Crack shape reconstruction in ferromagnetic materials using a novel fast numerical simulation method

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Crack Shape Reconstruction in Ferromagnetic Materials Using a Novel Fast Numerical Simulation Method

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Abstract—This paper describes research on crack shape reconstruction in ferromagnetic materials using a novel fast numerical simulation method. The fast numerical method developed here, which can treat ferromagnetic materials, is an extension of a precomputed database approach based on the reduced magnetic vector potential method. It provides a fast forward simulator that is about 80 times faster than a conventional one, even in the case of ferromagnetic materials, without losing accuracy. The fast simulator is applied to the inverse problem of eddy current testing (ECT), crack shape reconstruction, and results of some electric discharge machining (EDM) cracks on a ferromagnetic plate are shown.

Index Terms—Eddy current testing, ferromagnetic material, inverse problem, precomputed database approach.

I. INTRODUCTION

Eddy current testing (ECT) is a nondestructive testing method of metal materials. Numerical analysis methods are applied to predict the ECT signals, to aid in the design of ECT probes, and to reconstruct crack shapes from their ECT signals. Research into forward and inverse problems of ECT has been carried out over the last few years, and significant progress has been made.

For nonferromagnetic materials, the high accuracy of some numerical simulation techniques has been demonstrated and several fast computational methods are presented. Chen and Miya [1] have presented a fast analysis method using a small part of the inverse matrix of the coefficient matrix and FEM-BEM. Huang and Takagi [2] have also presented a precomputed database approach based on the edge-based finite element and reduced magnetic vector potential ($A_r$) method [3]. The ECT signals can be achieved in a short amount of CPU time with high precision, which allows the inverse problems to be solved in a practical amount of time. At this time, inverse problems from ECT signals of a single electric discharge machining (EDM) crack of nonferromagnetic materials have been solved; though some problems associated with natural cracks and multiple cracks remain. However, few papers have solved the ECT inverse problem in ferromagnetic materials.

The numerical analysis method based on the edge-based finite element method (FEM) and $A_r$ method [3] can be applied to ferromagnetic material problems and is verified using an axisymmetric FEM program, as well as benchmark problems [4] proposed by the Japan Society of Applied Electromagnetics and Mechanics (JSAEM). However, the numerical analysis of ferromagnetic materials remains a difficult and time-consuming job. The fast analysis method [2] based on the $A_r$ method has been extended to solve the forward and inverse problems when a ferromagnetic noise source exists [5]. However, this research [1], [2], [5] is restricted to only cracks that exist in nonferromagnetic materials.

Recently, a novel fast method [6] has been proposed that extends the precomputed database approach based on the $A_r$ method. This is different from the improvement suggested in [5], as not only the governing equations but also the expressions of ECT signals are newly developed. This method can be applied to the ECT of ferromagnetic materials, and the computing time is significantly reduced. In this paper, the novel fast simulator is applied to the ECT inverse problem of ferromagnetic materials. For the first time, the crack sizing in ferromagnetic materials is considered and excellent reconstruction results are shown.

II. FAST SIMULATION METHOD OF FERROMAGNETIC MATERIALS

A. Governing Equations of Ferromagnetic Materials

Following the same procedures as described in [1] and [2], systems with and without cracks are considered, as shown in Fig. 1, where the source in the figure is the exciting current in the coil, and the conductor is the sample to be tested.

Defining a new magnetic vector potential $A'$ as the potential difference arising from the presence of cracks as $A' = A - A^u$ and subtracting the governing equations of the cracked and crack-free cases, the governing equations can be obtained, as follows:

\[
\nabla \times \left( \frac{1}{\mu} \nabla \times A' \right) + j \omega \sigma A = 0 \quad \text{in the conductor} \quad (1)
\]

\[
\nabla \times \left( \frac{1}{\mu} \nabla \times A' \right) + j \omega \sigma A' = j \omega \sigma (A^u + A') + \nabla \times \left( \frac{1}{\mu_0} - \frac{1}{\mu} \right) \nabla \times (A^u + A') \quad \text{in the crack} \quad (2)
\]

\[
\nabla \times \frac{1}{\mu_0} \nabla \times A' = 0 \quad \text{in air} \quad (3)
\]
where
\[ A \] magnetic vector potential when cracks exist;
\[ A^u \] magnetic vector potential for the crack-free case;
\[ \sigma \] conductivity of the crack-free conductor;
\[ \mu_0 \] permeability of air;
\[ \mu \] permeability of the conductor.

B. Suspect Region and Algebraic Equations

Based on the reciprocity theorem, which will be discussed later in this paper, only the results inside the crack region are needed to compute the ECT signals. When considering the inverse problem, the crack region is unknown, so a slightly larger region (the “suspect region”) is considered. Following the Galerkin method, the weak formulations of (1)–(3) can be expressed as follows:

\[
\frac{1}{\mu} \int_{\Omega_c} \nabla \times A^f \cdot \nabla \times A^f \, dV + j \omega \sigma \int_{\Omega_c} A^f \cdot A^f \, dV = 0
\]

\[
\frac{1}{\mu} \int_{\Omega_f} \nabla \times A^f \cdot \nabla \times A^f \, dV + j \omega \sigma \int_{\Omega_f} A^f \cdot A^f \, dV = 0
\]

\[
\frac{1}{\mu_0} \int_{\Omega_a} \nabla \times A^f \cdot \nabla \times A^f \, dV = 0
\]

where
\[ \Omega \] whole region;
\[ \Omega_a \] air region;
\[ \Omega_c \] conductor region;
\[ \Omega_f \] crack region.

Comparing (1)–(3) with the governing equations of the crack-free case, one can find that the left-hand sides are similar. That is to say, using the Galerkin method, the coefficient matrices are the same. Considering the right-hand sides, only (2) is nonzero.

Instead of a crack, secondary sources are introduced in this method. The secondary source includes two parts when considering ferromagnetic materials: a secondary electric current source due to the change in conductivity, and a secondary magnetic current source due to the change in permeability. Following the same steps carried out in the conventional edge-based FEM in [3], the algebraic equation can be obtained, as follows:

\[
[K + j \omega L](A^f) = [K' + j \omega L'](A^f + A^u),
\]

To increase the computing speed, this fast simulator considers the suspect region rather than the whole of the model, including the conductor, the exciting coil, and the air. That is to say, we want to solve the equations that contain only the unknowns inside the suspect region. All of the unknowns are divided into two parts: unknowns inside the suspect region are denoted by subscript 1, and the others are denoted by subscript 2. Equation (7) can be rewritten using the subscripts as

\[
\begin{pmatrix}
P_{11} & P_{12} \\
P_{21} & P_{22}
\end{pmatrix}
\begin{pmatrix}
A^f_1 \\
A^u_2
\end{pmatrix} =
\begin{pmatrix}
Q_{11} & 0 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
A^f_1 + A^u_1 \\
A^f_2 + A^u_2
\end{pmatrix},
\]

Most elements in matrix \([Q]\) are zero, and only the elements related to the cracks are nonzero. By multiplying by the matrix \([R]\), which is the inverse matrix of \([P]\), into (8), we obtain

\[
\begin{pmatrix}
A^f_1 \\
A^u_2
\end{pmatrix} =
\begin{pmatrix}
R_{11} & R_{12} \\
R_{21} & R_{22}
\end{pmatrix}
\begin{pmatrix}
Q_{11} & 0 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
A^f_1 + A^u_1 \\
A^f_2 + A^u_2
\end{pmatrix}.
\]

Equation (9) can be divided into two independent equations. After the arrangement of the equations concerning the suspect region, equations with much smaller degrees of freedom are shown as

\[
\{ A^f_1 \} = [R_{11}][Q_{11}]\{ A^f_1 + A^u_1 \}
\]

which can also be expressed as

\[
[I - R_{11}Q_{11}]\{ A^f_1 + A^u_1 \} = \{ A^f_1 \}.
\]

In (11), \([Q_{11}]\) is the matrix related to the suspect region, and \([R_{11}]\) is a small part of the inverse matrix from a common stiffness matrix. Both \([R_{11}]\) and \([A^f_1]\) can be computed in advance because they are independent of the cracks, and are used as the precomputed database. Depending on the size of the suspect region, it may be time consuming to prepare the precomputed matrix. However, it can be used repeatedly for various kinds of cracks in the suspect region, unless the geometry and material properties of the test specimens change. The equations can be solved by the Gauss elimination method because of their small numbers of degrees of freedom.

C. Reciprocity Theorem in Ferromagnetic Materials

The reciprocity theorem was introduced into the analysis of magnetic fields by Harrington [7] in 1961. In the 1980s, Auld et al. [8] presented a method of computing impedance changes of ECT probes using reciprocity and the Born approximation. In order to apply the reciprocity theorem to ferromagnetic materials, we consider any two sets \((a \text{ and } b)\) of alternating current sources \(J^a_e, J^a_m\) and \(J^b_e, J^b_m\) of the same frequency, existing in the same linear medium, where the subscript \(m\) indicates the magnetic current source and the subscript \(e\) indicates the electric current source.

In ECT problems, \(J^e\) and \(J^m\) are the exciting electric and magnetic source current densities, respectively, and \(J^e_e, J^e_m\) and \(J^m_e, J^m_m\) are the secondary ones (in place of the cracks). From the reciprocity theorem, we have

\[
\int_{\Omega} (E^e \cdot J^e_e - H^e \cdot J^m_m)dV = \int_{\Omega} (E^m \cdot J^e_e - H^e \cdot J^m_m)dV.
\]
Denoting the excited source region by superscript $s$ and the crack region by superscript $f$, considering that $J^e_s$ exists only in $\Omega_s$ and $J^m_f$ is zero, we have

$$\int_{\Omega_s} E^f \cdot J^e_s dV = \int_{\Omega_f} E^s \cdot J^e_f dV - \int_{\Omega_f} H^s \cdot J^m_f dV. \quad (13)$$

The left-hand side of this equation is the change in energy due to the crack. Theoretically, we have

$$\int_{\Omega_3} E^f \cdot J^e_s dV = -I^2 \Delta Z. \quad (14)$$

where $\Delta Z$ is the impedance change due to the crack, the ECT signal of the absolute-type pancake coil that we are computing, and $I$ is the exciting current density.

In order to deduce the expression of the secondary electric and magnetic current source [6], [8], the generalized Maxwell equations including both electric and magnetic sources are considered. As a result, the expressions are given as follows:

$$J^f_e = -\sigma E = j\omega \sigma (A^f + A^u) \quad (15)$$

$$J^f_m = -j\omega (\mu - \mu_0) H = -j\omega \frac{(\mu - \mu_0)}{\mu_0} \nabla \times (A^f + A^u). \quad (16)$$

### III. INVERSE ANALYSIS METHOD

In the normal approach to solving inverse problems [2], [9], the least square error function between the estimated signals and the observed signals is minimized. The following evaluation function is used:

$$J(x) = \frac{1}{N_{\text{pos}}} \sum_{i=1}^{N_{\text{pos}}} \left[ S^i_{\text{comp}}(x) - S^i_{\text{obs}} \right]^2 \frac{1}{N_{\text{pos}}} \sum_{i=1}^{N_{\text{pos}}} \left[ S^i_{\text{obs}} \right]^2 \quad (17)$$

where $x$ is the vector characterizing the shape of the cracks, $S^i_{\text{comp}}(x)$ are the predicted signals related to the vector $x$, $S^i_{\text{obs}}$ are the observed signals, and $N_{\text{pos}}$ is the number of the observed points.

The initial shape here corresponds to the case when half of the depth of the suspect region is cracked. Crack shape reconstruction is performed by a combination of the fast forward ECT simulator and the steepest decent method. A flow chart of the inverse scheme is shown in Fig. 2.

The ECT signals used here are selected from the scanning line across the peak value along the direction of the crack. The parameter of the crack shape is the depth of one column in the suspect region. The evaluation function will decrease when the iteration number increases. The final reconstructions of the crack shape converge to the true shape when the stop criterion is satisfied. The stop criterion is $J \leq 0.001$ or $\Delta J \leq 0.0001$ where $\Delta J = |J_{i+1} - J_i|$ and $i$ is the iteration number.

### IV. RESULTS AND DISCUSSIONS

ECT applied to the inspection of a ferromagnetic metal plate is simulated by numerical methods. The size of the plate is $20 \times 20 \times 1.25 \text{ mm}^3$, and the size of the suspect region is $6 \times 0.2 \times 1.25 \text{ mm}^3$, where 0.2 mm is the width and 6 mm is the length. The relative permeability and conductivity of the conductor are 100 and $10^6 \text{ S/m}$, respectively. A pancake coil (140 turns) is used for the inspection, with inner diameter 1.2 mm, outer diameter 3.2 mm, and height 0.8 mm [4].

The results of the conventional $A_p$ method [3] are compared with those of the fast simulation method. Excellent agreements are shown in Fig. 3. Using an exciting frequency of 1.5 kHz, the pancake probe moves along the direction of the crack from -5 to 5 mm in steps of 1 mm. Comparisons of the computational costs are shown in Table I. The fast forward simulator presented here is about 80 times faster than a conventional one with the same computational accuracy.

By applying this fast forward analysis method, crack reconstruction is performed by a parameter estimation method. Using the steepest decent method, the number of iterations is large but the total amount of computation time will be short because of the fast simulator. Some fixed-width (0.2-mm) EDM cracks are considered within the suspect region. ECT signals from the forward numerical analysis are used. The reconstructed crack shapes are shown in Fig. 4. The FD and RD in the captions indicate cracks on the same side (facing defect) and on the opposite side (reverse defect) of the probe, respectively. Both the depth and length of the cracks are reconstructed satisfactorily compared with the true shapes, shown by the squares, within an error of 0.5% in one or two minutes.

**TABLE I**

<table>
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<td>Present Method</td>
<td>VT alpha667</td>
<td>84*</td>
<td>30*</td>
<td>8 s</td>
</tr>
<tr>
<td>Conventional Method</td>
<td>VT alpha667</td>
<td>6460</td>
<td>5472</td>
<td>660 s</td>
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ECT is applied to the inspection of an A533B ferromagnetic plate using a differential transmit–receive ECT probe [9]. To achieve larger skin depth, this probe has been modified to be adapted to a lower frequency of 10 kHz. Considering space resolution simultaneously, the diameter of the exciting coil changes to 4 mm and the pickup coil changes to 3 mm instead of the 2-mm size of the older ones. A533B is one of the materials used in nuclear plants. The size of the ferromagnetic plate is 100 \times 100 \times 1.85 \text{ mm}^3, and the size of the suspect region is 9 \times 0.2 \times 1.85 \text{ mm}^3, where 0.2 mm is the width and 9 mm is the length. The relative permeability and conductivity of the conductor are 36 and 3.5 \times 10^6 \text{ S/m}, respectively.

Using an exciting frequency of 10 kHz, the differential transmit–receive ECT probe moves along the direction of the crack from \(-10\) to 10 mm in steps of 1 mm. Four kinds of FDs with different depths (0.37, 0.74, 1.11, and 1.48 mm) are tested by ECT, and the reconstructed crack shapes from these experimental data are shown in Fig. 5. Both the depths and lengths of the cracks are reconstructed satisfactorily compared with the true shapes, as shown in Table II. Only the crack with a depth of 4 mm is estimated to be a little shallow compared with the true depth because of the skin effect.

### V. SUMMARY

A novel fast simulator and its applications to the ECT inverse problems of ferromagnetic materials are discussed.

1) The fast forward simulator of ECT of ferromagnetic materials is about 80 times faster than conventional methods but retains the same computational accuracy.
2) Crack sizing in ferromagnetic materials are performed. Satisfactory results are obtained in a practical amount of time by applying the fast simulator.
3) ECT of an A533B ferromagnetic plate is carried out. The reconstruction of surface EDM cracks from experimental data shows good agreement with the true shapes.

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### REFERENCES