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An Investigation on the Aftershocks of the Tokachi-oki Earthquake of 1968, (2) Statistical Study on Time Distribution

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Abstract: The temporal nature of the aftershock sequence is studied in detail for the aftershocks of the Tokachi-oki earthquake, 1968, based on the special observation at Karumai, Iwate Prefecture by means of a wide dynamic range system.

The decrease in daily number of aftershocks follows the modified Omori's law but the numerical value of the parameter p is smaller than the usual value. The minimum velocity amplitude in counting the earthquakes is taken as a measure of magnitude, and the statistics is made for various minimum amplitude. The p value is independent of magnitude. The distributions of time interval and the number of aftershocks in a unit time, as well as the decrease in daily number, indicate that the occurrence of aftershocks is stationary and random during a short period of time.

The secondary aftershocks were clearly observed after the second largest aftershock of June 12, 1968 ($M=7.2$). The daily number of these secondary aftershocks follows the modified Omori's law and the value of p is approximately the same as the usual value. The p value is independent of the minimum amplitude, whereas the c value in the modified Omori's formula, looks like dependent on the minimum amplitude. This dependency, however, is proved to be an apparent result due to the masking effect by successive occurrence of aftershocks.

Similar study is made for a special group of aftershocks, which occurred in a small area and showed an unusually small value of m in the Ishimoto-Iida's magnitude-frequency relation. The temporal distribution of these aftershocks indicates a strong tendency of group occurrence with respect to time.

1. Introduction

After the occurrence of the Tokachi-oki earthquake of 1968, a temporary observation with a wide dynamic range system was carried out in order to investigate the statistical character of the aftershock occurrence. The detailed description of observation was already reported in the previous paper (Hamaguchi and Hasegawa, 1970). Since the magnitude distribution of the aftershocks was already discussed, the temporal distribution is treated in detail in this paper.

2. Frequency Decrease with Time

A well known empirical relation holds for the decrease of number of aftershocks with the lapse of time. This relation, called the modified Omori's formula, is expressed by

$$n(t)dt = A\cdot(t+c)^{-p}dt\,, \tag{1}$$

where t is the time after the main shock, $n(t)$ the frequency of aftershocks in a unit time interval of $t \sim t+dt$, A, c and p are numerical constants (e.g. Utsu, 1961). The p and c values in Eq. (1) were estimated to be between 1.0 and 1.5 and between 0.1 and 1.0 day respectively, for many aftershock sequences of shallow earthquakes occurred in Japan during 1926 and 1968 (Utsu, 1969a). If $t \gg c$, Eq. (1) reduces to

$$n(t)dt = At^{-p}dt\,. \tag{2}$$

The daily number of aftershocks of the present Tokachi-oki earthquake is plotted on a doubly logarithmic diagram of Fig. 1. Three methods of expression are adopted in this figure to examine the relation between temporal and magnitude distributions. The solid circle, cross and hollow circle indicate the cases of the aftershocks having larger velocity amplitudes than 100, 1,000 and 5,000 microkines respectively. The lower limit of velocity amplitude in counting the data is provisionally called *the minimum amplitude* in this paper. As seen in Fig. 1, the modified Omori's relation holds very well in the present case, except the two plots after June 12, 1968, when the second largest aftershock of magnitude 7.2 occurred. The discrepancy of these two plots is reasonably thought to be the effect of secondary aftershocks of this second largest aftershock. In the calculation of p value, therefore, these two plots were excluded. The values of p in various cases of minimum amplitude are listed in Table

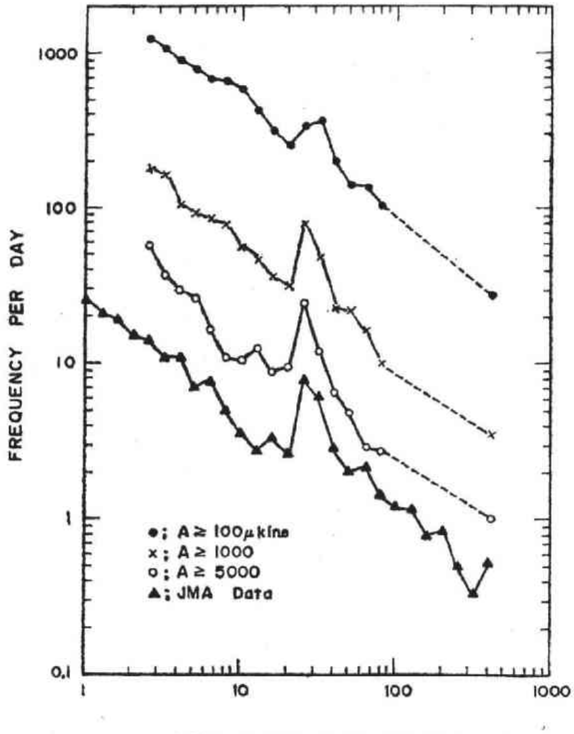


Fig. 1. Daily number of aftershocks versus time after the main shock. The sudden increase in number is due to the secondary aftershocks of the second largest aftershock ($M=7.2$) of June 12, 1968. The velocity amplitude in microkine is represented by A.

Table 1. The p value of the general aftershock sequence. The value in parenthesis is determined from the data before June 12.

Lower Limit of Velocity Amplitude in microkine	p Value
100	0.74 ± 0.02 (0.75 ± 0.05)
500	0.78 ± 0.04 (0.84 ± 0.09)
1,000	0.77 ± 0.03 (0.83 ± 0.03)
5,000	0.76 ± 0.04 (0.86 ± 0.09)

1. The accuracy of determination of c value is essentialy dependent on the data soon after the main shock. In our observation, however, the data for 60 hours just after the main shock were not observed and the good accuracy of c value cannot be expected even if Eq. (1) is used. Eq. (2) is therefore adopted in the computation.

The p values in Table 1 are smaller than the usual values estimated for many aftershock sequences in Japan. It may be considered, however, that the secondary aftershocks of the second largest aftershock still give some effect on p value even though the two points after June 12 are excluded. In order to check the effect, the p value for the data before June 12 is calculated. The results of calculation for various minimum amplitudes are shown in Table 1 with parenthesis. These values are still smaller than the normal value. The daily frequency of large aftershocks whose hypocenters were determined by JMA is taken for reference. As seen in Fig. 1, the decrease of daily number of these shocks is well represented by Eq. (1). The p value for these large aftershocks before June 12 is estimated to be 0.9, while the data during the same period as our observation gives 0.86 adopting Eq. (2) instead of Eq. (1). These two values also indicate that the p value is smaller than the normal value. Consequently it is confirmed that the sequence of the present aftershocks is characterized by the small p value. The Sanriku-oki earthquake of 1933 and the Tokachi-oki earthquake of 1952 occurred closely to the present shock. The p values in these two cases are estimated by Utsu (1969a) to be 1.4 and 1.1 respectively. According to Mogi (1962), the p values are 1.20 and 0.98. The difference may be due to the difference in the procedure of analysis (Utsu, 1969a). Anyway it is safely concluded that the present p value is smaller than these values.

Fig. 1 and Table 1 obviously show that the p value does not change with the minimum amplitude and the temporal distribution is independent of magnitude. In the previous paper (Hamaguchi and Hasegawa, 1970), it was concluded that the m value in the Ishimoto-Iida's relation between amplitude and number of aftershocks did not vary with time. It implies that the magnitude distribution is independent of time. The result of the present study is, therefore, compatible with the previous conclusion. The similar result was reported by Utsu (1962a) for three Alaskan aftershock sequences.

Utsu (1962a) pointed out that the c value also did not change with the lower limit of magnitude in counting the data. On the other hand, Yamakawa (1968) stated that the number of aftershocks did not start to decrease immediately after the main shock, but the decrease began with a time lag from the main shock. He considered that this time lag, of which measure was c in Eq. (1), corresponded to the time of fracturing process in the hypocentral region and depended on the lower limit of magnitude in counting the data. In general, however, it is difficult to obtain the sufficient data for the decisive conclusion. The data immediately after the main shock is essentialy important. This kind of data is hardly obtained in the case of temporary observation. Even in the case of routine work, the data are often not sufficient, because the number is hard to be counted owing to the overlapping of many successive aftershocks. In some cases, the seismometer is out of work by the big movement of the main shock.

In our observation, the activity of secondary aftershocks started on June 12 was fully covered and the wide dynamic range system was very useful to recognize each event separately. Our data, therefore, are suitable to solve the above mentioned problem. Even in the period after June 12, the primary aftershocks accompanied directly the main shock was still active and both the primary and secondary events were observed together. Since it is difficult to discriminate these two activities, we

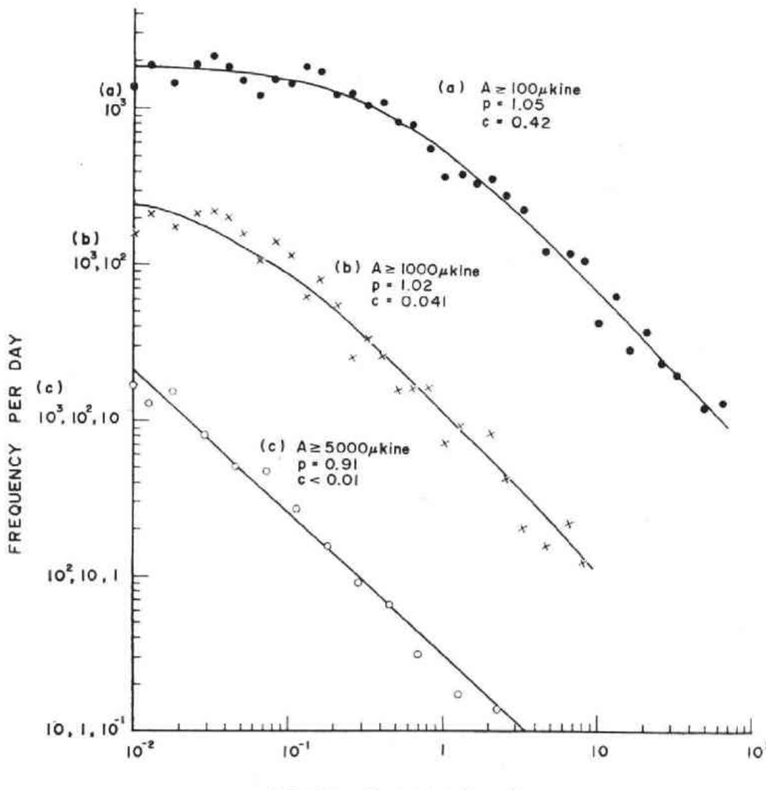


Fig. 2. Daily number of the secondary aftershocks accompanied the second largest aftershock of June 12, 1968. The mode of frequency decrease changes with the minimum amplitude. This change is proved to be an apparent result due to the masking effect (see text). p and c values in Eq. (1) are also shown in each case.

Table 2. The p and c values of the secondary aftershock sequence.

Lower Limit of Velocity Amplitude in microkine	p Value	c Value in day
5	0.84	0.99
100	1.05	0.42
500	1.18	0.12
1,000	1.02	0.041
5,000	0.91	<0.01

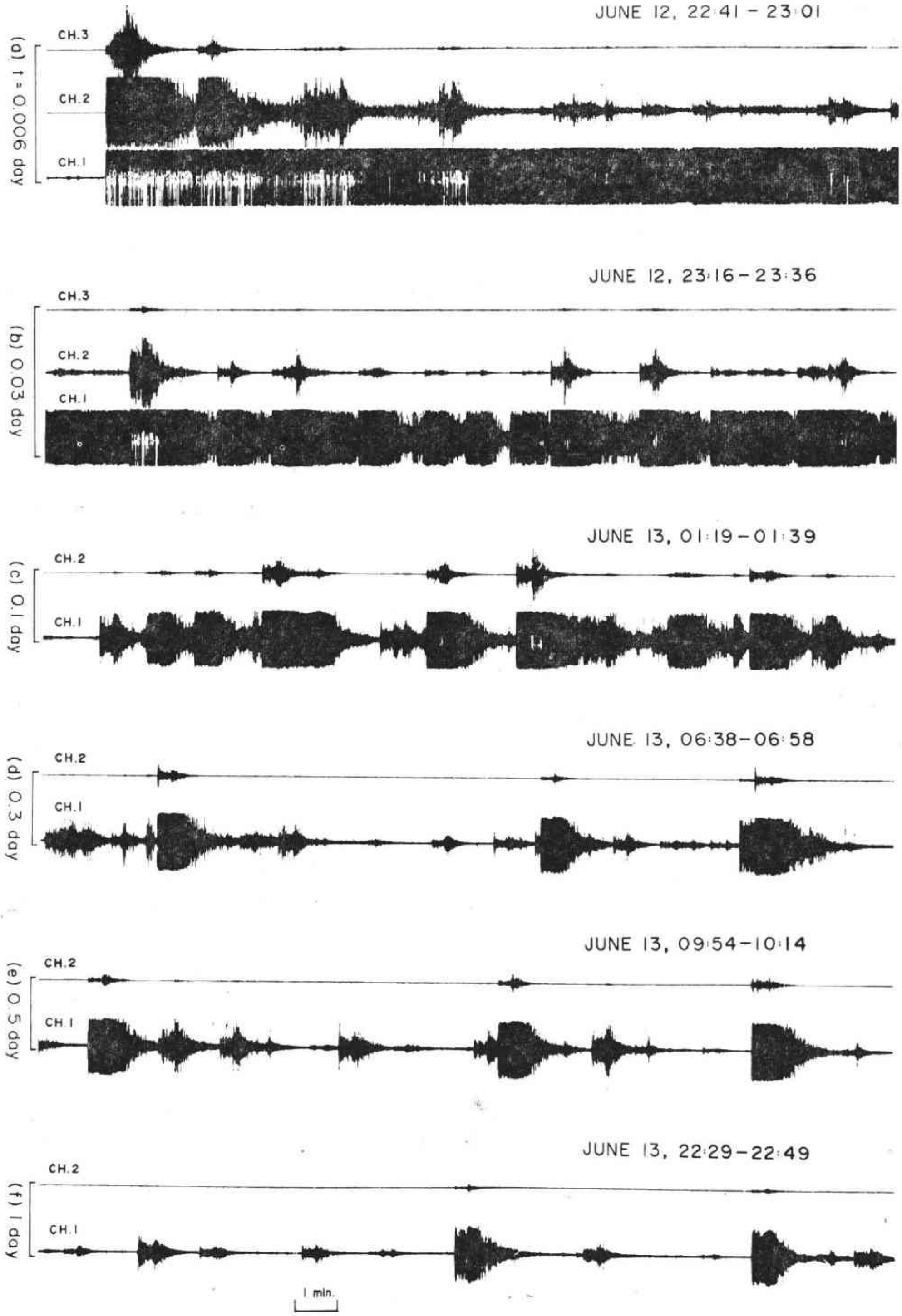
assume that the primary aftershock activity behaved according to the modified Omori's law. Then the number of secondary aftershocks can be estimated by subtraction of the assumed number of primary events from the total.

The temporal distribution of secondary aftershocks thus obtained is seen in Fig. 2. The solid circle, cross and open circle indicate the cases of the minimum amplitude of 100, 1,000 and 5,000 microkines respectively. These plots are well represented by the modified Omori's formula (1) and the secondary aftershocks are concluded to follow the same formula as the primary aftershocks with respect to their temporal distribution. The p and c values obtained by the method of least squares are listed in Table 2. It is found from this table that the c value decreases with increasing the minimum amplitude, while the p value is independent of the minimum amplitude. This fact could be misunderstood as if the smaller aftershocks are not so many as expected from Eq. (2) with the same p values as that for larger aftershocks as stated by Yamakawa (1968). However it is not true. The number of small shocks cannot be fully counted because of the successive occurrence of many events. As an example, Fig. 3 shows a seismogram by our wide dynamic range system immediately after the second largest aftershock of June 12. If we look at only the Ch. 1 trace, which is the high gain channel, the successive aftershocks are overlapped each other and it is very difficult especially for the small aftershocks to count the whole number of events. The usual system of observation, therefore, cannot give an adequate data for this sort of problem and the wide dynamic range system is requested.

The following consideration is made in order to examine whether the small number of aftershocks in the period immediately after the second largest shock is only an apparent result or not. The difference in number of aftershocks following Eq. (1) and Eq. (2) is evidently expressed by

$$\int_{t_1}^{t_2} A\left\{t^{-p}-(t+c)^{-p}\right\} dt\,, \tag{3}$$

for the period of $t_1 \leq t \leq t_2$. The numerical values of A, c and p have been obtained by the method of least squares as is listed in Table 2. Substituting these values into Eq. (3) and taking $t_1=0.01$ and $t_2=3c$, the difference in number is calculated to be approximately 3.2×10^3, 8.1×10^2 and 1.7×10^2 in the cases of the minimum amplitude of 100, 500 and 1,000 microkines respectively. Since the data in Fig. 2 are well represented by Eq. (1) with the numerical values of parameters in Table 2, this difference


JUNE 12, 22:41 - 23:01
CH.3
CH.2
CH.1
(a) t = 0.006 day
JUNE 12, 23:16-23:36
CH.3
CH.2
CH.1
(b) 0.03 day
JUNE 13, 01:19-01:39
CH.2
CH.1
(c) 0.1 day
JUNE 13, 06:38-06:58
CH.2
CH.1
(d) 0.3 day
JUNE 13, 09:54-10:14
CH.2
CH.1
(e) 0.5 day
JUNE 13, 22:29-22:49
CH.2
CH.1
(f) 1 day
1 min.

implies that the above quantity is the discrepancy of number of countable aftershocks from the expected value based on Eq. (2). The problem is, therefore, whether or not these values of discrepancy can be explained by the masking effect due to the successive occurrence of many aftershocks in this period. Two methods are adopted here to solve this problem.

If the Ishimoto-Iida's relation between amplitude and number of aftershocks holds good in this period, with a constant value of exponent m, the logarithm of number of aftershocks should be expressed by a straight line against the logarithm of amplitude. In practical case, however, the plots often show an upward concave curvature especially for small amplitude range and the number of aftershocks with small amplitude is apparently smaller than that expected from the Ishimoto-Iida's relation. If this discrepancy is considered to be due to the masking effect, the number of events which could not be counted are estimated by the discrepancy. The estimation gives that the numbers of masked aftershocks are about 3×10^3, 6×10^2 and 2×10^2 in the cases of the minimum amplitude of 100, 500 and 1,000 microkines respectively. These figures are compatible with the difference between the number expected from Eq. (2) and the counted number of aftershocks in the temporal distribution.

The second way is to estimate the masking effect from the time interval distribution. It is found to be due to the masking effect that the observed frequency of time intervals between two successive earthquakes is smaller than that expected from the exponential distribution for small time interval. The ground for this consideration is described in next paper (Hasegawa and Hamaguchi, 1970). The estimation according to this consideration results that the numbers of masked aftershocks are approximately 3×10^3, 4×10^2 and 2×10^2 for the minimum amplitude of 100, 500 and 1,000 microkines respectively. These figures are again compatible with the values derived from Eq. (3).

Above considerations lead us to the conclusion that the smaller number of events immediately after the main shock than that expected by Eq. (2) is an apparent result due to the overlapping of many shocks. It is also concluded that Eq. (2) holds good even for the period just after the main shock and that the c value in Eq. (1) is smaller than 0.01 day. It should be emphasized that the careful procedure is necessary to discuss a similar statistical problem to the present one. If the number of aftershocks, which cannot be separately counted, are taken as data, there is a danger of giving an erroneous conclusion.

Fig. 3. Example of the seismograms showing the successive occurrence of the secondary aftershocks of the second largest aftershock of June 12. The extremely high activity of the secondary aftershocks can be recognized from the seismograms. Each record of (a), (b), (c), (d), (e) and (f) shows the seismogram of 0.006, 0.03, 0.1, 0.3, 0.5 and 1 day after the second largest aftershock respectively. The recording time covered by one seismogram is 20 minutes. The first event in (a) is the principal shock. White parts in Ch. 1 show the saturated signal due to overmodulation in the magnetic data recorder. The records of Ch. 3 in the cases of (c), (d), (e) and (f) are omitted in this figure. It can be understood how many events could be missed in counting due to the overlapping or saturation, if only one record, say Ch. 1 or Ch. 2, is analyzed.

As stated in the previous paper, a cluster of aftershocks having the S-P times between 8 and 12 seconds at KAR station was observed in both observations in 1968 and 1969 separately from the major aftershock activity. These aftershocks were concentrated in a small region, which was provisionally called A 2 region in the previous paper, while the region where the general activity occurred was called A 3.

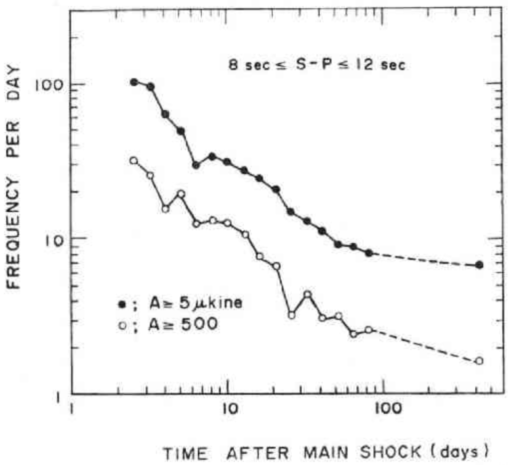


Fig. 4. Daily number of the aftershocks with S-P times between 8 and 12 seconds (A 2 region) versus time after the main shock. The velocity amplitude in microkine is represented by A.

The feature of this group of aftershocks is that the m value in the Ishimoto-Iida's relation is 1.58 and significantly smaller than the m value for general activity, which is 1.84. It is interesting, therefore, to compare the temporal distribution for this group with that for general aftershocks. The daily frequency of aftershocks with the S-P times between 8 and 12 seconds is seen in Fig. 4, abscissa being the lapse of time in logarithmic scale. The solid and open circles denote the cases of the minimum amplitude of 5 and 500 microkines respectively. The plots fit well for the modified Omori's formula Eq. (2), except for the data obtained in the observation in 1969, about one year after the main shock. The activity of this group, therefore, was not affected by the occurrence of the second largest aftershock of June 12. According to the data by JMA, a comparatively large aftershock ($M=5.6$) occurred in the region A 2 shortly before our observation in 1969. The discrepancy of the data in 1969 from Eq. (2), therefore, may be the effect of secondary aftershocks of this earthquake, although it is not decisively concluded.

Table 3 is the list of the p value for this group, which was determined by the data excluding those in 1969. The values in parenthesis in Table 3 are the p values obtained from the data before June 12. This table shows two important results. One is that the p value is independent of the minimum amplitude, or, in other words, that the temporal distribution is independent of the magnitude as well as in the case of general aftershocks. The second result is that the p value for this group is approximately the same as that for the general activity, in spite of that the m value in the Ishimoto-

Table 3. The p value of the aftershock sequence with S-P times between 8 and 12 seconds. The value in parenthesis is determined from the data before June 12.

Lower Limit of Velocity Amplitude in microkine	p Value
5	0.74 ± 0.04 (0.79 ± 0.07)
100	0.78 ± 0.04 (0.84 ± 0.07)
500	0.80 ± 0.05 (0.80 ± 0.09)
1,000	0.76 ± 0.06 (0.75 ± 0.11)

Iida's relation differs from each other. The similar result was reported by Utsu (1962a) for the three Alaskan aftershock sequences. Since this feature is generally valid for aftershock sequence, it may give an important clue to the mechanism of occurrence of aftershocks.

3. Time Interval Distribution

Since the study by Tomoda (1954), it has been observed in many aftershock sequences that the frequency distribution of time intervals between two successive aftershocks is well expressed by the inverse power distribution,

$$\phi(\tau)d\tau = \alpha\tau^{-\beta}d\tau\,, \tag{4}$$

where τ is the time interval, $\phi(\tau)$ is the frequency of time intervals in a unit time $d\tau$, and α and β are constants (e.g. Senshu, 1959; Utsu, 1962b; Research Group for Aftershocks (Tôhoku University), 1965; Comninakis *et al.*, 1968; Hirota, 1969). Senshu showed that the two equations of (2) and (4) are connected with the relation,

$$\beta = 2 - 1/p\,. \tag{5}$$

The distribution in the present case is seen in Fig. 5, in which the solid circle and the cross represent the distribution for general aftershock activity in the cases of the minimum amplitude of 100 and 1,000 microkines respectively and the open circle shows that for the aftershocks having the S-P times between 8 and 12 seconds. If Eq. (4) holds, the plots should be expressed by a straight line in doubly logarithmic diagram of Fig. 5. The distribution for general activity (solid circle and cross), however, does not follow this equation, as is obvious in the figure, but has an upward concave curvature. Similar tendency was reported by the Research Group for Aftershocks (1965) in the aftershocks of the Oga-oki earthquake of 1964.

It was proved by Utsu (1969b) that, if the occurrence of aftershocks is stationary and random within a short duration of time and the frequency of aftershocks changes as $n(t)$ with respect to time t, then the time interval distribution should be given by

$$\phi(\tau)d\tau = \int_{t_1}^{t_2} \{n(t)\}^2 \cdot e^{-n(t)\cdot\tau}\, dt \cdot d\tau\,, \tag{6}$$

for the data between t_1 and t_2. When $n(t)$ follows Eq. (2) and $t_1 = 0$ and $t_2 \to \infty$, the above equation reduces to Eq. (4). If Eq. (4) does not hold, therefore, it should be due

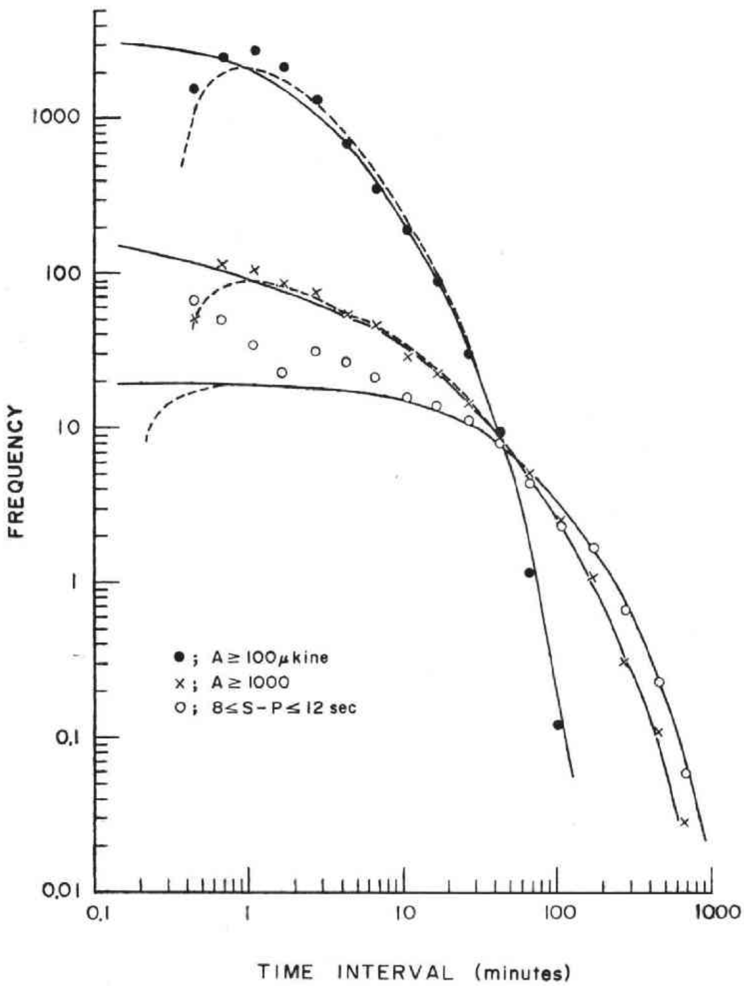


Fig. 5. Frequency distribution of the time intervals between successive aftershocks. Solid circle and cross denote the cases of the minimum amplitudes of 100 and 1,000 microkines respectively and open circle denotes the case of S-P times between 8 and 12 seconds. The solid lines are obtained by the numerical integration after Eq. (6). The dotted lines are obtained by the same operation, taking the miscount into consideration (see text).

to the condition (a) $n(t)$ is not expressed by Eq. (2), (b) t_1 is not sufficiently small and/or t_2 is not sufficiently large, and/or (c) the occurrence of aftershocks is unstationary and/or non-random even in a short period of time. In the present case, the daily number of aftershocks clearly shows that the whole aftershocks are consisted of two major sequences, the aftershocks of the main shock and those of the second largest aftershock on June 12. The decrease in daily number $n(t)$, therefore, does not follows Eq. (2) as a whole. Moreover, t_2 may not be sufficiently large, because the sensitivity of instrument is so high that many number of events were still observed at the end of the observation period. The direct calculation of Eq. (6) is carried out in order to check the effect of this situation.

Since each of the two sequences of aftershocks of the main and June 12 shocks is well represented by Eq. (1) or Eq. (2), as mentioned in the previous paragraph, the distribution $n(t)$ is safely assumed to have the form,

$$n(t)=\begin{cases} At^{-p} & \text{for } t<27.5 \\ At^{-p}+A'(t'+c')^{-p'} & \text{for } t\geq 27.5 \end{cases} \tag{7}$$

where $t=27.5$ means the time of occurrence of the second largest aftershock and t' stands for t-27.5. Adopting the values of A, p, A', p' and c' determined from the observation in the previous paragraph, the numerical integration of Eq. (6) is performed for the period of observation in 1968. The result is represented by the solid line in Fig. 5, which indicates that the observed values are interpreted fairly well by the solid line. This fact implies reasonably that the occurrence of both aftershock sequences is considered to be stationary and random within a short period of time. The numerical integration of Eq. (6) is performed for the period before June 12 ($t<27.5$) and it is found that the observed values, which have the upward concave curvature similarly to the case mentioned above, are represented fairly well by the expected values from Eq. (6). This correspondency implies that the time interval distribution has the upward concave curvature, even when the modified Omori's formula (2) holds good. Consequently the upward concave curvature is mainly due to the condition of (b) mentioned previously.

Exactly speaking, the observation for small τ between 0.7 and 4.0 minutes has a higher value than the calculation by Eq. (6), as seen in Fig. 5. This deviation may be perhaps due to the situation that the aftershocks in this domain have a tendency of group occurrence with respect to time to some extent and/or some large aftershocks are accompanied by their secondary aftershocks. For further smaller value of τ, the observed frequency is smaller than the calculated value. This is reasonably considered to be the masking effect of successive occurrence of many aftershocks. As stated in the previous paragraph, the detailed discussion on this effect is presented in next paper (Hasegawa and Hamaguchi, 1970). The correction of this masking effect can be done under some assumption and the corrected calculation is given by the dotted line in Fig. 5. The corrected value can interpret the observation fairly well.

The situation is different for the activity in A 2 region corresponding to the S-P times between 8 and 12 seconds. The time interval distribution of these events have the upwards concave curvature similarly to the general aftershock activity, while $n(t)$ in Fig. 4 is well expressed by Eq. (2). If we insert $n(t)$ in Eq. (6) and integrate numerically, the solid line in Fig. 5 is obtained. Taking the masking effect into consideration, the calculation results the dotted curve. As seen in Fig. 5, the calculated curves do not fit the data, especially those having small value of τ. This discrepancy, therefore, cannot be attributed the conditions of (a) and (b). It is consequently concluded that the occurrence of aftershocks in this group is unstationary and/or non-random even for the short period of time. This is a rare case of aftershocks studied so far. This conclusion is very interesting, if the fact that the m value for this activity is significantly different from the normal value is taken into account.

4. Frequency Distribution of Number of Aftershocks in a Unit Duration of Time

Now we take the number of events n in a unit duration of time Δt. If the occurrence of aftershocks is assumed to be stationary and random, the distribution of n should follow the Poisson distribution,

$$P(n) = \{(\mu \cdot \Delta t)^n / n!\} e^{-\mu \cdot \Delta t}, \tag{8}$$

where μ is a constant. If μ changes with t in the form of $\mu(t)$, although the occurrence is stationary and random within the short duration of time, then the distribution of n for the total data from t_1 to t_2 is given by

$$P(n) = \int_{t_1}^{t_2} \left[\{\mu(t) \cdot \Delta t\}^n / n! \right] e^{-\mu(t) \cdot \Delta t} \, dt / \Delta t, \tag{9}$$

as shown by Utsu (1969b). When $\mu(t)$ follows Eq. (2) and $t_1 = 0$, $t_2 \to \infty$ and $n > 1/p$, Eq. (9) reduces to

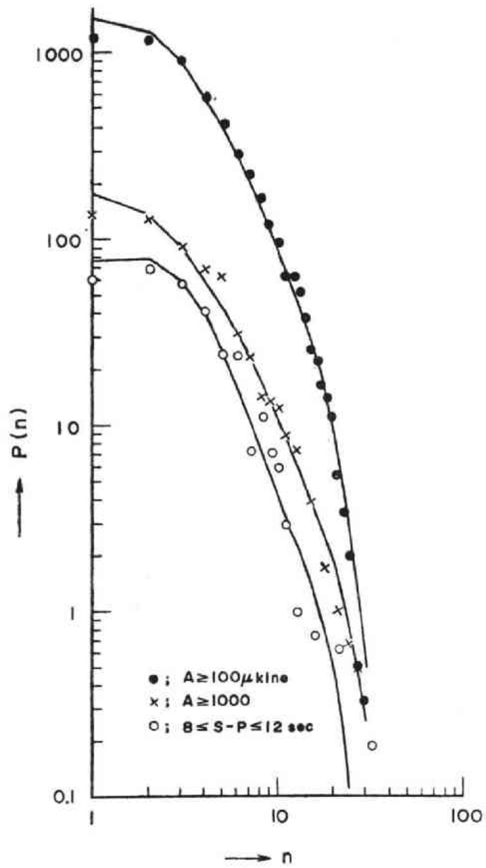


Fig. 6. Frequency distribution of the numbers of aftershocks in a unit time. The symbols have the same meaning as in Fig. 5. Solid lines show the results of the numerical integration after Eq. (9).

$$P(n) = K \cdot \Gamma(n-1/p)/\Gamma(n+1) , \tag{10}$$

where

$$K = (A \cdot \Delta t)^{1/p}/(p \cdot \Delta t) . \tag{11}$$

If it is further assumed that n is large enough, the distribution approaches the inverse power distribution as

$$P(n) \rightarrow Kn^{-(1+1/p)} . \tag{12}$$

The distribution $P(n)$ of the present aftershocks is seen in Fig. 6. The solid circle and cross represent the distribution for the general activity, when the minimum amplitude are taken to be 100 and 1,000 microkines respectively. The open circle shows the case of aftershocks in the region A 2. In each case, Δt is taken as 1/3, 3 and 6 hours respectively. These plots do not obviously follow the inverse power distribution of Eq. (12) but represent the upward concave curvature in doubly logarithmic diagram, similarly to the case of time interval distribution in Fig. 5. The reason of this discrepancy is attributed to (a) $n(t)$ does not satisfy Eq. (2), (b) $t_1 \neq 0$ and/or t_2 is not sufficiently large, (c) n is not large enough, and/or (d) the occurrence is unstationary and/or non-random. In order to check which is the main reason in the present case, the integration of Eq. (9) is numerically made adopting the values of $\mu(t)$, t_1 and t_2. The result of the numerical integration is shown by the solid line in Fig. 6, which indicates the satisfactory agreement between the observed value and Eq. (9). This implies the reason of discrepancy from the inverse power distribution is mainly because of (b) and/or (c), and the occurrence of general aftershocks is safely concluded to be stationary and random in the short duration of time. For the activity in A 2 region, however, the calculation does not fit the observed value, but the observation gives the larger value than Eq. (9) for large value of n, as seen in Fig. 6. This is the similar situation to the case of time interval distribution stated in the previous paragraph. The aftershock sequence in A 2 region is concluded to be unstationary and/or non-random, but to have the tendency of group occurrence with respect to time, in contrast to the general aftershock activity.

5. Discussion and Conclusions

The statistical investigation of temporal distribution is made based on the aftershocks of the Tokachi-oki earthquake of 1968. The observation of aftershocks was made at KAR station, Iwate Prefecture by a wide dynamic range system, which was very powerful for the exact discussion of statistical feature of the aftershock sequence. The following results are obtained:

(1) The daily number of general aftershocks decreases with time according to the modified Omori's formula (2). The p values for various minimum amplitude in Table 1 are somewhat smaller than the normal value obtained in many aftershock sequences. The frequency distribution of time intervals between two successive aftershocks is not expressed by a straight line in doubly logarithmic diagram, but has an upwards concave curvature. This is proved to be due to the condition that the period of observation is

too short to be accounted as the period of $0 \sim \infty$ and some other reasons. If the correction for these conditions is made, the observed data are well explained by Eq. (6) and, therefore, it is concluded that the occurrence of general aftershocks is stationary and random in a short period of time as a first approximation.

It should be noted that the inverse power distribution does not hold for the time interval distribution, if the decrease of number of aftershocks does not follow the modified Omori's formula, or, if the period of observation $t_1 \sim t_2$ is not long enough to be approximately taken as $0 \sim \infty$.

Exactly speaking, a small deviation from the stationary and random occurrence is seen in Fig. 5 even after the above mentioned correction is made. This implies that the aftershocks have the tendency of group occurrence to some extent. This may be the reason why the value of β in Eq. (4) of the time interval distribution is a little larger than the calculation by Eq. (5), as pointed out by Utsu (1969b).

The frequency distribution of numbers of events in a unit time is also studied. When the period of observation is finite, the distribution should be expressed by Eq. (9), if the occurrence is stationary and random. The calculation by Eq. (9) interprets the observed value fairly well and it again gives a good evidence for the stationary and random occurrence of the general aftershocks.

(2) The secondary aftershocks of the second largest aftershock on June 12 were observed from the very beginning of its sequence. The variation of daily number of these shocks is well expressed by the modified Omori's formula (1). The p and c values obtained are listed in Table 2. The c value in the table looks as if it depends on the minimum amplitude. However this dependency is an apparent result due to the masking effect by the successive occurrence. It is proved by the two methods based on the magnitude distribution and on the time interval distribution. Correcting the effect of masked number, it is shown that the c value does not depend on the minimum amplitude. The c value thus obtained is smaller than 0.01 day. It should be again emphasized that the careful consideration is necessary on the masking effect to make an exact discussion on statistical problem and the wide dynamic range system is very useful for this kind of problem.

(3) A cluster of aftershocks was observed at the range of S-P times between 8 and 12 seconds at KAR station. These aftershocks were separately treated from the general activity. This group is characterized by the fact that the m value in the Ishimoto-Iida's relation is smaller than the normal value.

The p value for this sequence is almost the same as that for the general activity. The time interval distribution for this group shows that the observed value for small time intervals is larger than the calculation by Eq. (6), even when the change in daily frequency and the finite duration of observation period are taken into account. It is concluded, therefore, that the aftershocks in this group are unstationary and/or non-random in their occurrence.

The distribution of number in a unit time also indicates the same result, and the group occurrence of these shocks is established. This is very important feature of this cluster of aftershocks, taking the unusual m value into consideration. Tomoda (1954) stated the good correlation between m and β values, and our result is compatible with his conclusion. It is considered to be necessary to examine in detail the relation between magnitude and temporal distributions in the statistical study on the occurrence of aftershocks.

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