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論　文　内　容　要　旨

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Abstract

This thesis consists of two parts. The first part concentrates on the query complexity of algorithms for game trees. In particularly, we treat multi-branching trees, and study eigen-distribution over independent distributions and correlated distributions on assignments for them. The second part is to investigate some mathematical results which are used to prove several game properties in the context of reverse mathematics, including arguments of fixed point theorems and continuous games.

Part I: Game Trees

Game trees are simple models of computation to analyze some difficult problems, and have been extensively investigated from various aspects in many literatures. In the first part, we mainly study the query complexity of algorithms on game trees. Saks and Wigderson [6] first studied the query complexity that computes game trees over randomized algorithms, which is called randomized complexity. It is usually difficult to directly analyze randomized algorithms. Thanks to Yao's principle [11], which implies the equivalence between randomized complexity and distributional complexity, we can instead study the distributional complexity, which runs over deterministic algorithms and easier to analyze. For this reason, Liu and Tanaka [1] proposed the concept of eigen-distribution on assignments, which achieves the distributional complexity computing uniform binary trees. They showed that an independent distribution on assignments is an independent identical distribution if it achieves the distributional complexity. Suzuki and Niida [10] extended their study by fixing the probability of root of a uniform binary tree.

Compared with previous research in this area, we concentrate on multi-branching trees and eigen-distribution over independent distributions and correlated distributions for them.

We begin with reviewing the background and some basic definitions about game trees in Chapter 3. In Chapter 4, we deal with eigen-distribution for balanced multi-branching trees. At first we will give some technical lemmas to prove that the average cost on 1-set is larger than that on other closed sets. Based on these results, we show that an eigen-distribution is equivalent to E^1-distribution for any AND-OR balanced multi-branching tree with respect to all closed subset of all alpha-beta pruning algorithms. Such a method is quite different from the proof given by Suzuki and Nakamura [9] for uniform binary trees. Then we mainly show that the uniqueness of eigen-distribution holds for the set of all alpha-beta pruning algorithms, although the uniqueness does not

hold for the set of directional algorithms.

In Chapter 5, we treat eigen-distribution over independent distributions (ID) for multi-branching trees. For any ID d, we define a directional algorithm DIR_d, and show it is optimal among all the depth-first algorithms with respect to d. Applying this result, we show that, for any ID d, there exists an IID d' such that the expected cost with d is not larger than that with d' following DIR_d. Then we extend the Liu-Tanaka's results for uniform binary trees to balanced multi-branching case, which has been left open since it was posed in [1].

The results represented in Chapters 4 have been published in [2] and [3], and the results in Chapter 5 appeared in [4].

Part II: Reverse Mathematics

Reverse mathematics is a well-known research program in the foundations of mathematics. It was founded by Harvey Friedman and Stephen Simpson in the 1970's and developed in many publications. A basic goal of reverse mathematics is to classify mathematical theorems according to the set existence axioms used in their proofs [7]. Though fruitful reverse mathematical results have been achieved in various fields, game theory is so far one of the undeveloped lands. In the second part of my thesis, we mainly investigate some mathematical results which are used to prove several game properties in the context of reverse mathematics.

Since we know that fixed point theorems are used to prove the existence of Nash equilibria of games, we first investigate the logical strength of some fixed point theorems in the context of reverse mathematics. Note that the logical strength of Brouwer fixed point theorem has been studied by Shioji and Tanaka [8]. They showed that it is equivalent over RCA_0 to WKL_0, which implies that the existence of Nash equilibria for finite games is provable in WKL_0. Then, we move on to the study on reverse mathematics and continuous game, which is a mathematical generalization of finite games. A continuous game is usually defined as an infinite game in which the strategy sets are compact and the payoff functions are continuous.

In Chapter 8, we investigate the logical strength of two types of fixed point theorems in the context of reverse mathematics. One is concerned with extensions of Banach contraction principle. Among theorems in this type, we mainly show that Caristi fixed point theorem is equivalent to ACA_0 over RCA_0. The other is dedicated to topological fixed point theorems like Brouwer fixed point theorem. We introduce the variants of Fan-Browder fixed point theorem and Kakutani fixed point theorem, say FBFP and KFP respectively. Then we show that FBFP is equivalent to WKL_0 and KFP is equivalent to ACA_0, over RCA_0. In addition, we also study some applications of Fan-Browder fixed point theorem to game systems.

In Chapter 9, we study reverse mathematics and continuous games. We present a new version of Stone-Weierstrass theorem, which can define a product measure from a given mixed profile. Working in RCA_0, we show that any sequence of probability measures on a compact space that has a weak convergent subsequence is equivalent to ACA_0. Using this result, we show that the existence of mixed Nash equilibria for any continuous game is provable in ACA_0.

The results in Chapter 8 appeared in [5].

Reference

[1]. C.G. Liu and K. Tanaka: Eigen-distribution on random assignments for game trees, Information Processing Letters, vol. 104, no. 2, pp. 73-77, 2007.

[2]. W. Peng, S. Okisaka, W. Li, and K. Tanaka: The Eigen-distribution for multi-branching trees, in: Proc. The International MultiConference of Engineers and Computer Scientists, IMECS, 16-18 March, Hong Kong, pp. 88-93, 2016.

[3]. W. Peng, S. Okisaka, W. Li, and K. Tanaka: The uniqueness of eigen-distribution under non-directional algorithms, IAENG International Journal of Computer Science, vol. 43, no. 3, 2016.

[4]. W. Peng, N. Peng, K.M. Ng, K. Tanaka, and Y. Yang: Optimal depth-first algorithms and equilibria of independent distributions on multi-branching trees, Submitted to Information Processing Letters.

[5]. W. Peng and T. Yamazaki: Two kinds of fixed point theorems and reverse mathematics, Submitted to Mathematical Logic Quarterly.

[6]. M. Saks and A. Wigderson: Probabilistic Boolean decision trees and the complexity of evaluating game trees, in: Proc. 27th Annual IEEE Symposium on Foundations of Computer Science (FOCS), pp. 29-38, 1986.

[7]. S.G. Simpson: Subsystems of Second Order Arithmetic, Cambridge University Press, 1999.

[8]. N. Shioji and K. Tanaka: Fixed point theory in weak second order arithmetic, Annals of Pure and Applied Logic, vol. 47, pp. 167-188, 1990.

[9]. T. Suzuki and R. Nakamura: The eigen distribution of an AND-OR tree under directional algorithms, IAENG International Journal of Applied Mathematics, vol. 42, no. 2, pp.122-128, 2012.

[10]. T. Suzuki and Y. Niida: Equilibrium points of an AND-OR tree: under constraints on probability, Annals of Pure and Applied Logic, vol. 166, no. 11, pp. 1150-1164, 2015.

[11]. A.C.-C. Yao: Probabilistic computations: toward a unified measure of complexity, in: Proc. 18th Annual IEEE Symposium on Foundations of Computer Science (FOCS), pp. 222-227, 1977.

別　紙

論文審査の結果の要旨

本博士論文は，ゲームの理論的諸相に対して論理的な視点から探査するものである．その内容は２部に分かれる．

第１部は，ランダム入力をもつブール木の決定アルゴリズムの研究である．アルゴリズムのコストは，ブール木の値の決定に必要となる，葉(変数)に割り与えられる値へのクエリ数で測られる．当然コストを小さくするようなアルゴリズムが望ましく，そのような最小コストを逆に最大にするような入力の確率分布を劉-田中は「固有分布」と呼んだ．そして，AND-OR 二分木に対する固有分布は，ブール木の値を１にして，かつ葉に割り当てられる１の個数が少ないような割り当ての集合に対する一様分布になることを示した．他方，非分岐な(directional)アルゴリズムに制限した場合には，固有分布は多数存在することも鈴木-中村が示している．本研究は，これらの先行結果を多分岐均整木に一般化したものである．また，最小コストを最大にするような i.d.分布は i.i.d.分布になるという二分木に対する先行研究(鈴木-仁井田ら)に対しても技巧的な手法で多分岐均整木への一般化を成功させた．

第２部は，数学基礎論の逆数学プログラムに基づく，種々の不動点定理とその応用としての Nash の均衡の存在定理についての論理的分析である．塩路-田中の先行研究により，Brouwer の不動点定理の証明に必要十分な公理系が WKLo であることが知られていたが，本論文では Fan-Brouder の不動点定理にも WKLo が必要十分であることが示され，それにより，制限付きの連続ゲームの均衡解の存在が WKLo で示された．これ以外にも不動点定理の様々なバリエーションについて逆数学的に考察した．

以上のことは，論文提出者が自立して研究活動を行うに必要な高度の研究能力と学識を有することを示している．したがって，彭偉光提出の博士論文は，博士（理学）の学位論文として合格と認める．