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The Masking Effect on Magnitude Distribution by the Successive Occurrence of Earthquakes

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Abstract: Since many earthquakes with small amplitudes would be masked by larger ones owing to the overlapping of successively occurring shocks, the m value in the Ishimoto-Iida's relation between amplitude and frequency of shocks is probably estimated to be smaller than the real value. Taking this effect into consideration, the amplitude versus frequency distribution is theoretically deduced similarly to the previous work (Hasegawa and Hamaguchi, 1970). The general expression is presented for the relation between the decrease in the m value due to the masking effect and the number of earthquake occurrences in a unit time. The numerical result is applied to the aftershock sequence of the Tokachi-oki earthquake of 1968. The apparent interdependence between the m value and the number of aftershocks in a unit time is removed by this correction; the variation of the m value ($1.80 \sim 1.97$) after the correction becomes smaller than that of the uncorrected value ($1.53 \sim 1.94$). This confirms the conclusion that the m value in the Ishimoto-Iida's relation did not change significantly in the whole period of observation.

It should be emphasized that the masking effect due to the successive occurrence of earthquakes must be significant especially in the case where a large number of shocks are observed in a short duration of time, such as the aftershock observation just after the main shock.

1. Introduction

It is well known in the earthquake statistics that the relation between the logarithm of the number of shocks and the logarithm of the amplitude or the magnitude can be expressed by a straight line so-called Ishimoto-Iida's or Gutenberg-Richter's relation. It has been frequently observed, however, that the number of shocks with small amplitudes or magnitudes is smaller than that expected by the above mentioned straight line. This causes the downward concave feature in the above stated distribution, as is frequently observed in actual cases. However, this may not be the real nature but may be due to missing of small shocks. For this reason, the parameter of m or b in the Ishimoto-Iida's or Gutenberg-Richter's relation is usually determined by assigning a straight line to the data in a linear range, that is, in a large amplitude range. However, the masking due to overlapping of the shocks should occur more or less even in the larger amplitude range. It is probable, therefore, that the value of m or b thus estimated is smaller than the real value, even when the larger amplitude events are exclusively used as data.

Recently the microearthquake observation with a high sensitivity was carried out and a large number of events could be detected in a short time. This situation would

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give an incorrect value of m or b owing to the missing of smaller shocks occurring in succession. It is important for the accurate discussion of statistical seismology, therefore, to evaluate the masking effect on the value of m or b.

The method used in this paper is based on the same concept as that stated in the previous paper (Hasegawa and Hamaguchi, 1970), which treats the discrepancy of the observed distribution of time interval between two shocks from the exponential distribution deduced from the theory, taking the masking effect into account.

2. Formulation

The following four items are assumed in our simple formulation.

1) The envelope of each seismogram can be symbolized as a simple form shown in Fig. 1, in which A is the maximum amplitude of an earthquake, T the total time of

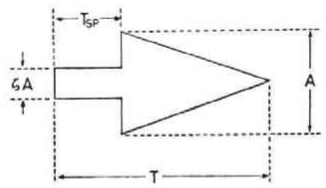

Fig. 1. Assumed envelope of wave form in a seismogram. The maximum amplitude, the ratio of P wave amplitude to the maximum amplitude, the duration time of oscillation and the S-P times are denoted by A, ζ, T and T_{sp} respectively.

wave duration, T_{sp} the S-P times of the shock and ζ the ratio of the amplitude of P waves to the maximum one: For the first approximation, the values of T_{sp} and ζ are assumed to be the same for all the shocks respectively.

2) The relation between A and T,

$$A \propto T^{\varepsilon}, \tag{1}$$

is adopted for all the events (Hasegawa and Hamaguchi, 1970), where ε is a constant.

3) The distribution of the maximum amplitude is independent of that of the time interval between two shocks.

4) It is not always that an earthquake cannot be counted, even when the earthquake is overlapped by other events which occur before or after the earthquake. If the earthquake is big enough, this can be recognized separately from the overlapping events. In counting the number of events, therefore, the earthquake in concern is regarded as a signal overlapped by noise of other events. The capability of counting is naturally thought to depend on the S/N ratio. In the present study the critical S/N ratio is expressed by n; an event is miscounted if the S/N ratio is smaller than n.

Many kinds of masking phenomena are considered as mentioned below. A variaty of combinations of maximum amplitude and time interval between two shocks should be taken into account (see Fig. 2) according to various cases: The case (A) indicates that the earthquake S is masked by the shock S_-' just before S; the case (B) the shock

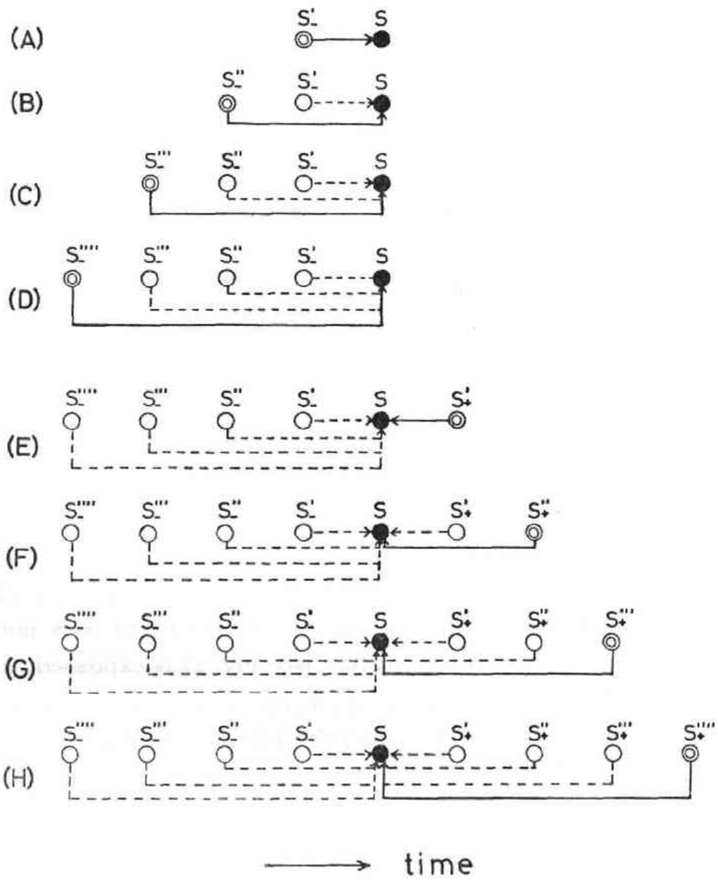

Fig. 2. Illustration of various combinations of earthquake occurrence. The earthquake S (solid circle) is assumed to be missed in counting by the masking effect of the earthquake (double circle) which is combined with S by the solid arrow and not to be masked by the earthquakes (open circles) which are combined with S by the dotted arrows.

S is not masked by S_-' but is masked by S_-'': Many other cases can theoretically be presumed as shown in Fig. 2. The probability that the shock is missed in counting by the masking effect is calculated in each case as follows.

Suppose the case (A) where an earthquake S is masked by S_-', i.e., the last shock before S. The maximum amplitude and the total duration time of oscillation are denoted by A and T and A_-' and T_-' for the shocks of S and S_-' respectively. The time interval between the two shocks is represented by τ. The notations are illustrated in Fig. 2. Then the probability of occurrence of these two shocks is expressed as

$$n(A_-')\,dA_-'\,\phi(\tau)\,d\tau\,, \tag{2}$$

where $n(A)$ and $\phi(\tau)$ are relative frequency distributions of the maximum amplitude and the time interval respectively. The earthquake S should be masked by the preceding shock S_-', when the two shocks are connected by the following relation:

$$A \leq n\cdot\{A_-'-\tau\cdot(A_-'-A_{\min})/(T_-'-T_{sp})\}\,, \tag{3}$$

where $A_{\max}$ and $A_{\min}$ mean the maximum and minimum values of A in population. Consequently the probability $R_1(A)$ that an earthquake S with maximum amplitude A is masked by the preceding shocks S_-' (case (A)) is expressed as

$$R_1(A)=\int_A^{A_{\max}} n(A_-')\,dA_-'\int_0^{\theta(A,A_-')}\phi(\tau)\,d\tau\,, \tag{4}$$

where

$$\theta(A,A_-')=(T_-'-T_{sp})\cdot(A_-'-A/n)/(A_-'-A_{\min})\,. \tag{5}$$

The earthquake already masked by the larger (preceding) event cannot contribute to mask the following one, so that Eq. (4) should be replaced by

$$R_1(A)=\int_A^{A_{\max}} n_{ob}(A_-')\,dA_-'\cdot\int_0^{\theta(A,A_-')}\phi(\tau)\,d\tau\,, \tag{6}$$

where $n_{ob}(A)$ is the distribution of the maximum amplitudes when the masking effect is taken into consideration. Eq. (6) shows the probability that an earthquake is missed in counting by the masking effect of the last one before the event (case (A)).

Next, the probability $R_2(A)$ of the case (B) that the event S is not masked by the shock S_-' but is masked by the event S_-'' (see Fig. 2) is expressed as follows.

$$R_2(A)=\int_A^{A_{\max}} n_{ob}(A_-'')dA_-''\iint^{(\tau+\tau')\leq\theta(A,A_-'')}\phi(\tau)\cdot\phi(\tau')\,d\tau\cdot d\tau'\cdot\{1-R_1(A)\}, \tag{7}$$

where A_-'' is the maximum amplitude of the shock S_-'' and τ' the time interval between S_-' and S_-''.

The probability $R_4(A)$ of the case (C) that the shock S is masked by the event S_-''' but neither by S_-' nor by S_-'' is written as

$$R_3(A)=\int_A^{A_{\max}} n_{ob}(A_-''')\,dA_-'''\iiint^{(\tau+\tau'+\tau'')\leq\theta(A,A_-''')}\phi(\tau)\cdot\phi(\tau')\cdot\phi(\tau'')\,d\tau\cdot d\tau'\cdot d\tau''$$
$$\times\{1-R_1(A)-R_2(A)\}, \tag{8}$$

where A_-''' denotes the maximum amplitude of S_-''' and τ'' means the time interval between S_-'' and S_-'''. Similarly we have

$$R_4(A)=\int_A^{A_{\max}} n_{ob}(A_-'''')\,dA''''\iiiint^{(\tau+\tau'+\tau''+\tau''')\leq\theta(A,A_-'''')}\phi(\tau)\cdot\phi(\tau')\cdot\phi(\tau'')\cdot\phi(\tau''')\,d\tau\cdot d\tau'\cdot d\tau''\cdot d\tau'''$$
$$\times\{1-R_1(A)-R_2(A)-R_3(A)\}\,, \tag{9}$$

using the similar notations to those in the previous cases.

Consider the masking effect by the events S_+', S_+'', S_+''', S_+'''', $\cdots$, which imply the first, second, third, fourth, $\cdots$ events following the shock S. The probabilities $Q_1(A)$, $Q_2(A)$, $Q_3(A)$, $Q_4(A)$, $\cdots$ of masking effects by S_+', S_+'', S_+''', S_+'''', $\cdots$ are expressed as follows by the similar deduction to the above cases.

$$Q_1(A)=\int_{A/\zeta}^{A_{\max}} n_{ob}(A_+')\cdot dA_+' \int_0^{T_{sp}} \phi(\tau)\cdot d\tau$$
$$\times\{1-R_1(A)-R_2(A)-R_3(A)-R_4(A)-\cdots\}, \quad (10)$$

$$Q_2(A)=\int_{A/\zeta}^{A_{\max}} n_{ob}(A_+'')\cdot dA_+'' \iint^{(\tau+\tau')\leq T_{sp}} \phi(\tau)\cdot\phi(\tau')\cdot d\tau\cdot d\tau'$$
$$\times\{1-R_1(A)-R_2(A)-R_3(A)-R_4(A)-\cdots-Q_1(A)\}, \quad (11)$$

$$Q_3(A)=\int_{A/\zeta}^{A_{\max}} n_{ob}(A_+''')\, dA_+''' \iiint^{(\tau+\tau'+\tau'')\leq T_{sp}} \phi(\tau)\cdot\phi(\tau')\cdot\phi(\tau'')d\tau\cdot d\tau'\cdot d\tau''$$
$$\times\{1-R_1(A)-R_2(A)-R_3(A)-R_4(A)-\cdots\cdots-Q_1(A)-Q_2(A)\}, \quad (12)$$

and

$$Q_4(A)=\int_{A/\zeta}^{A_{\max}} n_{ob}(A_+'''')dA_+'''' \iiiint^{(\tau+\tau'+\tau''+\tau''')\leq T_{sp}} \phi(\tau)\cdot\phi(\tau')\cdot\phi(\tau'')\cdot\phi(\tau''')\cdot d\tau\cdot d\tau'\cdot d\tau''\cdot d\tau'''$$
$$\times\{1-R_1(A)-R_2(A)-R_3(A)-R_4(A)-\cdots\cdots-Q_1(A)-Q_2(A)-Q_3(A)\}, \quad (13)$$

where A_+', A_+'', A_+''', and A_+'''' are the maximum amplitudes of earthquakes S_+', S_+'', S_+''' and S_+'''' respectively, and τ, τ', τ'' and τ''' are the time intervals between the pairs of (S, S_+'), (S_+', S_+''), (S_+'', S_+''') and (S_+''', S_+'''') respectively. Eqs. (10), (11), (12) and (13) apparently correspond to the probabilities of the cases (E), (F), (G) and (H) in Fig. 2.

If the miscount of events by the masking effect is taken into account, therefore, the frequency disitrbution of maximum amplitude is expressed by

$$n_{ob}(A)dA=n(A)dA\cdot\{1-R_1(A)-R_2(A)-R_3(A)-R_4(A)-\cdots\cdots$$
$$-Q_1(A)-Q_2(A)-Q_3(A)-Q_4(A)-\cdots\cdots\}, \quad (14)$$

using Eqs. $(6)\sim(13)$.

3. Numerical Results

The evaluation of Eq. (14) is performed by numerical integration: The functional form of the time interval distribution $\phi(\tau)$ is assumed to be the exponential form,

$$\phi(\tau)\, d\tau = \alpha\, e^{-\mu\tau}\, d\tau\,, \quad (15)$$

where μ is the number of earthquakes in a unit time and α a numerical constant. The functional form of the distribution of the maximum amplitude $n(A)$ is taken to be expressed by the so-called Ishimoto-Iida's relation;

$$n(A)\, dA = kA^{-m}\, dA\,, \quad (16)$$

where m and k are numerical constants and m is assumed here to be 2.0. The constants ε, n and ζ are assumed to be 2.25/1.25, 2 and 1/3 respectively as adopted in the previous paper (Hasegawa and Hamaguchi, 1970). The higher terms than $R_4(A)$ and

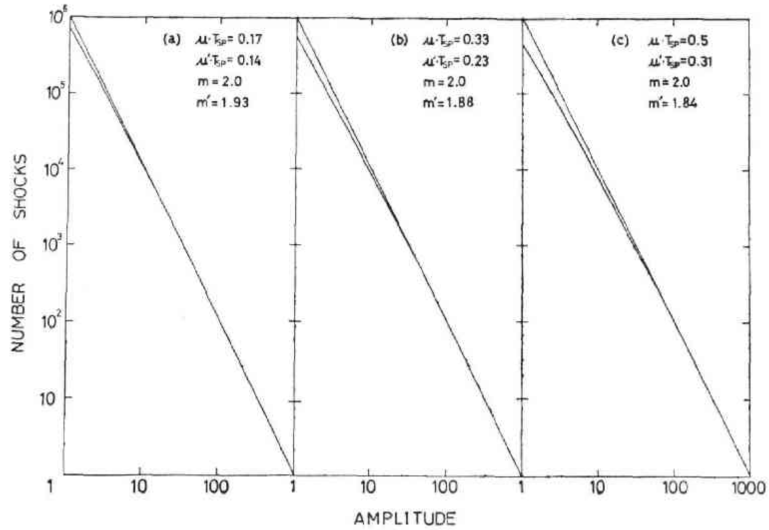

Fig. 3. Modifications of the amplitude-frequency relation by the masking effect are presented for some examples. The upper straight lines show the assumed Ishimoto-Iida's amplitude distribution (16) with $m=2.0$. The lower curves are obtained from Eq. (14) using the parameters μT_{sp} and m shown in each case. The values of $\mu' T_{sp}$ and m' are determined from the lower curves.

$Q_4(A)$ are neglected in the computation because of their small contribution.

The upper straight lines in Fig. 3 (a), (b) and (c) show the amplitude distributions $n(A)$ in Eq. (16) in the cases of $\mu T_{sp}=0.17$, 0.33 and 0.5 respectively and $m=2.0$; the lower curve in each case represents the amplitude distribution $n_{ob}(A)$ in Eq. (14) where the masking effect is taken into account. The distribution $n_{ob}(A)$ is clearly concaved downward in the region of small amplitude. If the straight line expressed by Eq. (16) is applied to each curve ignoring the masking effect, the apparent values of m' are obtained 1.93, 1.88 and 1.84 for the cases of (a), (b) and (c) in Fig. 3 instead of the true value of $m=2.0$, when the maximum likelihood method is adopted. This result shows that the m' value of the amplitude distribution in Eq. (14) is considerably smaller than the true value especially for a large value of μT_{sp}.

The value of μ, the probability of earthquake occurrence in a unit time, must be affected by the masking effect; the product μT_{sp} is estimated by the integration of amplitude distribution $n(A)$. The apparent values of $\mu' T_{sp}$ estimated in the cases of (a), (b) and (c) in Fig. 3 are 0.14, 0.23 and 0.31, while the true values are 0.17, 0.33, 0.5 respectively. These results indicate, for example, that the apparently estimated values of m and μ are 1.88 and 0.69 for the true values of 2.0 and 1.0 for $T_{sp}=20$ seconds. This difference cannot be neglected in the precise discussions in statistical seismology.

The relation between m' and μT_{sp} for various values of m is seen in Fig. 4, the m value ranging from 1.6 to 2.4 with the interval of 0.1. The difference in the m' values for $\mu T_{sp}=0$ and 0.5 is as large as 10%, which may sometimes be enough to lead us to

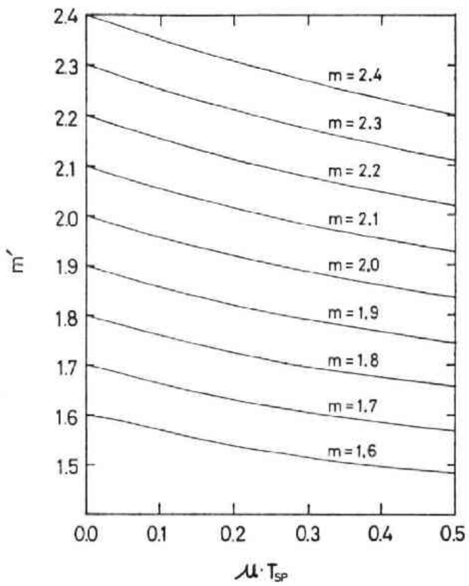

Fig. 4. Apparent m value, which is denoted by m', versus the product μT_{sp}. Each curve corresponds to the case when the real m value is 2.4, 2.3, 2.2, 2.1, 2.0, 1.9, 1.8, 1.7, or 1.6 respectively.

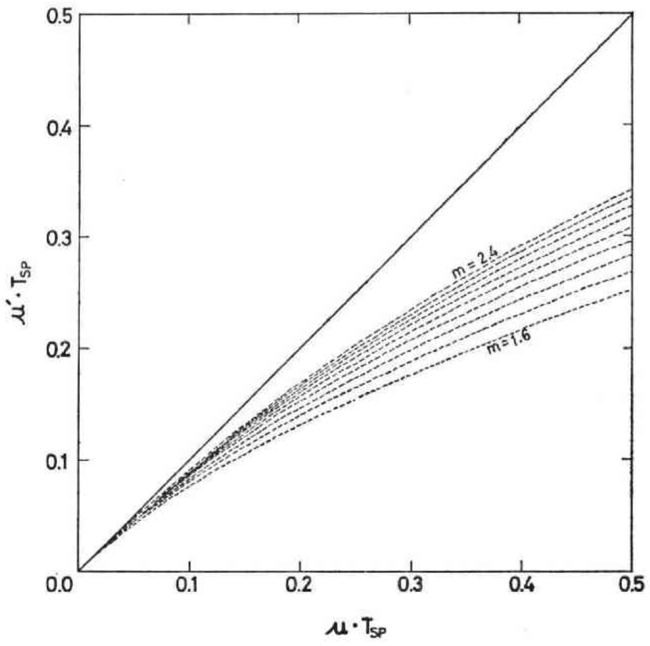

Fig. 5. The product μT_{sp} versus the product $\mu' T_{sp}$ for the various values of m. The dotted curves, from upper to lower, denote the cases where the real m takes the values of 2.4, 2.3, 2.2, 2.1, 2.0, 1.9, 1.8, 1.7, and 1.6 respectively. The solid line indicates $\mu=\mu'$.

an erroneous conclusion in the discussion of m value in population. Fig. 5 shows the relation betwen μT_{sp} and $\mu' T_{sp}$ for the various values of m from 1.6 to 2.4. This figure implies the difference between the counted number of earthquakes and the true number of earthquakes in a unit time, i.e., the number of shocks if the every shock is counted without missing. Figs. 4 and 5 are useful also for the correction of the apparent values of m' and μ' in order to estimate the true value m and μ from the observed data.

4. Example and Discussion

A good example for the discussion on the masking effect is obtained in the special observation of the aftershocks of the Tokachi-oki earthquake, 1968 (Hamaguchi and Hasegawa, 1970). The basic data in this case are summarized in Table 1. Fig. 6 (a) and (b) show the time variation of μ' and m', the time unit for μ' being taken as one minute. The abcissas of these figures correspond to the group number shown in Table 1, and the plot at the right end means the value determined from the observation made in 1969. The open circles indicate the corrected value of m calculated from μ' and m' by means of Figs. 4 and 5 under the assumption of $T_{sp}=20$ seconds. An increase in No. 12 was due to the occurrence of the secondary aftershocks accompany-

Table 1. Summary of the basic data for groups of 2,000 aftershocks and the corrected m values.

No	Date and time		Period (Hour)	Total Number of Shocks	m' Value	m Value
	Begin	End				
1	May 18 23^h 26^m	May 20 08^h 13^m	32.8	2000	1.71	1.86
2	May 20 08 13	May 21 23 26	39.2	2000	1.78	1.90
3	May 21 23 28	May 23 20 16	45.3	2000	1.74	1.84
4	May 23 20 49	May 25 22 07	49.3	2000	1.77	1.86
5	May 25 22 10	May 28 06 22	56.2	2000	1.77	1.84
6	May 28 06 24	May 30 20 30	62.1	2000	1.78	1.84
7	May 30 20 30	Jun 02 10 58	62.2	2000	1.87	1.91
8	Jun 02 11 00	Jun 04 23 30	60.5	2000	1.86	1.90
9	Jun 04 23 30	Jun 07 11 43	60.2	2000	1.87	1.90
10	Jun 07 11 45	Jun 10 08 27	68.7	2000	1.83	1.86
11	Jun 10 08 27	Jun 12 22 39	62.2	1578	1.87	1.90
12	Jun 12 22 42	Jun 14 10 01	35.3	2000	1.53	1.91
13	Jun 14 10 02	Jun 16 15 02	53.0	2000	1.83	1.89
14	Jun 16 15 04	Jun 19 00 35	57.6	2000	1.84	1.88
15	Jun 19 00 33	Jun 21 17 53	65.3	2000	1.88	1.91
16	Jun 21 17 56	Jun 24 09 32	63.6	2000	1.94	1.97
17	Jun 24 09 38	Jun 27 07 54	70.4	2000	1.89	1.92
18	Jun 27 07 54	Jul 01 09 57	98.0	2000	1.90	1.93
19	Jul 01 09 51	Jul 05 00 51	86.9	2000	1.84	1.86
20	Jul 05 00 52	Jul 09 02 50	97.9	2000	1.86	1.88
21	Jul 10 18 56	Jul 15 04 34	105.7	2000	1.78	1.81
22	Jul 15 04 47	Jul 19 06 35	97.9	2000	1.85	1.87
23	Jul 19 06 37	Jul 24 06 52	120.4	2000	1.86	1.88
24	Jul 24 06 57	Aug 02 06 13	167.3	2000	1.79	1.80
25	Aug 02 06 17	Aug 10 04 51	190.6	2000	1.89	1.90
26	Aug 10 04 53	Aug 19 23 01	234.1	2000	1.81	1.82
1969	Jun 28 18 40	Jul 12 18 47	336.1	1135	1.84	1.84

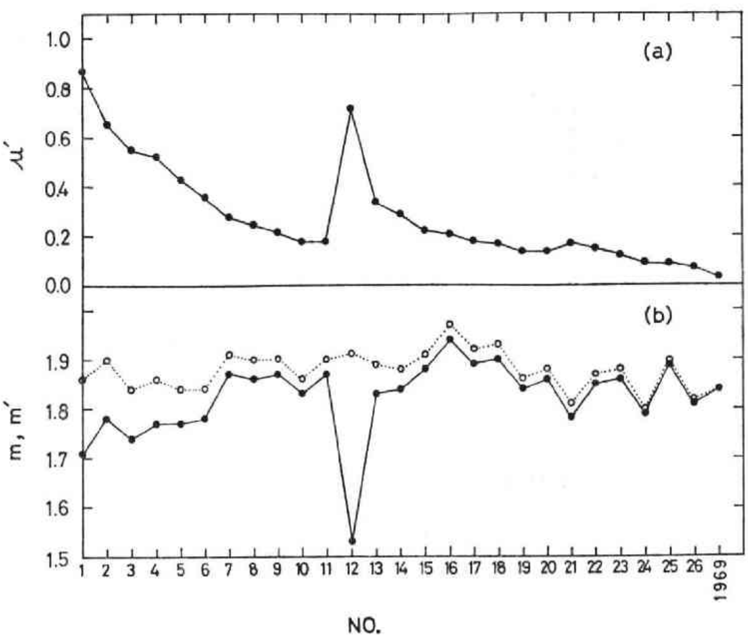

Fig. 6. (a) The variation of the apparent number of aftershock occurrences per minute with respect to the time after the main shock in the case of the Tokachi-oki earthquake of 1968.

(b) The variation of the m value with respect to time. Solid circle denotes the observed value m' and open circle denotes the corrected value m. The numerals of the abscissa are the ordinal number of each group shown in Table 1. The results in 1969, which were obtained from the observation of about one year after the main shock, are also plotted on the right hand side.

ing the large aftershock of M=7.2. The sudden increase and decrease of the activity in this period (June 12, 22^h42^m ~ June 14, 10^h01^m) show the unstationary nature of earthquake occurrence, and it is not consistent with the assumption that the time interval distribution is expressed by Eq. (15). The earthquakes in this period, therefore, were divided into nine small groups so that the above assumption held goods in each group. The value of m is calculated in each period and the mean value of them is plotted in Fig. 6 (b). The negative correlation betwen μ' and m' is clearly seen in Fig. 6, before the correction of the masking effect is made. The correlation is not so prominent between μ' and the corrected value m. This implies that the close correlation between μ' and m' is only an apparent result by the masking effect. The uncorrected values of m' are scattered between 1.53 and 1.94 but they are smoothed by the correction to converge within the range betwen 1.80 and 1.97. It is, therefore, concluded that the true value of m did not vary so much in the period of our observation.

Hamada (1968) reported that a clear negative correlation between the daily number of events and Ishimoto-Iida's coefficient was seen in the Matsushiro earthquake swarm.

The coefficient adopted in his paper is the value of m' according to our definition. He stated that the m' value decreased to $1.4 \sim 1.6$ after the two maxima of activities on April and August in 1966, whereas m' value was between 1.8 and 2.4 before the maxima. However, this negative correlation is quite similar to the case of Tokachi-oki earthquake discussed above, and it is very likely to be explained by the masking effect. Nishida (1970) reported that the m' value for the Matsushiro swarm was small in the range of small amplitudes in comparison with the value in the range of larger amplitudes. Yamakawa (1968) discussed on the small m' value immediately after a main shock. However, these facts may be explained by the masking effect at least qualitatively according to the present study.

The above examples reveals that the masking effect on m and μ values by the successive occurrence of many earthquakes must be taken into consideration in the precise discussion of statistical seismology as the problem of the variation of m value. A careful procedure is necessary in data processing to avoid this effect especially when the earthquake activity is very high.

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REFERENCES

Hamada, K., 1968: Ultra micro-earthquakes in the area aorund Matsushiro, Bull. Earthq. Res. Inst. Univ. of Tokyo, **46**, 271–318.

Hamaguchi, H. and A. Hasegawa, 1970: An investigation on the aftershocks of the Tokachi-oki earthquake of 1968, (1) Statistical study on magnitude distribution, Sci. Rep. Tôhoku Univ., Ser. **5**, Geophys., **20**, 85–105.

Hasegawa, A. and H. Hamaguchi, 1970: On the masking effect by the successive occurrence of earthquakes on time interval distribution, Sci. Rep. Tohoku Univ., Ser. **5**, Geophys., **20**, 135–143.

Nishida, R., 1970: Variation of the parameter "m" in the Ishimoto-Iida's relation on the Matsushiro earthquake swarm, Zisin (J. Seism. Soc. Japan), Ser. **2**, **23**, 142–151, (in Japanese with English abstract).

Yamakawa, N., 1968: Foreshocks, aftershocks and earthquake swarms, (IV) Frequency decrease of aftershock activity in its initial and later stages, Pap. Met. Geophys., **19**, 109–119.