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Recurrent Preemption Games

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Recurrent Preemption Games*

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Abstract

I consider a new model of an infinitely repeated preemption game with random matching, termed *the recurrent preemption game*, wherein each player's discount factor depends on whether she wins the current game. This model describes sequential strategic technology adoptions in which a company becomes outdated by failing to maintain a position at the forefront of innovation. Assuming incomplete information about the presence of a rival, I clarify how the prominence of the innovator's dilemma influences the degree of excessive competition in preemption. I also reveal interesting properties demonstrated by the unique symmetric Nash equilibrium of the recurrent preemption game.

Keywords: Recurrent Preemption Game, Strategic Technology Adoption, Innovator's Dilemma, Unique Equilibrium, Random Technology.

JEL Classification Numbers: C72, C73, L13, O30, O31

1. Introduction

This study investigates the *recurrent preemption game*, which is defined as a new version of an infinitely repeated game with random matching. The component game is a simple form of Bertrand duopoly that I term *the preemption game*. I assume *incomplete information* in that each player does not know whether her rival is present or absent. This game mainly departs from existing repeated games in that the discount factor (or the survival rate) of each player is not constant across component games and instead depends on the state and the consequence of the current component game, such as whether a player's rival is present and whether she wins the current game. This study shows that the recurrent preemption game demonstrates the *unique* symmetric Nash equilibrium, and reveals various important properties of this equilibrium.

This study situates the recurrent preemption game within the following dynamics of sequential strategic technology adoptions. A company attempts to develop and adopt a new technology within a limited time interval. If the company successfully adopts the technology earlier than its rival, then it earns the corresponding time-dependent adoption value as the winner's payoff; otherwise, it earns nothing. By postponing the timing of adoption, the company can further develop the technology and increase this adoption value, but the risk of being preempted by its rival increases at the same time.

Just after the end of the time interval, the company can enter the next preemption game with a positive probability; the rival in the next game is different from the rival in the current game due to the randomness of matching. Importantly, only one of the two companies in the current game will progress to the next game. Hence, the two companies are rivals in two respects: 1) they are rivals in the scramble over the next opportunity for technology adoption, and 2) they are rivals in the competition for preempting the adoption of the current technology.

I specifically focus on the situation in which the likelihood of the winner entering the next game is not the same as the likelihood of the loser entering the next game. The likeliness of each outcome crucially depends on the relative importance between the following winner's advantage and disadvantage. The case in which the loser is more likely to enter the next game than the winner corresponds to the prominence of the winner's disadvantage and is termed the *innovator's dilemma* (Christensen 1997; Igami 2017): the winner is constrained by the current technology, and therefore tends to be behind of the loser in starting the development of the next technology. Meanwhile, the case in which the winner is more likely to enter the next game than the loser corresponds to the prominence of the winner's (i.e., the innovator's) advantage: the winner outweighs the loser in technical and information-gathering skills. This study clarifies that the relative importance between the innovator's dilemma and advantage is crucial for understanding strategic behavior.

To be more concrete, I envision the following scenario behind the recurrent preemption game. A commercial company generally wants to ensure its long-term survival in emerging industries. To do so, it must constantly ride the tide of innovations that create new markets through competition across existing industries. However, the presence of a rival company can threaten its ability to remain at the forefront of the next innovation. Once a company loses its forefront position, its growth can quickly slow and it can become outdated.

Importantly, the company that wins the current race of technology adoption can also succeed in developing talent with the ability to discover and develop the next technology. However, the innovator's dilemma recalls that the winner of the current adoption race may not necessarily win the next opportunity—the loser of the current adoption race may *headhunt* the talent developed by the winner. Moreover, the company that successfully enters the next opportunity may still have a rival in the form of either a new venture or a company that previously worked in a different industry.

With this scenario in mind, the recurrent preemption game sharply distinguishes between the following two conflicts between the rivals in the current technology adoption. The first conflict is about meeting the next opportunity for technology adoption, which is expressed by the dependence of each player's discount factor on whether she wins the current game. The second conflict is the first-come, first-served style of the race for current technology adoption, which is expressed by the current, one-shot preemption game. This study is the first theoretical attempt to incorporate this first conflict into strategic technology adoption.

For our purposes, it is important to note that this study is related to existing literature on timing games (and specifically preemption games) in which various aspects of incomplete information are incorporated. Notable works here include those by Fudenberg and Tirole (1986), Abreu and Brunnermeier (2003), Brunnermeier and Morgan (2010), Hopenhayn and Squintani (2011), Bobtcheff and Mariotti (2012), and Matsushima (2013; forthcoming). In contrast to this study, these works commonly investigate static games.

The theoretical advantage of this study's model is that it involves the *unique* symmetric Nash equilibrium. In this respect, this study is closely related to Matsushima's (2013; forthcoming), which assumed incomplete information implying that each player expects that her rival is absent (or irrational) with a positive probability, and showed the uniqueness of the Nash equilibrium. This study extends this uniqueness to the recurrent preemption game by adding the symmetry constraint.

The unique symmetric Nash equilibrium in the recurrent preemption game makes clear that the above-mentioned first conflict has the following significant effects on strategic behavior. The prominence of the innovator's dilemma eases preemptive competition and consequently promotes more efficient technology adoption, while the prominence of the innovator's advantage worsens preemptive competition and consequently promotes less efficient technology adoption. The players' degrees of competitiveness from the viewpoint of the second conflict depend on the difference between the level of the current technology's adoption value and the average level. Importantly, the manner of this dependence substantially differs based on the prominences of the innovator's dilemma and advantage. If the innovator's dilemma is prominent, then player competitiveness in the equilibrium will increase as the level of adoption value increases. On the other hand, if the innovator's advantage is prominent, then player competitiveness in the equilibrium will decrease as the level of adoption value increases. This difference is due to the fact that the higher the quality of the current technology, the less important the first conflict.

In this respect, the work of Rotemberg and Saloner (1986) is related to this study, although it does differ substantially. More specifically, Rotemberg and Saloner studied a repeated oligopoly game with business fluctuations and showed that excessive price competition (e.g., price war) takes place in a boom because business fluctuations influence the relative importance between the instantaneous gains and the punishments in future games. In contrast, this study does not consider such future punishments, and instead considers the dependence of a player's discount factor on whether she wins the current component game: the excessive preemption competition takes place with a high level of adoption value if and only if the innovator's dilemma is prominent.

This study assumes that the rival in the current game is different from the rival in the next game. In this respect, this study is related to studies of repeated games such as those by Kandori (1992) and Deb, Sugaya, and Wolitzky (forthcoming), wherein players are randomly matched across component games. These works investigate community enforcements by assuming that a player can obtain information concerning the opponent's past history of play. In contrast, this study does not make this assumption: a player cannot even confirm that her rival is really present until the current game ends.

This study assumes that the development and adoption of the next technology do not affect the adoption value of the current technology. Accordingly, this study does not consider the replacement effect (Arrow 1962) that makes the incumbent hesitant to innovate—while the replacement effect may be one cause of the innovator's dilemma (Igami 2017), it is not the only cause (Christensen 1997). We conceptually distinguish between the innovator's dilemma and the replacement effect based on the above-mentioned headhunting scenario.

The organization of this study is as follows. Section 2 introduces the basic formulation of the recurrent preemption game, where I assume that the level of adoption value is constant across component games. Section 3 incorporates uncertainty about future technologies into the recurrent preemption game. Section 4 generalizes the formulation of the winner's payoffs without substantial changes to the contents outlined in Sections 2 and 3. Section 5 concludes the paper.

2. Basic Model

2.1. Formulation

I investigated a two-player infinitely repeated game with random matching that I termed the *recurrent preemption game*. The component game is given by (N, A, u), where $N = \{1,2\}$, $A = A_1 \times A_2$, $u = (u_1, u_2)$, and $u_i : A \to R$ for each $i \in \{1,2\}$. Each player $i \in \{1,2\}$ simultaneously selects an action $a_i \in A_i$, and obtains the instantaneous payoff $u_i(a)$, where $a = (a_1, a_2)$. I specify the component game by

$$A_1 = A_2 = [0,1],$$

$$u_i(a) = \theta a_i \text{ and } u_j(a) = 0 \quad \text{if } a_i < a_j,$$

and

$$u_1(a) = u_2(a) = \frac{\theta t}{2} \qquad \text{if } t = a_1 = a_2,$$

where $j \neq i$ and $\theta > 0$. I call this the *preemption game*, and it corresponds to the simple Bertrand duopoly. Each player i wins and obtains the winner's payoff θa_i if her action choice a_i is smaller than the rival's action choice a_j.

The preemption game involves the unique Nash equilibrium. The unique Nash equilibrium incentivizes each player to select $a_i = 0$, which exhausts her potential benefit. To calm this inefficiency, I assume *incomplete information*: each player $i \in N$ does not know if her rival j is present or absent; she expects that with a positive probability $\varepsilon > 0$, her rival $j \neq i$ is absent and therefore she automatically wins and obtains θa_i. With this incomplete information, the preemption game demonstrates the unique mixed strategy Nash equilibrium. This unique equilibrium incentivizes each player to select positive actions with certainty.

A mixed strategy—or, to it put less formally for my purpose, a *strategy*—for player i is defined as a cumulative distribution function $q_i : A_i \to [0,1]$, where $q_i(t)$ is non-decreasing and right continuous in $t \in [0,1]$. I simply wrote $q_i = t$ when player i selects $a_i = t$ with certainty. A strategy profile (q_1, q_2) is said to be *symmetric* if $q_1 = q_2$. I simply wrote q instead of (q_1, q_2) when $q_1 = q_2 = q$. According to my earlier works (2013; forthcoming), the following strategy is the unique Nash equilibrium in one-shot preemption games:

$$q^*(t) = q^*(t \mid \varepsilon) = 1 - \frac{\varepsilon(1-t)}{(1-\varepsilon)t} \text{ for all } t \in [\varepsilon, 1],$$

and

$$q^*(t) = q^*(t \mid \varepsilon) = 0 \quad \text{for all} \quad t \in [0, \varepsilon].$$

The interpretation of the preemption game is *strategic technology adoption*. A company has an opportunity to develop and adopt a technology within a time interval $[0,1]$. If the company successfully adopts the technology at a time $t \in [0,1]$, then it earns θt, where θ expresses the *level of adoption value*. Meanwhile, if the company's rival adopts the technology before it does, then the company earns nothing. While postponing the time of adoption allows a company to further develop its technology, it increases the risk that the company's rival will win the adoption race.

I introduce the *recurrent structure* as follows. Just after terminal time 1, the company can enter the next preemption game with a positive probability. I assume that the rival in the next game is not the rival in the current game. To be sure, if there is no rival in the current game, then the company enters the next game with certainty. However, if there is a rival in the current game, then the company enters the next game with a probability of $\delta_W \in [0,1]$ if it wins the current game, and a probability of $\delta_L \equiv 1 - \delta_W \in [0,1]$ if it loses the current game.

Hence, the recurrent preemption game is a substantial modification of a standard infinitely repeated game with random matching in that the discount factor of a player depends on whether she wins the current game and also on whether there is a rival in the current game. To be more precise, the discount factor equals 1 if the rival is absent in the current game, δ_W if the rival is present and the player wins the current game, and $1 - \delta_W$ $(= \delta_L)$ if the rival is present and wins the current game.

In the context of strategic technology adoption, the innovator (i.e., the winner) is at risk of her competitor (i.e., the loser) taking her opportunity to participate in the next race for technology adoption. The case that the loser will enter the next game more likely than the winner ($\delta_W < 1/2$) corresponds to the prominence of the innovator's dilemma that the winner is constrained by the current technology, and therefore tends to be behind of the loser in starting the development of the next technology. Meanwhile, the case in which the winner is more likely to enter the next game than the loser

($\delta_W > \frac{1}{2}$) corresponds to the prominence of the innovator's advantage, that is, that the winner obtains better technical and information-gathering skills than the loser.

I use $V(q) = V(q \mid \theta, \delta_W, \varepsilon)$ to denote the expected payoff for a player in the recurrent preemption game when all players commonly follow a strategy q. The winner and the loser receive

$$\theta t + \delta_W V(q) \text{ and } (1-\delta_W)V(q),$$

as their respective payoffs, where t denotes the winning time. If a player selects $t \in [0,1]$, then she wins the current game against her rival with a probability of $1 - \frac{q(t) + \lim_{\tau \uparrow t} q(\tau)}{2}$ and therefore receives the expected payoff given by

$$V(t,q) \equiv (1-\varepsilon)\left[\{\theta t + \delta_W V(q)\}\left\{1 - \frac{q(t) + \lim_{\tau \uparrow t} q(\tau)}{2}\right\}\right.$$

$$\left. +(1-\delta_W)V(q)\frac{q(t) + \lim_{\tau \uparrow t} q(\tau)}{2}\right] + \varepsilon\{\theta t + V(q)\},$$

that is,

(1) $$V(t,q) = \theta t\left\{1 - (1-\varepsilon)\frac{q(t) + \lim_{\tau \uparrow t} q(\tau)}{2}\right\}$$

$$+\left[(1-\varepsilon)\left\{\delta_W + (1-2\delta_W)\frac{q(t) + \lim_{\tau \uparrow t} q(\tau)}{2}\right\} + \varepsilon\right]V(q).$$

From (1), I define the expected payoff in the recurrent preemption game as

$$V(q) \equiv \int_{t=0}^{1} V(t,q)dq(t) = \int_{t=0}^{1} \theta t\left\{1 - (1-\varepsilon)\frac{q(t) + \lim_{\tau \uparrow t} q(\tau)}{2}\right\}dq(t)$$

$$+V(q)\int_{t=0}^{1}\left[(1-\varepsilon)\left\{\delta_W + (1-2\delta_W)\frac{q(t) + \lim_{\tau \uparrow t} q(\tau)}{2}\right\} + \varepsilon\right]dq(t),$$

that is,

(2) $$V(q)=\frac{\int_{t=0}^{1}\theta t\left\{1-(1-\varepsilon)\frac{q(t)+\lim_{\tau\uparrow t}q(\tau)}{2}\right\}dq(t)}{1-\int_{t=0}^{1}\left[(1-\varepsilon)\left\{\delta_W+(1-2\delta_W)\frac{q(t)+\lim_{\tau\uparrow t}q(\tau)}{2}\right\}+\varepsilon\right]dq(t)}.$$

Definition 1: A strategy q is said to be a *symmetric Nash equilibrium in the recurrent preemption game* if

$$V(q)\geq V(t,q) \text{ for all } t\in[0,1].$$

2.2. Specification and Uniqueness

I specify a strategy $\hat{q}=\hat{q}(\cdot\,|\,\delta_W,\varepsilon)$ as follows:

(3) $$\hat{q}(t)=1-\frac{\varepsilon\delta_W(1-t)}{(1-\varepsilon)\delta_W t-\varepsilon(1-2\delta_W)} \text{ for all } t\in[\hat{t},1],$$

and

$$\hat{q}(t)=0 \text{ for all } t\in[0,\hat{t}),$$

where the critical time $\hat{t}=\hat{t}(\delta_W,\varepsilon)\in(0,1)$ is defined as

(4) $$\hat{t}=\frac{\varepsilon(1-\delta_W)}{\delta_W} \qquad \text{if } \frac{\varepsilon}{1+\varepsilon}\leq\delta_W<1,$$

and

$$\hat{t}=1 \qquad \text{if } 0<\delta_W<\frac{\varepsilon}{1+\varepsilon}.$$

Note that $\hat{q}$ is independent of θ. The following theorem shows that $\hat{q}$ is the unique symmetric Nash equilibrium irrespective of θ. Since a player has an incentive to select terminal time 1, I reason that

$$V(\hat{q})=V(1,\hat{q})=\varepsilon\{\theta+V(\hat{q})\}+(1-\varepsilon)(1-\delta_W)V(\hat{q}),$$

that is,

(5) $$V(\hat{q})=\frac{\varepsilon\theta}{(1-\varepsilon)\delta_W}.$$

Theorem 1: *The strategy* $\hat{q}$ *is the unique symmetric Nash equilibrium in the recurrent preemption game.*

Proof: Consider an arbitrary symmetric Nash equilibrium q, where I assume that $q(t)$ is continuous in $t \in [0,1]$, and there exists $\tilde{t} \in [0,1)$ such that $q(t)$ is increasing in $t \in [\tilde{t},1]$ and $q(\tilde{t}) = 0$. From (1) and $q(t) = \lim_{\tau \uparrow t} q(\tau)$, the following first order condition is necessary and sufficient for the Nash equilibrium: for every $t \in [\tilde{t},1]$,

$$\frac{\partial}{\partial t} V(t,q) = \theta\{1-(1-\varepsilon)q(t)\} - \left\{\theta t - (1-2\delta_W)V(q)\right\}(1-\varepsilon)q'(t) = 0,$$

that is,

$$\frac{(1-\varepsilon)q'(t)}{1-(1-\varepsilon)q(t)} = \frac{\theta}{\theta t - (1-2\delta_W)V(q)}.$$

From $q(1) = 1$, for every $t \in [\tilde{t},1)$, I can derive

$$q(t) = 1 - \frac{\varepsilon\theta(1-t)}{(1-\varepsilon)\left\{\theta t - (1-2\delta_W)V(q)\right\}},$$

which, along with (3), (4), and (5), implies that

$$q(t) = 1 - \frac{\varepsilon\delta_W(1-t)}{(1-\varepsilon)\delta_W t - \varepsilon(1-2\delta_W)} = \hat{q}(t) \quad \text{for all} \quad t \in [\tilde{t},1),$$

and

$$\tilde{t} = \hat{t}.$$

Hence, $\hat{q}$ is the unique Nash equilibrium that satisfies continuity and increasingness.

I can prove that if a strategy q is a symmetric Nash equilibrium, then $q(t)$ is continuous in $t \in [0,1]$, and there exists $\tilde{t} \in [0,1)$ such that $q(t)$ is increasing in $t \in [\tilde{t},1]$ and $q(\tilde{t}) = 0$. Meanwhile, if $q(t)$ is not continuous, then there exists $\tau' > 0$ such that $\lim_{\tau \uparrow \tau'} q(\tau) < q(\tau')$. By selecting a time slightly earlier than τ', a player can dramatically increase her probability of winning. This implies that she does not select τ', which is a contradiction. Hence, $q(t)$ is continuous.

From continuity, I can define

$$\tau = \max\left\{\tau \in [0,1) : q(\tau) = q(0)\right\}.$$

Meanwhile, if $q(t)$ is not increasing in $[\tau,1]$, then, there exist $\tau' \in [\tau,1]$ and $\tau'' \in [\tau,1]$ such that $\tau' < \tau''$, $q(\tau') = q(\tau'')$, and the selection of τ' is a best response. Since a player does not select any τ in the open interval (τ',τ''), it follows from continuity that by selecting τ'' instead of τ', she can increase the winner's payoff without decreasing her winning probability, which is also a contradiction. Hence, $q(t)$ is increasing in $t \in [\tau,1]$, where I can set $\tau = \tilde{t}$.

Q.E.D.

2.3. Discussion

The unique equilibrium strategy $\hat{q}$ depends on the frequency with which the winner enters the next game, δ_W. The following proposition states that the more frequently the winner enters the next game (i.e., the greater δ_W), the more competitively the players behave in equilibrium; that is, the less efficiently technologies are adopted. Hence, the prominence of the innovator's dilemma eases preemptive competition and consequently promotes more efficient technology adoption.

Proposition 2: *For every* $\delta_W \in (\frac{\varepsilon}{1+\varepsilon},1)$,

$$\frac{\partial \hat{q}(t \mid \delta_W, \varepsilon)}{\partial \delta_W} > 0 \quad \textit{for all} \quad t \in (\tilde{t},1),$$

$$\frac{\partial \hat{t}(\delta_W, \varepsilon)}{\partial \delta_W} < 0 \text{, } \textit{and} \quad \frac{\partial V(\hat{q}(\cdot \mid \delta_W, \varepsilon))}{\partial \delta_W} < 0 .$$

Proof: From (3),

$$\frac{\partial \hat{q}(t \mid \delta_W, \varepsilon)}{\partial \delta_W} = \frac{\varepsilon^2 (1-t)}{\{(1-\varepsilon)\delta_W t - \varepsilon(1-2\delta_W)\}^2} > 0,$$

implying that $\hat{q}(t \mid \delta_W, \varepsilon)$ is increasing in α. From (4) and (5), both $\hat{t}(\delta_W, \varepsilon)$ and $V(\hat{q} \mid \theta, \delta_W, \varepsilon)$ are decreasing in δ_W.

Q.E.D.

From (3) and (4), in the medium case in which the winner and the loser have the same opportunity to enter the next game ($\delta_W = \frac{1}{2}$), the corresponding unique equilibrium strategy is given by

$$\hat{q}(t) = \hat{q}(t \mid \tfrac{1}{2}, \varepsilon) = 1 - \frac{\varepsilon(1-t)}{(1-\varepsilon)t} \quad \text{for all} \quad t \in [\varepsilon, 1].$$

This is the same as the unique equilibrium strategy in the one-shot preemption game, that is, I have $\hat{q} = q^*$ in the medium case. From the specification of $\hat{q} = \hat{q}(\cdot \mid \delta_W, \varepsilon)$, I have

$$\hat{q}(t \mid \delta_W, \varepsilon) > q^*(t \mid \varepsilon) \qquad \text{for all} \quad t \in (\hat{t}, 1) \qquad \text{if} \quad \delta_W > \tfrac{1}{2},$$

while

$$\hat{q}(t \mid \delta_W, \varepsilon) < q^*(t \mid, \varepsilon) \qquad \text{for all} \quad t \in (0, 1) \qquad \text{if} \quad \delta_W < \tfrac{1}{2}.$$

Hence, if $\delta_W > \frac{1}{2}$, then the introduction of recurrent structure makes players more competitive, while if $\delta_W < \frac{1}{2}$, then the introduction of recurrent structure makes players less competitive.

The following proposition shows that the greater possibility of the rival being absent (i.e., the greater ε) makes technology adoption more efficient.

Proposition 3: *For every* $\varepsilon \in (0,1)$ *and* $\delta_W \in (\frac{\varepsilon}{1+\varepsilon}, 1)$,

$$\frac{\partial \hat{q}(t \mid \delta_W, \varepsilon)}{\partial \varepsilon} < 0 \quad \textit{for all} \quad t \in (\hat{t}, 1),$$

$$\frac{\partial \hat{t}(\delta_W, \varepsilon)}{\partial \varepsilon} > 0, \textit{ and } \frac{\partial V(\hat{q}(\cdot \mid \delta_W, \varepsilon))}{\partial \varepsilon} > 0.$$

Proof: From (3), (4), and (5), it is clear that $\hat{q}(t \mid \delta_W, \varepsilon)$ is decreasing, $\hat{t}(\delta_W, \varepsilon)$ is increasing, and $V(\hat{q} \mid \theta, \delta_W, \varepsilon)$ is decreasing in ε.

Q.E.D.

From the specification of $\hat{q}$, I find that

$$\lim_{\varepsilon \downarrow 0} \hat{q}(t \mid \delta_W, \varepsilon) = 1 \text{ for all } \delta_W \in (0,1] \text{ and } t \in [0,1].$$

This implies full inefficiency in that the technology is immediately adopted without development if a player expects that a rival is (almost) certainly present. From the specification of $\hat{q}$,

$$\lim_{\varepsilon \uparrow 1} \hat{q}(t \mid \delta_W, \varepsilon) = 0 \text{ for all } \delta_W \in (0,1] \text{ and } t \in [0,1).$$

This implies full efficiency in that the technology is adopted after it is fully ripe, if a player expects that a rival is (almost) certainly absent.

3. Random Technology

3.1. Formulation

I have assumed that the level of adoption value θ is constant across component games. This section considers the situation in which θ is determined randomly and independently across component games according to a cumulative distribution function $p:(0,\infty) \to [0,1]$, where $p(\theta)$ is differentiable and increasing in θ, $\lim_{\theta \downarrow 0} p(\theta) = 0$, and $\lim_{\theta \uparrow \infty} p(\theta) = 1$. Players are informed of the adoption value of the current technology but not informed about the future.

In the recurrent preemption game with random technology, I define a strategy as $r = (r(\cdot \mid \theta))_{\theta \in (0,\infty)}$, where $r(\cdot \mid \theta) = r(\cdot \mid \theta, \delta_W, \varepsilon):[0,1] \to [0,1]$ denotes a cumulative distribution function according to which a player selects a time in any preemption game associated with θ. I denote by $V(r) = V(r \mid \delta_W, \varepsilon)$ the ex-ante expected payoff in the recurrent preemption game with random technology. If the current game is associated with θ and a player selects $t \in [0,1]$, then she obtains the expected payoff given by

$$V(t, r, \theta) \equiv \theta t \left\{ 1 - (1-\varepsilon) \frac{r(t \mid \theta) + \lim_{\tau \uparrow t} r(\tau \mid \theta)}{2} \right\} \tag{6}$$

$$+\left[(1-\varepsilon)\left\{\delta_W+(1-2\delta_W)\frac{r(t\,|\,\theta)+\lim_{\tau\uparrow t} r(\tau\,|\,\theta)}{2}\right\}+\varepsilon\right]V(r).$$

Let $V(r,\theta)$ denote the expected payoff associated with θ when all players commonly follow q, which is defined as

(7) $$V(r,\theta)\equiv\int_{t=0}^{1}V(t,r,\theta)dr(t\,|\,\theta)$$

$$=\int_{t=0}^{1}\theta t\left\{1-(1-\varepsilon)\frac{r(t\,|\,\theta)+\lim_{\tau\uparrow t} r(\tau\,|\,\theta)}{2}\right\}dr(t\,|\,\theta)$$

$$+V(r)\int_{t=0}^{1}\left[(1-\varepsilon)\left\{\delta_W+(1-2\delta_W)\frac{r(t\,|\,\theta)+\lim_{\tau\uparrow t} r(\tau\,|\,\theta)}{2}\right\}+\varepsilon\right]dr(t\,|\,\theta).$$

I define the ex-ante expected payoff in the recurrent preemption game with random technology as

(8) $$V(r)\equiv E_\theta[V(r,\theta)]$$

$$=\frac{E_\theta\left[\int_{t=0}^{1}\theta t\left\{1-(1-\varepsilon)\frac{r(t\,|\,\theta)+\lim_{\tau\uparrow t} r(\tau\,|\,\theta)}{2}\right\}dr(t)\right]}{1-E_\theta\left[\int_{t=0}^{1}\left[(1-\varepsilon)\left\{\delta_W+(1-2\delta_W)\frac{r(t\,|\,\theta)+\lim_{\tau\uparrow t} r(\tau\,|\,\theta)}{2}\right\}+\varepsilon\right]dr(t)\right]}.$$

Definition 2: A strategy r is said to be a *symmetric Nash equilibrium in the recurrent preemption game with random technology* if for every $\theta>0$,

$$V(r,\theta)\geq V(t,r,\theta)\quad\text{for all }t\in[0,1].$$

3.2. Specification and Uniqueness

I specify a strategy $\hat{r}=(\hat{r}(\cdot\,|\,\theta))_{\theta\in(0,1]}$ as follows. Let us specify $\hat{t}(\theta)\in[0,1]$, $V(\hat{r})\geq 0$, and $\hat{\theta}>0$ according to the following five equations:

(9) $$\hat{t}(\theta)=\varepsilon+\frac{(1-\varepsilon)(1-2\delta_W)V(\hat{r})}{\theta},$$

(10) $$V(\hat{r}) = \frac{2\varepsilon\int_{\hat{\theta}}^{\infty}\theta dp(\theta)}{(1-\varepsilon)\left[1-(1-2\delta_W)\{1-p(\hat{\theta})\}\right]} \qquad \text{if } \delta_W > 1/2,$$

(11) $$V(\hat{r}) = \frac{(1+\varepsilon)\int_{0}^{\hat{\theta}}\theta dp(\theta) + 2\varepsilon\int_{\hat{\theta}}^{\infty}\theta dp(\theta)}{(1-\varepsilon)\left[1-(1-2\delta_W)\{1-p(\hat{\theta})\}\right]} \qquad \text{if } \delta_W < 1/2,$$

(12) $$\hat{\theta} = \frac{(1-\varepsilon)(2\delta_W - 1)V(\hat{r})}{\varepsilon} \qquad \text{if } \delta_W > 1/2,$$

and

(13) $$\hat{\theta} = (1-2\delta_W)V(\hat{r}) \qquad \text{if } \delta_W < 1/2.$$

Based on these specifications, for every $\theta > \hat{\theta}$, let

(14) $$\hat{r}(t \mid \theta) = 1 - \frac{\varepsilon\theta(1-t)}{(1-\varepsilon)\{\theta t - (1-2\delta_W)V(\hat{r})\}} \quad \text{for all } t \in (\hat{t}(\theta), 1],$$

and

$$\hat{r}(t \mid \theta) = 0 \quad \text{for all } t \in [0, \hat{t}(\theta)].$$

For every $\theta \in (0, \hat{\theta}]$, let

(15) $$\hat{r}(t \mid \theta) = 1 \quad \text{for all } t \in [0,1] \qquad \text{if } \delta_W > 1/2,$$

and

(16) $$\hat{r}(t \mid \theta) = 0 \quad \text{for all } t \in [0,1) \qquad \text{if } \delta_W < 1/2.$$

The following theorem shows that it is the unique symmetric Nash equilibrium.

Theorem 4: *The specified strategy $\hat{r}$ uniquely exists, and it is the unique symmetric Nash equilibrium in the recurrent preemption game with random technology.*

Proof: I show that $\hat{r}$ uniquely exists. For each $\theta > \hat{\theta}$, $\hat{t}(\theta)$ must satisfy the following equation:

$$\hat{r}(\hat{t}(\theta) \mid \theta) = 1 - \frac{\varepsilon\theta\{1-\hat{t}(\theta)\}}{(1-\varepsilon)\{\theta\hat{t}(\theta) - (1-2\delta_W)V(\hat{r})\}} = 0,$$

which implies (9). For $\hat{r}$ to satisfy the Nash equilibrium property, the selection of terminal time 1 must be the best response to $\hat{r}(\cdot \mid \theta)$ for each $\theta > \hat{\theta}$. Hence,

(17) $$V(\hat{r},\theta)=V(1,\hat{r},\theta)=\varepsilon\theta+\left\{(1-\varepsilon)(1-\delta_W)+\varepsilon\right\}V(\hat{r}).$$

Consider $\delta_W > 1/2$. Equation (15) makes clear that for each $\theta\in(0,\hat{\theta}]$, a player must have an incentive to select 0, which implies

$$V(\hat{r},\theta)=V(0,\hat{r},\theta)=\frac{1+\varepsilon}{2}V(\hat{r}).$$

Hence, from (17),

$$V(\hat{r})=V(\hat{r}\mid\delta_W,\varepsilon)=E_\theta[V(\hat{r},\theta)]$$

$$=\varepsilon\int_{\hat{\theta}}^{\infty}\theta dp(\theta)+V(\hat{r})\left[\frac{1+\varepsilon}{2}p(\hat{\theta})+\left\{(1-\varepsilon)(1-\delta_W)+\varepsilon\right\}\left\{1-p(\hat{\theta})\right\}\right],$$

which implies (10). Since $\hat{t}(\hat{\theta})=0$ and (9), I have (12). From (10) and (12),

(18) $$\hat{\theta}\left[1-(1-2\delta_W)\{1-p(\hat{\theta})\}\right]+2(1-2\delta_W)\int_{\hat{\theta}}^{\infty}\theta dp(\theta)=0,$$

which determines $\hat{\theta}$. Such $\hat{\theta}$ uniquely exists because the left-hand side of (18) is continuous in $\hat{\theta}$, increasing in $\hat{\theta}$, negative for $\hat{\theta}=0$, and positive for sufficient $\hat{\theta}$. Hence, as per Equations (9), (10), and (12), $\hat{t}(\theta)$, $V(\hat{r})$, and $\hat{r}$ are all uniquely defined.

Consider $\delta_W < 1/2$. Equation (16) makes clear that for each $\theta\in(0,\hat{\theta})$, no player must have an incentive for a time before 1, which implies

$$V(\hat{r},\theta)=V(1,\hat{r},\theta)=\frac{1+\varepsilon}{2}\{\theta+V(\hat{r})\}.$$

Hence, from (A-1),

$$V(\hat{r})=V(\hat{r}\mid\delta_W,\varepsilon)=E_\theta\left[V(\hat{r},\theta)\right]=\frac{(1+\varepsilon)\int_0^{\hat{\theta}}\theta dp(\theta)+2\varepsilon\int_{\hat{\theta}}^{\infty}\theta dp(\theta)}{(1-\varepsilon)\left[1-(1-2\delta_W)\{1-p(\hat{\theta})\}\right]},$$

which implies (11). Since $\hat{t}(\hat{\theta})=1$ and (9), I have (13). From (11) and (13),

(19) $$\hat{\theta}(1-\varepsilon)\left[1-(1-2\delta_W)\left\{1-p(\hat{\theta})\right\}\right]$$

$$-(1-2\delta_W)\left[(1+\varepsilon)\int_0^{\hat{\theta}}\theta dp(\theta)+2\varepsilon\int_{\hat{\theta}}^{\infty}\theta dp(\theta)\right]=0,$$

which determines $\hat{\theta}$. Such $\hat{\theta}$ uniquely exists because the left-hand side of (19) is continuous in $\hat{\theta}$, increasing in $\hat{\theta}$, negative for $\hat{\theta}=0$, and positive for sufficient $\hat{\theta}$. Hence, as per Equations (9), (11), and (13), $\hat{t}(\theta)$, $V(\hat{q})$, and $\hat{r}$ are all uniquely defined. Note that $\hat{r}(t\mid\theta)$ is continuous in $t\in[0,1]$, irrespective of θ.

The above observations prove that $\hat{r}$ uniquely exists.

Notably, $\hat{r}$ is the unique symmetric Nash equilibrium. Suppose $\delta_W > \frac{1}{2}$ and consider an arbitrary symmetric Nash equilibrium r that I assume is continuous in $t\in[0,1)$ and there exists $\tilde{\theta}\geq 0$ such that

$$r(0\mid\theta)=1 \text{ for all } \theta\in[0,\tilde{\theta}),$$

and, moreover, for each $\theta>\tilde{\theta}$, there exists $\tilde{t}(\theta)\in[0,1)$ such that $r(t\mid\theta)$ is increasing in $t\in[\tilde{t},1]$ and $r(\tilde{t}(\theta)\mid\theta)=0$. Consider $\theta>\tilde{\theta}$. From Equation (6) and $r(t\mid\theta)=\lim_{\tau\uparrow t} r(\tau\mid\theta)$, the following first order condition is necessary and sufficient for the Nash equilibrium: for every $t\in[\tilde{t}(\theta),1]$,

$$\frac{\partial}{\partial t}V(t,r,\theta)=\theta\{1-(1-\varepsilon)r(t\mid\theta)\}$$
$$-\{\theta t-(1-2\delta_W)V(r)\}(1-\varepsilon)r'(t\mid\theta)=0.$$

As in the proof for Theorem 1, it follows from the unique specification of $\hat{r}$ that

$$V(r)=V(\hat{r}),$$

for every $t\in[\tilde{t}(\theta),1)$,

$$r(t\mid\theta)=1-\frac{\varepsilon\theta(1-t)}{(1-\varepsilon)\{\theta t-(1-2\delta_W)V(r)\}}=\hat{r}(t\mid\theta),$$

and $\tilde{t}(\theta)=\hat{t}(\theta)$, where, since a player has an incentive to select 1 if and only if $\theta\geq\hat{\theta}$, we have $\tilde{\theta}=\hat{\theta}$. These observations imply that $\hat{r}$ is the unique symmetric Nash equilibrium.

Suppose $\delta_W < \frac{1}{2}$ and consider an arbitrary symmetric Nash equilibrium r that I assume is continuous in $t\in[0,1)$ and there exists $\tilde{\theta}\geq 0$ such that

$$r(t\mid\theta)=0 \text{ for all } \theta\in[0,\tilde{\theta}) \text{ and } t\in[0,1),$$

and, for each $\theta > \tilde{\theta}$, there exists $\tilde{t}(\theta) \in [0,1)$ such that $r(t \mid \theta)$ is increasing in $t \in [\tilde{t},1]$ and $r(\tilde{t}(\theta) \mid \theta) = 0$. In the same manner as in the case of $\delta_W > 1/2$,

$$r(\cdot \mid \theta) = \hat{r}(\cdot \mid \theta) \text{ for all } \theta > \tilde{\theta},$$

where, since a player has an incentive to select 1 irrespective of her rival's strategy if and only if $\theta < \hat{\theta}$, we have $\tilde{\theta} = \hat{\theta}$, and $r(t \mid \theta) = 0$ for all $t < 1$ and $\theta < \hat{\theta}$. These observations imply that $\hat{r}$ is the unique symmetric Nash equilibrium.

As in the proof for Theorem 1, any symmetric Nash equilibrium satisfies continuity and increasingness. Hence, the above work proves Theorem 4.

Q.E.D.

3.3. Discussion

The unique equilibrium strategy $\hat{r}(\cdot \mid \theta)$ crucially depends on θ. The reason for this dependence is that the relative importance of the future expected payoff after entering the next game, compared to the winner's payoff in the current game, decreases as the current level of adoption value increases. Importantly, the manner of dependence of $\hat{r}(\cdot \mid \theta)$ on θ substantially differs between $\delta_W > 1/2$ and $\delta_W < 1/2$. This subsection will show that if $\delta_W > 1/2$, then the greater the level of adoption value θ, the less competitive a player will behave; meanwhile, if $\delta_W < 1/2$, then the greater the level of adoption value θ, the more competitive a player behaves.

Suppose that the level of adoption value is insufficient, that is, $\theta < \hat{\theta}$. Accordingly, Equation (15) suggests that if $\delta_W > 1/2$, then players are so competitive, act immediately, and therefore adopt the technology without development. On the other hand, Equation (16) suggests that if $\delta_W < 1/2$, then players are so uncompetitive, postpone the timing to terminal time 1, and therefore adopt the technology with full development. The following proposition shows that this tendency holds even if the level of adoption value is sufficient.

Proposition 5: *Suppose* $\theta > \hat{\theta}$. *Then,*

$$\left[\delta_W > \frac{1}{2}\right] \Rightarrow \left[\frac{\partial}{\partial\theta}\hat{r}(t\,|\,\theta) < 0 \quad and \quad \frac{\partial}{\partial\theta}\hat{t}(\theta) > 0\right],$$

and

$$\left[\delta_W < \frac{1}{2}\right] \Rightarrow \left[\frac{\partial}{\partial\theta}\hat{r}(t\,|\,\theta) > 0 \quad and \quad \frac{\partial}{\partial\theta}\hat{t}(\theta) < 0\right].$$

Proof: From (14),

$$\frac{\partial}{\partial\theta}\hat{r}(t\,|\,\theta) = \frac{\varepsilon(1-t)^2(1-2\delta_W)V(\hat{r})}{(1-\varepsilon)\{\theta t-(1-2\delta_W)V(\hat{r})\}^2},$$

which is negative (positive) if $\delta_W > 1/2$ ($\delta_W < 1/2$, respectively). From (9),

$$\frac{\partial}{\partial\theta}\hat{t}(\theta) = -\frac{(1-\varepsilon)(1-2\delta_W)V(\hat{r})}{\theta^2},$$

which is positive (negative) if $\delta_W > 1/2$ ($\delta_W < 1/2$, respectively).

Q.E.D.

Consider the medium case, that is, $\delta_W = 1/2$. Note that $\hat{r}(t\,|\,\theta)$ is independent of θ, and

$$\hat{r}(\cdot\,|\,\theta) = q^* \quad \text{for all} \quad \theta > 0.$$

In other words, the unique equilibrium strategy in the medium case is the same as the unique equilibrium strategy in the one-shot game. Hence, the incorporation of random technology does not influence the players' equilibrium behavior in the medium case. Moreover, it is helpful to note that

$$\lim_{\theta\uparrow\infty}\hat{r}(t\,|\,\theta,\delta_W,\varepsilon) = q^* \quad \text{for all} \quad \delta_W \in (0,1) \quad \text{and} \quad \varepsilon \in (0,1).$$

The equilibrium strategy associated with a sufficiently high level θ is approximated by the equilibrium strategy in the one-shot preemption game. As in Section 2, if $\delta_W > 1/2$, then the incorporation of recurrent structure makes players more competitive;

meanwhile if $\delta_W < 1/2$, then the incorporation of recurrent structure makes players less competitive.

From the specification of $\hat{r}$ and $V(\hat{r}) > 0$, the following proposition holds. Notably, this proposition implies that the prominence of the innovator's dilemma promotes more efficient technology adoption. I write $\hat{r}(\cdot \mid \theta) = \hat{r}(\cdot \mid \theta, \delta_W, \varepsilon)$.

Proposition 6: *If* $\delta_W > 1/2$, *then*

$$\hat{r}(t \mid \theta, \delta_W, \varepsilon) > q^*(t) \ \ \textit{for all} \ \ t \in (0,1) \ \ \textit{and} \ \ \theta \in (0,\infty),$$

while if $\delta_W < 1/2$, *then*

$$\hat{q}(t \mid \theta, \delta_W, \varepsilon) < q^*(t) \ \ \textit{for all} \ \ t \in (0,1) \ \ \textit{and} \ \ \theta \in (0,\infty).$$

In the same manner as in Subsection 2.3, the positivity of ε plays a central role in making players delay technology adoption. For every $\theta < 0$ and $\delta_W \in (0,1]$,

$$\lim_{\varepsilon \downarrow 0} \hat{r}(t \mid \theta, \delta_W, \varepsilon) = 1 \ \ \text{for all} \ \ t \in [0,1],$$

which implies full inefficiency in that the technology is immediately adopted without development if a player expects that a rival is (almost) certainly present. For every $\theta < 0$ and $\delta_W \in (0,1]$,

$$\lim_{\varepsilon \uparrow 1} \hat{r}(t \mid \theta, \delta_W, \varepsilon) = 0 \ \ \text{for all} \ \ t \in [0,1),$$

which implies full efficiency in that the technology is adopted with full development if a player expects that a rival is (almost) certainly absent.

4. Generalization

I can generalize the winner's payoffs by replacing θt with $v(t \mid \theta)$ without any substantial change, where I assume that $v(t \mid \theta)$ is continuous and increasing in $t \in [0,1]$ and $\theta \in (0,\infty)$, $v(0 \mid \theta) = 0$, and $v(1 \mid \theta) = \theta$. In the recurrent preemption game without random technology, I specify $q^+ = q^+(\cdot \mid \delta, \varepsilon, v(\cdot \mid \theta))$ by

$$q^{+}(t)=\hat{q}(\frac{v(t\,|\,\theta)}{\theta}) \quad \text{for all} \quad t\in[0,1].$$

As in Theorem 1, q^{+} is the unique symmetric Nash equilibrium, and the corresponding expected payoff is the same as $V(\hat{q})$. In the recurrent preemption game with random technology, I specify r^{+} by

$$r^{+}(t\,|\,\theta)=\hat{r}(\frac{v(t\,|\,\theta)}{\theta}\,|\,\theta) \quad \text{for all} \quad t\in[0,1] \quad \text{and} \quad \theta\in(0,\infty).$$

As in Theorem 4, r^{+} is the unique symmetric Nash equilibrium, and the corresponding expected payoff is the same as $V(\hat{r})$.

5. Conclusion

This study introduced and investigated the *recurrent preemption game* as the infinitely repeated game with random matching, wherein each player's discount factor depends on whether her rival is present in the current game and whether she wins the current game. Notably, the recurrent preemption game involves the unique symmetric Nash equilibrium. This paper clarified the impact of the innovator's dilemma on equilibrium behavior from various viewpoints such as the degree of competitiveness and its dependence on the quality of technology.

It will be important for future research to generalize this study's analysis in various respects. For instance, the recurrent preemption game can be replaced with a richer model that details a company's internal organization and explains how the company addresses the innovator's dilemma. Moreover, in conversation with previous literature on strategic technology adoption, my component game can be replaced with more general games that incorporate various aspects of asymmetry, such as the replacement effect (Arrow 1962), the informational spillover (Hoppe 2000; Mariotti 1992; Awaya and Krishna 2019), the preemption effect (Gilbert and Newbery; 1982), the imitation (Katz and Shapiro 1987; Hendricks 1992), and the uncertainty of profitability (Jensen 1982). To be sure, sound policymaking requires such future studies, and I hope that my study of the recurrent preemption game provides a solid basis for such work.

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