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On the Masking Effect by the Successive Occurrence of Earthquakes on Time Interval Distribution

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Abstract: When many earthquakes occur successively, some events are often masked by the preceding one, and the distribution of time intervals between two successive shocks are sometimes distorted. The distribution of time interval is theoretically deduced taking the masking effect into account. The result is successfully applied to the actual example of aftershocks of the Tokachi-oki earthquake, 1968.

The deviation of observed frequency of time intervals from the constant probability of earthquake occurrence is fairly explained by this theory. It is shown that this deviation is not the real nature but only an apparent phenomenon due to the masking effect.

1. Introduction

It is well known that, if the earthquake occurrence is stationary and random with respect to time, the frequency distribution of time intervals between two successive earthquakes follows the exponential distribution,

$$\phi(\tau)d\tau = \mu e^{-\mu\tau}d\tau\,, \tag{1}$$

where τ is the time interval between two successive shocks, $\phi(\tau)$ the number of time intervals between τ and $\tau+d\tau$, and μ a numerical constant (Watanabe, 1936). It has been confirmed in many cases that the observed distribution agrees with the ditribution (1) at least as the first approximation for large value of τ (e.g. Utsu, 1969). For small value of τ, however, the observed distribution often has the smaller value than that expected by the theoretical formula (1), as was pointed out by Senshu (1959). He showed that the deviation of the observed distribution from Eq. (1) could be explained successfully under the following assumption:

The non-conditional probability μ of earthquake occurrence in a unit time and the time interval τ between two successive shocks are connected by the relation for small value of τ,

$$\mu = a\tau\,, \tag{2}$$

where a is a positive constant, while the probability μ tends to constant for larger value of τ. His discussion, however, did not take the effect of miscount of frequency due to the overlapping of successively occurring earthquakes into consideration. The better fit of his theory to the data may be, at least to some extent, the natural result due to the increase in number of parameters. In the exact discussion on this kind of problem, the masking effect by the successive occurrence of earthquakes should be taken into account.

This paper deals theoretically with the masking effect on the time interval distribution.

2. Theoretical Formulation

If an earthquake occurs during the oscillation of the preceding earthquake and the amplitude is not so large compared to that of the preceding shock, the later event would be masked by the oscillation of the former and it could not be counted as an event. The probability of the masking will be theoretically formulated in this section.

First, we will consider the probability that an earthquake takes place before the amplitude of the preceding one diminishes down to a critical value, over which the earthquake can be counted. Assuming that the distribution of amplitude is independent of the temporal distribution, the probability of the occurrence of earthquake with amplitude A at the lapse of time τ after the occurrence of the preceding one is given by

$$\frac{\phi(\tau)}{N_t} d\tau \cdot \frac{n(A)}{N_t} dA \; , \tag{3}$$

where $\phi(\tau)$ is the frequency of time intervals τ between two successive shocks, $n(A)$ the frequency of earthquakes of which the amplitudes are between A and $A+dA$, and N_t the total number of events.

The relation between the magnitude M of an earthquake and the duration time T_{FP} of oscillation of the earthquake can be represented by

$$10^M \propto T_{FP}{}^{\gamma} \; , \tag{4}$$

where γ is a constant, when the epicentral distances of the earthquakes in concern are not so much different from each other, as in the case of aftershocks (e.g. Tsumura, 1967). The velocity amplitude for near earthquakes follows the relation,

$$A^{\delta} \propto 10^M \; , \tag{5}$$

where M is the magnitude and δ a constant (e.g. Muramatu, 1964). Eliminating the magnitude M from Eqs. (4) and (5), we obtain

$$A \propto T_{FP}{}^{\varepsilon} \; , \tag{6}$$

where

$$\varepsilon = \gamma / \delta \; . \tag{7}$$

If T denotes the time of oscillation until its amplitude decreases to the limit over which the earthquakes can be counted, the relation between T and the velocity amplitude A may be reasonably assumed to be given by

$$A \propto T^{\varepsilon} \; , \tag{8}$$

similarly to the relation between T_{FP} and A. Using Eq. (8), Eq. (3) is transformed to

$$\frac{\phi(\tau)}{N_t} d\tau \cdot \frac{q(T)}{N_t} dT \; , \tag{9}$$

where the frequency distribution of T is denoted by $q(T)$, which can be converted from $n(A)$ by means of Eq. (8). The probability, that an event occurs before the oscillation of preceding one decays to the critical value of amplitude, is consequently given by

$$\frac{\phi(\tau)}{N_t}\,d\tau\cdot\int_{\tau}^{T_1}\frac{q(T)}{N_t}\,dT\,, \tag{10}$$

where T_1 is the maximum of T. The integral is defined so as to be zero when $\tau > T_1$. This definition will be adopted throughout the present paper.

Even if an earthquake occurs during the oscillation of the preceding one, it can be counted as an event when the later shock has an amplitude large enough. We consider the probability of such a case as the second step. The probability that the amplitude A' of the later event is larger than n times of the amplitude A of the preceding event is expressed by

$$\int_{nA}^{A_1}\frac{n(A')}{N_t}\,dA'\,, \tag{11}$$

where A_1 is the maximum of A. Converting A and A' into T and T' respectively, it is transformed as

$$\int_{n'T}^{T_1}\frac{q(T')}{N_t}\,dT'\,, \tag{12}$$

where

$$n' = n^{1/2}\,. \tag{13}$$

Connecting (10) with (12), we obtain the probability that an earthquake occurs within the oscillation of the preceding one but the later event is not masked by the former because its amplitude is larger than n times of the amplitude of the former event as

$$\frac{\phi(\tau)}{N_t}\,d\tau\cdot\int_{\tau}^{T_1/n'}\left[\frac{q(T)}{N_t}\int_{n'T}^{T_1}\frac{q(T')}{N_t}\,dT'\right]dT\,. \tag{14}$$

If an event is missed in counting, it naturally results an effect on the distribution of time intervals between two successive shocks. The effect is next investigated. Consider a series of events and denote the time intervals from the first to the second and from the first to the third events as τ' and τ respectively. When the second event is missed, then the time intervals is taken as τ instead of two intevals of τ' and $\tau-\tau'$. The probability of this kind of miscounting of time interval is expressed by

$$\int_0^{\tau}\left[\frac{\phi(\tau')}{N_t}\cdot\frac{\phi(\tau-\tau')}{N_t}\cdot\{K_1(\tau')-K_2(\tau')\}\right]d\tau'\,, \tag{15}$$

where

$$K_1(\tau) = \int_{\tau}^{T_1}\frac{q(T)}{N_t}\,dT\,, \tag{16}$$

$$K_2(\tau) = \int_{\tau}^{T_1/n'}\left[\frac{q(T)}{N_t}\int_{n'T}^{T_1}\frac{q(T')}{N_t}\,dT'\right]dT\,. \tag{17}$$

Furthermore, it is possible that two or more earthquakes are masked by the preceding one. The probability of occurrence of such cases can be deduced in a similar way, and the time interval distribution $\phi_{ob}(\tau)$ taking the masking effect into account is consequently given by

$$\phi_{ob}(\tau) = \phi(\tau)\cdot\{1-K_1(\tau)+K_2(\tau)-K_3(\tau)+K_4(\tau)-\cdots\}$$
$$\times\{1+S_1(\tau)-S_2(\tau)+S_3(\tau)-S_4(\tau)+\cdots\}, \qquad (18)$$

where

$$K_3(\tau) = \int_0^{T_1-\tau}\left[\frac{\phi(\tau')}{N_t}\int_{\tau+\tau'}^{T_1}\frac{q(T'')}{N_t}\,dT''\right]d\tau', \qquad (19)$$

$$K_4(\tau) = \int_0^{T_1-\tau}\left[\frac{\phi(\tau')}{N_t}\int_{\tau+\tau'}^{T_1/n'}\left\{\frac{q(T'')}{N_t}\int_{n'T''}^{T_1}\frac{q(T')}{N_t}\,dT'\right\}dT''\right]d\tau', \qquad (20)$$

$$S_1(\tau) = \frac{1}{N_t\cdot\phi(\tau)}\int_0^{\tau}[\phi(\tau')\cdot\phi(\tau-\tau')\cdot K_1(\tau')]\,d\tau', \qquad (21)$$

$$S_2(\tau) = \frac{1}{N_t\cdot\phi(\tau)}\int_0^{\tau}[\phi(\tau')\cdot\phi(\tau-\tau')\cdot K_2(\tau')]d\tau', \qquad (22)$$

$$S_3(\tau) = \frac{1}{N_t^2\cdot\phi(\tau)}\int_0^{\tau}d\tau'\int_0^{\tau-\tau'}d\tau''\cdot[\phi(\tau')\cdot\phi(\tau'')\cdot\phi(\tau-\tau'-\tau'')\cdot K_3(\tau'+\tau'')], \qquad (23)$$

$$S_4(\tau) = \frac{1}{N_t^2\cdot\phi(\tau)}\int_0^{\tau}d\tau'\int_0^{\tau-\tau'}d\tau''\cdot[\phi(\tau')\cdot\phi(\tau'')\cdot\phi(\tau-\tau'-\tau'')\cdot K_4(\tau'+\tau'')]. \qquad (24)$$

In the deduction of Eq. (18), the amplitude-frequency relation,

$$n(A)dA = kA^{-m}dA, \qquad (25)$$

is adopted, where k and m are constants. From Eqs. (25) and (8), $q(T)$ in Eqs. (16), (17), (19) and (20) is expressed as

$$q(T)dT = k'T^{-m'}dT, \qquad (26)$$

where k' and m' are constants. The exponent m' in Eq. (26) is connected with m and ε by the relation as

$$m' = (m-1)\cdot\varepsilon+1. \qquad (27)$$

3. Actual Example and Discussion

The numerical evaluation of Eq. (18) is made for an actual example. The values of constants of γ and δ is adopted to be 2.25 and 1.25 according to Solov'ev (1965) and Muramatu (1964). The value of m is taken to be 2.0, which is the normal value in many cases. The value of n is fixed to be 2. This implies that the later event can be picked up as an event if the amplitude of the later event is larger than twice of that of the preceding event, even when the later event occurs in the duration time of oscilla-

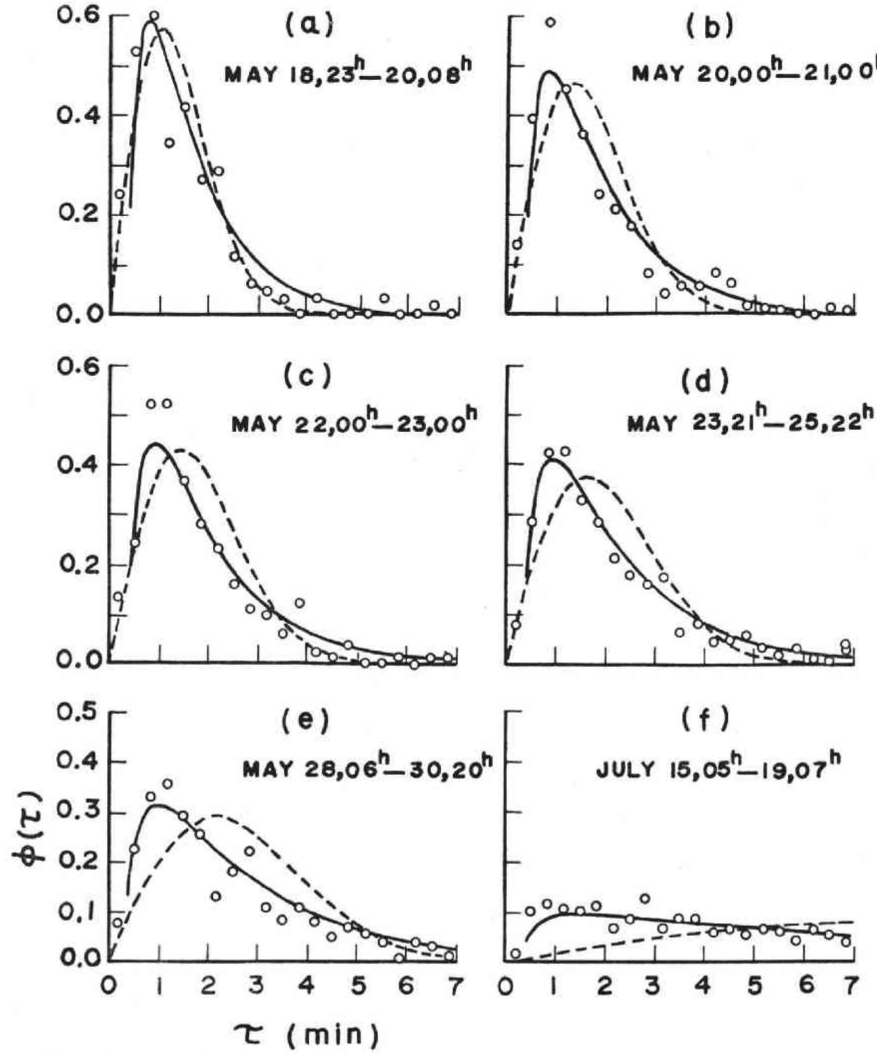

Fig. 1. Time interval distribution of the aftershocks of the Tokachi-oki earthquake of 1968. Open circles indicate the observed values, solid curves are obtained from Eq. (18), and dotted curves show the Senshu's distribution (28). The solid curves (Eq. (18)) fit the observed data better than the dotted curves (Eq. (28)) do.

tion of the preceding one. The distribution function of Eq. (1), which was introduced by Watanabe (1936), is adopted as $\phi(\tau)$ in Eq. (18).

The actual case of aftershocks of the Tokachi-oki earthquake, 1968 is taken as an example. The data were observed at Karumai (KAR), Iwate Prefecture in northern Honshu, Japan. The observation was made by means of a wide dynamic range system. The detailed description of the observation was made in another paper (Hamaguchi and Hasegawa, 1970a, b). The circles in Fig. 1 show the time interval distribution of these aftershocks, the period of observation being written in the figure.

Adopting the values above mentioned, the integral in Eq. (18) is numerically evaluated and the result is shown in Fig. **1** by solid line, which fits the observed data fairly well. The numerical calculation was not done for the smaller value of τ than 20 seconds which is the mean S-P times, because the simple form is assumed implicitly for the envelope of seismic signal. The higher order terms in Eq. (18) than fifth order are neglected in the numerical calculation. The contribution of each term

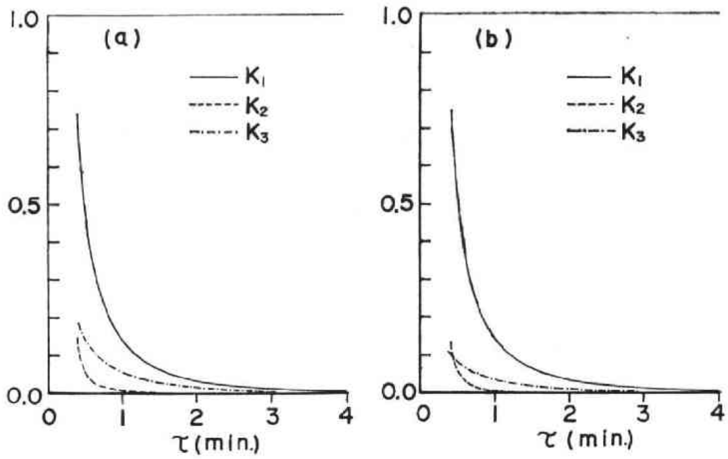

Fig. 2. The contributions of the first, second, third, and fourth terms are separately shown. (a) and (b) correspond to (a) and (e) in Fig. 1 respectively.

in Eq. (18) is separately shown in Fig. 2. The symbols of (a) and (b) in this figure correspond to the cases of (a) and (e) in Fig. 1, respectively. The contribution of higher order term decreases rapidly with increasing order and the above neglection is justified. The effect of the assumed numerical values of constants is also investigated. The solid curve in Fig. 3 represents the case where the numerical values are assumed as mentioned previously and hence this curve is the same as that in Fig. 1. The dotted line shows the case where the value of n is taken as 1 instead of 2. The chain line is the case where γ is assumed to be 2.85 after Tsumura (1967) instead of 2.25. Other constants are kept the same in each case. Fig. 3 shows that the change in numerical values of parameters does not cause the essential effect on the conclusion, taking the scatter of observed data into consideration.

Senshu (1959) stated that the time interval distribution especially for small value of τ is expressed by

$$\phi(\tau)d\tau = a\tau e^{-a\tau^2/2}\, d\tau\,, \tag{28}$$

where a is a constant, better than by Eq. (1). This distribution is introduced under the assumption that the non-conditional probability μ of earthquake occurrence is given by Eq. (2), which implies that μ increases with the lapse of time after the preceding event. His discussion, however, neglected the masking effect due to the successive occurrence of earthquakes. In order to check the effect, Eq. (18) is applied

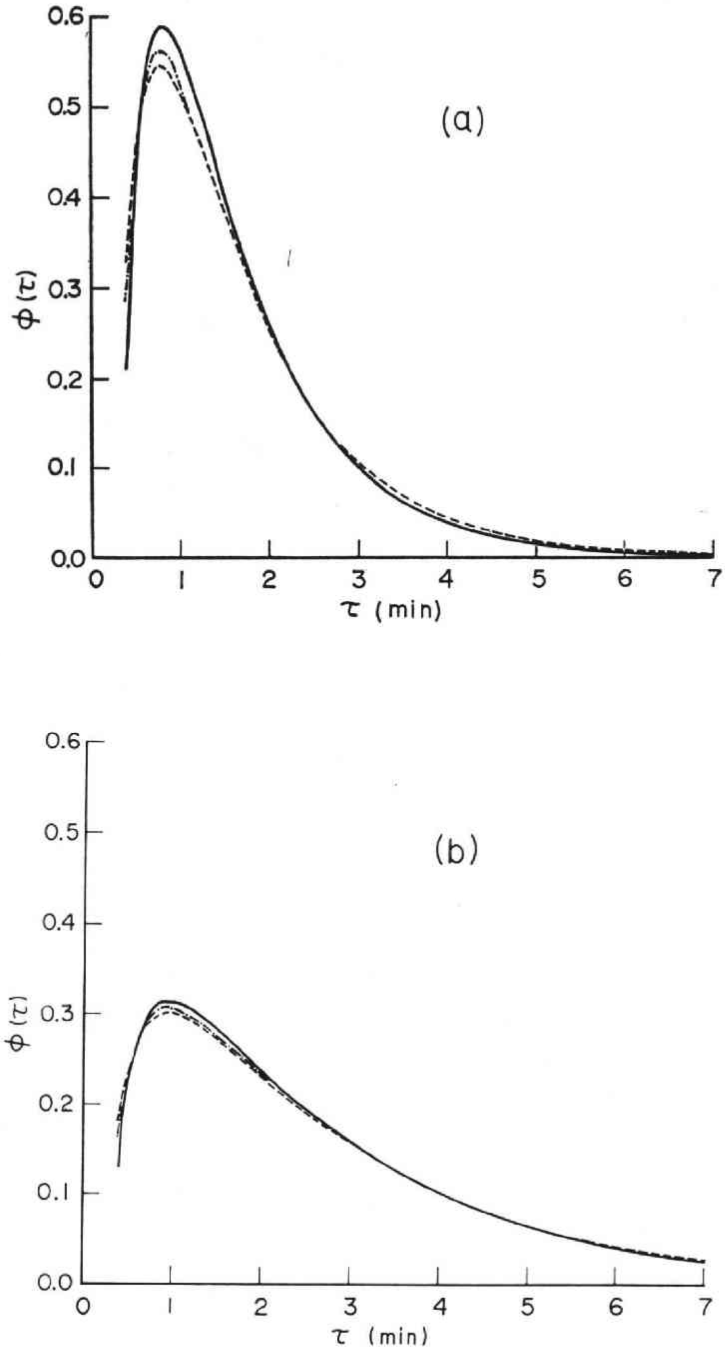

Fig. 3. Calculated time interval distribution for various values of constants. The solid curve; $m=2$, $n=2$, $\gamma=2.25$ and $\delta=1.25$, the dotted curve; $m=2$, $n=1$, $\gamma=2.25$ and $\delta=1.25$, the chain curve; $m=2$, $n=2$, $\gamma=2.85$ and $\delta=1.25$. (a) and (b) correspond to (a) and (e) in Fig. 1 respectively.

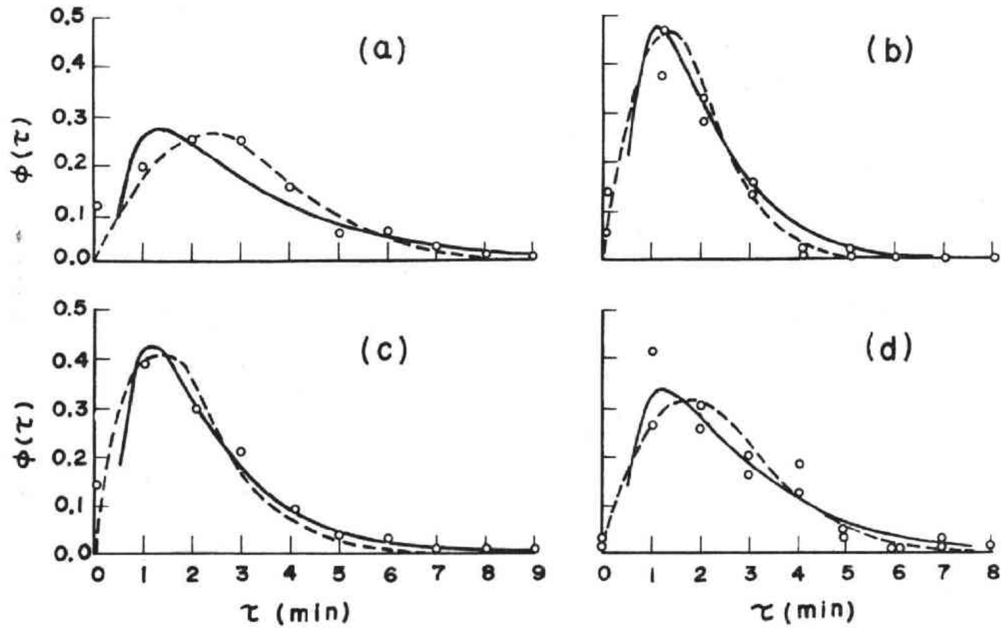

Fig. 4. Time interval distribution of the aftershocks of the Sanriku-oki earthquake of 1933 (redrafted from Senshu's paper (1959)). (a), (b), (c) and (d) correspond to Figs. 18, 19, 20 and 21 in Senshu's paper. Open circles, solid curves and dotted curves are the same as in Fig. 1.

to the data used by him. Some examples of the results are seen in Figs. 4 (a), (b), (c) and (d), which correspond to the cases of Figs. 18, 19, 20 and 21 in his paper respectively. The dotted line in the figure represents the calculation based on Senshu's idea, while the solid line indicates the result by Eq. (18) according to the authors' idea. Our result can explain the observed data at least as well as the Senshu's result.

On the other hand, the Senshu's idea is applied to our data of the aftershocks of the Tokachi-oki earthquake, 1968, as seen in Fig. 1. The dotted curve shows the distribution of time intervals due to the Senshu's formula of Eq. (28). It is evident in this case that the present consideration fits the data better than the Senshu's one. For the large value of average time interval as seen in Fig. 1 (f), the discrepancy of the Senshu's distribution from the observed data is naturally expected because of the following situation: Eq. (2) was adopted even for large value of τ, although Senshu's thought is that μ increases with time for small τ but tends to a constant value for large τ, as illustrated in Fig. 5 by solid line. This effect is naturally large for the large value of average time interval.

From above consideration, it is consequently concluded that the discrepancy of observed distribution of time interval between two successive earthquakes from the exponential distribution is mostly due to the masking effect discussed in the present study. It does not indicate, therefore, the real nature of occurrence but only an apparent phenomenon due to the successive occurrence of many events. Accordingly

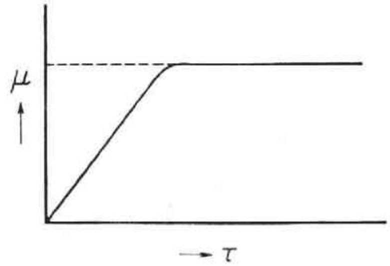

Fig. 5. Non-conditional probability of earthquake occurrence versus time interval. The solid line proposed by Senshu (1959) shows that the probability μ gradually increases just after the occurrence of an event and reaches a constant value. The reduction of μ at a small value of τ, however, can be interpreted as the apparent phenomenon due to the masking effect by the successive occurrence of earthquakes. Constant μ model is proposed as shown by dotted line.

the non-conditional probability μ of earthquake occurrence is safely considered to be kept constant even for very small value of time ineterval as illustrated in Fig. 5 by dotted line.

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