Workshop on Irregular Stellar Pulsation at Mizusawa*)

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A workshop on stellar pulsation theory was held at the conference room of Mizusawa Astrogeodynamics Observatory on 22 August, 1989. The workshop was opened by the address of Professor I. Okamoto. The introductory talk was delivered by Professor M. Takeuti. Five talks were presented. The following is the abstracts of talks. The workshop was financially supported by National Astronomical Observatory of Japan.

Introduction

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Since the stars are the dynamical framework of galaxies and the principal agent to synthesize heavy elements, the studies of structures and evolutionary stages of stars are very important. Studies of stellar pulsations are useful because the features of stellar variability may depend on the details of stellar structures. The period-density relationship have been obtained based on linear pulsation theory. Combined with evolutionary models of supergiant stars, investigators succeeded in estimating the masses of classical cepheids. Hydrodynamical models of RR Lyrae stars give us the helium abundance.

The theoretical studies of pulsating stars have proceeded in several ways. We can estimate linear adiabatic periods from linear adiabatic study (LA). Linear nonadiabatic studies (LNA) gives us more reliable periods and growth rates of pulsations. Hydrodynamic models which are the numerical simulation based on the time-dependent hydrodynamic equations are useful to study the time variations of observed quantities. The hydrodynamical studies require a lot of computing time, however. Therefore, a qualitative nonlinear study is useful for preparing such an expensive investigation. Nonlinear amplitude equations based on an assumption that oscillations can be decomposed to several harmonic waves were investigated. Nonlinear equations without any assumption on the separation of variables were also studied. Baker's one-zone stellar model is so simple that we cannot reproduce their luminosity changes, but quite useful for studying the exciting processes of critical ionization layers in stellar envelopes.

*) Communicated from Professor M. Takeuti.

The qualitative properties of nonlinear stellar pulsations found in recent papers (Saito et al., Seya et al.) are as follows:

1) one-zone models which have only one mode show a period-doubling sequence proceeding to chaotic oscillations when convenient conditions are fulfilled in opacity changes;
2) nonlinear coupling of a model cepheid in a two-mode case including self-exciting processes shows period-doubling bifurcation;
3) with the changes of parameters, oscillations synchronized in a single mode move to double-mode oscillations in phase-locking.

Not only the period-doubling bifurcation but also chaotic intermittency are found in hydrodynamic simulations. The period-doubling in nearly resonant case is also confirmed. Nevertheless, the double-mode case in phase-locking has not been confirmed yet in hydrodynamical studies except for a few example of double-mode oscillations.

At the end of my introductory talk, I wish to express my hope that the attendants pay sufficient attention to dominating physical processes in pulsating stars, even though dynamics treat the action and reaction by ignoring their physical nature in general. The essential contribution to astrophysics is usually come from the discovery of essential physical processes controlling the phenomena.

On linear nonadiabatic pulsations of yellow bright stars

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Pulsations of yellow bright stars sometimes show semiirregular properties. So that, the study of these stars is very important. The investigation is useful also to clarify the parent stars of planetary nebulae. Compared with its importance, linear nonadiabatic periods which give basic data for pulsating stars have not been clearly established.

Takeuti and Aikawa (1986: Sci. Reports Tohoku Univ. Eighth Ser. 7 109) calculated linear nonadiabatic periods of yellow bright stars. Since Worrell's careful study (1986: Mon. Not. R. astr. Soc. 223 787) is not identical with their results, it seems necessary to investigate again the linear nonadiabatic periods by using much more detailed calculations. We try in determining them using 400 zoning models. We examined Zalewski's unpublished procedure which indicates the connection between adiabatic modes and nonadiabatic ones.

We compared standard method originally derived by Castor (1971: Astrophys. J. 166 109). Takeuti and Aikawa's procedure, Worrell's one, and Zalewski's one. The results show Zalewski's one is the best for solving strongly nonadiabatic pulsations. So we use this one for calculating the pulsations properties of the models investigated by Takeuti and Aikawa (loc. cit.). Two procedures give nearly identical periods for the fundamental mode oscillations. Worrell's B and C modes seem the nonadiabatic cases derived from the first and second overtone modes when we use Zalewski's procedure. The real role of these modes in stellar pulsations has not been resolved till now.

Chaotic behavior in nonlinear two-mode coupling stellar models

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The time developments of two-mode coupling models with the Van der Pol type force of radial stellar oscillations (M. Takeuti, 1984: in Non-Linear Phenomena in Stellar Outer-Layers, Tohoku Univ.) are studied. The equations are as follows:

$$d^2x_1/dt^2 = -\sigma_1^2 x_1 + \sigma_1^2 [((1/2)C_{111}x_1 + C_{112}x_2)x_1 + \varepsilon_1(1-\alpha_1^2 x_1^2)(dx_1/dt) + (1/2)C_{122}x_2^2]$$

$$d^2x_2/dt^2 = -\sigma_2^2 x_2 + \sigma_2^2 [((1/2)C_{222}x_2 + C_{212}x_1)x_2 + \varepsilon_2(1-\alpha_2^2 x_2^2)(dx_2/dt) + (1/2)C_{211}x_1^2]$$

As the coupling coefficients C_{ijk}, the values for a model classical cepheid (M. Takeuti and T. Aikawa, 1981, Science Reports Tohoku Univ.) are adopted. Since these values are around 3 in Mode 1 and around 6 in Mode 2, the modal coupling is rather strong.

We choose $\sigma_1^2 = 1$, $\sigma_2^2 = 1.9$ and $\varepsilon_1 = 0.1$, and take α_1^2 $(i = 1,2)$ and ε_2 as variables. Three cases of α_1^2 are chosen: (I) $\alpha_1^2 > \alpha_2^2$, (II) $\alpha_1^2 = \alpha_2^2$ and (III) $\alpha_1^2 < \alpha_2^2$. We have already obtained complex beats, phase-locking and period-doubling bifurcation (K. Seya, Y. Tanaka and M. Takeuti, 1989: Submitted to Publ. Astron. Soc. Japan). We now show Poincare sections for $\varepsilon_2 = 0.15$ for the cases I and II which are given in Fig. 1(a) and (b), respectively. The points at which the values of x_1 change from negative to positive are plotted on the $(x_2 - dx_2/dt)$ plane. Since a closed curve is formed in the case I, we can see that the trajectory moves on the surface of torus (i.e. quasi-periodic). In the case II, the surface is folded, which corresponds to the chaotic behavior.

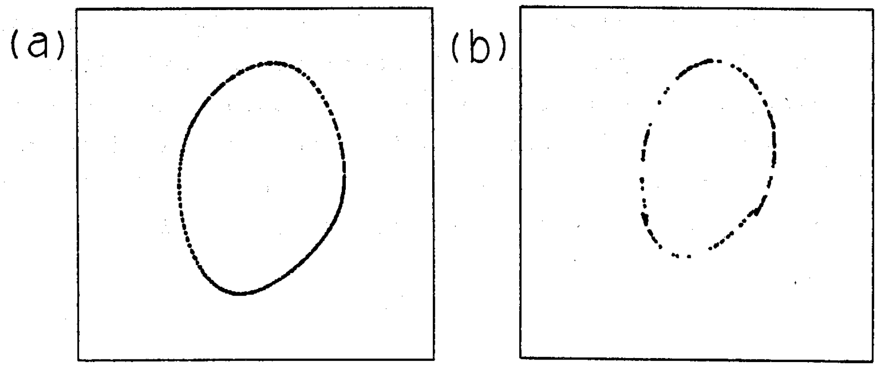

Fig. 1

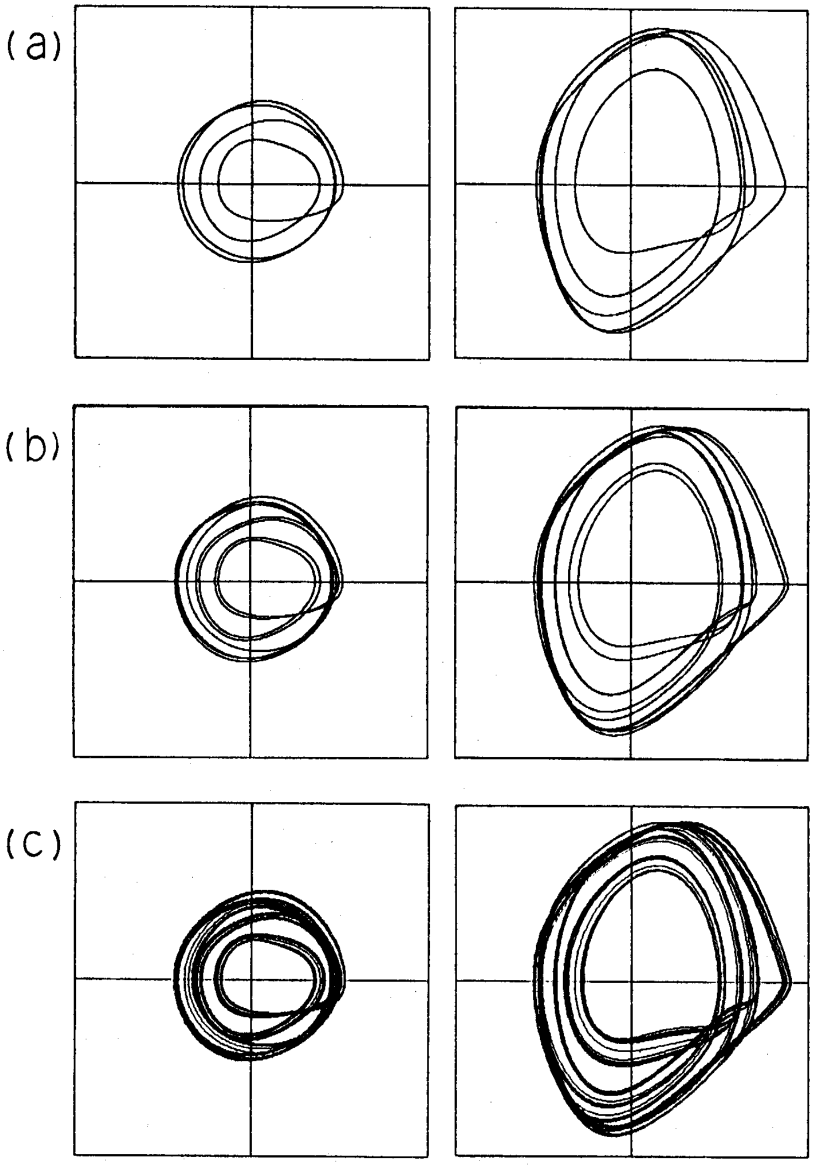

Fig. 2

On the Poincare section for the case III, a closed curve which is far smaller than that in the case I is formed.

Next, period-doubling bifurcation can be seen after phase-locking with the change of α_2^2 for $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.24$ and $\alpha_1^2 = 800$. An example of this is shown in Fig. 2: (a) $\alpha_2^2 = 510$, period 1; (b) $\alpha_2^2 = 500$, period 2; (c) $\alpha_2^2 = 489$, chaos. The subpanels on the left are the orbits on the $(x_1 - dx_1/dt)$ planes, and those on the right are the orbits for Mode 2. We regard the phase-locking with 4:5 as period 1 here.

In conclusion, we found out that the coupled nonlinear model oscillators show quasi-periodic, phase-locking, period-doubling and chaos. It is emphasized that the phase-locking with 5:7 is obtained (K. Seya et al., loc. cit.)

Three mode nonresonant coupling in stellar pulsation

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Recent observations have shown the existence of many multi-mode pulsating stars. Theoretically, there are two approaches to study modal coupling in stellar pulsation. One is to study the oscillator model equations as Takeuti and Aikawa (1981: Sci. Rep. Tohoku Univ. 8th Ser. 2 106). The other is to study the amplitude equations as Dziembowski and Kovács (1984: Mon. Not. R. astr. Soc. 206 497). Some properties of pulsating stars are explained by their researches. However, no one studied the three mode nonresonant coupling case. As three dimensional systems have essential difference from two dimensional ones, especially on the occurence of chaotic behaviors, it is important to study three mode nonresonant coupling as a general case.

In present study, we qualitatively investigated the amplitude equations for nonresonantly coupling three mode case. An example of the three mode interacting hydrodynamic model of classical Cepheid is also presented.

The amplitude equations which studied here are followings:

$$\frac{dX}{dt} = \varepsilon_F(1 - \alpha_{11}X - \alpha_{12}Y - \alpha_{13}Z)X ,$$

$$\frac{dY}{dt} = \varepsilon_{10}(1 - \alpha_{21}X - \alpha_{22}Y - \alpha_{23}Z)Y ,$$

$$\frac{dZ}{dt} = \varepsilon_{20}(1 - \alpha_{31}X - \alpha_{32}Y - \alpha_{33}Z)Z ,$$

where $X = A_F^2$, $Y = A_{10}^2$, $Z = A_{20}^2$ and, A_F, A_{10}, A_{20}, ε_F, ε_{10}, ε_{20} are the amplitudes and the linear growth rates of the fundametal, the first overtone, and the second overtone, respectively. $\alpha_{ij}(i,j=1,2,3)$ are the mode coupling coefficients.

Next, we studied the properties of singularities by stability analysis. The result shows that in a certain case the three mode fixed point has possibility to being an unstable focus and the other four points are unstable. Therefore, there is the possibility of chaotic behavior of three amplitudes.

On the other hand, we found an example of three mode nonresonantly interacting hydrodynamic model of classical Cepheid. Model parameters are $L=1500L_{\odot}$, $T_{eff}=6000K$, and $M=5.0\ M_{\odot}$. Linear nonadiabatic periods for the fundamental, the first overtone, and the second overtone are 3.60944, 2.65190, and 2.11051 (days), respectively. There is no significant resonance and all modes are pulsationally unstable. The hydrodynamical model shows complicated behavior, and none of three modes is negligible. Although the final state of this model is unclear for the limited computation time, obviously the nonresonant three mode coupling has an imoortant role.

In conclusion, irregular behavior by the change of mode coupling coefficients, which has never been considered, can occur in three mode nonresonant coupling amplitude equations. We also presented an example of three mode nonresonantly interacting hydrodynamic model of classical Cepheid.

Chaotic oscillation in hydrodynamic models for less-massive supergiant stars

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Chaotic behaviors of oscillation are commonly observed in hydrodynamic models for less-massive supergiant variable stars. Takeuti (1987: Astrophys. Space Sci. 136, 129) has suggested that the irregular oscillation is closely related with the deterministic chaos. In fact, Aikawa (1987: Astrophys. Space Sci. 139, 281) has shown that the type of the transition from limit cycles to the irregular oscillations in less-massive supergiant stars is in agreement with the intermittency in deterministic chaos.

Meantime, Kovacs and Buchler (1988: Astrophys. J. 344, 971) have investigated the transition in less-massive supergiant stars with a wider range of stellar parameters, and found the intermittency for luminous stars, and the period-doubling cascade for lesser luminous stars as types of the transition.

In this report, we investigate the results of Kovacs and Buchler using the TGRID code (Simon and Aikawa, 1986: Astrophys. J. 304, 249) which was used in Aikawa (1987).

The stellar parameters used in the present study are as follows:

$$M = 0.6\ M_{\odot}\ ,$$

$$L = 400,\ 500,\ 600\ L_{\odot}\ ,$$

and the chemical abundances of Population II are assumed with $x = 0.745$, $z = 0.005$. The effective temperature is a control parameter. For each of the three luminosities, we thus constructed a model sequence with changing the effective temperature. We run each model in the sequence for about 400 pulsation periods to see the behavior of oscillation in non-linear regime.

We demonstrate the summary of the present study on the HR diagram (Fig.1). For $L = 400\ L_{\odot}$ the models show limit cycles for a wide range of effective temperature. The model at $T_e = 5000$ K has a periodic oscillation with period 2. We thus expect the first subharmonic bifurcation between $T_e = 5100$ and 5000 K models.

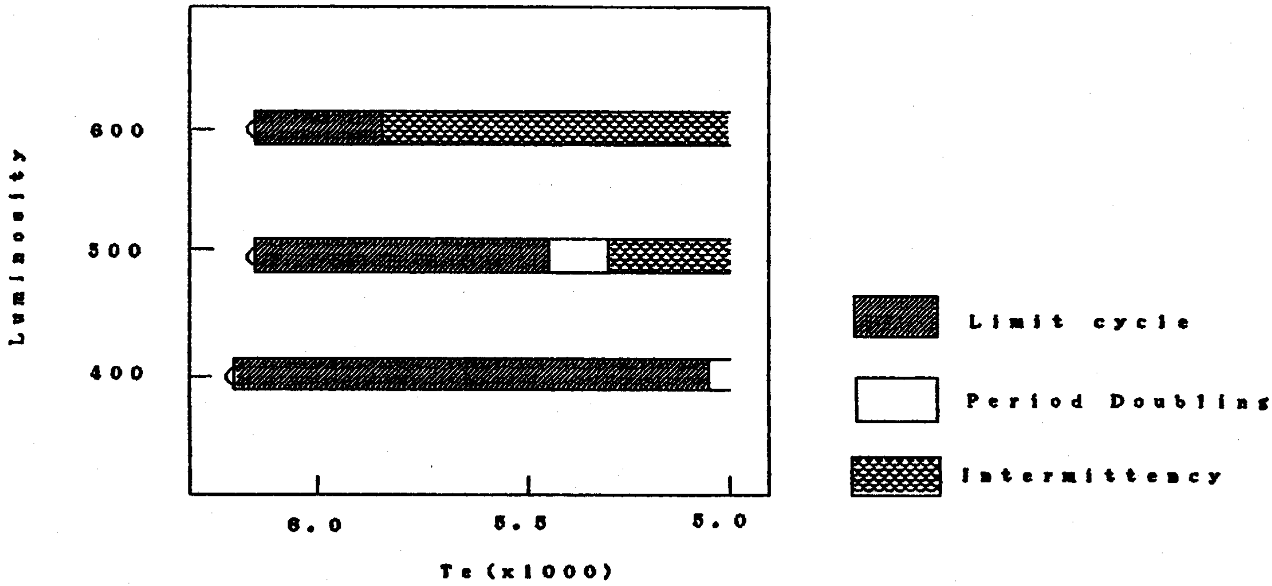

Fig. 1 Summary of the behavior of oscillation in nonlinear regime presented on the HR diagram. We use fifferent symbols for the region of limit cycle, period-doubling and intermittency. A small circle at the edge of each sequence indicates the blue edge of the fundamental mode.

For $L = 500\ L_{\odot}$, the situation is much more complicated. We confirm the subharmonic bifurcation started at about $T_e = 5360$ K, up to the second subharmonic bifurcation. The period 4 oscillation generated from the final bifurcation, however, goes to irregular oscillation thought the intermittency abruptly, as the effective temperature is decreased. This means that the intermittency takes place in the way of the period-doubling cascade.

For $L = 600\ L_\odot$, the intermittency takes place directly through the limit cycle. This sequence thus has no subharmonic bifurcation.

We have shown that the types of bifurcation from the limit cycle in hydrodynamic models for less-massive supergiant stars are the intermittency for luminous stars, and the subharmonic bifurcation for lesser luminous stars. The conclusion is qualitatively in agreement with that of Kovacs and Buchler (1988). The present results, however, suggest transition from regular to irregular oscillations are caused by the intermittency for both the cases.