as 40-to 50 in every quarter during the year 1998-2017.

After obtaining $Z_i$ values we fitted $t$ distribution and obtained degrees of freedom $(df)$. Fig 3.46 illustrates the cross sectional distribution of $df$ for N225 stocks.

**Figure 3.46**

![Df of t distribution fitted into R residuals, N225](image)

Undoubtedly, residuals do not have a similar distribution but it varies depending on the stocks from companies. Nonetheless, two distinct clusterings are emerged, one is centered around 20 $df$ and the other around 100 $df$.

### 3.6.2 Asymptotic variance

We obtain a parametric form of asymptotic variance for LS and R estimates of $\beta$ by substituting the functional form of the density of $t$-distribution into the Eq. (3.6) and (3.7) in section 3.2. It shows that the asymptotic variances are functions of the degrees of freedom.

Specifically, for each quarter we estimated asymptotic variance for LS and R cases, respectively. Since, we used the $t$ - distribution for estimation, its degrees of freedom changed depending on the quarter and stock.

Following the estimation of asymptotic variances we took the ratio of $\hat{\sigma}^2(\hat{\beta}_{LS})/\hat{\sigma}^2(\hat{\beta}_R)$ for further analysis. Especially, variance ratio is higher when the degrees of freedom is smaller than approximately 20 (the tail is heavier than the Normal distribution and close to the Logistic). This behavior reverses when the degrees of freedom bigger than 20 (it is close to Normal distribution).

Fig. 3.47 - 3.48 illustrate cross sectional scatter plots of degrees of freedom and variance ratio. Besides, plots are colored based on the estimated $\hat{\theta}$.

Thus, each dots in the plot represents statistics for individual stock for a given quarter, respectively. As we mentioned earlier, we fitted $t$ distribution into residuals to estimate the suitable degrees of freedom, obtained variance ratio and also estimated $\hat{\theta}$. Hence, plots represents all these statistics together for comparison.
Figure 3.47

Degrees of freedom, Variance ratio, Theta

Figure 3.48

Degrees of freedom, Variance ratio, Theta
Figure 3.49

Degrees of freedom, Variance ratio, Theta

Figure 3.50

Degrees of freedom, Variance ratio, Theta
Figure 3.51

Figure 3.52
Clearly, LS is less efficient than R estimates (variance ratio is higher than 1) when underlying $t$ distribution’s degrees of freedom is smaller than approximately 20. Moreover, R estimate is still more efficient (red dots) when variance ratio is smaller than one and but has asymmetrical distribution. However, this behavior changes from quarter to quarter as displayed by the plots for distinct periods.

R-estimate is efficient when the degrees of freedom is smaller than 20. Small degrees of freedom indicates a heavy tailed distribution than Normal. Our results indicate the efficiency of R-estimates when residuals have heavier tails. Moreover, findings are in line with Lehmann’s (1983) results given in table in chapter 5. For instance, it shows the relative efficiency ratio to be 1.24 (for $t$-distribution with $df = 5$) which our findings confirm this as well.

Similarly, figures 3.49 - 3.50 illustrate the same relationship between 3 parameters ($\theta$, variance ratio and degrees of freedom) and also displays the industry of companies, respectively. Interestingly, we can see a close patterns of stocks in one industry. For instance, “Capital Goods” stocks have high $\theta$ and high variance ratio but “Materials” stocks have high $\theta$ and lower variance ratio in Fig. (3.49).

To investigate further by industry-wise, we estimated the average degrees of freedom and variance ratio for industries, respectively. As Fig. 3.51 - 3.52 illustrate, “Financial” and “Transportation” company stocks have distinct patterns than the rest of industries. More importantly, figures reveal that for most of the industries, the observed residuals are asymmetric on average and it has been increasing significantly in the last 5 years.
3.7 Conclusion

Undeniably, asset pricing is one of the central questions for most of the researches in finance. This paper focused on a simple market model and applied two distinct approaches to estimate $\beta$, least squares and rank, respectively. LS is a common method in social sciences and based on a numerous assumptions, on the contrary, rank is a distribution free as well as robust approach. Thus, CAPM $\beta$ is estimated by both techniques and residuals are used for further tests.

Indeed, observed residuals from a simple regression rejected normality assumptions and have asymmetry. Empirical tests supported our prior expectations. Thus, R is more accurate than LS when error term is not from a family of normal distributions and have a significant deformations in its distribution.

Following a regression, the GLAM method is applied onto observed residuals to investigate deformation in unsymmetrical distribution and this is captured by the parameter $\theta$ as well as the location estimated by the parameter $\mu$. Estimated $\theta$ and $\mu$ have a noticeable fluctuation over the sample period across stocks in Nikkei 225, FTSE 100 and S&P 500. In addition, we applied parametric approach skew-t distribution to fit into residuals and to estimate a skewness parameter. Location $\xi$ and skewness $\gamma$ from skew-t exhibit a significant fluctuations as well across N225 stocks and supported our previous results from semi-parametric approach.

Thus, CAPM $\beta$ does not completely explain the abnormality in stock returns. Possible explanation could be that the model leaves unobservable firm and industry related factors in error term.

Asymmetry parameter $\theta$ is statistically significant to explain $\alpha$ from a simple linear regression. Especially, during the crisis periods (2007-2009) sensitiveness of $\alpha$ to $\theta$ more than tripled in Japanese stocks. Hence, decomposition of Jensen’s Alpha proved that depending on the magnitude of asymmetry in error terms $\alpha$ could vary. In addition, $\alpha$ also explained by $\xi$ and $\gamma$ from skew-t distribution, nonetheless, results are unclear in case of symmetrical error term distribution. This also possibly could serve as a proof that GLAM is suitable than parametric approach - skew-t to estimate skewness.

Our research sheds light on analyzing Jensen’s Alpha from prospective of asymmetry in error term distribution and applying robust non-parametric approaches to estimate stock $\beta$ when residual distribution is heavy tailed and asymmetric.
3.8 Reference


3.9 Appendix

This section mathematically derives score functions for $\theta$ and $\mu$ assuming that error terms follow logistic distribution, $F(\frac{x-\mu}{s})$.

\[
F(\frac{x-\mu}{s}) = \frac{1}{1 + e^{-\frac{x-\mu}{s}}} \tag{3.58}
\]

First, the derivative from an empirical distribution with respect to $x$ is taken.

\[
g(x : \mu, \theta) = \frac{dG(x : \mu, \theta)}{dx} = -\theta (1 - F(\frac{x-\mu}{s}))^{\theta-1} f\left(\frac{x-\mu}{s}\right) = -\theta (1 - F(\frac{x-\mu}{s}))^{\theta-1} f(\frac{x-\mu}{s}) \tag{3.59}
\]

Here, $f(\frac{x-\mu}{s})$ is a probability density function.

Next, we take derivative from $g(x : \mu, \theta)$ with respect to $\theta$. In order to derive $g_\theta(x : \mu, \theta)$, we first take the natural logarithm of $g(x : \mu, \theta)$.

\[
\ln(g(x : \mu, \theta)) = \ln(\theta) + (\theta - 1) \ln(1 - F(\frac{x-\mu}{s})) + \ln(f(\frac{x-\mu}{s})) \tag{3.61}
\]

\[
g_\theta(x : \mu, \theta) = \frac{d\ln(g(x : \mu, \theta))}{d\theta} = \frac{1}{\theta} + \ln(1 - F(\frac{x-\mu}{s})) \quad \frac{d\ln(g(x : \mu, \theta))}{d\theta} \tag{3.62}
\]

\[
g_\theta(x : \mu, \theta) = \frac{dg(x : \mu, \theta)}{d\theta} = \left[ \frac{1}{\theta} + \ln(1 - F(\frac{x-\mu}{s})) \right] \frac{dg(x : \mu, \theta)}{d\ln(g(x : \mu, \theta))} \tag{3.63}
\]

\[
g_\theta(x : \mu, \theta) = \frac{dg(x : \mu, \theta)}{d\theta} = \left[ \frac{1}{\theta} + \ln(1 - F(\frac{x-\mu}{s})) \right] g(x : \mu, \theta) \tag{3.64}
\]

$g_\mu(x : \mu, \theta)$ is also obtained by taking the derivative from $g(x : \mu, \theta)$ with respect to $\mu$.

\[
g_\mu(x : \mu, \theta) = \frac{dg(x : \mu, \theta)}{d\mu} = -\left[ \theta (\theta - 1)(1 - F(\frac{x-\mu}{s}))^{\theta-2} f(\frac{x-\mu}{s})(-1)f(\frac{x-\mu}{s}) \right] \tag{3.65}
\]

Here, derivation of $\frac{df(\frac{x-\mu}{s})}{d\mu}$ is as following:

\[
\frac{df(\frac{x-\mu}{s})}{d\mu} = \left[ \theta (1 - F(\frac{x-\mu}{s}))^{\theta-1} \frac{df(\frac{x-\mu}{s})}{d\mu} \right]
\]
Using Eq. (3.66), we replace $df(x-\mu)/d\mu$ with:

$$\frac{df(x-\mu)/d\mu}{s} = e^{-\frac{x-\mu}{s}} \left[ \frac{1}{s(1 + e^{-\frac{x-\mu}{s}})^2} + \frac{e^{-\frac{x-\mu}{s}}}{s} (-2)(1 + e^{-\frac{x-\mu}{s}})^{-3} \frac{e^{-\frac{x-\mu}{s}}}{s} = \right.$$  

$$\frac{1}{s} \left[ \frac{e^{-\frac{x-\mu}{s}}}{s(1 + e^{-\frac{x-\mu}{s}})^2} \left[ 1 - 2 \frac{e^{-\frac{x-\mu}{s}}}{1 + e^{-\frac{x-\mu}{s}}} \right] \right] =$$  

$$\left[ \frac{e^{-\frac{x-\mu}{s}}}{s(1 + e^{-\frac{x-\mu}{s}})^2} \right] \left[ \frac{1}{s} - 2 \frac{e^{-\frac{x-\mu}{s}}}{s(1 + e^{-\frac{x-\mu}{s}})^2} (1 + e^{-\frac{x-\mu}{s}}) \right] =$$  

$$f\left(\frac{x-\mu}{s}\right) \left[ \frac{1}{s} - 2f\left(\frac{x-\mu}{s}\right) \frac{1}{F\left(\frac{x-\mu}{s}\right)} \right]$$  

(3.66)

Using Eq. (3.66), we replace $df(x-\mu)/d\mu$ in (3.65) and take out the common factor.

$$g_{\mu}(x : \mu, \theta) = \theta(1 - F\left(\frac{x-\mu}{s}\right)^{\theta-1} f\left(\frac{x-\mu}{s}\right) \left[ (\theta - 1)(-1) \frac{f\left(\frac{x-\mu}{s}\right)}{1 - F\left(\frac{x-\mu}{s}\right)} + \frac{1}{s} - 2 \frac{f\left(\frac{x-\mu}{s}\right)}{F\left(\frac{x-\mu}{s}\right)} \right]$$  

(3.67)

Now, score functions for $\theta$ and $\mu$ are estimated as following:

$$J_{\mu}(F\left(\frac{x-\mu}{s}\right)) = \frac{g_{\mu}(x : \mu, \theta)}{g(x : \mu, \theta)} = \frac{\theta(1 - F\left(\frac{x-\mu}{s}\right))^{\theta-1} f\left(\frac{x-\mu}{s}\right) \left[ (\theta - 1)(-1) \frac{f\left(\frac{x-\mu}{s}\right)}{1 - F\left(\frac{x-\mu}{s}\right)} + \frac{1}{s} - 2 \frac{f\left(\frac{x-\mu}{s}\right)}{F\left(\frac{x-\mu}{s}\right)} \right]}{\theta(1 - F\left(\frac{x-\mu}{s}\right))^{\theta-1} f\left(\frac{x-\mu}{s}\right)} =$$  

$$\left[ (\theta - 1)(-1) \frac{f\left(\frac{x-\mu}{s}\right)}{1 - F\left(\frac{x-\mu}{s}\right)} + \frac{1}{s} - 2 \frac{f\left(\frac{x-\mu}{s}\right)}{F\left(\frac{x-\mu}{s}\right)} \right]$$  

(3.68)

To simplify further we define:

$$f\left(\frac{x-\mu}{s}\right) = \frac{e^{-\frac{x-\mu}{s}}}{s(1 + e^{-\frac{x-\mu}{s}})^2} = \frac{1 + e^{-\frac{x-\mu}{s}} - 1}{s(1 + e^{-\frac{x-\mu}{s}})^2} = \frac{1 + e^{-\frac{x-\mu}{s}}}{s(1 + e^{-\frac{x-\mu}{s}})^2} + \frac{-1}{s(1 + e^{-\frac{x-\mu}{s}})^2} =$$  

$$\frac{1}{s} F\left(\frac{x-\mu}{s}\right) - \frac{1}{s} F\left(\frac{x-\mu}{s}\right)^2 = \frac{1}{s} F\left(\frac{x-\mu}{s}\right) (1 - F\left(\frac{x-\mu}{s}\right))$$  

(3.69)

We replace $f\left(\frac{x-\mu}{s}\right)$ in (3.68) with (3.69). $J_{\mu}(F\left(\frac{x-\mu}{s}\right))$ gets simpler form.

$$J_{\mu}(F\left(\frac{x-\mu}{s}\right)) = \frac{1}{s} \left[ (\theta - 1)(-1) \frac{F\left(\frac{x-\mu}{s}\right)}{1 - F\left(\frac{x-\mu}{s}\right)} + 1 - 2\left(1 - F\left(\frac{x-\mu}{s}\right) \right) \right]$$  

(3.70)
In order to derive $J_\mu(t)$ and $J_\theta(t)$ we have to find the inverse of $h^{-1}(F\left(\frac{x-\mu}{s}\right); \theta)$. However, $h(F\left(\frac{x-\mu}{s}\right); \theta)$ has been omitted during the derivation process. Thus, we use functional form of $h$ as in (3.72).

$$G_{\mu,\theta}^{-1}(t) = h^{-1}(F\left(\frac{x-\mu}{s}\right); \theta)$$ (3.71)

$$h^{-1}(F\left(\frac{x-\mu}{s}\right); \theta) = 1 - (1 - t)^{1/\theta}$$ (3.72)

Derivation the inverse of $h(F\left(\frac{x-\mu}{s}\right); \theta)$ is as following.

$$t = G_{\mu,\theta}(x) = h(F\left(\frac{x-\mu}{s}\right); \theta) = 1 - (1 - F\left(\frac{x-\mu}{s}\right))^\theta$$ (3.73)

$$F\left(\frac{x-\mu}{s}\right) = 1 - (1 - t)^{1/\theta}$$ (3.74)

We replace $F\left(\frac{x-\mu}{s}\right)$ in (3.70) with $1 - (1 - t)^{1/\theta}$.

$$J_\mu(t) = -\frac{1}{s} \left[ (\theta - 1)(-1) \left[ 1 - (1 - t)^{1/\theta} \right] + 1 - 2(1 - t)^{1/\theta} \right]$$ (3.75)

For $J_\theta(t)$ derivation we follow the same process. First we estimate $J_\theta(F\left(\frac{x-\mu}{s}\right))$ as in (3.76).

$$J_\theta(F\left(\frac{x-\mu}{s}\right)) = \frac{g_\theta(x : \mu, \theta)}{g(x : \mu, \theta)} = \frac{\frac{1}{\theta} + \ln(1 - F\left(\frac{x-\mu}{s}\right))}{g(x : \mu, \theta)} =$$

$$\frac{1}{\theta} + \ln(1 - F\left(\frac{x-\mu}{s}\right))$$ (3.76)

After replacing $F\left(\frac{x-\mu}{s}\right)$ in (3.76) with $1 - (1 - t)^{1/\theta}$ we obtain $J_\theta(t)$.

$$J_\theta(t) = \frac{1}{\theta} + \ln(1 - [1 - (1 - t)^{1/\theta}]) = \frac{1}{\theta} + \ln(1 - t)^{1/\theta}$$ (3.77)
Figure 3.53
Residual histogram Canon

Figure 3.54
Residual histogram Canon

Figure 3.55
Residual histogram Canon

Figure 3.56
Residual histogram Canon

Figure 3.57
Lag Beta Canon

Figure 3.58
Lag Beta Canon

Figure 3.59
Lag Theta Canon

Figure 3.60
Lag Theta Canon
Chapter 4

Fund manager performance and investment strategy
Abstract

Surge in a number and types of investment funds attracted a great deal of attention by researchers to evaluate manager and fund performance. Since the classic paper by Jensen (1968), diverse set of tools and methods have been developed and put into test to extract a part of the return contribution made by the manager. This study also focuses on measuring manager performance for closed mutual funds - Unit Trusts. In comparison with other studies, here I employ Total Return Index which is accepted as a better and more suitable indicator to analyze close end fund performance. By focusing on monthly data from 1970 until 2015, performance (regression $\alpha$) evaluated based on non-parametric Rank statistics and Least Squares. R estimates are found to be more efficient than LS counterpart for most of the periods. To separate R estimate of alpha based on manager skill alone, a cross sectional bootstrap method is used. Observed $\eta_i$ ($\alpha$ included residuals) are further analyzed by fitting skew-normal distribution (Azzalini (1986)) and non-parametric Generalized Lehmann’s Alternative Model (Miura and Tsukahara (1993)) to obtain parameters for skewness and location. By looking at the performance persistence I found that alpha due to manager skill persists together with skewness for long horizon. This study clearly shows that skill of the manager and portfolio allocation of fund is reflected in asymmetry of error terms ($\eta_i$, $\alpha$ added) from the regression which is used to evaluate the performance.
4.1 Introduction

Evaluation of manager performance is a research topic in finance that has been addressed quite often. Since the pioneering work of Jensen (1969) a number of studies increased with an application of various methodologies to assess the manager’s stock picking ability and style. Furthermore, a surge in the number and types of funds for investment also attracted a great amount of attention.

Majority of studies evaluated manager performance based on multi factor models and as risk factors were employed various benchmark portfolios (Carhart (1997), Lynch and Musto (2003), Berk and Green (2004), Christopherson et al. (1998)). Usual method Ordinary Least Squares (LS) is used to estimate model parameters, even though, recently some studies employed other approaches as well (Kosowski et al. 2006, 2007). Nonetheless, literature lacks of studies with an application of robust non-parametric methods in comparison to LS.

First contribution of this paper is an application of Rank (R) statistics to assess the performance. Theoretical development of R statistics is found in studies by Jureckova (1971), Jaeckel (1972) and Hettmansperger and McKean (1977). This method is commonly employed in statistics field and has a great advantage compared to LS (Onder and Zaman (2003), (2005)). It is insensitive to outliers in data and does not make prior assumptions regarding the distribution of error terms.

Shortfalls of parametric methods other than LS also has been investigated as well, such as Hettmansperger and Sheather (1992) showed that the Least Median Squares (LMS) is unstable when central location of data changes. In addition, recently Denhere and Bindele (2016) compared Rank based estimation with LS and Least Absolute Deviation (LAD) estimators, and found that R estimators are robust compared to parametric methods when data has outlying observations and fat-tailed error distribution.

Secondly this paper focuses on the shape of residual distribution. Rather than measuring \( \alpha \) as a mean value of observed residuals from regression and assuming normality of error terms, a skew-normal distribution (Azzalini, 1985) is fitted into observed residuals (\( \alpha \) included) to estimate the skewness and location of distribution by Maximum Likelihood method (MLE). Here, the location parameter from skew-normal fitting and regression \( \alpha \) serve for the same purpose which is a measure of performance. In addition, Generalized Lehmann’s Alternative Model is also used for modeling and to obtain asymmetry and location parameters.

Furthermore, estimated \( \alpha \) are separated into two groups due to manager skill and luck based on cross sectional bootstrapping of residuals. Recently, bootstrap is frequently used to study the performance of mutual and hedge fund manager (Kosowski et al. 2006, 2007).

Persistence of skill is studied by cross sectional regressions of compounded return on estimated \( \alpha \) together with skewness and location parameters from skew-normal fitting (Fama and Macbeth (1973), Christopherson et al. (1998)). Study found that skewness and location of residual distribution for the case of skillful manager persists for 5 year period but not in shorter horizons. This finding is in line with previous studies on mutual and pension funds that focused just on \( \alpha \) assuming symmetrical residual distribution.

In this study, I employ Total Return Index (TRI) of unit trusts. TRI assumes that any capital gains such as cash distributions and dividend yields are reinvested back into the fund and considered more precise measurement approach of active management performance. In addition, TRI becoming popular in financial industry in comparison with traditional method - net returns.

Research is constructed as follows. Section 4.2 presents data and TRI description. Section 4.3 meticulously explains manager performance evaluation, cross sectional bootstrap method and
asymmetry estimation by skew-t and GLAM methods. All estimated results are given in section 4.4. Next, section 4.5 discusses main findings and the last section 4.6 sums up this study with directions for future research.

4.2 Data

This study employs 1097 Unit Trust data and obtained from Thomson Reuters Database. Data is in monthly frequency and covers the time period from January 1963 until January 2016. In addition, Fama and French (2015) 5 factors which is constructed for US markets are also employed and risk free rate is 3 month T-bill rate. FF5 data is from Kenneth French data library \(^1\) and risk free rate is from Federal Reserve Bank of St.Louis\(^2\).

Data starts in different time periods for funds and to provide a fair comparison we divided data into non overlapping 5-year periods from 1965 until 2015 in total 10 sub-periods. It leaves us 60 monthly observation for each fund for each sub-periods which is the most commonly time span used for measuring the manager performance (Kosowski et al., (2006), (2007), Christopherson et al. (1998)).

TRI is employed as a measure of the return for investment. It expects that any capital gains such as cash distributions and dividend yields are reinvested back into the fund. Total Return Index is estimated as given in Eq (4.1).

\[
RI_t = RI_{t-1} \ast \frac{PI_t}{PI_{t-1}} \ast (1 + \frac{DY_t}{100} \frac{1}{N})
\]

(4.1)

Here, \(RI_t\) return index at \(t\), \(PI_t\) price index at \(t\), \(DY_t\) at \(t\) is dividend yield and \(N\) number of trading days per year since annualized \(DY\) is used. The rate of change in index is used as dependent variable in Eq (4.2).

4.3 Methodology

4.3.1 Indicator of manager performance

Manager performance is analyzed through commonly employed multi-factor model - Fama and French (2015) 5 factor model as shown in Eq (4.2).

\[
r_{i,t} = \alpha_i + \beta_i \text{RMRF}_t + s_i \text{SMB}_t + h_i \text{HML}_t + r_i \text{RMW}_t + c_i \text{CMA}_t + \epsilon_{i,t}
\]

(4.2)

for \(t=1,...,T\) period and \(i=1,...,n\) funds.

Here, \(r_{i,t}\) is excess return for fund \(i\) and for month \(t\). \text{RMRF}\ - market excess return, \text{SMB} \ - small minus big, \text{HML} \ - high minus low, \text{RMW} \ - robust minus weak and \text{CMA} \ - conservative minus aggressive risk factor portfolios (Fama and French factors).

We apply this model for each fund data in a separately and non-overlapping 60-month period. Parameters estimated by OLS and R \(\{\hat{\alpha}, \hat{\beta}, \hat{s}, \hat{h}, \hat{r}, \hat{c}\}\), and observed residuals \(\{\hat{\epsilon}\}\) from the first step are saved for cross sectional bootstrap procedure.

\(^1\)http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Benchmarks
\(^2\)https://fred.stlouisfed.org/
Rank approach

Model coefficients are estimated by R approach as well based on non-overlapping 60-month data. In comparison with OLS, R minimizes the sum of dispersion as shown in Eq. (4.3).

\[ D_i = \sum_{t=1}^{T} \left( \frac{R_{\epsilon_{i,t}}}{T+1} - \frac{1}{2} \right) \left( r_{i,t} - \beta_i \text{RMRF}_t - s_i \text{SMB}_t - h_i \text{HML}_t - r_i \text{RMW}_t - c_i \text{CMA}_t \right) \] (4.3)

Here, \( D_i \) - sum of dispersion, \( R_{\epsilon_{i,t}} \) - rank of \( \epsilon_{t} \) and \( W_T(R_{\eta}) \) - Wilcoxon scores. In other words, in Eq. (4.3) dispersion is observed as a weighted sum of residuals. Thus, residuals resulting from outliers in data should have a low rank and a small effect on dispersion. Clearly, R approach is based on rank of observed residuals and estimates coefficients which produce the lowest dispersion value.

4.3.2 Cross sectional bootstrap

Following the estimation of model parameters, we employ a bootstrap method to extract alphas based on the skill rather than luck (or bad luck). We followed Kosowski et al. (2006, 2007) for bootstrapping t-values of estimated \( \alpha \) in Eq. (4.2). Sample of residuals is taken with replacement from observed residuals which are saved in the first step after estimation of model parameters in Eq. (4.2). For bootstrap \( b \) and for the period \( t \), \( \hat{\epsilon}_{i,t}^b \) is re-sampled from residuals of other funds for the identical time \( t \).

Following the re-sampling, returns are created artificially as in Eq. (4.4) by employing previously saved parameters \( \{ \hat{\alpha}, \hat{\beta}, \hat{s}, \hat{h}, \hat{r}, \hat{c} \} \).

\[ \hat{r}_{i,t}^b = \hat{\alpha} \cdot \text{RMRF}_t + \hat{s} \cdot \text{SMB}_t + \hat{h} \cdot \text{HML}_t + \hat{r} \cdot \text{RMW}_t + \hat{c} \cdot \text{CMA}_t + \hat{\epsilon}_{i,t}^b \] (4.4)

Next, \( \hat{r}_{i,t}^b \) re-regressed on 5 factors and obtained t statistics of \( \alpha_i^b \). This process is repeated 500 times and resulted a distribution of t statistics to test the significance of \( \alpha \) which is estimated in the first step by Eq. (4.2). If bootstrapped t values are less then obtained t statistics in the first step standardized by the number of bootstrap iterations, I assume manager’s skill exists otherwise it is due to a luck (Efron and Tibshirani, 1994).

4.3.3 Asymmetry

\[ \eta_{i,t} = \epsilon_{i,t} + \alpha_{i} = r_{i,t} - \beta_i \text{RMRF}_t - s_i \text{SMB}_t - h_i \text{HML}_t - r_i \text{RMW}_t - c_i \text{CMA}_t \] (4.5)

In comparison with Eq. (4.4), residuals include \( \alpha \) from the regression in Eq. (4.5). Intercept term in Eq. (4.4) simply a location of residual distribution and by including it in Eq.(4.5), shape of residuals distribution is not affected.

\[ \eta \sim SN(\xi, w^2, \gamma) \] (4.6)

Here, \( \eta \) are observed residuals and follows the skew-normal distribution (Azzalini, 1985). Parameter \( \xi \) is a location of residuals and \( \gamma \) is asymmetry of distribution. If residuals follow normal distribution and do not display any deformation, \( \gamma \) is equal to zero and location is a
simple mean of residuals which is the same estimate from LS. Thus, when the distribution has any asymmetric shape it will captured by $\gamma$.

Moreover, GLAM method is employed to estimate the location and asymmetry as well. This is semi-parametric method allows the distribution of $\eta$ to be unknown but symmetric around zero. In comparison with skew-normal distribution, GLAM fits well when $\eta$ have heavy tails. To obtain $\mu$ and $\theta$ parameters this study followed the same estimation algorithm employed in previous chapter as in Eq. (4.7).

$$\eta \sim G(x - \alpha) \quad (4.7)$$

### 4.4 Results

#### 4.4.1 Estimated alpha

Table 4.1 presents estimated alphas from Eq (4.2). Starting from means of alpha in percentage from LS and R approaches, they are not statistically significant but only in 1996 - 2000 (high $t$-ratio). Besides, means of alpha from LS and R approaches not significantly different and similar. This gives an expectation that basically both methods yield similar results. But before moving onto comparison, normality of residuals should be checked. Mean of skewness and kurtosis of residual distribution, resulting from regression estimation based on LS, is different from normal distribution case. Jarque and Bera (1987) test result supports this evidence. Especially, in the last period, the number of funds with non-normal residual distribution increased up to 65%. Clearly, this supports the view that non-parametric and distribution free approach needed for estimation of $\alpha$ (Kosowski et al. 2006).

Table 4.1: Estimated alpha and normality test

<table>
<thead>
<tr>
<th>Period</th>
<th>Funds</th>
<th>LS $\alpha$ (%)</th>
<th>LS $\alpha$ t-stat</th>
<th>R $\alpha$ t-stat</th>
<th>R $\alpha$</th>
<th>Skewness (LS)</th>
<th>Kurtosis (LS)</th>
<th>JB test (LS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966 - 1970</td>
<td>24</td>
<td>-0.27</td>
<td>-0.60</td>
<td>-0.22</td>
<td>-0.59</td>
<td>-0.18</td>
<td>2.29</td>
<td>0.27</td>
</tr>
<tr>
<td>1971 - 1975</td>
<td>32</td>
<td>-0.04</td>
<td>-0.13</td>
<td>-0.06</td>
<td>-0.17</td>
<td>0.14</td>
<td>0.20</td>
<td>0.26</td>
</tr>
<tr>
<td>1976 - 1980</td>
<td>201</td>
<td>-0.12</td>
<td>-0.73</td>
<td>-0.16</td>
<td>-0.97</td>
<td>-0.22</td>
<td>3.83</td>
<td>0.26</td>
</tr>
<tr>
<td>1981 - 1985</td>
<td>321</td>
<td>-0.35</td>
<td>-0.65</td>
<td>-0.28</td>
<td>-0.66</td>
<td>-0.27</td>
<td>4.44</td>
<td>0.28</td>
</tr>
<tr>
<td>1986 - 1990</td>
<td>736</td>
<td>0.25</td>
<td>-0.05</td>
<td>0.20</td>
<td>-0.12</td>
<td>-0.27</td>
<td>3.92</td>
<td>0.45</td>
</tr>
<tr>
<td>1991 - 1996</td>
<td>1097</td>
<td>0.19</td>
<td>-0.25</td>
<td>0.17</td>
<td>-0.28</td>
<td>-0.11</td>
<td>2.85</td>
<td>0.24</td>
</tr>
<tr>
<td>1996 - 2000</td>
<td>1097</td>
<td>0.06</td>
<td>-2.38</td>
<td>0.05</td>
<td>-2.67</td>
<td>-0.14</td>
<td>2.51</td>
<td>0.23</td>
</tr>
<tr>
<td>2001 - 2005</td>
<td>1097</td>
<td>-0.00</td>
<td>-1.18</td>
<td>-0.03</td>
<td>-1.70</td>
<td>0.12</td>
<td>0.84</td>
<td>0.45</td>
</tr>
<tr>
<td>2006 - 2010</td>
<td>1097</td>
<td>0.19</td>
<td>0.19</td>
<td>0.21</td>
<td>-0.09</td>
<td>-0.01</td>
<td>1.61</td>
<td>0.57</td>
</tr>
<tr>
<td>2011 - 2015</td>
<td>1097</td>
<td>0.12</td>
<td>0.26</td>
<td>0.14</td>
<td>1.26</td>
<td>-0.50</td>
<td>3.44</td>
<td>0.65</td>
</tr>
</tbody>
</table>

This table reports the number of funds available for analysis, mean of estimated $\alpha$ in percentage by both approaches, mean of $t$-ratios and statistics related to observed residual by time periods. First and second column shows time period and number of funds. Third and fourth column shows mean of LS estimate $\alpha$ in percentage and mean of $t$-ratios across funds. Similarly, in fourth and fifth columns given mean of R estimate $\alpha$ in percentage and mean of $t$-ratios across funds. Seventh and eighth column present mean of skewness and kurtosis of residuals from regression in Eq. (4.2) by LS method. The last column shows the percentage of funds which residuals rejected Jarque and Bera (1987) normality test.

Table 4.1 presents mean value of LS and R estimate of alphas for each period. Mean coefficient and $t$-ratios are very close for both approaches. To have a clear understanding, study looks at the difference of alpha estimates at tails of the distribution.

In order to look for the difference between R and LS estimate of $\alpha$, pair-wise analysis is conducted. Table 4.2 illustrates a clear dissimilarity of LS and R estimates of $\alpha$ by looking into mean difference of $\alpha$ and standard deviation. Obviously, LS $\alpha$ overestimated in top and underestimated in bottom of the distribution for most of the periods. In the first two periods difference is in other way, perhaps, due to a small number of funds. In addition, the difference
Table 4.2: Group wise comparison of LS and R estimate of alphas

Table presents group wise difference between LS and R estimates of $\alpha$ for 10 different periods. Second and third column give the difference between the mean of LS $\alpha$ and R $\alpha$ (ranked by LS $\alpha$) in top 10% and bottom 10% groups. Third and fourth columns show the difference between the mean of LS $\alpha$ standard error and R $\alpha$ standard error (ranked by LS $\alpha$) in top 10% and bottom 10% groups.

<table>
<thead>
<tr>
<th>Period</th>
<th>LS $\alpha$ - R $\alpha$ (%) bottom</th>
<th>LS $\alpha$ - R $\alpha$ (%) top</th>
<th>LS $\alpha$ s.e - R $\alpha$ s.e bottom</th>
<th>LS $\alpha$ s.e - R $\alpha$ s.e bottom</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966 - 1970</td>
<td>-0.3129</td>
<td>-0.0459</td>
<td>-0.0000</td>
<td>-0.0000</td>
</tr>
<tr>
<td>1971 - 1975</td>
<td>0.0028</td>
<td>-0.1257</td>
<td>-0.0003</td>
<td>0.0002</td>
</tr>
<tr>
<td>1976 - 1980</td>
<td>-0.6236</td>
<td>1.1042</td>
<td>0.0039</td>
<td>0.0052</td>
</tr>
<tr>
<td>1981 - 1985</td>
<td>-0.9031</td>
<td>0.5823</td>
<td>0.0076</td>
<td>0.0032</td>
</tr>
<tr>
<td>1986 - 1990</td>
<td>-0.3210</td>
<td>1.0281</td>
<td>0.0021</td>
<td>0.0054</td>
</tr>
<tr>
<td>1991 - 1995</td>
<td>-0.7384</td>
<td>1.0192</td>
<td>0.0097</td>
<td>0.0143</td>
</tr>
<tr>
<td>1996 - 2000</td>
<td>-0.2013</td>
<td>0.3846</td>
<td>0.0013</td>
<td>0.0033</td>
</tr>
<tr>
<td>2001 - 2005</td>
<td>0.0092</td>
<td>0.1010</td>
<td>0.0004</td>
<td>0.0002</td>
</tr>
<tr>
<td>2006 - 2010</td>
<td>-0.0141</td>
<td>-0.0576</td>
<td>0.0005</td>
<td>0.0008</td>
</tr>
<tr>
<td>2011 - 2015</td>
<td>-0.1898</td>
<td>0.2039</td>
<td>0.0024</td>
<td>0.0044</td>
</tr>
</tbody>
</table>

of mean standard errors are presented in the third and fourth columns. Results show that LS $\alpha$ has higher deviation than R estimate of $\alpha$ for majority of periods.

To sum up, R method is better at producing more precise estimate than LS counterpart. This behavior of LS estimates is found in other studies based on mutual and hedge fund data. Kosowski et al. (2006) found that LS overestimates $\alpha$ than Bayesian estimate and underestimates for bottom funds. Thus, for further analysis is used R estimate of $\alpha$ and residuals.

4.4.2 Bootstrapping and manager skill

R estimate of $\alpha$ is further investigated by bootstrap method and details of this approach are presented in section 4.3.2. Based on null hypothesis Eq. (4.4) should produce $\alpha$ equal to zero. If significantly large number of non-zero $\alpha$ are estimated by Eq. (4.4) I conclude that fund’s $\alpha$ estimated by Eq. (4.2) was due to a luck (or bad luck in case of negative $\alpha$) but not manager’s skill.

Next, all estimated $\alpha$ separated into a group where $\alpha$ is proved to be a significant based on the standard and bootstrap $p$-values. Another group includes $\alpha$ with significant $p$-value from standard test but not based on bootstrap. Thus, I assume that second group $\alpha$ are based on only luck.

Table 4.3 presents estimate of $\alpha$ together with standard and bootstrap $p$-values for funds in the top and bottom of $\alpha$ distribution. Different periods are given in rows and funds by rank are given in columns. Focusing on the first period only, all 9 funds have insignificant $\alpha$ based on both $p$-values, but the last fund. It has a significant $\alpha$ based on standard $p$-value but bootstrap method indicates that this is due to a luck ($p$-value is smaller than 0.05). Similar behavior is observed for the second fund from the bottom of distribution in the next period. Moving to the third period, second fund in the top has a significant $\alpha$ based on both tests and considered as a manager skill. This process is continued to extract funds with $\alpha$ based on skill for the rest of periods.
Table 4.3: R estimate of alpha for funds in the top and the bottom of distribution by periods.

Table presents R estimate of alpha, standard and bootstrap p-value for each period. Periods are given in rows and in columns is given 5 funds in the top and the bottom of alpha distribution.

<table>
<thead>
<tr>
<th>Period</th>
<th>Statistic</th>
<th>Top1</th>
<th>Top2</th>
<th>Top3</th>
<th>Top4</th>
<th>Top5</th>
<th>Bottom5</th>
<th>Bottom4</th>
<th>Bottom3</th>
<th>Bottom2</th>
<th>Bottom1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966-1970</td>
<td>R Alpha</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.00</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>Standard pv</td>
<td>0.38</td>
<td>0.50</td>
<td>0.56</td>
<td>0.58</td>
<td>0.59</td>
<td>0.58</td>
<td>0.40</td>
<td>0.32</td>
<td>0.30</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>Bootstrap pv</td>
<td>0.11</td>
<td>0.34</td>
<td>0.47</td>
<td>0.49</td>
<td>0.55</td>
<td>0.71</td>
<td>0.90</td>
<td>0.73</td>
<td>0.77</td>
<td>1.00</td>
</tr>
<tr>
<td>1971-1975</td>
<td>R Alpha</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>Standard pv</td>
<td>0.17</td>
<td>0.53</td>
<td>0.32</td>
<td>0.61</td>
<td>0.63</td>
<td>0.43</td>
<td>0.44</td>
<td>0.44</td>
<td>0.08</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>Bootstrap pv</td>
<td>0.12</td>
<td>0.11</td>
<td>0.07</td>
<td>0.25</td>
<td>0.26</td>
<td>0.82</td>
<td>0.78</td>
<td>0.79</td>
<td>0.60</td>
<td>0.75</td>
</tr>
<tr>
<td>1976-1980</td>
<td>R Alpha</td>
<td>0.06</td>
<td>0.02</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>Standard pv</td>
<td>0.18</td>
<td>0.03</td>
<td>0.33</td>
<td>0.37</td>
<td>0.33</td>
<td>0.01</td>
<td>0.06</td>
<td>0.12</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Bootstrap pv</td>
<td>0.02</td>
<td>0.08</td>
<td>0.08</td>
<td>0.12</td>
<td>0.23</td>
<td>1.00</td>
<td>0.91</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>1981-1985</td>
<td>R Alpha</td>
<td>0.06</td>
<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.05</td>
</tr>
<tr>
<td></td>
<td>Standard pv</td>
<td>0.01</td>
<td>0.00</td>
<td>0.01</td>
<td>0.27</td>
<td>0.36</td>
<td>0.07</td>
<td>0.03</td>
<td>0.02</td>
<td>0.10</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Bootstrap pv</td>
<td>0.01</td>
<td>0.02</td>
<td>0.00</td>
<td>0.14</td>
<td>0.27</td>
<td>0.99</td>
<td>0.99</td>
<td>0.88</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>1986-1990</td>
<td>R Alpha</td>
<td>0.05</td>
<td>0.04</td>
<td>0.01</td>
<td>0.20</td>
<td>0.17</td>
<td>0.00</td>
<td>0.10</td>
<td>0.01</td>
<td>0.08</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Standard pv</td>
<td>0.05</td>
<td>0.01</td>
<td>0.01</td>
<td>0.34</td>
<td>0.08</td>
<td>1.00</td>
<td>0.97</td>
<td>0.99</td>
<td>0.97</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Bootstrap pv</td>
<td>0.13</td>
<td>0.02</td>
<td>0.04</td>
<td>0.08</td>
<td>0.09</td>
<td>0.94</td>
<td>0.93</td>
<td>1.00</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>1991-1995</td>
<td>R Alpha</td>
<td>0.03</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>-0.01</td>
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<td>-0.02</td>
<td>-0.02</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>Standard pv</td>
<td>0.07</td>
<td>0.02</td>
<td>0.02</td>
<td>0.14</td>
<td>0.02</td>
<td>0.17</td>
<td>0.41</td>
<td>0.14</td>
<td>0.00</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>Bootstrap pv</td>
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<td>0.00</td>
<td>0.06</td>
<td>0.10</td>
<td>0.00</td>
<td>0.94</td>
<td>0.93</td>
<td>1.00</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>1996-2000</td>
<td>R Alpha</td>
<td>0.03</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>Standard pv</td>
<td>0.04</td>
<td>0.06</td>
<td>0.03</td>
<td>0.07</td>
<td>0.06</td>
<td>0.03</td>
<td>0.33</td>
<td>0.07</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Bootstrap pv</td>
<td>0.01</td>
<td>0.00</td>
<td>0.03</td>
<td>0.14</td>
<td>0.06</td>
<td>1.00</td>
<td>0.77</td>
<td>0.99</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>2001-2005</td>
<td>R Alpha</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>Standard pv</td>
<td>0.16</td>
<td>0.04</td>
<td>0.15</td>
<td>0.10</td>
<td>0.01</td>
<td>0.05</td>
<td>0.01</td>
<td>0.05</td>
<td>0.04</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>Bootstrap pv</td>
<td>0.02</td>
<td>0.10</td>
<td>0.06</td>
<td>0.15</td>
<td>0.00</td>
<td>0.97</td>
<td>1.00</td>
<td>0.99</td>
<td>1.00</td>
<td>0.95</td>
</tr>
<tr>
<td>2006-2010</td>
<td>R Alpha</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>Standard pv</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
<td>0.03</td>
<td>0.03</td>
<td>0.19</td>
<td>0.19</td>
<td>0.32</td>
<td>0.31</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>Bootstrap pv</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.90</td>
<td>0.88</td>
<td>0.82</td>
<td>0.92</td>
<td>0.99</td>
</tr>
<tr>
<td>2011-2015</td>
<td>R Alpha</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>-0.03</td>
<td>-0.03</td>
<td>-0.03</td>
<td>-0.03</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>Standard pv</td>
<td>0.11</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.09</td>
<td>0.01</td>
<td>0.08</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>Bootstrap pv</td>
<td>0.06</td>
<td>0.00</td>
<td>0.05</td>
<td>0.01</td>
<td>0.00</td>
<td>0.38</td>
<td>1.00</td>
<td>0.98</td>
<td>1.00</td>
<td>0.99</td>
</tr>
</tbody>
</table>
4.4.3 Performance persistence

In order to test the persistence of skill, Fama and MacBeth (1973) cross sectional regression is employed by regressing compounded future return $r_{i,t+1,\tau}$ on observed alpha which are estimated during previous 10 consecutive 5-year periods. Time periods for compounding returns and for estimation of alpha do not overlap and it is a predictive regression.

$$r_{i,t+1,\tau} = \beta_0 + \beta_1 \hat{\alpha}_{i,t} + \epsilon_{i,t+1,\tau} \quad (4.8)$$

Here, $r_{i,t+1,\tau}$ is compounded return for $i$ fund for $\tau$ horizons ($\tau = 1,3,6,12,24,36,48$ and $60$ months) starting from $t + 1$ month. $\hat{\alpha}_{i,t}$ is estimated based on the previous 60 months period and $t$ denotes the last month of the corresponding 5-year period (Only 10 periods). This method is commonly used to assess the fund manager performance (Christopherson et al. (1998)).

For Eq. (4.8) Weighted Least Squares is applied and inverse of standard deviation of residuals from Eq. (4.4) are used as weights. This approach is usual for cross sectional regression and preferred over LS (Roll and Ross (1994), Kandel and Stambaugh (1995), Christopherson et al. (1998)). Standard deviations of residuals are obtained when the return for the fund is regressed on 5 factors in the first step.

Next, observed skewness ($\hat{\gamma}, \hat{\theta}$) and location ($\hat{\xi}, \hat{\mu}$) parameters of skew-normal distribution and GLAM are employed as explanatory variables in Eq. (4.9).

$$r_{i,t+1,\tau} = \beta_0^* + \beta_1^* \hat{\gamma}_{i,t} + \beta_2^* \hat{\xi}_{i,t} + \epsilon_{i,t+1,\tau}$$

$$r_{i,t+1,\tau} = \beta_0^* + \beta_1^* \hat{\theta}_{i,t} + \beta_2^* \hat{\mu}_{i,t} + \epsilon_{i,t+1,\tau} \quad (4.9)$$

Here, $\hat{\gamma}_{i,t}, \hat{\xi}_{i,t}, \hat{\theta}_{i,t}$ and $\hat{\mu}_{i,t}$ are obtained from regression residuals in Eq. (4.4) similarly to Eq. (4.8). Thus, explanatory variables obtained based on the previous 60 months period and $t$ denotes the last month of the corresponding 5-year period.

Due to the low $R^2$ of cross sectional regressions only $t$ statistics of estimated $\beta_1$ ($\beta_1^*$) and $\beta_2$ ($\beta_2^*$) from 10 regressions are presented (Fama and Macbeth (1973), Christopherson et al. (1998)).

Results are given in Table (4.4). In Panel A are given $\beta_1$ and $\beta_2$ for $\alpha$ from full sample, skill and luck cases. It is found that manager skill is persistent for long horizon but in short periods. Clearly, $t$ statistic is significant to explain 4 and 5 year compounded return but fails to explain compounded return for previous periods.

Similarly, Panel B presents the results of estimation based on Eq (4.9). Location is persistent in short and long term horizons, in comparison with asymmetry. It clearly supports previous results in Panel A. Thus, manager performance measured as a mean value of residuals or location parameter of $\eta$ yield similar results. However, it could be misleading if not taken into account the bootstrap results. By bootstrapping residuals we separated $\alpha$ based on skill or luck. From results in Panel A, it is clear that manager skill is not persistent in short horizon but in long horizon it seems significant to affect the return of the fund. This result is in line with previous studies which found the persistence of manager skill in longer horizons for winning funds. Similarly, Panel B also illustrates the results for both types of managers. For skilled managers case, asymmetry and location parameters are found to be significant for longer horizons than shorter cases.

A possible explanation of the significance of asymmetry indicator is the manager’s style of investing or fund’s strategy. Certainly, unit trusts have different investment approaches and managers which could have affected on residual distribution’s location ($\alpha$ in case of OLS) or skewness.
Table 4.4: t-statistics of estimated coefficients from cross sectional regression

Table reports t-ratios of estimated coefficients from Eq.(4.8) in Panel A and Eq.(4.9) in Panel B. Rows in Panel A shows the types of $\alpha$ used for cross sectional regression in Eq. (4.8). First, all significant $\alpha$, which are obtained from the time series regression in Eq. (4.2), are employed as independent variable in Eq. (4.8). Following the estimation, to obtain t-ratios, the mean of estimated coefficients for different time periods is divided by the standard deviation over square root of number of periods. Thus, columns present types of compounded returns that is used for regression, respectively. Second row is for $\alpha$ that found to be significant based on times series regression $p$-value and bootstrapped $p$-value. Third row is for $\alpha$ that is found to be significant based on times series regression $p$-value but not based on bootstrapped $p$-value. Panel B is for results from Eq. (4.9). Similarly, columns indicate the compounded return horizons used as a dependent variable and rows report types of location and asymmetry used as independent variables in Eq. (4.9). First and second row show t-ratios for skewness and location parameters when the respective $\alpha$ is found to be a significant based on standard of $p$ value. Third and fourth row show t-ratios for skewness and location parameters when the respective $\alpha$ is found to be significant based on time series regression of $p$ value and bootstrapped $p$-value. Fifth and sixth rows for the case of skewness and location when the respective $\alpha$ is found to be based on luck. Similarly, last rows are for the case of $\theta$ and $\mu$.

### Panel A. t-ratios of coefficients from Eq. (4.8)

<table>
<thead>
<tr>
<th></th>
<th>1 month</th>
<th>3 month</th>
<th>6 month</th>
<th>1 year</th>
<th>2 year</th>
<th>3 year</th>
<th>4 year</th>
<th>5 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full sample $\alpha$ ($\beta_1^*$)</td>
<td>1.77</td>
<td>2.24</td>
<td>2.94</td>
<td>3.54</td>
<td>3.58</td>
<td>2.87</td>
<td>3.55</td>
<td>3.68</td>
</tr>
<tr>
<td>Skill alpha ($\beta_1^*$)</td>
<td>0.59</td>
<td>0.93</td>
<td>0.67</td>
<td>0.95</td>
<td>1.13</td>
<td>0.85</td>
<td>2.65</td>
<td>3.67</td>
</tr>
<tr>
<td>Luck alpha ($\beta_1^*$)</td>
<td>1.61</td>
<td>0.28</td>
<td>1.22</td>
<td>2.12</td>
<td>2.31</td>
<td>1.53</td>
<td>2.25</td>
<td>1.59</td>
</tr>
</tbody>
</table>

### Panel B. t-ratios of coefficients from Eq. (4.9)

<table>
<thead>
<tr>
<th></th>
<th>1 month</th>
<th>3 month</th>
<th>6 month</th>
<th>1 year</th>
<th>2 year</th>
<th>3 year</th>
<th>4 year</th>
<th>5 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skewness ($\beta_1$)</td>
<td>1.26</td>
<td>0.42</td>
<td>0.65</td>
<td>0.85</td>
<td>-0.43</td>
<td>-0.03</td>
<td>0.59</td>
<td>-0.01</td>
</tr>
<tr>
<td>Location ($\beta_2$)</td>
<td>1.74</td>
<td>1.18</td>
<td>1.85</td>
<td>4.21</td>
<td>4.61</td>
<td>5.58</td>
<td>3.02</td>
<td>2.87</td>
</tr>
<tr>
<td>Skill skewness ($\beta_1$)</td>
<td>0.42</td>
<td>-1.50</td>
<td>-0.26</td>
<td>-1.21</td>
<td>-1.17</td>
<td>-0.91</td>
<td>-1.86</td>
<td>-2.47</td>
</tr>
<tr>
<td>Skill location ($\beta_2$)</td>
<td>0.78</td>
<td>0.73</td>
<td>0.86</td>
<td>0.59</td>
<td>0.86</td>
<td>0.82</td>
<td>1.81</td>
<td>2.24</td>
</tr>
<tr>
<td>Luck skewness ($\beta_1$)</td>
<td>0.99</td>
<td>1.11</td>
<td>1.03</td>
<td>1.05</td>
<td>-0.55</td>
<td>-0.41</td>
<td>0.88</td>
<td>0.41</td>
</tr>
<tr>
<td>Luck location ($\beta_2$)</td>
<td>2.17</td>
<td>0.37</td>
<td>1.21</td>
<td>1.92</td>
<td>2.51</td>
<td>2.18</td>
<td>2.19</td>
<td>1.89</td>
</tr>
<tr>
<td>Skill theta ($\beta_1^*$)</td>
<td>-0.216</td>
<td>0.141</td>
<td>-0.227</td>
<td>0.783</td>
<td>0.757</td>
<td>0.969</td>
<td>2.306</td>
<td>2.399</td>
</tr>
<tr>
<td>Skill mu ($\beta_2^*$)</td>
<td>0.772</td>
<td>0.926</td>
<td>1.138</td>
<td>0.924</td>
<td>1.103</td>
<td>0.928</td>
<td>1.966</td>
<td>2.757</td>
</tr>
<tr>
<td>Luck theta ($\beta_1^*$)</td>
<td>1.547</td>
<td>-0.615</td>
<td>-0.464</td>
<td>0.051</td>
<td>0.784</td>
<td>0.854</td>
<td>0.925</td>
<td>1.007</td>
</tr>
<tr>
<td>Luck mu ($\beta_2^*$)</td>
<td>1.484</td>
<td>0.072</td>
<td>1.001</td>
<td>1.931</td>
<td>2.672</td>
<td>1.548</td>
<td>2.188</td>
<td>1.660</td>
</tr>
</tbody>
</table>
4.4.4 Alpha decomposition

Following the analysis of persistence of the manager ability, this section presents the decomposition of estimated $\hat{\alpha}$ into two sources, location and asymmetry as shown in Eq.(4.10).

$$\hat{\alpha}_{i,t} = \kappa_0 + \kappa_1 \hat{\theta}_{i,t} + \kappa_2 \hat{\mu}_{i,t} + \epsilon_{i,t}$$  \hspace{1cm} (4.10)

Here, $i = 1...1097$ funds and $t = 1...10$ periods.

Eq. (4.10) repeatedly applied into unit trust data for each period, respectively. Initially, funds with a statistically significant $\alpha$ are focused and the result is presented in Panel A of Table 4.5. Moreover, results table divided based on the investment strategy of unit trust. Panel B presents cross sectional results for unit trust that found to have a lucky manager and Panel C presents results for unit trusts with skilled manager which are found by the application of bootstrapping in the previous sections.

Table 4.5: Alpha decomposition results

Table reports cross sectional regression results for 3 different cases from Eq. (4.10). Panel A is based on full sample of unit trusts with significant $\alpha$. Rows show the period of cross-sectional regressions and columns show three different types of unit trusts with sub-columns representing regression coefficients ($\kappa_0, \kappa_1, \kappa_2$). Panel B reports results for unit trusts with skilled managers. Panel C reports results for unit trusts with unskilled managers. All estimates are significant.

<table>
<thead>
<tr>
<th>Panel A. Unit trusts with significant $\alpha$</th>
<th>Bond</th>
<th>Equity</th>
<th>Money Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Periods</td>
<td>$\kappa_0$</td>
<td>$\kappa_1$</td>
<td>$\kappa_2$</td>
</tr>
<tr>
<td>1976-1980</td>
<td>-0.0330</td>
<td>0.0330</td>
<td>0.9800</td>
</tr>
<tr>
<td>1981-1985</td>
<td>-0.0150</td>
<td>0.0150</td>
<td>0.9910</td>
</tr>
<tr>
<td>1986-1990</td>
<td>-0.0140</td>
<td>0.0140</td>
<td>1.0160</td>
</tr>
<tr>
<td>1991-1995</td>
<td>-0.0080</td>
<td>0.0080</td>
<td>0.9280</td>
</tr>
<tr>
<td>1995-2000</td>
<td>-0.0110</td>
<td>0.0110</td>
<td>1.1060</td>
</tr>
<tr>
<td>2001-2005</td>
<td>-0.0210</td>
<td>0.0210</td>
<td>1.0030</td>
</tr>
<tr>
<td>2006-2010</td>
<td>-0.0100</td>
<td>0.0090</td>
<td>1.0410</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B. Unit trusts with skillful manager case</th>
<th>Bond</th>
<th>Equity</th>
<th>Money Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Periods</td>
<td>$\kappa_0$</td>
<td>$\kappa_1$</td>
<td>$\kappa_2$</td>
</tr>
<tr>
<td>1976-1980</td>
<td>-0.0030</td>
<td>0.0030</td>
<td>1.0020</td>
</tr>
<tr>
<td>1981-1985</td>
<td>-0.0490</td>
<td>0.0470</td>
<td>1.1250</td>
</tr>
<tr>
<td>1996-1995</td>
<td>-0.0370</td>
<td>0.0360</td>
<td>0.9880</td>
</tr>
<tr>
<td>1996-2000</td>
<td>-0.0130</td>
<td>0.0120</td>
<td>1.1030</td>
</tr>
<tr>
<td>2001-2005</td>
<td>-0.0090</td>
<td>0.0090</td>
<td>1.0420</td>
</tr>
<tr>
<td>2006-2010</td>
<td>-0.0100</td>
<td>0.0090</td>
<td>1.0720</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C. Unit trusts with unskilled manager case</th>
<th>Bond</th>
<th>Equity</th>
<th>Money Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Periods</td>
<td>$\kappa_0$</td>
<td>$\kappa_1$</td>
<td>$\kappa_2$</td>
</tr>
<tr>
<td>1976-1980</td>
<td>-0.0340</td>
<td>0.0340</td>
<td>0.9440</td>
</tr>
<tr>
<td>1981-1985</td>
<td>-0.0030</td>
<td>0.0030</td>
<td>1.0000</td>
</tr>
<tr>
<td>1986-1990</td>
<td>-0.0030</td>
<td>0.0030</td>
<td>1.0000</td>
</tr>
<tr>
<td>1991-1995</td>
<td>-0.0030</td>
<td>0.0030</td>
<td>1.0000</td>
</tr>
<tr>
<td>1995-2000</td>
<td>-0.0030</td>
<td>0.0030</td>
<td>1.0000</td>
</tr>
<tr>
<td>2001-2005</td>
<td>-0.0290</td>
<td>0.0280</td>
<td>0.8580</td>
</tr>
<tr>
<td>2006-2010</td>
<td>-0.0010</td>
<td>0.0010</td>
<td>0.9820</td>
</tr>
</tbody>
</table>

Results clearly show the importance of asymmetry to explain $\alpha$ by various categories and by types of managers of funds. First interesting result is that $\alpha$ of money market trusts relatively not related to asymmetry no matter on what type of manager controls them. Asymmetry parameters ($\kappa_0$ and $\kappa_1$) are very close to zero (All results in table are found to be highly significant) and only $\kappa_2$ has a bigger coefficient, relatively.
Moving into bond and equity trusts for skillful and unskilful managers’ columns, we could see that results are strikingly different. In Panel B and Panel C, $\kappa_0$ and $\kappa_1$ for bond trusts are always smaller in magnitude than equity trusts. So, clearly equity trusts’ $\alpha$ are explained more by asymmetry than bond and money market trusts.

One possible explanation for symmetric distribution of observed residuals is the type of assets under the management of unit trusts. Money market funds invest into highly liquid assets such as short term bonds and bond trusts hold long-term bonds. In contrast, traditional equity trusts invest larger part of the fund (60%) into equities and smaller part (40%) into bond market. Hence, uncertainty and liquidity in bond, as well as equity markets contribute into the shape of $\eta_i$ and eventually to $\alpha$ generation.

To further investigate the difference Fig. 4.1 illustrate average $\theta$ across types of unit trusts along time periods. Obviously, “Money market” funds are quite distinct and have almost symmetrical distribution. This is in line with our results in Table (4.5).

Figure 4.1

![Average theta across unit trusts (skill)](image)

Fig. 4.2 illustrate box plot of $\theta$ for different funds, respectively. In case of funds with skilled manager 4.2, bond and equity funds’ asymmetry estimate has skewed and deformed distribution as can be seen from unequal median and mean values. This is an indication of various portfolio strategies being applied by bond and equity funds’ managers. Money and mixed market funds have quite narrow box plots, especially money market funds.

4.5 Discussion

Result clarifies that manager performance should not be evaluated based on only $\alpha$ (usually mean of observed residuals in case of OLS) but location and asymmetry ($\eta_i$ distribution) should

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be considered. Focusing on $\alpha$ alone does not fully reveal the fund performance and lacks of clarity. As this study found, skewness is persistent in long horizons together with location.

Long horizon persistent of manager performance is well supported by investment objective of funds. Normally, open or close end funds target to have a portfolio return 5% higher than interbank offered rate over five year period or 60 monthly data. Our findings support this view in a sense that manager performance becomes clear by looking at longer period of time. This is also supported by other studies such as Lynch and Musto (2003) and Berk and Green (2004) where they found the persistence of winning funds in longer periods and losing funds in shorter horizons.

Our result documents the importance of asymmetry which is consistent together with location parameter to explain compounded return for 5 year period. Study employed 5 factor model to capture all the possible factors in the market that could affect fund performance. Still, asymmetry is found to be persistent for long horizon and undoubtedly it should be included into consideration to evaluate the manager performance.

Moreover, $\theta$ is found to be different based on the type of funds. Equity funds exhibit stronger asymmetry than bond or mixed assets funds. Thus, diversity in funds’ portfolio allocation is well captured by the $\eta_i$ deformation. Possible explanation could be that asymmetry reflects the financial health of the stocks under the management and overall uncertainty in stock markets. In comparison, fixed income assets are considered to be risk free and secure. Hence, funds with high investments in bond markets exhibit relatively symmetric $\eta_i$ distribution.
4.6 Conclusion

This study looked into manager performance from a different point of view in comparison with previous studies. To our knowledge, this is the first study that applies robust non-parametric methods to measure manager performance and focuses on error term distribution to extract additional information.

Study showed that manager skill (if exists) is not only excess return over the benchmark portfolios (measured as a mean of observed residuals in case of LS) but asymmetry of $\eta_i$ as well. Study also found that asymmetry is related to fund’s management style and objective. In addition, study found that when excess return is observed by luck (based on bootstrap method) asymmetry is not persistent and insignificant. Only location of $\eta_i$ is significant for short horizons. Thus, I conclude that excess return due to a luck do not have a significant asymmetry in $\eta_i$ that persists for long periods. Only in case of skillful manager, asymmetry is persistent and it captures the uncertainty and risk related to assets under the management of the fund.

Moreover, as we found asymmetry relates to mutual fund’s strategy and investment style. When $\alpha$ is decomposed into location and asymmetry parts, equity focused funds are found to have higher asymmetry sensitivity than bond (highly secure) market focused funds. Thus, this shows that uncertainty related to assets under the management is well captured by $\theta$. 
4.7 Reference


Chapter 5

Conclusion

Thesis consisted of three main parts and each chapter focused on specific research topics related to equities from overall stock market performance to individual stock itself.

First, we investigated the importance of financial network on stock market performance. As mentioned earlier, due to the interconnectedness of economies, the high priority of domestic investment decreased and international portfolio investments increased. Financial networks are constructed based on bilateral equity, long-term and short-term debt investment flows among economies. By applying spatial model we found that markets are affected by neighboring economies as well as by centrality of the economy itself in a network of debt flows.

Secondly, we focused on daily stock price data. By applying two distinct approaches to estimate CAPM $\beta$ we found that on top (bottom) of $\beta$ distribution, which are pair-wise ranked based on LS estimate, LS consistently overestimates (underestimates) than R estimate of $\beta$. As a result, we obtained quite different residuals from both approaches.

Furthermore, we found that residuals are, indeed, skewed and GLAM model estimated a significant deviation of residuals from the state of being symmetrical across all stocks, especially during the financially stressful periods. Furthermore, we decomposed $\alpha$ into location and asymmetry part. By looking into the cross sectional quarterly regressions, we found that $\theta$ has a statistically significant relation to $\alpha$. Especially during the financially unstable periods (2007-2009) the sensitivity of $\alpha$ to $\theta$ increased than peaceful quarters. Possible explanation is that estimated $\theta$ is an indicator of uncertainty existant in the market that increases during crisis periods.

Lastly, thesis focused on performance evaluational of mutual funds’ managers. Our study found that portfolios focusing on equity itself experience significant asymmetry but not funds which are focused on debt market. Certainly, portfolio is the aggregate level representation of assets under the management. Thus, mutual funds which invest mainly into equities, manage portfolios of the stocks and portfolio performance summarizes the performance of all the underlying assets (stocks). Knowing that individual stocks experience asymmetric residuals following the simple linear regression analysis in second study, it is clear that equity based portfolios as well summarize this phenomenon.

Research found that $\theta$ varies depending on the stocks’ industry and time period. $\theta$ also increases during the financially stressed periods and decreases during the calm market. Thus, I assume $\theta$ is a stock specific uncertainty associated indicator. Thus, median $\theta$ positively associated with volatility index also known as uncertainty index and negatively with quarterly market index rate of return. Moreover, this is supported by our first study as well. Economy attracts
debt inflows (centrality of the economy increased) when economy related uncertainty is lower and this centrality is positively related to market performance. Hence, uncertainty and market performance have a negative association which is also supported by negative relation of median $\theta$ and quarterly index rate of return in our second study.

To sum up, due to the technological advancements financial markets are well interconnected and centrality of the economy in debt based financial networks became significant to explain market performance. Meaning that economies with low uncertainty (high inflow in debt markets) have higher stock market performance (due to a spillover effect). In case of individual stocks, they experience different levels of uncertainty as captured by $\theta$ parameter from GLAM model which is fitted into observed residuals. More importantly, $\theta$ shows how $\alpha$ is being generated during the diverse set of periods. In addition, unit trusts with high investment into equities only also experience higher levels of $\theta$ than bond market focused mutual funds meaning that $\theta$ is, indeed, uncertainty related indicator since bond markets are considered as highly secure markets.