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Trade with Search Frictions: Identifying New Gains from Trade

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Abstract

This paper develops a dynamic industry model to study the effect of search frictions on industry structure and aggregate welfare. We consider a search-theoretic setting with two types of agents, firms and suppliers. To customize inputs, each firm needs to find a supplier but search is costly and does not always end in success. Matched firms use customized inputs obtained from matched suppliers to enhance production efficiency, while unmatched firms use generic inputs obtained from a competitive input market. In equilibrium the number of unmatched and matched firms is endogenous. We use this model to contrast the implications of two forms of economic integration: integration of final-good markets allowing firms to export varieties to another market, and integration of matching markets allowing firms to seek suppliers from another market. We show that the former form of integration can amplify the welfare gains from trade by improving firms' matching frequency associated with resource reallocations from unmatched firms to matched firms. In contrast, the latter might cause welfare losses by hindering the resource-reallocation process of firms.

Keywords: Search, matching, gains from trade, firm heterogeneity, economic integration
JEL Classification Numbers: F12, F15

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1 Introduction

Firms often search for suppliers to procure specialized inputs in manufacturing processes. Many anecdotes have documented that although a few core inputs to final production are made in-house, other non-core inputs are largely purchased from outside in order to take advantage of suppliers' specialization in arm's-length dealing. Such transactions require suppliers to invest substantial amounts in input customization for the needs of firms, creating the value of inputs applicable only to particular buyer–seller relationships. Moreover, recent advances in information technology reduce search frictions and make it easier to seek suppliers not only within borders but also across borders. Apple's sourcing strategy widely known as "Designed by Apple in California Assembled in China" is portrayed as a symbol of this economic phenomenon; it clearly indicates that successful matching with compatible suppliers is integral for firms to keep productive exchanges of inputs and hence is considered an important source of firm productivity.

Search and matching are not the only one option for firms to purchase inputs from outside. Like final goods, intermediate inputs are also frequently sold on an organized exchange or reference priced in trade publications (Rauch, 1999). The prevalence of two kinds of input procurement, however, varies from one industry to another. Nunn (2007) finds that firms mostly buy inputs by market transactions (sold on an exchange or reference priced) in industries that intensively use primary inputs; while firms mostly buy inputs by non-market transactions (such as search and matching) in industries that intensively use inputs requiring relationship-specific investments. The cross-industry difference implies that, in the latter class of industries, firms are more likely to use high-quality inputs customized by suitable suppliers, a key feature of firms that produce high-quality products and maintain high productivity (Kugler and Verhoogen, 2012; Bastos et al., 2018; Fan et al., 2018). Given that, firms' choices between market and non-market activities may account for the predominant difference in aggregate productivity at the industry level as well as aggregate welfare at the country level.

This paper develops a dynamic industry model to analyze the effect of search frictions on industry structure and aggregate welfare. We consider a search-theoretic setting in which firms seek suppliers to obtain specialized inputs they need, but search is costly and does not always end in success. Thus the status of firms and suppliers is either unmatched or matched. When agents fail to find partners, unmatched suppliers have no choice but to sell inputs without any customization to unmatched firms, in which case inputs are "generic" or "standardized" and are transacted via market activities across anonymous unmatched agents. When agents find partners and agree on provision of specialized inputs, in contrast, matched suppliers incur an investment cost to customize inputs for the needs of matched firms, in which case inputs are "customized" to matched firms and are transacted via non-market activities within particular matched agents. Due to the investment conducted by matched suppliers, matched firms can produce goods at lower cost relative to unmatched firms.[1] In this way, successful matching serves as a vehicle to enhance matched firms' production efficiency, which proves useful in shedding new light on the role of search frictions in industry characterization and gains from trade.

We first consider an autarky version of the model to explain the cross-industry difference between market and non-market activities above. Each industry entails the same search technology allowing agents to meet partners randomly. The importance of matches differs by industry, however, because the degree of input customization differs by industry: firms use inputs customized by suppliers in some industries more than in others. As a result, the extent to which matched firms have the cost advantage over unmatched firms differs by industry. One of the key features of our model is that the number of all types of agents in the industry is endogenously determined by

[1] Since productivity and product quality are isomorphic under the assumption of CES preferences and monopolistic competition, we mainly assume that customized inputs allow matched firms to produce final goods of the same quality at lower cost (rather than to produce final goods of higher quality at the same cost).

both free entry and search technology in equilibrium. Exploiting the model's property, we show that the ratio of suppliers to firms increases with the degree of input customization, so that the market thickness of intermediate inputs in the industry is larger, the greater the degree of input customization of the industry. At the same time, the ratio of unmatched firms to matched firms decreases with the degree of input customization, so that input procurement via market activities dominates in the industry with the low degree of input customization, whereas a majority of firms engage in non-market activities in the industry with the high degree of input customization. Hence, focusing on the effect of search frictions on industry structure, we are able to account for Nunn (2007)'s empirical patterns between market versus non-market activities.

We use this model to contrast the implications of two forms of economic integration: integration of final-good markets allowing firms to export varieties to another market and integration of matching markets allowing firms to seek suppliers from another market. This paper refers to these distinct forms of integration as X-integration and M-integration respectively.[2] As in Antràs and Costinot (2011), the former form of economic integration aims to derive the implications of convergence of goods price indices across countries, while the latter seeks to capture the consequence of entry of foreign agents into domestic matching markets, such as deregulation of cross-border share in domestic stocks. Despite the fact that two forms of economic integration help internationalize trading opportunities, we find that X-integration generates welfare gains but M-integration causes welfare losses for two trading countries so long as they are symmetric in terms of labor endowments and technology. More important is that our model uncovers a new source of welfare gains or losses from trade in the presence of search frictions: X-integration can amplify the welfare gains from trade by improving firms' matching frequency associated with resource reallocations from unmatched firms to matched firms. In contrast, the welfare effect of M-integration arises from hindering the resource-reallocation process of firms.

At this point, it is worth stressing that if search frictions are so large that firms cannot search for suppliers, our model collapses to Krugman (1980)'s trade model. Welfare gains are due solely to increased product variety, which are in turn captured solely by the domestic expenditure share and trade elasticity (Arkolakis et al., 2012). If search frictions are not so large, however, costly trade affects the probability of matches and firm heterogeneity in matching status matters for welfare gains beyond two sufficient statistics: firms' matching frequency improved by X-integration entails first-order significance for welfare gains. The finding is along the lines of recent research showing that welfare gains are greater in the heterogeneous firm model than in the homogeneous firm model due to *endogenous firm selection* that is absent in the latter (Melitz and Redding, 2015). The argument applies to this paper where firm heterogeneity is driven by matching status. Then it is not surprising to see that welfare gains are greater in our model than in Krugman (1980) due to *endogenous firm matches* that are absent in the latter. Our new welfare channel is also of quantitative relevance. We find that the gains from trade, measured as welfare changes moving from autarky to costly trade, are 0.9 percent without search (a comparable magnitude reported in Arkolakis et al. (2012)) but such gains increase to 3.1 percent with search.

We focus (for the most part) on a symmetric-country setting to illustrate the contrasting welfare implications and the extra adjustment margin through firms' matching frequency. However, it is possible to incorporate some of country asymmetry into the baseline model. In so doing, we provide interesting implications in more realistic cases where agents face search frictions in obtaining specialized inputs from different types of countries. First, when the relative abundance of firms and suppliers is different between countries in autarky, the extent to which a country gains or loses from economic integration may vary. We show that there are larger welfare gains from

[2] These two forms of integration are first introduced by Antràs and Costinot (2011) into a perfectly competitive Ricardian model. As they study Walrasian markets where homogenous goods are transacted, the former is referred to as W-integration in their paper. We instead focus on monopolistically competitive markets where product differentiation is present and hence refer to it differently.

X-integration and smaller welfare losses from M-integration in a country where firms providing core inputs are relatively more abundant. Second, when production technology (or labor endowments) differs across countries, M-integration may lead to welfare gains in one of the countries. We show that a country with more advanced technology (or larger market size) is more likely to gain from M-integration, while a country with less advanced technology (or smaller market size) is more likely to suffer from this integration. Thus policymakers need to pay closer attention to trade liberalization in a less developed country especially when such liberalization facilitates cross-border matches.

We contribute to a growing literature that examines the role of search and matching in international trade. By explicitly introducing frictions associated with search and matching among agents in production processes, the literature has found interesting insights in otherwise standard models. For example, Grossman and Helpman (2002, 2005) study firms' organizational and locational choices in a monopolistically competitive environment, Antràs and Costinot (2011) investigate economic integration with intermediation in a standard Ricardian model, and Felbermayr et al. (2011) analyze the selection effect on labor markets in a model with heterogenous firms. Among existing work, the analysis in our paper is most closely related to that in Antràs and Costinot (2011). As described above, we consider, like them, the welfare consequences of two distinct forms of economic integration. There are, however, two crucial differences in the analysis. First, we develop a monopolistic competition model which is regarded as a strict generalization of Krugman (1980): if search frictions are prohibitively large so that firms cannot find suppliers, the equilibrium of our model becomes identical to that of Krugman (1980). Second, intermediation plays no role in our analysis: firms have direct access to markets where varieties are exchanged. Instead firms must seek suppliers to procure inputs whose production can be offshored. Our contribution to the literature is to show that, notwithstanding the key underlying differences, search frictions lead to the contrasting welfare implications between integration of goods markets and that of matching markets. In addition, calibrating the model's parameters, we provide the quantitative relevance of our welfare results.[3]

Our results on the effect of search frictions are closely related to some of the existing results on the effect of contractual frictions. Papers in this strand of the literature can be broadly categorized into the following two classes. First is to model the effect of contractual frictions on firms' choices between integration and outsourcing in procuring specialized inputs (e.g., Antràs, 2003; Antràs and Helpman, 2004; Ornelas and Turner, 2008, 2012). Second is to investigate the effect of contractual frictions on countries' productivity and comparative advantage (e.g., Acemoglu et al., 2007; Costinot, 2009; Levchenko, 2007; Nunn, 2007). We show that search frictions can also lead to a well-defined tradeoff between firms' choices (although our focus is on market versus non-market activities) similar to but distinct from that driven by contractual frictions. In that respect, our contribution to this literature is to demonstrate that the effect of search frictions on firms' choices might help to obtain a better understanding of industry structure and aggregate welfare. For example, our model is useful in appreciating the welfare implications of global sourcing in which firms decide whether to offshore some of production processes to a country with different characteristics. Extending our model to the Antràs and Helpman (2004) framework and thereby interpreting the equilibrium as that of M-integration, we find that if a wage difference is sufficiently large relative to a technological difference, global sourcing with search frictions can generate welfare gains for the developed North. Under the same condition, however, it might simultaneously cause welfare losses for the less developed South.

[3]This paper is also related, though less closely, to the recent literature that explores production networks in international trade. See, for example, Chaney (2014), Bernard et al. (2022), and Eaton et al. (2022). Motivated by the fact on buyer–seller relationships, these papers show that the variation in firm sales is largely accounted for by the way firms search for and match with customers. As a result, changes in search frictions can have an impact as great as those in trade frictions on the sales variation. Their analysis, however, remains silent about the welfare consequences of two distinct forms of economic integration.

2 Setup

2.1 Demand

Consider an industry that is populated by L units of identical agents who consume a continuum of varieties. The preferences of a representative agent are given by a CES utility function:

$$U = \left(\int_0^{N^F} y(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1,$$

where $y(\omega)$ is consumption of variety ω and N^F is the number (measure) of varieties produced in the industry. Each firm produces a single variety and N^F is also the number of firms in the industry. Utility maximization subject to budget constraint yields the optimal consumption and expenditure for variety ω:

$$y(\omega) = Ap(\omega)^{-\sigma},$$
$$r(\omega) = Ap(\omega)^{1-\sigma},$$

where $p(\omega)$ is the price of variety ω and A is the index of industry demand. Using $P = \left(\int p(\omega)^{1-\sigma} d\omega\right)^{1/(1-\sigma)}$ and $R = \int r(\omega) d\omega$ to denote the price index and aggregate expenditure (or revenue), the utility function implies $A = RP^{\sigma-1}$ which is treated as a constant by an individual firm in the industry.

2.2 Production

Production of good $y(\omega)$ requires a development of two variety-specific intermediate inputs, which are combined by a Cobb-Douglas production function (hereafter a variety index ω is dropped from relevant variables):

$$y = \alpha \left(\frac{x^F}{\eta} \right)^{\eta} \left(\frac{x^S}{1-\eta} \right)^{1-\eta}, \quad 0 < \eta < 1,$$

where α is a productivity parameter. There are two types of agents that are involved in final-good production: firms who provide input x^F and suppliers who provide input x^S. We think of x^F as a core input that must be developed at firms' own expense and x^S as a non-core input that can be outsourced to independent suppliers. Hereafter superscripts F and S are attached to variables relevant to firms and suppliers respectively.

Following the literature on firm importing (e.g., Antràs and Helpman, 2004; Antràs et al., 2017), we make a simplifying assumption that all inputs are competitively produced and sold at marginal cost. More specifically, input x^F requires a^F units of labor per unit of output in a country in which firms locate, and input x^S requires a^S units of labor per unit of output in a country in which suppliers locate. If firms and suppliers operate in the same country so that inputs are produced within borders (which we will assume until Section 4), the marginal cost of final-good production is $c = \left(a^F w\right)^{\eta} \left(a^S w\right)^{1-\eta} / \alpha$ where w is a common wage rate. Using the two inputs, all final goods are produced and sold in a monopolistically competitive market.

Each firm seeks a potential supplier and vice versa, but search is costly and does not always end in success. Depending on the matching status of firms and suppliers, the following differences arise. Unmatched suppliers have to produce inputs that every unmatched firm can use without any customization. As inputs are produced under conditions of perfect competition, they sell at the price $a^S w$ and earn zero profit. Unmatched firms buy such generic inputs via "market" activity to produce the final goods according to the technology with $\alpha = 1$.

Because of monopolistic competition among firms, however, unmatched firms earn profit. For the most part of the paper, we normalize the unit labor requirements of two inputs to one ($a^F = a^S = 1$) for analytical simplicity and hence the marginal cost of unmatched firms is given by $c = w$.[4] In contrast, matched suppliers incur an investment cost to customize inputs for the needs of matched firms. This input customization makes it possible for matched suppliers to sell their inputs at the price that is naturally higher than the competitive price and to earn profit. Matched firms obtain such customized inputs via "non-market" activity to produce the final goods according to the technology with $\alpha > 1$, implying that the marginal cost of matched firms is given by $c = w/\alpha$. As a result, matched firms produce with better efficiency and earn higher profit than unmatched firms.

A few points are in order for the parameter α that governs the cost difference between two types of firms. First, α is assumed to differ by industry depending on how matches are important for production in the industry. This variable can be calibrated to match observable moments in the data, such as shipment differences between unmatched and matched firms, as we will do later. We refer to α as the degree of input customization hereafter. Second, while α is assumed to be an exogenous variable for firms, it might be an endogenous variable for them. Consider a variant of our setup where only matched firms combine a bundle of non-core inputs produced by all suppliers in a CES fashion. In this model, the larger the number of non-core inputs in the industry, the greater the cost advantage of matched firms over unmatched firms via input variety expansion (Acemoglu et al., 2007), yielding equilibrium outcomes similar to ours. Finally, while α is assumed to be common to all matched firms, it might vary across them. We can show that when α is assumed to be Pareto distributed, a shape parameter of the distribution plays a qualitatively similar role in the analysis (Ara and Furusawa, 2020).

To highlight the effect of search frictions on the industry characterization and aggregate welfare more sharply, we restrict attention to the case of *complete* contracts between firms and suppliers regardless of matching status. As is well known, agents are able to sign enforceable contracts specifying the purchase of variety-specific inputs in that case, allowing them to set an amount of inputs x^F, x^S cooperatively that maximizes the variable profit $r - wx^F - wx^S$ without facing any underinvestment (holdup) problem for provision of these inputs. Subsequently, the variable profit is distributed to firms and suppliers by taking into account the future probability of matches, as will be described in Section 2.4. For now, consider the optimal behavior of agents. From consumer demand and firm technology, we can write the potential revenue from sales of the final good as a function of two inputs. As firms choose an amount of x^F and suppliers choose an amount of x^S so as to maximize the variable profit, combining the first-order conditions for profit maximization by each type of agents yields the equilibrium price:

$$p(\alpha) = \frac{\sigma}{\sigma - 1}\frac{w}{\alpha}.$$

As is usual with CES preferences and monopolistic competition, the optimal pricing for each firm is to charge a constant markup over marginal cost. Note however that matched firms set a lower price than unmatched firms by $1/\alpha$ in this model. From the equilibrium price, the equilibrium output and revenue are respectively given by

$$y(\alpha) = A\left(\frac{\sigma - 1}{\sigma}\frac{\alpha}{w}\right)^{\sigma},$$

$$r(\alpha) = A\left(\frac{\sigma - 1}{\sigma}\frac{\alpha}{w}\right)^{\sigma - 1},$$

and the equilibrium variable profit (excluding any fixed cost) is given as $r(\alpha)/\sigma$ with CES preferences. Hereafter the common wage rate is normalized to one by choosing labor as the numéraire.

[4] The difference in these requirements are introduced when examining welfare implications of global sourcing (see Section 6).

Let $r \equiv r(1)$ denote the equilibrium revenue of unmatched firms, and r/σ is the equilibrium variable profit of these firms. Then, the ratio of equilibrium revenue (and the equilibrium profit) of matched firms to that of unmatched firms depends only on the degree of input customization α:

$$\frac{r(\alpha)}{r} = \alpha^{\sigma-1}. \tag{1}$$

Thus the higher degree of input customization is associated with the larger difference in the equilibrium revenue (and the equilibrium profit) between unmatched firms and matched firms.

2.3 Search and Matching

Firms need to find potential suppliers to customize inputs whereas suppliers need to find potential firms to sell such customized inputs. This search process involves search frictions and one-to-one random matching between unmatched firms and unmatched suppliers, where so-called directed search is not possible for any agents.

Upon incurring one-time fixed entry costs F_e^F, F_e^S (measured in units of labor), new firms and new suppliers enter the industry with being unmatched. Following Grossman and Helpman (2002), fixed costs include resources needed not only to enter the industry but also to search for partners. Unmatched firms and unmatched suppliers seeking potential partners meet together randomly. We denote by N_e^F, N_e^S the number of newly entered agents, N^F, N^S the number of all (unmatched and matched) agents, and u^F, u^S the number of unmatched agents per unit of time in the industry. Hence $N^F - u^F, N^S - u^S$ denote the number of matched agents per unit of time in the industry. The number of matches per unit of time in the industry is given by a matching function $m(u^F, u^S)$, which is increasing, concave, homogeneous of degree one and satisfies standard Inada conditions. As a result, there are constant returns to scale in matching, e.g., a doubling in the number of unmatched agents results in a doubling in the number of matched agents.

Using the matching function, we define the rate at which matches randomly occur across unmatched agents. The rate at which unmatched firms meet unmatched suppliers is equal to $\mu^F \equiv m(u^F, u^S)/u^F = m(1, \theta)$ where $\theta \equiv u^S/u^F$ is the ratio of unmatched suppliers to unmatched firms in the industry. Similarly, the rate at which unmatched suppliers meet unmatched firms is equal to $\mu^S \equiv m(u^F, u^S)/u^S = m(1/\theta, 1) = \mu^F/\theta$. From the property of the matching function, the probability μ^F increases with θ but the probability μ^S decreases with θ. Later we impose free entry so that any agent entering as a firm or a supplier ends up with zero expected profit. This means that the number of agents N^F, N^S is endogenously determined by free entry, while the number of matched agents $N^F - u^F, N^S - u^S$ as well as the ratio of unmatched agents θ are endogenously determined by search technology. Moreover, regardless of matching status, all existing agents face an exogenous probability δ of a bad shock that forces them to exit the industry at every point in time.

This paper considers a dynamic industry model in which matching and exiting simultaneously occur among unmatched and matched agents. Figure 1 shows the search process of firms that arises at every point in time. Upon incurring a fixed entry cost F_e^F, the number N_e^F of new firms enters the unmatched pool at which point they do not go straight to search for potential suppliers. Among the number u^F of existing unmatched firms, a fraction of them randomly finds their partners at the rate of μ^F, and hence the number $\mu^F u^F$ of firms enters the matched pool at every point in time. However, among the number u^F of existing unmatched firms and the number $N^F - u^F$ of existing matched firms, a fraction of them is hit by a bad shock at the rate of δ. As the number δu^F of unmatched firms and the number $\delta(N^F - u^F)$ of matched firms are hit by this shock, in total, the number δN^F of firms exits the industry at every point in time.

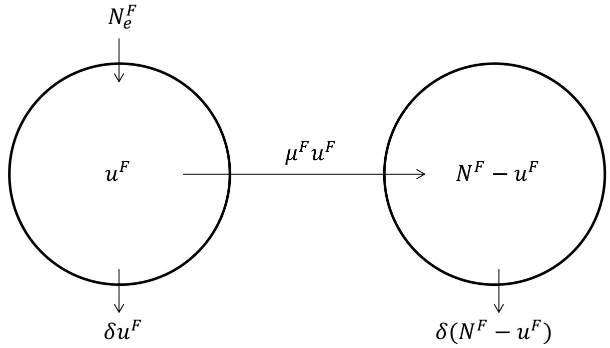

Figure 1 – Search process of firms

2.4 Bargaining

Matched agents negotiate the division of variable profit. Firms and suppliers of matched status are assumed to have complete information that makes bargaining bilaterally efficient. In our dynamic industry model, they take into account future profit flows in this negotiation as they may be hit by a bad shock in the future, in which case the outside option obtained when being unmatched must also be reflected by the future probability of matches. Thus the surplus generated by bilateral relationships is divided to maximize the net value of matches.

Let V^F and V^S denote the value function of unmatched firms and unmatched suppliers, while let $V^F(\alpha)$ and $V^S(\alpha)$ denote the value function of matched firms and matched suppliers. As will be shown in the next section, these value functions include the variable profit obtained at every point in time as well as the probabilities of matches and a bad shock that may occur in the future. Let $r^F(\alpha)/\sigma$ and $r^S(\alpha)/\sigma$ denote the variable profit of matched firms and matched suppliers. Clearly these two variable profits sum to $r(\alpha)/\sigma$ in (1) and are included in the value functions of matched agents introduced above. We formalize the bargaining within matched agents as symmetric Nash bargaining in which matched firms and matched suppliers capture an equal share of the ex-post surplus generated from bilateral relationships:

$$\max_{\frac{r^F(\alpha)}{\sigma}, \frac{r^S(\alpha)}{\sigma}} \Big(V^F(\alpha) - V^F\Big)\Big(V^S(\alpha) - F_d - V^S\Big),$$

subject to $r^F(\alpha)/\sigma + r^S(\alpha)/\sigma = r(\alpha)/\sigma$ where F_d is a one-time fixed cost (measured by units of labor) that matched suppliers incur to customize inputs for the need of matched firms when the bargaining is successful. From the fact that matched suppliers earn profit, it follows that the bilateral input price implicitly determined at this profit sharing must be higher than the competitive one.

2.5 Timing

The timing of events is composed of four periods. In the first period, new agents enter the industry as either firms or suppliers with being unmatched at which point they incur a portion of entry costs. In the second period, existing agents seek potential partners at which point they incur the remaining entry costs and find matching status. In the third period, suppliers produce and sell inputs to firms via either market or non-market activity at which point matched suppliers incur an investment cost. In the last period, firms produce and sell final goods at which point new matches are formed among unmatched agents and some existing agents are forced to exit the industry.

3 Autarky

3.1 Equilibrium Conditions

Denoting a common discount rate by γ, the value functions must satisfy the following Bellman equations that characterize the expected profit of any type of agents:

$$\begin{aligned}
\gamma V^F &= \frac{r}{\sigma} + \mu^F\Big(V^F(\alpha) - V^F\Big) - \delta V^F + \dot{V}^F, \\
\gamma V^F(\alpha) &= \frac{r^F(\alpha)}{\sigma} - \delta V^F(\alpha) + \dot{V}^F(\alpha), \\
\gamma V^S &= \mu^S\Big(V^S(\alpha) - F_d - V^S\Big) - \delta V^S + \dot{V}^S, \\
\gamma V^S(\alpha) &= \frac{r^S(\alpha)}{\sigma} - \delta V^S(\alpha) + \dot{V}^S(\alpha).
\end{aligned} \tag{2}$$

The first equation shows that unmatched firms obtain a gain r/σ, and become matched at the rate μ^F at which point they obtain a gain $V^F(\alpha) - V^F$ but become inactive at the rate δ at which point they suffer a loss V^F from exiting the market, along with a potential gain or loss $\dot{V}^F$ from remaining unmatched. On the other hand, the second equation shows that matched firms obtain a gain $r^F(\alpha)/\sigma$ and become inactive at the same rate δ at which point they suffer a loss $V^F(\alpha)$ from exiting the market, along with a potential gain or loss $\dot{V}^F(\alpha)$ from remaining matched. While a similar interpretation applies to the third and fourth equations of suppliers, unmatched suppliers obtain zero profit (due to the competitive input market) and become matched at the rate μ^S at which point they make a one-time investment F_d for the needs of matched firms.

We can describe how Nash bargaining between firms and suppliers affects the division of surplus. As formally derived in Appendix A.1, symmetric Nash bargaining imposes the following condition at any point in time:

$$V^F(\alpha) - V^F = \frac{1}{2}\Big(V^F(\alpha) - V^F + V^S(\alpha) - F_d - V^S\Big), \tag{3}$$

which states that, under symmetric bargaining power and complete information between two types of agents, matched firms obtain half of surplus at the bargaining stage.

The number of agents evolves according to the following law of motion in the matched pool:

$$\begin{aligned}
\dot{u}^F &= \delta(N^F - u^F) - \mu^F u^F, \\
\dot{u}^S &= \delta(N^S - u^S) - \mu^S u^S,
\end{aligned} \tag{4}$$

where the number of matched firms must be equal to the number of matched suppliers:

$$N^F - u^F = N^S - u^S \equiv n.$$

While we mainly use (4), the law of motion also applies to the unmatched pool $\dot{N}_e^F = (\delta + \mu^F)u^F - N_e^F$, $\dot{N}_e^S = (\delta + \mu^S)u^S - N_e^S$ and the industry as a whole $\dot{N}^F = \delta N^F - N_e^F$, $\dot{N}^S = \delta N^S - N_e^S$ (see Figure 1).

Free entry into any production activity ensures that the net value of entry must be zero at all points in time. Since any agents first enter the industry with being unmatched, the condition must hold for unmatched agents. Let $V_e^F \equiv V^F - F_e^F$, $V_e^S \equiv V^S - F_e^S$ denote the net value of entry for unmatched agents. Then we have

$$V_e^F = V_e^S = 0. \tag{5}$$

3.2 Equilibrium Characterization

This paper only considers a steady state equilibrium in which the aggregate variables remain constant over time, which means no potential gain or loss from any production activity and thus $\dot{V}^F = \dot{V}^F(\alpha) = \dot{V}^S = \dot{V}^S(\alpha) = 0$. Moreover, the discount rate is assumed to be zero because (i) the probability of a bad shock introduces an effect similar to time discounting; and (ii) the aggregate profit would not equal the aggregate investment cost at any point in time in equilibrium with a positive discount rate.[5]

Setting $\gamma = 0$ and rearranging, we can express (2) as

$$\begin{aligned} V^F &= \frac{r}{\delta\sigma} + \left(\frac{\mu^F}{\delta + \mu^F}\right)\left(\frac{r^F(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma}\right), \\ V^F(\alpha) &= \frac{r^F(\alpha)}{\delta\sigma}, \\ V^S &= \left(\frac{\mu^S}{\delta + \mu^S}\right)\left(\frac{r^S(\alpha)}{\delta\sigma} - F_d\right), \\ V^S(\alpha) - F_d &= \frac{r^S(\alpha)}{\delta\sigma} - F_d. \end{aligned} \tag{6}$$

As the probability of a bad shock δ works as time discounting in our dynamic model, the profit divided by δ represents the present value of profit flows. Then, (6) shows that the value of each type of agents consists of the present value of outside option flows obtained when being unmatched plus that of net profit flows additionally obtained when being matched. In order to ensure an enough incentive for unmatched agents to seek partners in the presence of search frictions, we assume $r^F(\alpha)/\delta\sigma - r/\delta\sigma > 0,\ r^S(\alpha)/\delta\sigma - F_d > 0$, which in turn implies $V^F(\alpha) - V^F > 0,\ V^S(\alpha) - F_d - V^S > 0$ and the net value of matched agents is greater than that of unmatched agents. In this dynamic model, the expectation of future positive profit flows is the only reason that suppliers consider sinking the investment cost F_d, which is also shared by firms at the bargaining stage.

Solving the bargaining condition (3) in light of (6), we get the following optimal sharing rule:

$$\begin{aligned} \frac{r^F(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} &= \beta\left(\frac{r(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d\right), \\ \frac{r^S(\alpha)}{\delta\sigma} - F_d &= (1-\beta)\left(\frac{r(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d\right), \end{aligned} \tag{7}$$

where

$$\beta \equiv \frac{\delta + \mu^F}{2\delta + \mu^F + \mu^S} \tag{8}$$

is "effective" bargaining power of firms. From this sharing rule, we can derive important implications that arise when search and matching are present.

On the one hand, (7) shows that while the one-time fixed cost F_d is initially incurred by matched suppliers, some portion is eventually shared by matched firms through the Nash bargaining above; the extent to which this fixed cost is incurred by firms and suppliers is determined by effective bargaining power β. On the other hand, (8) shows that while primitive bargaining power is symmetric between firms and suppliers, effective bargaining power is endogenously determined by the probability of a bad shock δ and the possibility of matches μ^F, μ^S where the latter is influenced by the number of firms and suppliers N^F, N^S in the industry. In particular, from the features of our matching function, it follows immediately that: (i) β is increasing in $\theta = (N^S - n)/(N^F - n)$;

[5] These points are the same as those in Melitz (2003, footnote 16). See also Appendix A.2 for a formal proof in this paper.

and (ii) β is greater than 1/2 if and only if N^F is smaller than N^S. As is known in the bargaining literature, thus, (8) says that each firm's bargaining power effectively increases with the number of suppliers but it effectively decreases with the number of firms: other things equal, the acquisition of alternative sellers improves a buyer's bargaining position but the invitation of competing buyers worsens its bargaining position.[6]

Setting $\dot{u}^F = \dot{u}^S = 0$ in (4) and using $\mu^F = \mu^S/\theta$, the number of matched agents satisfies the following equality in the steady state equilibrium:

$$n = \left(\frac{\mu^F}{\delta + \mu^F}\right) N^F = \left(\frac{\mu^S}{\delta + \mu^S}\right) N^S. \tag{9}$$

(9) describes how the number of matched agents n is tied to the exogenous rate of a bad shock δ as well as the endogenous rate of matches, μ^F, μ^S among the total number of agents N^F, N^S in the industry. For example, the ratio of matched firms n/N^F increases with the ratio of unmatched agents θ but decreases with the probability of a bad shock. While the steady-state relationship is derived by setting $\dot{u}^F = \dot{u}^S = 0$, the same relationship is also derived by setting $\dot{N}_e^F = \dot{N}_e^S = 0$ and $\dot{N}^F = \dot{N}^S = 0$ and rearranging them.

Finally, using (6), (7), (8) and (9), the free entry condition in (5) can be written as

$$\begin{aligned} &\frac{r}{\sigma} + \frac{n}{N^F}\beta\left(\frac{r(\alpha)}{\sigma} - \frac{r}{\sigma} - f_d\right) - f_e^F = 0, \\ &\frac{n}{N^S}(1-\beta)\left(\frac{r(\alpha)}{\sigma} - \frac{r}{\sigma} - f_d\right) - f_e^S = 0, \end{aligned} \tag{10}$$

where $f_d \equiv \delta F_d$, $f_e^F \equiv \delta F_e^F$ and $f_e^S \equiv \delta F_e^S$ respectively denote the amortized per-period portion of the one-time fixed cost incurred by relevant agents at every point in time.

To understand this, consider the first equality in (10) which relates to the free entry condition of firms. When being unmatched, firms earn the variable profit r/σ at every point in time as their outside option. Moreover, the fraction n/N^F of firms finds suppliers and additionally earns the economic rent $r(\alpha)/\sigma - r/\sigma - f_d (> 0)$ weighted by effective bargaining power β. Hence the first two terms represent firms' expected profit. Let $\phi^F \equiv (n/N^F)\beta$ denote the expected share of economic rent that firms obtain in the Nash bargaining. From (8) and (9), this share is $\mu^F/(2\delta + \mu^F + \mu^S)$. Then the free entry condition of firms means that the expected profit at every point in time consists of the outside option r/σ plus the economic rent weighted by the expected share ϕ^F, which must equal the amortized per-period portion of a fixed entry cost f_e^F. A similar interpretation also applies to the free entry condition of suppliers given in the second equality of (10), in the sense that the expected profit at every point in time consists of the economic rent weighted by the expected share $\phi^S \equiv (n/N^S)(1-\beta) = \mu^S/(2\delta + \mu^F + \mu^S)$ where their outside option is zero as they earn zero profit when being unmatched.

(10) characterizes the model's equilibrium as follows. First, it pins down the profit of unmatched firms r/σ. While both unmatched and matched firms earn positive profits, unmatched firms' profits are exactly offset by the entry and investment costs so that the net profit of unmatched firms reduces to zero. Second, together with the matching function, it determines the number of matched agents n and the total number of agents N^F, N^S, which in turn determines the ratio of unmatched agents $\theta = (N^S - n)/(N^F - n)$. Finally, together with the labor market clearing condition, it ensures that the aggregate revenue R equals the aggregate payment to labor L and thus is exogenously fixed by the index of market size (see Appendix A.2).

[6] See Wolinsky (1987) in a closed-economy model. Applying this insight to an open-economy model, Ara and Ghosh (2016) show that when firms offshore some production processes to suppliers abroad, endogenous bargaining power might have a serious impact on optimal trade policy via the division of surplus between agents. While existing work mostly uses a partial-equilibrium setting, we extend to a general-equilibrium setting and uncover a new source of the gains from trade operating through the channel.

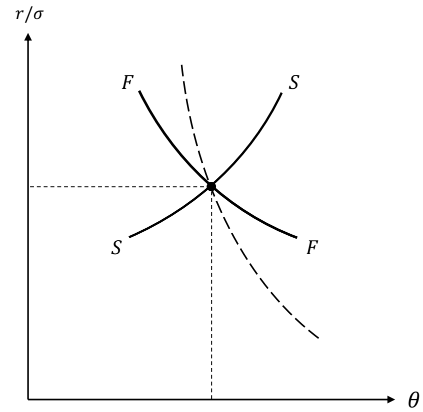

Figure 2 – Autarky equilibrium

3.3 Existence and Uniqueness

The equilibrium revenue in (1) implies that the economic rent depends on the degree of input customization α:

$$\frac{r(\alpha)}{\sigma} - \frac{r}{\sigma} - f_d = \frac{\left(\alpha^{\sigma-1} - 1\right) r}{\sigma} - f_d.$$

Using this in (10), we can solve for the variable profit of unmatched firms given as the following equalities:

$$\begin{aligned} \frac{r}{\sigma} &= \frac{f_e^F + f_d \phi^F}{1 + (\alpha^{\sigma-1} - 1)\phi^F}, \\ \frac{r}{\sigma} &= \frac{f_e^S + f_d \phi^S}{(\alpha^{\sigma-1} - 1)\phi^S}, \end{aligned} \tag{11}$$

where the dependence of (11) on θ is understood from ϕ^F, ϕ^S. Since the investment cost f_d is shared between matched firms and suppliers with the expected shares ϕ^F, ϕ^S at the bargaining stage, the right-hand side of (11) can be interpreted as the expected fixed cost of firms and suppliers under free entry. With this interpretation, (11) shows that any agents who enter the industry as firms or suppliers earn zero net expected profit.

The model's equilibrium can be characterized by (11) which is the system of two equations that endogenously pin down two unknowns, θ and r/σ. The solid curves in Figure 2 illustrate the relationship of (11), labeled as FF and SS, in the $(\theta, r/\sigma)$ space. Since μ^F is increasing in θ, the first equality of (11) is decreasing in θ and thus the FF curve is downward sloping.[7] Similarly, since μ^S is decreasing in θ, the second equality of (11) is increasing in θ and thus the SS curve is upward sloping. The economic intuition behind the downward FF curve and the upward SS curve is very simple. For firms, the higher θ, the higher the probability of matches and the higher the expected profit through the expected share ϕ^F. Under free entry, however, this *ex ante* expected profit induces further entry and drives down the *ex post* profitability for firms. It is clear to see that an increase in θ has an opposite effect on the expected profit for suppliers. These features of the FF and SS curves ensure the existence and uniqueness of autarky equilibrium. The intersection of two curves simultaneously determines the ratio of unmatched agents θ and the profit of unmatched firms r/σ that are both consistent with free entry, which is graphically represented in Figure 2.

[7] Differentiating the first equality of (11) with respect to θ, we also need $(\alpha^{\sigma-1} - 1)f_e^F - f_d > 0$; however, this is shown to hold by plugging the first equality into the condition under which the economic rent is positive, i.e., $(\alpha^{\sigma-1} - 1)r/\sigma - f_d > 0$.

Once these two endogenous variables are determined, other endogenous variables are written as a function of them. The variable profit of matched firms is $r(\alpha)/\sigma = \alpha^{\sigma-1}r/\sigma$ from (1). This implies that their equilibrium net profit is strictly positive because free entry is imposed on unmatched agents, as given in (5). The number of agents is determined by both free entry and search technology in the present model where the former pins down N^F, N^S and the latter pins down n from the steady-state relationship in (9). Let f denote the expected fixed cost under free entry in autarky, given as the right-hand side of (11). Noting that free entry requires $r/\sigma = f$, the number of each type of agents is expressed as (see Appendix A.3)

$$n = \left(\frac{\mu^F}{\Xi}\right)L, \quad N^F = \left(\frac{\delta + \mu^F}{\Xi}\right)L, \quad N^S = \left(\frac{\delta\theta + \mu^F}{\Xi}\right)L, \tag{12}$$

where $\Xi \equiv \sigma f(\delta + \alpha^{\sigma-1}\mu^F)$. Using (12) in the price index, welfare per worker equivalent to the real wage is

$$W = \frac{\sigma - 1}{\sigma}\left(\frac{L}{\sigma f}\right)^{\frac{1}{\sigma-1}}. \tag{13}$$

This completes the characterization of autarky equilibrium.

It is important to emphasize that market size L has no effect on the key equilibrium variables of the model, θ and r/σ. The result follows from inspecting that the free entry condition in (11) does not involve L, and hence the equilibrium represented by the intersection of FF and SS curves is not affected by market size. Intuitively, when market size increases, the number of firms and suppliers increases proportionately, and so does the number of matched agents under constant returns to scale in matching (see (12)). Since an increase in market size leads to a proportionate increase in all numbers of agents n, N^F, N^S in the industry, the ratio of unmatched agents $\theta = (N^S - n)/(N^F - n)$ remains fixed. The variable profit of unmatched firms r/σ remains fixed as well since an increase in the aggregate expenditure is exactly offset by a fall in the price index associated with an increase in the number of agents. Though market size does not affect the two key endogenous variables, (13) shows that an increase in market size nonetheless leads to welfare gains due to increased product variety in the industry.

3.4 Industry Equilibrium

Building on the equilibrium characterization, we examine how the endogenous variables are affected by industry-specific exogenous variables of the model. Before the comparative statics, we need to stress that if search frictions are prohibitively large, our model collapses to a standard monopolistic competition model of Krugman (1980). To see this point, imagine what would happen if firms were not able to search for suppliers. As the number of matches is zero ($n = 0$) in that case, the first equality of (10) is the only relevant free entry condition. Further, as the expected share of economic rent is zero ($\phi^F = 0$) in the first equality of (11), the number of firms in (12) reduces to $N^F = L/\sigma f$ (from $\mu^F = 0$), while welfare in (13) must be evaluated at $f = f_e^F$ (from $r/\sigma = f$). This equilibrium characterization is exactly the same as that derived by Krugman (1980). If search frictions are not so large that firms are able to search for suppliers, in contrast, both of these aggregate variables are critically affected by the ratio of unmatched agents seeking partners θ as seen in (12) and (13). We will show in Sections 4 and 5 that search opportunities matter for the gains or losses from trade by affecting the ratio of unmatched agents. Our model also features firm heterogeneity similar to Melitz (2003) in the sense that firms produce with different production efficiencies depending on their matching status. One of the key departures from his model, however, is that the firm distribution is *binomial* (i.e., firm status is only either unmatched or matched) which varies *endogenously* with industry characteristics.

Suppose, then, that all exogenous variables of the model differ by industry. Below we focus on the degree of input customization α and examine its impact on industry equilibrium; however the impact of other exogenous variables can be similarly examined.[8] Simple inspection of (11) reveals that an increase in α shifts down the FF and SS curves in Figure 2 and thus decreases the variable profit of unmatched firms r/σ. In contrast, from (1), the variable profit of matched firm $r(\alpha)/\sigma$ relatively increases with α, implying that the profit is relatively more skewed toward matched firms in the industry with the higher degree of input customization. In addition, this downward shift in the two curves always increases the ratio of unmatched agents seeking partners θ, since the following equilibrium relationship holds from cancelling out the common term from (10):

$$\frac{r}{\sigma} = f_e^F - f_e^S\theta. \tag{14}$$

The negative relationship between θ and r/σ derived from (14) is illustrated by the dashed curve in Figure 2, summarizing the locus of equilibria of different sets of industries. Hence, the more important input customization of the industry, the higher the ratio of unmatched agents and the lower the variable profit of unmatched firms. From this cross-industry difference, (9) shows that an increase in α raises the ratio of matched firms n/N^F but reduces the ratio of matched suppliers n/N^S. In contrast, (12) shows that an increase in α does not necessarily increase or decrease the number of agents n, N^F, N^S. As an increase in α always leads to a decrease in $f = r/\sigma$, however, (13) shows that welfare increases with the degree of input customization of the industry.

Intuition behind the results is explained by a differential impact of an increase in α on firms and suppliers. An increase in the economic rent (associated with high α) raises the *ex ante* expected profit, which induces new entry of firms and suppliers under free entry and drives down the *ex post* profitability of agents in the industry. In Figure 2, this reflects that the two curves shift downward by an increase in α. The competitive pressure is weaker for firms than for suppliers, since further entry decreases the outside option of firms (r/σ) while it has no effect on the outside option of suppliers (zero), which dampens an entry incentive of firms relative to suppliers. In Figure 2, this implies that the downward shift is smaller in the FF curve than in the SS curve. As suppliers enter relatively more than firms, the probability of matches improves for firms but worsens for suppliers, and the ratio of matched firms rises but the ratio of matched suppliers falls. However, the number of agents takes a balance between an improvement of the matching probability (i.e., increased μ^F) and a decline of the *ex post* profitability (i.e., increased α), which leads to an ambiguous effect on the number of agents.

As for welfare, we first stress that search – even though it is costly – always raises welfare relative to standard monopolistic competition models. Let $\tilde{W}$ denote welfare when search frictions are so large that any firms cannot seek suppliers, keeping W to denote welfare when such frictions are small enough. From (13), the welfare ratio $W/\tilde{W}$ depends only on the (expected) fixed cost f, which in turn equals the variable profit r/σ under free entry. In light of (14), $\tilde{W}$ must be evaluated at $f = f_e^F$ (as $\theta = 0$ with prohibitively large search frictions) and hence

$$\frac{W}{\tilde{W}} = \left(\frac{f_e^F}{f_e^F - f_e^S\theta}\right)^{\frac{1}{\sigma-1}}.$$

The fact $W/\tilde{W} > 1$ follows from $\theta > 0$ in our model. Then, the above comparative statics suggest that the welfare ratio is higher, the higher the degree of input customization of the industry. Intuitively, an increase in α raises the number of suppliers N^S relative to that of firms N^F in the industry (see (12)). This market structure allows

[8] To be more precise, we need to assume that a representative agent has a two-tier utility function defined over multiple industries where the upper tier is Cobb-Douglas and the lower tier is Dixit-Stiglitz for comparative statics. Though our paper can start with this general setting, we choose a baseline model easily comparable to Krugman (1980), letting the preference structure be understood.

firms to find suppliers more easily (reflected by the higher ratio of unmatched agents $\theta = (N^S - n)/(N^F - n)$), and varieties are relatively more likely produced by matched firms than unmatched firms. At the same time, resources are relatively more allocated from unmatched firms to matched firms (reflected by the lower profit of unmatched firms r/σ). In that sense, the difference in firms' choices between market and non-market activities can account for the predominant difference in aggregate productivity at the industry level.

Our prediction for cross-industry variations can be empirically examined by computing the expenditure share on final goods produced by unmatched firms in the industry. Let $s \equiv (N^F - n)r/R$ denote the expenditure share on final goods produced by unmatched firms where the aggregate expenditure R equals market size L. Observe that market size has no impact on the expenditure share because all of the variables consisting of that share increase proportionately with market size in the present model with constant returns to scale in matching. In fact, using (1) and (9) in $R = (N^F - n)r + nr(\alpha)$, the expenditure share is expressed as

$$s = \frac{\delta}{\delta + \alpha^{\sigma-1}\mu^F}, \tag{15}$$

where μ^F is not affected by market size (as θ is independent of L). The expenditure share, however, is critically affected by the degree of input customization α in a way such that the share is lower, the higher the degree of input customization of the industry. This reflects that the expenditure share is relatively more allocated from unmatched firms to matched firms in the industry where input customization is more important.

The result can be alternatively examined in terms of the relative expenditure share $s/(1-s)$. Noting that the expenditure share on matched firms' products is $1 - s = nr(\alpha)/R$, (15) implies

$$\frac{s}{1-s} = \frac{\delta}{\alpha^{\sigma-1}\mu^F},$$

which is further decomposed into the intensive margin ratio $r/r(\alpha) = 1/\alpha^{\sigma-1}$ and the extensive margin ratio $(N^F - n)/n = \delta/\mu^F$ from (1) and (12) respectively. It can be easily seen that the ratio is decreasing in α not only through the *exogenous* intensive margin ratio but also through the *endogenous* extensive margin ratio. Thus the model predicts that the higher degree of input customization of the industry entails the lower ratio of goods produced by unmatched firms to goods produced by matched firms in the industry, both because unmatched firms produce at a lower level of output (and earn lower revenue) in the industry and because a smaller set of unmatched firms produce in the industry.

Proposition 1: *The ratio of market transactions across unmatched agents to non-market transactions within matched agents is smaller, the higher the degree of input customization of the industry.*

Our theoretical finding is consistent with empirical evidence found by Nunn (2007) who constructs a variable that measures, for each product defined at the industry level, the proportion of its intermediate inputs transacted on a non-organized exchange (broadly speaking "non-market" activity). To explain why the proportion is higher in an industry with higher relationship-specificity, Nunn (2007) stresses contractual frictions between agents as a main deriving force, but we show that search frictions can alternatively provide a theoretical foundation of his measure. In our model, the measure is given by $1 - s$, which summarizes the expenditure share on final goods produced by matched firms who procure customized inputs via non-market activity such as search, matching and bargaining. This measure differs by industry because the number of buyers and sellers ("market thickness") endogenously varies with industry characteristics, thereby affecting the matching probability of agents.

4 Integration of Final-good Markets

4.1 Assumptions

We turn to considering a world economy composed of two countries of the type described in Section 3. There is no difference in labor endowments and technology between these countries, so that $L = L^*$ and $a^F = a^{F^*} = a^S = a^{S^*} (= 1)$, where asterisks are attached to foreign variables. Further, they have the same matching function and the same primitive bargaining power of agents (the latter is assumed symmetric between firms and suppliers). As a result, not only is the number of firms and suppliers but the number of unmatched and matched agents is also the same between the two countries. Although the countries are assumed symmetric, they are not exactly identical as each firm produces a variety differentiated at home and abroad.

This section first studies economic integration of final-good markets referred to as *X-integration* in our paper, which allows firms to sell final goods across borders, maintaining the assumption that firms search for suppliers only within borders. In this integration, a firm wishing to export must incur an iceberg transport cost τ_x and a one-time fixed cost F_x (measured in units of labor) where the latter can be regarded as an investment that is required to enter the export market as modeled by Melitz (2003). Our assumptions of X-integration imply that the two countries have the same rate at which unmatched agents meet partners, given by μ^F and $\mu^S = \mu^F/\theta$ where $\theta \equiv (N^S - n)/(N^F - n)$ as before. In this setting, the aggregate revenue equals the aggregate payment to labor, where the common wage rate is normalized to one.

When X-integration takes place, there are three possible cases associated with different levels of trade costs: (i) no firm exports; (ii) only matched firms export; or (iii) both unmatched firms and matched firms export.[9] For the expositional purpose, we mainly focus on the case where the level of trade costs is intermediate so that matched firms export but unmatched firms do not. As will be clear shortly, the level of trade costs is immaterial to our analysis, and the results of X-integration essentially remain valid even in the case where trade costs are so low that unmatched firms also export. In relation to this point, if there are no trade costs and all firms freely export, search frictions do not play any role in examining the impact of X-integration. In that case, the impact of X-integration is the same as that of market size, in the sense that welfare gains come solely from increased product variety without affecting the two key endogenous variables of the model, θ and r/σ. This impact is very similar to that described by Krugman (1980) and Melitz (2003), even though they do not model search frictions. If firms incur trade costs, however, X-integration has a critical impact on these two variables so as to reinforce welfare gains by improving the matching frequency of firms and reallocating resources from unmatched firms to matched firms. As the equilibrium conditions in X-integration are similar to those in autarky, we provide only important equations pertaining to this integration below and relegate others to Appendix A.4.

4.2 Equilibrium Characterization

This section mainly characterizes and solves for X-integration equilibrium at home due to country symmetry. It is clear that the equilibrium price, revenue and profit are the same as those in autarky in the home market for both types of firms. In addition, matched firms export in X-integration and set the higher equilibrium price in the foreign market due to the increased marginal cost τ_x:

$$p_x(\alpha) = \frac{\sigma}{\sigma - 1} \frac{\tau_x}{\alpha}.$$

[9] Since unmatched firms are less efficient than matched firms, there is no equilibrium where only unmatched firms export.

Given that unmatched firms do not export, their revenue (and profit) is earned only from the home market and the equilibrium domestic revenue is the same as that in autarky, which is denoted by r as before. In contrast, the equilibrium domestic revenue of matched firms is $r_d(\alpha) = \alpha^{\sigma-1}r$, while the equilibrium export revenue of these firms is $r_x(\alpha) = \tau_x^{1-\sigma} r_d(\alpha)$ with CES preferences and the optimal pricing above. This in turn yields the equilibrium revenue $r_d(\alpha) + r_x(\alpha) = r(\alpha)$ as well as the equilibrium profit $r(\alpha)/\sigma$. Similarly to (1) in autarky, the equilibrium export revenue for matched firms must satisfy the following equality in X-integration:

$$\frac{r_x(\alpha)}{r} = \left(\frac{\alpha}{\tau_x}\right)^{\sigma-1}. \tag{16}$$

We next specify the equilibrium conditions of X-integration. Under the assumption that only matched firms export, the Bellman equations in X-integration entail the following differences. First, unmatched firms become matched at the rate μ^F at which point they obtain a gain $V^F(\alpha) - F_x - V^F$ where matched firms make a one-time investment F_x for entry into the foreign market. Second, matched firms obtain the variable profit from not only the home market $r_d^F(\alpha)/\sigma$ but also the foreign market $r_x^F(\alpha)/\sigma$, which sum to $r^F(\alpha)/\sigma$. While the Bellman conditions for suppliers are the same as (2), matched suppliers also obtain the variable profit from the home market $r_d^S(\alpha)/\sigma$ and the foreign market $r_x^S(\alpha)/\sigma$, which sum to $r^S(\alpha)/\sigma$. As in autarky, Nash bargaining determines how to split the total variable profit $r(\alpha)/\sigma = r^F(\alpha)/\sigma + r^S(\alpha)/\sigma$.

Setting $\gamma = 0$ as well as $\dot{V}^F = \dot{V}^F(\alpha) = 0$, we get the value functions of firms in X-integration. We assume $r^F(\alpha)/\delta\sigma - r/\delta\sigma - F_x > 0$ in order to ensure that matched firms have an enough incentive to export goods in the presence of search frictions. Under this condition, $V^F(\alpha) - F_x - V^F > 0$ so that the net value of matched firms is greater than that of unmatched firms. As in F_d sunk by suppliers, this is the only reason that firms consider sinking the investment cost F_x. This also implies that the only difference from autarky in the profit sharing is the ex-post gains for matched firms which are replaced by $V^F(\alpha) - F_x - V^F$ in X-integration. From the fact that the constraint of (3) similarly holds in X-integration, the solution to the bargaining problem subject to $r^F(\alpha)/\sigma + r^S(\alpha)/\sigma = r(\alpha)/\sigma$ gives us the following profit sharing rule:

$$\begin{aligned}
\frac{r^F(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_x &= \beta\left(\frac{r(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d - F_x\right), \\
\frac{r^S(\alpha)}{\delta\sigma} - F_d &= (1-\beta)\left(\frac{r(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d - F_x\right),
\end{aligned}$$

where the effective bargaining power β is the same as (8) in autarky (for a given ratio θ) as the opportunities to search for partners are restricted only within borders in X-integration. For the same reason, the steady-state number of agents is also the same as (9) in autarky.

Finally, using the optimal profit sharing, the free entry condition in (5) can be written as

$$\begin{aligned}
\frac{r}{\sigma} + \frac{n}{N^F}\beta\left(\frac{r(\alpha)}{\sigma} - \frac{r}{\sigma} - f_d - f_x\right) - f_e^F &= 0, \\
\frac{n}{N^S}(1-\beta)\left(\frac{r(\alpha)}{\sigma} - \frac{r}{\sigma} - f_d - f_x\right) - f_e^S &= 0,
\end{aligned} \tag{17}$$

where $f_x \equiv \delta F_x$ denotes the amortized per-period portion of the one-time fixed export cost incurred at every point in time. We can interpret (17) in X-integration in a similar way to (10) in autarky. Further, this condition characterizes the model's equilibrium by simultaneously determining the ratio of unmatched agents θ and the profit of unmatched firms r/σ that are both consistent with free entry in X-integration.

Though the free entry condition (17) looks similar to that in autarky (10), the economic rent is earned from the home and foreign final-good markets by matched firms in X-integration. It follows immediately from (16) that the economic rent of matched agents is expressed as

$$\frac{r(\alpha)}{\sigma} - \frac{r}{\sigma} - f_d - f_x = \frac{\left[\left(1+\tau_x^{1-\sigma}\right)\alpha^{\sigma-1} - 1\right] r}{\sigma} - f_d - f_x.$$

Hence the economic rent of matched agents depends not only on the degree of input customization α but also on the level of trade costs τ_x, f_x in X-integration. Plugging this into (17), we can solve for the variable profit of unmatched firms, which satisfies the following equalities:

$$\begin{aligned} \frac{r}{\sigma} &= \frac{f_e^F + (f_d + f_x)\phi^F}{1 + \left[\left(1+\tau_x^{1-\sigma}\right)\alpha^{\sigma-1} - 1\right]\phi^F}, \\ \frac{r}{\sigma} &= \frac{f_e^S + (f_d + f_x)\phi^S}{\left[\left(1+\tau_x^{1-\sigma}\right)\alpha^{\sigma-1} - 1\right]\phi^S}, \end{aligned} \tag{18}$$

where the expected shares of matched agents ϕ^F, ϕ^S are the same as those in Section 3. Then, (18) shows that the variable profit equals the expected fixed cost of firms and suppliers in X-integration. It is important to see, however, that the expected fixed cost includes trade costs τ_x, f_x, even though we have derived the equilibrium where unmatched firms do not export in X-integration. This reflects the dynamics where unmatched agents may become matched in the future at which point they share trade costs at the bargaining stage with the expected shares ϕ^F, ϕ^S.

(18) can be illustrated in the $(\theta, r/\sigma)$ space. The argument similar to Figure 2 applies here to establish the existence and uniqueness of X-integration equilibrium, represented by the intersection of FF and SS curves. The questions that remain to ask are: (i) how the equilibrium variables are affected by X-integration; and (ii) how changes in the equilibrium variables are related to the gains from trade in X-integration. In the follows, we first address the impact of X-integration in Section 4.3 and next consider the gains from trade in X-integration in Section 4.4. Finally, we calibrate the model in order to assess the quantitative relevance in Section 4.5.

4.3 Impact of X-integration

To examine the impact of X-integration, we consider how the free entry condition (17) is affected by the level of trade costs associated with this integration. In equilibrium where only matched firms export, matched agents earn higher profit in X-integration than in autarky ($(1+\tau_x^{1-\sigma})\alpha^{\sigma-1}r/\sigma - f_d - f_x > \alpha^{\sigma-1}r/\sigma - f_d$), but unmatched agents do not ($(1+\tau_x^{1-\sigma})r/\sigma - f_x < r/\sigma$). Simplifying these inequalities, we find that trade costs fall into the following intermediate level in such equilibrium:

$$\frac{r}{\sigma} < \tau_x^{\sigma-1} f_x < \frac{\alpha^{\sigma-1} r}{\sigma}. \tag{19}$$

Comparing (10) and (17) under (19) reveals that the economic rent of matched agents is greater in X-integration ($\left[\left(1+\tau_x^{1-\sigma}\right)\alpha^{\sigma-1} - 1\right] r/\sigma - f_d - f_x$) than in autarky ($\left(\alpha^{\sigma-1} - 1\right) r/\sigma - f_d$) for given expected shares ϕ^F, ϕ^S. This raises the *ex ante* expected profit and induces further entry of firms and suppliers under free entry, which in turn decreases the *ex post* profit r/σ in X-integration relative to autarky. In fact, comparing (11) and (18) under (19) reveals that the FF and SS curves in X-integration are always located below those in autarky for given θ. Further, the equilibrium relationship (14) continues to hold in X-integration from rearrangement of (17), so that a decrease in r/σ always leads to an increase in θ which occurs only when the downward shift is smaller in the FF curve than in the SS curve. These changes in two curves imply that X-integration not only enlarges the profit differential between matched and unmatched firms but also increases the ratio of unmatched agents θ and improves the matching frequency of firms μ^F. Although we have derived the impact of X-integration by comparing equilibria between autarky and this integration, simple inspection of (18) shows that reductions in trade costs (either variable τ_x or fixed f_x) induce a similar impact on the two curves and hence the two key endogenous variables.

Intuitively, the downward shifts in FF and SS curves are triggered by an increase in the economic rent in X-integration, as seen above. Note, however, that this increase only requires the second inequality of (19), i.e., matched firms export in this integration. The extent to which the economic rent increases differs between firms and suppliers. From the free entry condition of firms given in the first equality of (17), X-integration has two effects on their expected profit. First, this integration allows foreign firms to penetrate the home market, which drives down the *ex post* profitability of firms there and decreases their outside option (first term). Second, this integration allows home firms to have direct access to the foreign market, which gives them an additional opportunity to earn the export profit and increases the economic rent earned by matched firms (second term). While the latter dominates the former under (19), the opposing effects dampen the entry incentive of firms. As for the free entry condition of suppliers, in contrast, suppliers receive no negative effect on the expected profit from foreign firms' penetration of the home market because the outside option of being unmatched is zero. Thus X-integration raises the *ex ante* expected profit of suppliers relative to firms, which leads to the *ex post* large competitive pressure on suppliers relative to firms. As the number of suppliers rises relative to firms, the ratio $\theta = (N^S - n)/(N^F - n)$ rises in X-integration relative to that in autarky.

This intuition suggests that, even if trade costs are so low that both unmatched and matched firms export, the impact of X-integration on the key endogenous variables is similar. This can be confirmed by observing that the second inequality in (19) is only relevant to an increase in the expected profit that results in the downward shift in the two curves. When unmatched firms also export, the first inequality of (19) is reversed but matched firms nonetheless earn revenue and profit relatively more than unmatched firms in X-integration, which in turn raises the expected profit and induces the downward shift in two curves. In addition, the effect of X-integration on the expected profit is larger for suppliers than for firms as the outside option of suppliers is zero before and after X-integration even in this case. The differential effect induces further entry of suppliers relative to firms, leading to the greater downward shift for the SS curve than for the FF curve. Thus, unmatched firms' profit falls and the ratio of unmatched agents rises even in this equilibrium. The only difference is that this decrease applies to the *domestic* profit of unmatched firms (r_d/σ). In contrast, the *total* profit increases since they earn the export profit as well ($r/\sigma = (1 + \tau_x^{1-\sigma})r_d/\sigma$), though matched firms earn larger total profit.[10]

Once these two endogenous variables are determined, other endogenous variables are written as a function of them. The number of agents is expressed in a similar way to (12) where $\Xi \equiv \sigma f \left[\delta + \left(1 + \tau_x^{1-\sigma}\right) \alpha^{\sigma-1} \mu^F\right]$ and f is the expected fixed cost under free entry of X-integration, given as the right-hand side of (18). Welfare per worker is also expressed in a similar way to (13). The impact of X-integration yields the following implications for these variables. First, from the fact that θ rises in X-integration, the ratio of matched firms is larger but the ratio of matched suppliers is smaller in X-integration than in autarky. Second, from the fact that $r/\sigma (= f)$ falls in X-integration, welfare is higher in X-integration than in autarky. Thus $n/N^F > n_a/N_a^F$, $n/N^S < n_a/N_a^S$, $W > W_a$ where the subscript a is attached to autarky variables. In contrast, the number of agents can increase or decrease

[10]See Appendix A.4 for a detailed analysis of equilibrium where both unmatched and matched firms export. We show there that without fixed export cost ($f_x = 0$), any level of transport cost $\tau_x > 0$ does not affect the two endogenous variables in X-integration.

by X-integration. The effect of X-integration on the number of agents and welfare is similar even in the case where both unmatched and matched firms export.

4.4 Gains from Trade in X-integration

The impact of X-integration on two key endogenous variables allows us to highlight a new mechanism through which economic integration of final-good markets generates the gains from trade. Recall that if search frictions are prohibitively large, our model reduces to Krugman (1980)'s trade model. To make the point, suppose that all firms export as in Krugman (1980). This corresponds to equilibrium where both unmatched and matched firms export in our model in which case the free entry condition similar to (17) also holds (see Appendix A.4). If firms cannot seek suppliers in that equilibrium, then, the number of matched agents is zero ($n = 0$) and the variable profit must equal the fixed entry cost ($r/\sigma = f_e^F$) as before. As the free entry condition pinning down the number of firms remains the same as that in autarky, the number of varieties N^F also remains the same. However, since all firms export at a cost of τ_x, the total number of varieties available in each market increases to $(1 + \tau_x^{1-\sigma})N^F$. This impact of trade on equilibrium variables is identical to that shown by Krugman (1980): although the number of varieties produced is not affected by trade, welfare is higher in X-integration than in autarky because the number of varieties consumed rises by trade.

If search frictions are not prohibitively large as in our model, in contrast, the number of firms is affected by trade as the expected fixed cost $f(= r/\sigma)$ varies from autarky to X-integration. This in turn alters the impact of trade on welfare through an extra adjustment margin of trade, namely *improved matching frequency* of firms that is absent in a standard trade model. On the one hand, the fact that unmatched firms' profit falls as a result of X-integration ($r/\sigma < r_a/\sigma$) implies that resources are reallocated from less efficient unmatched firms to more efficient matched firms within the industry. On the other hand, the fact that the ratio of unmatched agents rises as a result of this integration ($\theta > \theta_a$) implies that firms have the higher probability to meet suppliers, enhancing the overall production efficiency of the industry. These two features of our model jointly offer a new channel through which countries enjoy the gains from trade in the presence of search frictions.

To see the new channel of our model, we find it useful to consider how X-integration affects the expenditure share on final goods produced by unmatched firms in X-integration, given as $s = (N^F - n)r/R$ as in autarky. While the aggregate expenditure R includes expenditure spent on foreign goods in X-integration, it is still equal to the exogenously fixed labor endowment L, which means that X-integration only reallocates the expenditure share on final goods produced by unmatched and matched firms. From (9) and (16), this share is given by

$$s = \frac{\delta}{\delta + (1 + \tau_x^{1-\sigma})\alpha^{\sigma-1}\mu^F}.$$

Compared to (15), the expenditure share falls in X-integration. In this model, a decrease in this share is directly related to the gains from trade because this change implies that consumers can access relatively more varieties that are produced by customized inputs in X-integration than in autarky. Further, the relative expenditure share $s/(1-s)$ can be decomposed into the intensive margin ratio $r/r(\alpha) = 1/(1+\tau_x^{1-\sigma})\alpha^{\sigma-1}$ and the extensive margin ratio $(N^F - n)/n = \delta/\mu^F$. Clearly these two ratios fall in X-integration relative to those in autarky, reflecting that unmatched firms shrink and matched firms expand in this integration. The impact of X-integration on resource reallocations between firms has a similar flavor to that in Melitz (2003), but our novelty is that such reallocations are associated with an increase in the ratio of unmatched agents seeking partners θ that improves the matching frequency of firms μ^F, which additionally contributes to the gains from trade.

Proposition 2: *X-integration increases welfare in both countries by improving the matching frequency of firms associated with resource reallocations from unmatched firms to matched firms.*

A number of features of Proposition 2 are worth noting. First, the theoretical prediction can be empirically tested by comparing Nunn (2007)'s measure before and after final-good trade liberalization. The implication of this proposition is that the measure ($1 - s$ in our model) rises after such liberalization. This change is thought of as summarizing the matching frequency of firms improved by X-integration, as more varieties are made by matched firms via "non-market" activity. In light of Proposition 1, we can also say that the measure is higher in X-integration than in autarky, the higher the degree of input customization of the industry.

Second, the matching frequency improved by X-integration can amplify the gains from trade in our model relative to a standard trade model. From (13), the welfare ratio between X-integration and autarky is expressed in terms of expected fixed costs: $W/W_a = (f/f_a)^{-1/(\sigma-1)}$. Using $f = r/\sigma,\ f_a = r_a/\sigma$ and rearranging,

$$\frac{W}{W_a} = \left[\left(\frac{N_a^F + (\alpha^{\sigma-1} - 1)n_a}{N^F + (\alpha^{\sigma-1} - 1)n}\right)\lambda\right]^{-\frac{1}{\sigma-1}}, \tag{20}$$

where $\lambda \equiv [(N^F - n)r + nr(\alpha)]/R$ is the expenditure share on domestic goods. If search frictions are prohibitively large, the number of matches is zero ($n = n_a = 0$) and the number of firms is not affected by trade ($N^F = N_a^F$). In that case, (20) reduces to $W/W_a = \lambda^{-1/(\sigma-1)}$ and the gains from trade can be captured only by the domestic expenditure share and trade elasticity (Arkolakis et al., 2012). If search frictions are not prohibitively large, however, the number of firms changes so that the ratio of matched firms rises by X-integration ($n/N^F > n_a/N_a^F$). In that case, firm heterogeneity also matters for the gains from trade beyond the two sufficient statistics. The result is along the lines of recent research showing that the gains from trade are greater in Melitz (2003) than in Krugman (1980) due to *endogenous firm selection* that is absent in the latter (Melitz and Redding, 2015). The argument similarly applies to our paper by noting that firm heterogeneity is driven by matching status. Hence, the gains from trade can be greater in our model than in Krugman (1980) due to *endogenous firm matches* that are absent in the latter.

Finally, our model is shown to be constrained efficient in both autarky and X-integration, in the sense that a world social planner faced with the same technology would choose the same distribution of quantities and the same number of agents as those in Sections 3 and 4.[11] This efficiency property of equilibrium suggests that our welfare results come from generating an extra adjustment margin of trade, but do *not* come from adjusting the insufficient/excessive entry of firms.

4.5 Numerical Solutions

To get a sense of the magnitude involved in X-integration, we parameterize the model and solve it numerically. In both autarky and X-integration, we compare equilibrium with search as in this model to that without search as in Krugman (1980). The following conditions are imposed to render this comparison meaningful and sharp. First, we contrast the outcomes in these two cases by assuming that exporting firms incur both variable and fixed trade costs in X-integration, as there is no fixed export cost in Krugman (1980). Second, we restrict attention to X-integration equilibrium where only matched firms export. As stressed above, however, our results are similar in X-integration equilibrium where both unmatched and matched firms export.

[11] See Melitz and Redding (2015) for a similar argument. In contrast, if we drop the assumption of CES preferences, the equilibria in Sections 3 and 4 would not be always efficient with firm heterogeneity (Dhingra and Morrow, 2019).

Table 1 – Quantitative impact of X-integration

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
	r/σ	$r_d(\alpha)/\sigma$	$r_x(\alpha)/\sigma$	θ	μ^F	μ^S	N^F	N^S	n	s	λ	W
Autarky	1.68	4.22	0	0.16	0.14	0.86	65	57	55	0.06	1	3.97
	2	0	0	0	0	0	125	0	0	0	1	3.75
X-integration	1.53	3.84	0.66	0.23	0.19	0.81	60	55	53	0.04	0.85	4.10
	2.27	0	0	0	0	0	109	0	0	0	0.85	3.78

Notes: The first row corresponds to values in our model, while the second row corresponds to values in the Krugman Benchmark in autarky and X-integration.

Following Grossman and Helpman (2002), we assume that the matching function is given by $m(u^F, u^S) = u^F u^S/(u^F + u^S)$. With this specification, the rate at which unmatched firms meet unmatched suppliers is equal to $\mu^F = \theta/(1+\theta)$, and the rate at which unmatched suppliers meet unmatched firms is equal to $\mu^S = 1/(1+\theta)$. Using these rates, the expected share of matched firms and suppliers is expressed as $\phi^F = \theta/(1+\theta)(2\delta+1)$ and $\phi^S = 1/(1+\theta)(2\delta+1)$ respectively. Substituting these shares into the free entry condition in autarky (11) and that in X-integration (18), we can explicitly solve for the ratio of unmatched agents in both cases. For example, combining the two equalities in (11) and rearranging, the closed-form solution of θ in autarky is

$$\theta = \frac{(\alpha^{\sigma-1} - 1)f_e^F - (2\delta + 1)f_e^S - f_d}{(\alpha^{\sigma-1} + 2\delta)f_e^S}.$$

The variable profit of unmatched firms is subsequently obtained by noting the equilibrium relationship between θ and r/σ in (14), which holds in both autarky and X-integration.

We choose standard values for the monopolistic competition model's parameters employed in the literature. We set the elasticity of substitution between varieties $\sigma = 4$. Market size L has no impact on two key endogenous variables of the model under constant returns to scale in matching (including the matching function above) and we simply choose $L = 1000$. Following Bernard et al. (2007b), we set the probability of a bad shock $\delta = 0.025$, the fixed production cost $f_d = 0.1$ and the fixed entry cost for firms $f_e^F = 2$. We also set the same level of fixed entry cost for suppliers $f_e^S = 2$. Regarding the fixed and variable export costs, when only matched firms export, the level of trade costs must be intermediate so as to satisfy (19). Thus we choose $f_x = 0.275$, $\tau_x = 1.8$ in the exercise below but our results would not be sensitive to these parameter values. Finally, we calibrate the degree of input customization α to match the average shipment advantage of exporting firms over non-exporting firms. As reported in Bernard et al. (2007a), exporting firms have on average 108 percent larger log shipments than non-exporting firms in US manufacturing (after including industry fixed effects). Substituting (1) and (16) into $\ln([r_d(\alpha) + r_x(\alpha)]/r) = 1.08$, we obtain $\alpha = 1.36$.

Table 1 summarizes quantitative comparison between autarky and X-integration computed under the above specifications and parameter values. In autarky and X-integration, the values in the first and second rows are those with search and without search respectively where the latter is labelled as the Krugman Benchmark below. Though we examine the impact of X-integration by comparing equilibria between autarky and this integration, the impact is similar when comparing high trade costs and low trade costs in X-integration.

Columns I–III show the variable profit of unmatched firms r/σ and that of matched firms $r_d(\alpha)/\sigma, r_x(\alpha)/\sigma$. In the Krugman Benchmark, the variable profit equals the fixed costs f_e^F in autarky and $f_e^F + f_x$ in X-integration. The profit level is higher in X-integration than in autarky since the introduction of fixed export costs raises average firm output and hence average firm profit as well. In our model, in contrast, the variable profit equals the expected fixed cost given as (11) in autarky and (18) in X-integration. The profit level of unmatched firms is smaller in X-integration than in autarky due to penetration of (more efficient) exporting firms from abroad. Though matched firms earn smaller profit from the domestic market, their total profit is larger in X-integration than in autarky. This indicates that resources are reallocated from unmatched firms to matched firms.

Columns IV–VI show the ratio of unmatched agents θ and the associated probability of matches μ^F, μ^S. In the Krugman Benchmark, the ratio is zero and hence the probability of matches is also zero. In our model, in contrast, the ratio is higher in X-integration than in autarky as this integration raises the *ex ante* expected profit of suppliers relative to firms, which leads to the *ex post* large competitive pressure on suppliers relative to firms, increasing the ratio of unmatched suppliers to unmatched firms. Due to trade-induced industry restructuring in market thickness, X-integration improves the probability of firms' matches by roughly 5 percent but it worsens the probability of suppliers' matches by the same amount.

Columns VII–IX then show the number of agents N^F, N^S as well as the number of matched firms n. In the Krugman Benchmark, the number of firms falls in X-integration as labor is used to fixed export costs f_x; but the total number of varieties (domestic plus foreign) rises to $(1 + \tau_x^{1-\sigma})N_F$. For our parameter values, this number is 128, which is greater than 125 in autarky. In our model, however, the number of firms is smaller than that in the Krugman Benchmark in both autarky and X-integration because labor is used to develop non-core inputs that are not directly related to the number of varieties. At the same time, such development also allows matched firms to access customized inputs, improving their production efficiency relative to unmatched firms. Hence, the search opportunity creates a tradeoff between decreased product variety and improved production efficiency of matched firms. The total number of varieties that takes into account the tradeoff is $N^F - n + (1 + \tau_x^{1-\sigma})\alpha^{\sigma-1}n$ in our model. This number is 163 in the numerical exercise, which is greater than 149 in autarky of our model and 128 in X-integration of the Krugman Benchmark.

Finally, Columns X–XII show the expenditure share on unmatched firms' goods s, the domestic expenditure share λ and welfare W. In the Krugman Benchmark, the gains from trade are due to increased product variety. For our parameter values, the welfare ratio between X-integration and autarky is 1.009 and thus the gains from trade are 0.9 percent, a comparable magnitude reported in Arkolakis et al. (2012). In our model, X-integration reduces unmatched firms' expenditure share by improving the matching frequency of firms, which additionally contributes to the gains from trade. The welfare ratio between the two regimes in (20) is 1.031 and thus the gains from trade are 3.1 percent.[12] Examining reductions in variable export costs from $\tau_x = 1.8$ in X-integration, we also find that such gains are roughly 2 percent higher in our model than those in the Krugman Benchmark. Finally, note that not only is the trade elasticity but also the domestic expenditure share is (almost) the same between the two models. This confirms our theoretical results that endogenous firm matches entail first-order significance for the gains from trade beyond the two sufficient statistics.

5 Integration of Matching Markets

5.1 Assumptions

We next study economic integration of matching markets referred to as *M-integration* in the paper, which allows firms to search for suppliers across borders (though they can search in only one country at any point in time),

[12] In the Krugman Benchmark, it can be easily shown that the welfare ratio is expressed as $W/W_a = \left[(f_e^F + f_x)\lambda/f_e^F\right]^{-1/(\sigma-1)}$. When there is no fixed export cost ($f_x = 0$), this ratio naturally reduces to (20) corresponding to Krugman (1980).

maintaining the assumption that final goods are traded within borders. For simplicity, this section assumes that the matching markets are integrated so that customized inputs are traded. On the other hand, the competitive markets are segregated so that generic inputs are non-traded. However our results of M-integration remain valid if generic inputs are freely traded via integrated markets as an "outside" good that is frequently assumed in the literature.

In order to meaningfully contrast the implications of X- and M-integration, we impose the same assumptions as in Section 4, i.e., there is no difference in labor endowments and technology between two trading countries. The structure of trade costs is also similar to those in Section 4, in the sense that a firm wishing to import from a supplier matched abroad must incur an iceberg transport cost τ_m and a one-time fixed cost F_m (measured by units of labor) in this integration. The latter includes both an investment cost to customize inputs just like F_d and a communication cost to reach foreign suppliers. Due to additional resources included in F_m, agents are matched with foreign partners must incur a higher fixed cost, $F_m > F_d$ (Antràs and Helpman, 2004).[13] While the matching function is assumed to be the same between domestic and cross-border matches, the number of matches per unit of time in each country becomes half (as matches occur both within and across borders with identical search technology). Thus, the rate at which unmatched firms and suppliers meet unmatched partners in one country reduces to $\mu^F/2$ and $\mu^S/2 = \mu^F/2\theta$. In Figure 1, the number $\mu^F u^F/2$ of firms enters the matched pool with being matched with domestic suppliers and the same number of firms enters with being matched with foreign suppliers. Though we define $\theta \equiv (N^S - n)/(N^F - n)$ as before, we denote by $n \equiv N^F - u^F = N^S - u^S$ the *total* number of matched agents which includes both types of matches. Finally, when agents find partners in one country, they stop searching in another country by implicitly assuming a large switching cost (Antràs and Costinot, 2011). Under these assumptions, the aggregate revenue still equals the aggregate payment to labor, where the common wage rate is normalized to one.

When M-integration takes place, there are two possible cases associated with different levels of trade costs: (*i*) no firm imports; or (*ii*) firms matched with foreign suppliers import. Below we focus on a more interesting case where cross-border matches are profitable. Clearly, such equilibrium occurs with the low level of trade costs, but M-integration entails not only trade costs but also search frictions that agents face to seek foreign partners. Autarky can be regarded as a special case in which such frictions are so prohibitively large that no agents are able to seek these partners. M-integration then allows firms to have an additional opportunity to source inputs from another country. This differs from X-integration which allows firms to have an additional opportunity to sell final goods to another country, though the two forms of economic integration help internationalize trading opportunities. With this difference in mind, we show that M-integration causes welfare losses for both countries by worsening the matching frequency of firms, which stands in sharp contrast to the impact of X-integration. (Just like X-integration, this contrasting outcome can occur in equilibrium only if firms incur some trade costs.) Assuming identical search technology between domestic and cross-border matches, we first analyze a simple case where search frictions are the same between the two types of matches. We later analyze an extended case where search frictions are larger for cross-border matches than for domestic matches. In both cases, the equilibrium conditions are similar to those in autarky and detailed conditions are relegated to Appendix A.5.

5.2 Equilibrium Characterization

We mainly characterize and solve for M-integration equilibrium at home as in X-integration. In M-integration, however, there are six types of agents at home at any point in time: (*i*) unmatched firms; (*ii*) firms matched

[13]From this reason, if there is no fixed cost associated with M-integration, agents matched with domestic and foreign suppliers incur the same fixed cost in this integration ($F_d = F_m$).

with home suppliers; (*iii*) firms matched with foreign suppliers; (*iv*) unmatched suppliers; (*v*) suppliers matched with home firms; and (*vi*) suppliers matched with foreign firms. It is clear that the behavior of agents classified as (*i*), (*ii*), (*iv*) and (*v*) is similar to that in autarky. In particular, denoting by $r_d(\alpha)$ the equilibrium revenue of firms matched with home suppliers (instead of $r(\alpha)$), this equilibrium revenue satisfies the relationship in (1). Regarding the behavior of agents matched with foreign partners classified as (*iii*) and (*vi*), on the other hand, together with the fact that firms matched with foreign suppliers source the input x^S by incurring a transport cost τ_m, the Cobb-Douglas production function implies the price of the final good equal to

$$p_m(\alpha) = \frac{\sigma}{\sigma - 1} \frac{\tau_m^{1-\eta}}{\alpha}.$$

From the equilibrium price, the equilibrium revenue of these firms is given by $r_m(\alpha) = \tau_m^{(1-\sigma)(1-\eta)} r_d(\alpha)$ where $r_d(\alpha)$ is the equilibrium revenue of firms matched with home suppliers introduced above. Hence, the equilibrium revenue of firms matched with foreign suppliers must satisfy

$$\frac{r_m(\alpha)}{r} = \left(\frac{\alpha}{\tau_m^{1-\eta}} \right)^{\sigma-1}, \tag{21}$$

which is very similar to (16) in X-integration.

We next specify the equilibrium conditions of M-integration. As there are six types of agents at any point in time, we need to introduce six types of value functions. Below we focus on the Bellman equations for firms. Let V^F, $V_d^F(\alpha)$, and $V_m^F(\alpha)$ denote the value function of unmatched firms, firms matched with home suppliers and firms matched with foreign suppliers respectively. Depending on the matching status of firms, we can express the Bellman equations of the three types of firms as follows:

$$\begin{aligned}
\gamma V^F &= \frac{r}{\sigma} + \frac{\mu^F}{2}\left(V_d^F(\alpha) - V^F\right) + \frac{\mu^F}{2}\left(V_m^F(\alpha) - V^F\right) - \delta V^F + \dot{V}^F, \\
\gamma V_d^F(\alpha) &= \frac{r_d^F(\alpha)}{\sigma} - \delta V_d^F(\alpha) + \dot{V}_d^F(\alpha), \\
\gamma V_m^F(\alpha) &= \frac{r_m^F(\alpha)}{\sigma} - \delta V_m^F(\alpha) + \dot{V}_m^F(\alpha).
\end{aligned}$$

Compared to those in autarky, the Bellman equations in M-integration have the following differences. The first equation shows that unmatched firms obtain a gain r/σ, and become matched with home or foreign suppliers at the same rate $\mu^F/2$ (due to identical search technology between two types of matches) at which point they obtain a gain $V_d^F(\alpha) - V^F$ or $V_m^F(\alpha) - V^F$. On the other hand, the third equation shows that firms matched with foreign suppliers obtain a gain $r_m^F(\alpha)/\sigma$ which includes a transport cost τ_m. Though a fixed trade cost F_m do not directly enter the Bellman equations for firms, this cost indirectly enters the expected profit of firms as it is shared with matched suppliers at the bargaining stage.

Setting the discount rate as well as all potential gains and losses to zero in the Bellman equations, the value functions can be expressed as those in autarky. The Bellman equations above indicate that the value functions of matched firms are given as the present value of profit flows, $V_d^F(\alpha) = r_d^F(\alpha)/\delta\sigma$, $V_m^F(\alpha) = r_m^F(\alpha)/\delta\sigma$ like (6). In contrast, those for unmatched firms are now given by

$$V^F = \frac{r}{\delta\sigma} + \left(\frac{\mu^F}{2(\delta + \mu^F)}\right)\left(\frac{r_d^F(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma}\right) + \left(\frac{\mu^F}{2(\delta + \mu^F)}\right)\left(\frac{r_m^F(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma}\right).$$

Note that the probability of matches is divided by 2 in the second and third terms. This is because M-integration allows foreign firms to penetrate the home matching market, which decreases the probability of matches in that market by half. At the same time, this integration allows home firms to penetrate the foreign matching market, which decreases the probability of matches in that market by half.[14]

Agents matched with home or foreign partners determine their profit sharing by symmetric Nash bargaining. This bargaining imposes the condition for two types of matched agents, which is similar to (3). Using the value functions derived above, we can obtain the optimal profit sharing rule within matched agents. We find however that the effective bargaining power is the same as (8) in autarky, even though cross-border matches are present. The reason is explained by recalling that bargaining power is endogenously determined by the number of agents. On the one hand, M-integration increases the number of available suppliers for home firms by allowing them to penetrate the foreign market, enhancing the effective bargaining power of home firms. On the other hand, M-integration increases the number of firms seeking home suppliers by allowing foreign firms to penetrate the home market, reducing the effective bargaining power. When penetration of each matching market occurs at the same probability, these forces exactly offset one another so as to leave effective bargaining power unaffected by the presence of cross-border matches. While the law of motion in (4) is also unaffected by it, the number of matched agents $n \equiv N^F - u^F = N^S - u^S$ represents the total number of matched agents. Since the steady-state number (9) holds in M-integration, the ratio of firms matched with either home or foreign suppliers is given by $n/2N^F = \mu^F/2(\delta + \mu^F)$. The above value function of unmatched firms then shows that the value of these firms consists of their outside option plus net profit flows obtained when being matched which occurs at this ratio.

Finally, using the steady-state relationships, the free entry condition in (5) can be written as follows:

$$
\begin{aligned}
&\frac{r}{\sigma} + \frac{n}{2N^F}\beta\left(\frac{r(\alpha)}{\sigma} - \frac{2r}{\sigma} - f_d - f_m\right) - f_e^F = 0,\\
&\frac{n}{2N^S}(1-\beta)\left(\frac{r(\alpha)}{\sigma} - \frac{2r}{\sigma} - f_d - f_m\right) - f_e^S = 0,
\end{aligned}
\tag{22}
$$

where $r(\alpha)/\sigma = r_d(\alpha)/\sigma + r_m(\alpha)/\sigma$ is the total variable profit of matched firms and $f_m \equiv \delta F_m$ denotes the amortized per-period portion of the one-time fixed import cost incurred at every point in time. Moreover, using (21) in (22), the variable profit of unmatched firms satisfies the following equalities:

$$
\begin{aligned}
\frac{r}{\sigma} &= \frac{f_e^F + (f_d + f_m)\frac{\phi^F}{2}}{1 + \left[\left(1 + \tau_m^{(1-\sigma)(1-\eta)}\right)\alpha^{\sigma-1} - 2\right]\frac{\phi^F}{2}},\\
\frac{r}{\sigma} &= \frac{f_e^S + (f_d + f_m)\frac{\phi^S}{2}}{\left[\left(1 + \tau_m^{(1-\sigma)(1-\eta)}\right)\alpha^{\sigma-1} - 2\right]\frac{\phi^S}{2}}.
\end{aligned}
\tag{23}
$$

As before, (23) shows that the variable profit equals the expected fixed cost of agents, which simultaneously determines the ratio of unmatched agents θ and the profit of unmatched firms r/σ in M-integration. Further, despite that the equilibrium conditions differ between autarky and M-integration, the FF and SS curves in (23) are respectively downward and upward sloping in the $(\theta, r/\sigma)$ space just as in Figure 2, which establishes the existence and uniqueness of M-integration equilibrium. In what follows, examining the impact of M-integration on key equilibrium variables, we address the welfare consequence of M-integration which stands in contrast to that of X-integration.

[14] As in autarky, we focus on equilibrium where firms matched with foreign suppliers consider sinking the investment cost F_m.

5.3 Losses from Trade in M-integration

To examine the impact of M-integration, we consider how the free entry condition (22) is affected by the level of trade costs associated with this integration. In equilibrium where matched firms source inputs from abroad, agents matched with foreign partners earn higher profit in M-integration than in autarky ($r_m(\alpha)/\sigma - f_m > r/\sigma$). Using (21), this condition – a counterpart to (19) in X-integration – is expressed as

$$\frac{\left(\alpha^{\sigma-1} - \tau_m^{(\sigma-1)(1-\eta)}\right) r}{\sigma} > \tau_m^{(\sigma-1)(1-\eta)} f_m. \tag{24}$$

If trade costs are so large that (24) is violated, firms matched with foreign suppliers would immediately dissolve their partnerships. Comparing (10) and (22) under (24) then reveals that the economic rent of matched agents is greater in M-integration than in autarky. This raises the expected profit and induces further entry of agents, which leads to downward shifts in the FF and SS curves.

In M-integration, however, there is another force for the expected profit: the expected share becomes half, because of penetration by foreign agents of the home matching market. This decreases the expected profit and deters further entry of agents, which leads to upward shifts in two curves. In equilibrium firms and suppliers strike a balance between these forces, one related to trade costs and another related to search frictions abroad. If the first dominates the second, the impact of M-integration is similar to that of X-integration; otherwise it is completely opposite to that of X-integration. The question is which forces dominate.

To answer this question, it suffices to consider the condition under which the expected profit in M-integration increases relative to autarky. From (10) and (22), it follows that this is equivalent with the following condition: the economic rent from cross-border matches is greater than that from domestic matches in M-integration ($r_d(\alpha)/\sigma - r/\sigma - f_d < r_m(\alpha)/\sigma - r/\sigma - f_m$). Using (21), the condition can be expressed in terms of trade costs:

$$\frac{\left(1 - \tau_m^{(1-\sigma)(1-\eta)}\right) \alpha^{\sigma-1} r}{\sigma} < f_d - f_m. \tag{25}$$

However, so long as the transport cost is an iceberg type ($\tau_m > 1$) and the fixed import cost is greater than the fixed overhead cost ($f_m > f_d$), we see that the inequality in (25) never holds, implying that an increase in the economic rent is always smaller than a decrease in the expected share. In fact, comparing (11) and (23) under the opposite inequality in (25) reveals that the FF and SS curves in M-integration are located above those in autarky for a given ratio θ and thus the variable profit of unmatched firms rises in M-integration ($r/\sigma > r_a/\sigma$). Further, the equilibrium relationship between θ and r/σ in (14) holds in M-integration from (22), implying that the ratio of unmatched agents falls in M-integration ($\theta < \theta_a$). These changes in the two endogenous variables suggest that M-integration not only shrinks the profit differential between matched and unmatched firms but also worsens the matching frequency of firms.

Intuition behind the results is explained by noting resource reallocations in M-integration. With our search technology, the number of domestic matches is the same as that of cross-border matches, implying that resources are reallocated from the most efficient firms matched with home suppliers to moderately efficient firms matched with foreign suppliers. As this weakens the degree of competition in the industry, resources also reallocated to the least efficient unmatched firms. In this way, M-integration hinders the resource-reallocation process of firms, which decreases welfare. To explore this channel, let us consider how M-integration affects the expenditure share on final goods produced by unmatched firms in M-integration. Although this share is given as $s = (N^F - n)r/R$ as before, the aggregate expenditure is now expressed as $R = (N^F - n)r + nr_d(\alpha)/2 + nr_m(\alpha)/2$ in M-integration.

Using (9) and (21) and rearranging, we obtain

$$s = \frac{\delta}{\delta + \left(1 + \tau_m^{(1-\sigma)(1-\eta)}\right) \frac{\alpha^{\sigma-1}\mu^F}{2}}.$$

Inspection of (15) shows that the share is higher in M-integration than in autarky. This is clearly seen in terms of the relative expenditure share $s/(1-s)$. Since $r(\alpha) = r_d(\alpha)/\sigma + r_m(\alpha)/\sigma$, simple average of revenue between two types of matched firms is $r(\alpha)/2$. Then, the relative share is decomposed into the intensive margin ratio $r/(r(\alpha)/2) = 2/\big(1 + \tau_m^{(1-\sigma)(1-\eta)}\big)\alpha^{\sigma-1}$ and the extensive margin ratio $(N^F - n)/n = \delta/\mu^F$, where both ratios rise relative to those in autarky. This fact reflects that M-integration allows the least efficient unmatched firms to expand both margins, which directly leads to welfare losses in this integration.

Proposition 3: *M-integration decreases welfare in both countries by worsening the matching frequency of firms associated with resource reallocations from matched firms to unmatched firms.*

5.4 Extensions

We have demonstrated that M-integration causes welfare losses. The results are derived, however, by assuming identical search technology between domestic and cross-border matches. To check the robustness, we extend to a natural case where cross-border matches are more difficult than domestic matches.

Consider search technology that allows for the different probability of domestic and cross-border matches. Let $m_d(u^F, u^S)$ and $m_m(u^F, u^S)$ denote the number of domestic matches and cross-border matches per unit of time respectively. To make the analysis tractable, we adopt the matching function $m_m(u^F, u^S) = \kappa m_d(u^F, u^S)$ where $\kappa(< 1)$ is the difficulty of cross-border matches relative to domestic matches. Let n_d and n_m denote the number of matched agents within and across borders respectively. Then the search technology implies $n_m = \kappa n_d$. Maintaining all of the other assumptions of our baseline model, the free entry condition corresponding to (22) in this extended case is given by the following expression (see Appendix A.5):

$$\frac{r}{\sigma} + \frac{n}{(1+\kappa)N^F}\beta\left(\frac{r(\alpha)}{\sigma} - \frac{(1+\kappa)r}{\sigma} - f_d - \kappa f_m\right) - f_e^F = 0,$$
$$\frac{n}{(1+\kappa)N^S}(1-\beta)\left(\frac{r(\alpha)}{\sigma} - \frac{(1+\kappa)r}{\sigma} - f_d - \kappa f_m\right) - f_e^S = 0,$$

where $n = n_d + n_m$ is the total number of matched agents as in the baseline case.

Several observations stand out. First, not surprisingly, when search frictions abroad are so sufficiently large that the probability of cross-border matches is zero ($\kappa = 0$), the free entry condition reduces to (10) in autarky. On the other hand, when such frictions are so sufficiently small that the probability of cross-border matches is equal to that of domestic matches ($\kappa = 1$), it reduces to (22) in M-integration of the baseline case.

Second, although κ enters the free entry condition above, the impact of M-integration in this extended case is qualitatively similar to that in the baseline case, with the additional comparative statics with respect to κ: the easier are cross-border matches (the higher κ), the greater the upward shifts in the FF and SS curves and hence the more significant changes in two key endogenous variables. Observe that the impact of search frictions abroad is completely opposite to that in trade costs: the lower are trade costs (the smaller τ_m, f_m), the greater the downward shifts in two curves. Thus reductions in search frictions abroad may generate different welfare implications from those in trade costs.

Table 2 – Quantitative impact of M-integration

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
	r/σ	$r_d(\alpha)/\sigma$	$r_m(\alpha)/\sigma$	θ	μ^F	μ^S	N^F	N^S	n	s	λ	W
Autarky	1.68	4.22	0	0.16	0.14	0.86	65	57	55	0.06	1	3.97
M-integration	1.84	4.61	3.20	0.08	0.08	0.92	69	53	52	0.12	1	3.85

Finally, perhaps more interestingly, κ does not affect the condition under which the expected profit rises in M-integration relative to autarky, given in (25). Intuition comes from that κ has opposing effects on expected profit. On the one hand, the smaller is κ, the more difficult is for foreign firms to penetrate the home market. Since foreign firms' penetration is less intense, the expected share of domestic matches is higher at home, which makes the expected profit greater. On the other hand, the smaller is κ, the more difficult is for home firms to penetrate the foreign market. Since home firms' penetration is less intense, the expected share of cross-border matches is lower, which makes the expected profit smaller. With the matching functions considered above, these two effects leave (25) independent of the relative difficulty of cross-border matches.

5.5 Numerical Solutions

To understand the impact of M-integration, we solve the model numerically with the specifications and parameter values as those we set in X-integration. As reported in Bernard et al. (2007a), importer and exporter premia are of similar size in US manufacturing, and we choose $\alpha = 1.36$ as before. In M-integration equilibrium where firms matched with foreign suppliers import, however, trade costs must be low enough to satisfy (24), which includes η as well as f_m, τ_m. To calibrate η, we follow the literature in relating it to measures of capital or skill intensity, and we use the estimate of log mean human-capital intensity of US manufacturing firms (-0.69 as reported in Antràs (2003)). In our complete-contract setting along with Cobb-Douglas technology, the ratio of two inputs is $x^F/x^S = \eta/(1-\eta)$. From $\ln(\eta/(1-\eta)) = -0.69$, we have $\eta = 0.33$ which leads us to set $f_m = 0.125, \tau_m = 1.2$.[15] In addition, we set κ in order to adjust the difficulty of cross-border matches. Toward that end, we use data on the average fraction of firms that import among firms that also export in US manufacturing (0.4 as reported in Bernard et al. (2007a)). This fraction is $n_m/n_d = \kappa$ from Section 5.4 and hence $\kappa = 0.4$.

Table 2 summarizes quantitative comparison between autarky and M-integration computed in this setting. Clearly, the values in autarky are the same as those in Table 1 but the values in M-integration are different from those in X-integration. Columns I–III show that the profit level of unmatched firms and that of firms matched with home suppliers are higher in M-integration than in autarky, where the profit level of firms matched with foreign suppliers is higher than that of unmatched firms. These changes imply that M-integration weakens the degree of competition by reallocating resources from the most efficient firms to moderately efficient firms within the industry. Columns IV–VI next show that the ratio of unmatched agents falls in M-integration as decreased competition in the industry raises the *ex post* profitability and induces a new entry of firms relative to suppliers. As a result, firms find it difficult to find suppliers and the probability of firms' matches falls but the probability of suppliers' matches rises. Columns VII–X show that the number of firms rises but the number of suppliers falls in M-integration reflecting the difference in entry patterns. While the number of matched firms also falls, it is split into domestic matches ($n_d = 37$) and cross-border matches ($n_m = 15$).

[15] The low level of variable import costs can be justified by the fact that input tariffs tend to be smaller than output tariffs. For example, output tariffs are more than four times as high as input tariffs in 2010–2017 in the United States (Grossman et al., 2023).

Finally, Columns X–XII show that: (*i*) the expenditure share spent on unmatched firms' goods rises; (*ii*) the domestic expenditure share is unity in both regimes as the final-good markets are segregated in M-integration; and (*iii*) welfare is smaller in M-integration than in autarky as this integration weakens the degree of competition. The welfare ratio between M-integration and autarky is 0.970 and hence welfare losses are 3.0 percent, a sizable magnitude. This result does not mean, however, that the welfare impact of M-integration is directly comparable to that of X-integration because the level of trade costs differs between the two forms of integration. In fact, if variable trade costs decrease from $\tau_x = 1.8$ to $\tau_x = 1.2$ in X-integration (keeping all of the other parameter values), unmatched firms also export in this integration and the gains from trade increase from 3.1 percent to 13.9 percent. Hence, although M-integration causes welfare losses for both countries, such losses are quantitatively smaller than welfare gains in X-integration conditional on the level of trade costs.

6 Country Asymmetry

Throughout the analysis, we have focused exclusively on the symmetric-country setting. It is possible, however, to incorporate country asymmetry into the baseline model. In so doing, we can provide interesting implications in a more realistic case where agents face search frictions in obtaining specialized inputs from different types of countries.

Consider a slightly different setting where the home country is developed "North" and the foreign country is less developed "South," each providing two distinct inputs which are now referred to as *headquarter services* and *manufactured components* respectively. Headquarter services entail high-tech manufacturing such as research and development, whereas manufacturing components entail low-tech manufacturing such as simple assembly. Moreover, headquarter services need to be produced in the same country in which firms locate, but production of manufacturing components is allowed to be geographically separated from the location of firms subject to search frictions. Finally, the vector of unit labor requirements $\{a^F, a^S, a^{F^*}, a^{S^*}\}$ is arranged such that: (*i*) the South has a comparative advantage in producing manufacturing components ($a^{S^*}/a^{F^*} < a^S/a^F$); and (*ii*) the North has an absolute advantage in producing the two inputs ($a^F < a^{F^*}$, $a^S < a^{S^*}$). Then, these countries can have a different ratio of suppliers to firms. In order to avoid a taxonomy of cases, we assume that firms are relatively more abundant in the North than in the South in autarky, $N_a^S/N_a^F < N_a^{S^*}/N_a^{F^*}$, which implies $\theta_a < \theta_a^*$ so long as search technology is identical between the countries. Observing that manufacturing components are the only tradable input, the first assumption works to reinforce northern firms' incentive to search for southern suppliers. On the other hand, the second assumption ensures that the wage rate is higher in the North than in the South ($w > w^*$). While we keep other variables symmetric, the wage outcome would be more likely when market size is larger in the North than in the South due to the "home market" effects on the wage rate (Krugman, 1980). Using this extended setting, we explore additional welfare implications for both X- and M-integration.

X-integration. X-integration generates the gains from trade to different degrees between the two countries. Our assumption leads to $\theta_a < \theta_a^*$ in autarky, which in turn leads to $r_a/\sigma > r_a^*/\sigma$ from (14). As northern firms find it harder to meet northern suppliers, the North has fewer varieties made by customized inputs and thus lower welfare relative to the South in autarky. If firms are allowed to export, X-integration decreases the price index and variable profit more sharply in the North than in the South (as the South has more matched firms in autarky). Furthermore, X-integration increases the ratio of unmatched agents more sharply in the North than in the South (as the equilibrium relationship in (14) holds in both autarky and X-integration). Comparing these variables between the two regimes reveals that while the countries enjoy the gains from trade in X-integration, such gains are greater for the North than for the South. Decreases are greater for r/σ than for r^*/σ, reflecting that X-integration reallocates resources among firms more significantly for the North for the South. Similarly, increases are greater for θ than for θ^*, reflecting that X-integration improves the matching frequency of firms more significantly for the North than for the South.

M-integration. Similarly to X-integration, the extent to which M-integration causes welfare losses is different between the two countries. However, M-integration allows for a movement of agents across borders and alters the ratio of unmatched agents first. Given that the South is relatively more abundant in suppliers in autarky, this integration relocates supplier from the South to the North, which decreases the ratio of unmatched agents more sharply in the South. Further, M-integration increases the variable profit of unmatched firms more sharply in the South than in the North (as (14) also holds in M-integration). Together with the findings in X-integration, we can say that there are larger welfare gains from X-integration and smaller welfare losses from M-integration for a country in which firms providing core inputs are relatively more abundant.

This welfare result closely relates to welfare consequences of global sourcing by Antràs and Helpman (2004). They show that firms with high productivity are more likely to offshore some production processes in a North-South model (whose theoretical prediction has received lots of empirical support), but the implications of their model for aggregate welfare have been left unanswered. To make this point in our model, note that the marginal cost of firms depends not only on firms' matching status but also on suppliers' location in global sourcing. For example, the marginal cost of northern firms matched with northern suppliers is $c_d = (a^F w)^\eta (a^S w)^{1-\eta}/\alpha$ and that of northern firms matched with southern suppliers is $c_m = (a^F w)^\eta (\tau_m a^{S^*} w^*)^{1-\eta}/\alpha$. In contrast to the baseline model, the latter is not necessarily higher than the former. Using these costs, we find that the ratio of equilibrium revenue in (1) remains unaffected but that in (21) changes to the following expression:

$$\frac{r_m(\alpha)}{r} = \left[\alpha \left(\frac{a^S w}{\tau_m a^{S^*} w^*}\right)^{1-\eta}\right]^{\sigma-1}.$$

Thus the revenue of offshoring firms in the North increases with the wage difference $w/w^*(>1)$ but decreases with the technological difference of manufacturing components $a^S/a^{S^*}(<1)$. Recall that, to know what happens to welfare as a result of M-integration, we just need to compare the economic rent from domestic and cross-border matches without solving for the FF and SS curves, given in (25). From the equilibrium revenue above,

$$\frac{\left[1 - \left(\frac{\tau_m a^{S^*} w^*}{a^S w}\right)^{(1-\sigma)(1-\eta)}\right] \alpha^{\sigma-1} r}{\sigma} < w\big(f_d - f_m\big).$$

The inequality can hold when the wage difference is sufficiently large relative to the technological difference. In fact, it always holds when offshoring firms earn higher profits than domestic firms, in which case M-integration generates welfare gains for the North. Under the same parameterization, however, the opposite is true for the South, creating an additional channel through which M-integration exacerbates welfare losses in global sourcing with search frictions.

One of the broad welfare implications from this extension is that a country with less advanced technology (or with smaller market size) is more likely to suffer from integration of matching markets. To alleviate welfare losses associated with this integration, policymakers need to pay closer attention to a less developed country so that integration of matching markets takes place along with integration of goods markets.

7 Conclusion

This paper has described and analyzed the effect of search frictions on the characterization of industry structure and aggregate welfare. The importance of search for firm performance and productivity has been documented in the era of globalization where drastic reductions in search frictions coupled with gradual reductions in trade frictions allow firms to profitably search for compatible suppliers across the globe. In fact, recent empirical work on search, networks and intermediation in international trade has extensively corroborated this importance using micro-level data on buyer–seller linkages from several countries.

We show that the introduction of search frictions into standard workhorse models of trade offers non-standard welfare results. In particular, depending upon whether globalization reduces trade or search frictions that firms must face, it generates the contrasting welfare implications by affecting industry structure in which firms operate. On the one hand, when globalization makes it easier for firms to ship products abroad, the gains from trade can be amplified relative to those without search opportunities through an additional adjustment margin of trade: costly trade affects industry structure in such a way that the number of suppliers rises relative to the number of firms, thereby thickening the market of intermediate inputs. This trade-induced industry restructuring improves the matching frequency of firms, while simultaneously giving rise to intra-industry resource reallocations from less efficient unmatched firms to more efficient matched firms. On the other hand, when globalization makes it easier for firms to match with suppliers abroad, costly trade leads to the opposite impacts on industry structure, worsening the matching frequency of firms and hindering the resource-reallocation processes among firms. As a result, countries may suffer from the losses from trade. We also demonstrate that these welfare changes triggered by trade-induced industry restructuring are quantitatively substantial.

Although our analysis reveals that welfare losses due to integration of matching markets are relatively smaller than welfare gains due to integration of goods markets, it also indicates that there is some room to circumvent these welfare losses through policies. What kind of policy implications can we derive from our analysis? These implications are, of course, not that local governments should ban or restrict integration of matching markets. One of the reasons is that such integration is more likely to generate welfare gains for a more advanced country (though it comes at the expense of a less advanced country), in which case restrictions on this integration could result in aggregate losses in global welfare. The policy implications from our model are, instead, that integration of matching markets may as well be designed with integration of goods markets in order to soften welfare losses. In that sense, our model highlights a more critical role played by traditional trade liberalization in globalization where firms search for and match with suppliers.

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A Online Appendix (Not for Publication)

A.1 Nash Bargaining Solution

We show that the solution to the Nash bargaining problem satisfies (3) at any point in time. While matched agents choose their variable profit $r^F(\alpha)/\sigma, r^S(\alpha)/\sigma$ subject to $r^F(\alpha)/\sigma + r^S(\alpha)/\sigma = r(\alpha)/\sigma$ in the main text, it is equivalent for them to choose their revenue $r^F(\alpha), r^S(\alpha)$ subject to $r^F(\alpha) + r^S(\alpha) = r(\alpha)$. Let λ^N denote the Lagrange multiplier associated with the Nash bargaining problem. It is clear that the first-order conditions associated with Nash bargaining are given by

$$\left(V^S(\alpha) - F_d - V^S\right)\frac{\partial V^F(\alpha)}{\partial r^F(\alpha)} = \lambda^N,$$
$$\left(V^F(\alpha) - V^F\right)\frac{\partial V^S(\alpha)}{\partial r^S(\alpha)} = \lambda^N.$$

Moreover, (2) implies $\partial V^F(\alpha)/\partial r^F(\alpha) = \partial V^S(\alpha)/\partial r^S(\alpha)$. Using this in the first-order conditions, we get

$$V^F(\alpha) - V^F = V^S(\alpha) - F_d - V^S.$$

Rearranging this equality yields the condition given in (3).

A.2 Labor Market Clearing Condition

We show that the aggregate revenue equals the aggregate payment to labor in the steady state equilibrium.

A.2.1 Autarky

Let L_e^F and L_e^S denote the aggregate labor used for entry by new firms and suppliers respectively at every point in time. Since we have chosen the common wage rate to one, they also denote the aggregate cost of investment incurred by each type of agents.

As for firms, there is the number $N_e^F(=\delta N^F)$ of new entrants that pays a fixed entry cost F_e^F at every point in time. Equivalently, there is the number N^F of incumbent firms that pays a fixed entry cost $f_e^F(=\delta F_e^F)$ at every point in time. In both cases, the aggregate labor used for entry at every point in time is $L_e^F = \delta N^F F_e^F$. Using free entry in (5) that requires $F_e^F = V^F$, the aggregate labor used for entry at any point in time is

$$\begin{aligned}
L_e^F &= \delta N^F V^F \\
&= N^F\left[\frac{r}{\sigma} + \left(\frac{\mu^F}{\delta+\mu^F}\right)\left(\frac{r^F(\alpha)}{\sigma} - \frac{r}{\sigma}\right)\right] && \text{(using (6))} \\
&= \left(\frac{\delta N^F}{\delta+\mu^F}\right)\frac{r}{\sigma} + \left(\frac{\mu^F N^F}{\delta+\mu^F}\right)\frac{r^F(\alpha)}{\sigma}.
\end{aligned}$$

From the steady-state relationship in (9), the number of unmatched firms satisfies $N^F - n = \delta N^F/(\delta+\mu^F)$ and the number of matched firms satisfies $n = \mu^F N^F/(\delta+\mu^F)$. Substituting these two equalities reveals that the aggregate labor equals the aggregate profit earned by firms:

$$L_e^F = \frac{(N^F - n)r}{\sigma} + \frac{n r^F(\alpha)}{\sigma}. \tag{A.1}$$

As for suppliers, there is the number $N_e^S(=\delta N^S)$ of new entrants that pays a fixed entry cost F_e^S at every point in time; equivalently, there is the number N^S of incumbent suppliers that pays a fixed entry cost $f_e^S(=\delta F_e^S)$ at every point in time. In addition, the number n of matched ones that pays a fixed investment cost δF^D at every point in time. Hence, the aggregate labor used for entry at every point in time is $L_e^S = \delta N^S F_e^S + \delta n F_d$. Using free entry in (5) that requires $F_e^S = V^S$ as well as (6), (7) and (9), we have

$$L_e^S = \frac{nr^S(\alpha)}{\sigma}. \tag{A.2}$$

Recall that matched firms' revenue satisfies $r(\alpha) = r^F(\alpha) + r^S(\alpha)$ while the aggregate revenue satisfies $R = (N^F - n)r + nr(\alpha)$. From (A.1) and (A.2), it follows immediately that

$$L_e^F + L_e^S = \frac{R}{\sigma}. \tag{A.3}$$

Thus, the aggregate investment labor equals the aggregate profit at every point in time. Since $L_e^F + L_e^S$ also represents the aggregate investment cost, (A.3) shows that there is no net investment income. Observe that (A.3) does hold with a positive discount factor, in the sense that the equality does not hold when $\gamma > 0$ in the Bellman equations in (2). This equilibrium property is identical to that described by Melitz (2003).

Next, consider labor used for production. Let L_p^F and L_p^S denote the aggregate labor used for production by firms and suppliers respectively, which includes production workers for both inputs and final goods. There is the number $N^F - n$ of unmatched firms who purchase generic inputs to incur a production cost cy, while there is the number n of matched firms who obtain customized inputs to incur a production cost $cy(\alpha)/\alpha$. From this, the aggregate labor used for production is $L_p^F + L_p^S = (N^F - n)cy + ncy(\alpha)/\alpha$. Using the optimal output levels $y, y(\alpha)$ as well as the number of firms n, N^F in (9), we obtain

$$L_p^F + L_p^S = \left(\frac{\sigma - 1}{\sigma}\right) R. \tag{A.4}$$

Finally, consider the aggregate labor used in the industry. Summing up (A.3) and (A.4),

$$L = (L_e^F + L_e^S) + (L_p^F + L_p^S) = R,$$

where L represents the aggregate payment to labor as the wage rate is one. This establishes the desired result.

A.2.2 X-Integration

As matched firms additionally incur a fixed export cost F_x in X-integration, the aggregate labor used for entry by firms at any point in time is $L_e^F = \delta N^F F_e^F + \delta n F_x$. Using the value functions of firms in Appendix A.4 and (9), we can express this aggregate labor as (A.1). In contrast, the Bellman equations for suppliers are the same as those in autarky and (A.2) holds. Thus (A.3) also holds in X-integration so that the aggregate investment labor equals the aggregate profit at every point in time.

Similarly, as matched firms additionally incur a transport cost τ_x in X-integration, the aggregate labor used for production is $L_p^F + L_p^S = (N^F - n)cy + ncy_d(\alpha)/\alpha + n\tau_x cy_x(\alpha)/\alpha$ where $y_x(\alpha) = \tau_x^{-\sigma} y_d(\alpha)$. Using the optimal output levels $y, y(\alpha)$ as well as the steady-state number of firms n, N^F in (9), the aggregate labor used for production also satisfies (A.4).

Finally, from (A.3) and (A.4), the equilibrium relationship $R = L$ holds in X-integration.

A.2.3 M-Integration

Using the value functions of firms given in (A.11) in Appendix A.5, we can show that the aggregate labor used for entry by firms is (A.1). In contrast, as suppliers matched with foreign firms incur the fixed import cost F_m in M-integration, the aggregate labor used for entry by suppliers is given by $L_e^S = \delta N^S F_e^S + \delta n F_d + \delta n F_m$ at any point in time, which can be written as (A.2) in light of (A.11). This implies that (A.3) holds in M-integration.

Similarly, as suppliers matched with foreign firms incur a transport cost τ_m in M-integration, the aggregate labor used for production is $L_p^F + L_p^S = (N^F - n)cy + ncy_d(\alpha)/\alpha + n\tau_m cy_m(\alpha)/\alpha$ where $y_m(\alpha) = \tau_m^{-\sigma(1-\eta)} y_d(\alpha)$. Using the optimal output levels $y, y_d(\alpha), y_m(\alpha)$ as well as the steady-state number of firms n, N^F in (9), the aggregate labor used for production satisfies (A.4).

Finally, from (A.3) and (A.4), the equilibrium relationship $R = L$ holds in M-integration.

A.3 Number of Agents and Welfare

We show detailed derivations of the number of agents and welfare.

A.3.1 Autarky

From the optimal pricing rule of unmatched firms and that of matched firms, the price index is

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma - 1}\right)^{1-\sigma} \left(N^F - n + \alpha^{\sigma-1} n\right). \tag{A.5}$$

Moreover, substituting $R = L$ and (A.5) into the optimal consumer expenditure $r = Ap^{1-\sigma}$ where $A = RP^{\sigma-1}$, the variable profit of unmatched firms is expressed as

$$\frac{r}{\sigma} = \frac{L}{\sigma(N^F - n + \alpha^{\sigma-1} n)}. \tag{A.6}$$

To derive the number of agents in (12), rewrite the steady-state relationship in (9) as $N^F - n = \left(\delta/\mu^F\right) n$. Using this equality and solving (A.6) for n and, we get

$$n = \frac{L}{r}\left(\frac{\mu^F}{\delta + \alpha^{\sigma-1}\mu^F}\right).$$

The number of matched agents is obtained by noting that the variable profit of unmatched firms given in the above equality must satisfy $r/\sigma = f$ in equilibrium. On the other hand, the total number of firms and suppliers in the industry is obtained by rewriting (9) as

$$N^F = \left(\frac{\delta + \mu^F}{\mu^F}\right) n, \quad N^S = \left(\frac{\delta + \mu^S}{\mu^S}\right) n.$$

Regarding welfare per worker in (13), substituting (A.6) into (A.5) and rearranging

$$\frac{1}{P} = \frac{\sigma - 1}{\sigma} \left(\frac{L}{r}\right)^{\frac{1}{\sigma-1}}.$$

Welfare per worker is obtained by noting that welfare is defined as an inverse of the price index with the CES preferences and the variable profit of unmatched firms given in (A.6) must satisfy $r/\sigma = f$ under free entry.

A.3.2 X-Integration

In X-integration, the price index in (A.5) is written as

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left[N^F - n + (1+\tau_x^{1-\sigma})\alpha^{\sigma-1} n\right].$$

Similarly, the variable profit of unmatched firms in (A.6) is written as

$$\frac{r}{\sigma} = \frac{L}{\sigma\left[N^F - n + (1+\tau_x^{1-\sigma})\alpha^{\sigma-1} n\right]}.$$

Using these two equalities and following similar procedures shown above, we can get the number of agents and welfare in the X-integration equilibrium.

A.3.3 M-Integration

In M-integration, the price index in (A.5) is written as

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left[N^F - n + \left(1+\tau_m^{(1-\sigma)(1-\eta)}\right)\alpha^{\sigma-1} \frac{n}{2}\right].$$

Similarly, the variable profit of unmatched firms in (A.6) is written as

$$\frac{r}{\sigma} = \frac{L}{\sigma\left[N^F - n + \left(1+\tau_m^{(1-\sigma)(1-\eta)}\right)\alpha^{\sigma-1} \frac{n}{2}\right]}.$$

Using these two equalities and following similar procedures shown above, we can get the number of agents and welfare in the M-integration equilibrium.

A.4 Free Entry Condition in X-integration

A.4.1 When Only Matched Firms Export

Consider the Bellman equations of agents under the assumption that only matched firms export. While the Bellman conditions for suppliers are the same as (2) the Bellman conditions for firms are given by

$$\begin{aligned}
\gamma V^F &= \frac{r}{\sigma} + \mu^F \left(V^F(\alpha) - F_x - V^F\right) - \delta V^F + \dot{V}^F, \\
\gamma V^F(\alpha) &= \frac{r^F(\alpha)}{\sigma} - \delta V^F(\alpha) + \dot{V}^F(\alpha).
\end{aligned}$$

Unmatched firms become matched at the rate μ^F at which point they obtain a gain $V^F(\alpha) - F_x - V^F$ where matched firms make a one-time investment F_x for entry into the foreign market. Further, matched firms' profit $r^F(\alpha)/\sigma$ includes that earned from the home market $r_d^F(\alpha)/\sigma$ and the foreign market $r_x^F(\alpha)/\sigma$.

Setting $\gamma = 0$ as well as $\dot{V}^F = \dot{V}^F(\alpha) = 0$, we get the value functions of firms corresponding to (6):

$$\begin{aligned}
V^F &= \frac{r}{\delta\sigma} + \left(\frac{\mu^F}{\delta+\mu^F}\right)\left(\frac{r^F(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_x\right), \\
V^F(\alpha) - F_x &= \frac{r^F(\alpha)}{\delta\sigma} - F_x.
\end{aligned}$$

Obviously the interpretation is similar to that of (6) in autarky, but we assume $r^F(\alpha)/\delta\sigma - r/\delta\sigma - F_x > 0$ in order to ensure that matched firms have an enough incentive to export in the presence of search frictions. Under the condition, the above value functions imply $V^F(\alpha) - F_x - V^F > 0$ so that the net value of matched firms is greater than that of unmatched firms. As before, this is the only reason that firms consider sinking the investment cost F_x, which is shared by suppliers at the bargaining stage.

We can describe how Nash bargaining between firms and suppliers affects the division of surplus. Following Appendix A.1, symmetric Nash bargaining imposes the following condition at any point in time:

$$V^F(\alpha) - F_x - V^F = \frac{1}{2}\Big(V^F(\alpha) - F_x - V^F + V^S(\alpha) - F_d - V^S\Big).$$

Using the value functions derived above for this bargaining constraint, the solution to the bargaining problem subject to $r^F(\alpha)/\sigma + r^S(\alpha)/\sigma = r(\alpha)/\sigma$ gives us the profit sharing rule. The result is obtained by noting that the effective bargaining power and the steady-state number of agents in X-integration are the same as those in autarky, given in (8) and (9) respectively.

A.4.2 When Both Unmatched and Matched Firms Export

Consider the Bellman conditions of agents under the assumption that both unmatched and matched firms export. In this case, unmatched firms earn the domestic revenue r_d and the export revenue $r_x = \tau_x^{1-\sigma} r_d$. The equilibrium revenue of unmatched firms is $r = (1 + \tau_x^{1-\sigma})r_d$ while that of matched firms is $r(\alpha) = (1 + \tau_x^{1-\sigma})\alpha^{\sigma-1} r_d$. Then the ratio of the equilibrium revenue (and hence profit) of matched firms to that of unmatched firms satisfies (1) as in autarky. In addition, unmatched firms earn the variable profit $r/\sigma = (1 + \tau_x^{1-\sigma})/\sigma$ at every point in time, but they incur the fixed export cost $f_x (= \delta F_x)$ to enter the foreign final-good market at every point in time. As a result, firms' outside option is given by $r/\sigma - f_x$.

It is clear that other equilibrium conditions do not change from the baseline case where only matched firms export. From the profit and trade costs seen above, the free entry condition in this equilibrium is given by

$$\begin{aligned} \frac{r}{\sigma} - f_x + \frac{n}{N^F}\beta\left(\frac{r(\alpha)}{\sigma} - \frac{r}{\sigma} - f_d\right) - f_e^F &= 0, \\ \frac{n}{N^S}(1-\beta)\left(\frac{r(\alpha)}{\sigma} - \frac{r}{\sigma} - f_d\right) - f_e^S &= 0. \end{aligned} \tag{A.7}$$

In (A.7), note that, even though we consider X-integration, the economic rent of matched agents does not include the fixed trade cost f_x, as the rent is defined as the difference in profits between matched and unmatched firms where both types of firms incur this fixed cost in equilibrium. The expression in (A.7) looks similar to the free entry condition in autarky (10), but there exist two differences in firms' outside option: (i) the total profit r/σ includes the domestic profit r_d/σ as well as the export profit r_x/σ; and (ii) the fixed export cost f_x is subtracted from r/σ as unmatched firms have to incur this fixed cost.

In the special case where there are no trade costs in X-integration ($\tau_x = 1, f_x = 0$), unmatched firms export and the free entry condition must be defined as (A.7), instead of (17) in the main text. However, (A.7) is almost identical with that in autarky (10) except that the export profit is included in the total profit r/σ. Intuitively, when there are no trade costs, X-integration is essentially the same as an increase in market size L. Even if there is the transport cost, so long as the fixed export cost is zero, (A.7) is the same as that in autarky (10). In that case, the transport cost τ_x affects only the distribution of the total profit earned from the home and foreign markets, where the equilibrium total profit is the same.

Next, consider the equilibrium characterization. Substituting $r(\alpha) = (1+\tau_x^{1-\sigma})\alpha^{\sigma-1}r_d$ and $r = (1+\tau_x^{1-\sigma})r_d$ into (A.7) and using the definition of expected shares $\phi^F \equiv \beta n/N^F, \phi^S \equiv (1-\beta)n/N^S$, we can solve the free entry condition (A.7) for the *domestic* variable profit of unmatched firms r_d/σ:

$$\begin{aligned} \frac{r_d}{\sigma} &= \frac{f_e^F + f_x + f_d\phi^F}{(1+\tau_x^{1-\sigma})\left[1+(\alpha^{\sigma-1}-1)\phi^F\right]}, \\ \frac{r_d}{\sigma} &= \frac{f_e^S + f_d\phi^S}{(1+\tau_x^{1-\sigma})(\alpha^{\sigma-1}-1)\phi^S}. \end{aligned} \tag{A.8}$$

(A.8) can be shown in the $(\theta, r_d/\sigma)$ space, where the first and second equalities are respectively downward- and upward-sloping and the intersection uniquely determines two endogenous variables, θ and r_d/σ.

To address the impact of X-integration, we only need to compare the free entry condition associated with different levels of trade costs. Note that, for both unmatched and matched firms to have an enough incentive to export their final goods, trade costs must be low enough to satisfy $(1+\tau_x^{1-\sigma})\alpha^{\sigma-1}r_d/\sigma - f_d - f_x > \alpha^{\sigma-1}r_d/\sigma - f_d$ for matched firms and $(1+\tau_x^{1-\sigma})r_d/\sigma - f_x > r_d/\sigma$ for unmatched firms, which are simplified as $(\tau_x/\alpha)^{1-\sigma}r_d/\sigma > f_x$ and $\tau_x^{1-\sigma}r_d/\sigma > f_x$ respectively. It is easy to see that when the latter holds, the former always hold. Thus

$$\frac{r_d}{\sigma} > \tau_x^{\sigma-1}f_x. \tag{A.9}$$

Comparing (10) and (A.7) under (A.9) reveals that the economic rent obtained by matched agents is greater in X-integration than in autarky for given expected shares of matched agents. Since these shares are the same (for a given θ), X-integration increases the *ex ante* expected profit and induces further entry of firms and suppliers under free entry. An increase in the expected profit, in turn, decreases the *ex post* profit r_d/σ in X-integration relative to autarky. In fact, comparing (11) and (A.8) under (A.9) reveals that both FF and SS curves in the X-integration are located below relative to those in autarky. Moreover, from (A.7), the negative relationship between r_d/σ and θ holds in this case, which implies that θ is higher while r_d/σ is lower in the X-integration equilibrium than those in the autarky equilibrium, just as in Section 4.

Although the domestic profit of unmatched firms always decreases by X-integration, the *total* profit of them $r/\sigma = r_d/\sigma + r_x/\sigma$ always increases by this integration when both types of firms export. Using $r = (1+\tau_x^{1-\sigma})r_d$, the free entry condition (A.8) can be alternatively written as

$$\begin{aligned} \frac{r}{\sigma} &= \frac{f_e^F + f_x + f_d\phi^F}{1+(\alpha^{\sigma-1}-1)\phi^F}, \\ \frac{r}{\sigma} &= \frac{f_e^S + f_d\phi^S}{(\alpha^{\sigma-1}-1)\phi^S}. \end{aligned} \tag{A.10}$$

Similarly to (A.8), we can show (A.10) in the $(\theta, r/\sigma)$ space, which uniquely determines θ and r/σ. However, comparing (11) and (A.10) under (A.9) reveals that the FF curve in X-integration is located *above* relative to that in autarky, while the SS curve is the same between the two regimes. Intuitively, if both unmatched and matched firms export, firms always have to incur the fixed export cost f_x regardless of their matching status. This implies that if agents enter as firms, their expected profit is small, which shifts the FF curve upwards. Further, a decrease in the expected profit deters further entry of firms and decreases the number of firms N^F, thereby increasing the ratio of unmatched agents $\theta = (N^S - n)/(N^F - n)$. Clearly, free entry (5) implies that the total profit of unmatched firms in the left-hand side of (A.10) must be offset by the expected fixed cost in the right-hand side of (A.10) in equilibrium.

Once these two endogenous variables of the model – either $(\theta, r_d/\sigma)$ from (A.8) or $(\theta, r/\sigma)$ from (A.10) – determined, other endogenous variables are written as a function of them. It is useful, however, to consider the latter to describe equilibrium since free entry implies $r/\sigma = f$ where f denotes the expected fixed cost that satisfies (A.10) in the case where all firms export. In such an equilibrium, the price index in (A.5) is

$$P^{1-\sigma} = (1+\tau_x^{1-\sigma})\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(N^F - n + \alpha^{\sigma-1}n\right).$$

Moreover, the *total* variable profit of unmatched firms is still written as (A.6) where the left-hand side includes the export profit. Using these relationships, we can solve for the CES price index as

$$\frac{1}{P} = \frac{\sigma-1}{\sigma}\left(\frac{(1+\tau_x^{1-\sigma})L}{r}\right)^{\frac{1}{\sigma-1}}$$

Note that an inverse of the price index includes the term $1+\tau_x^{1-\sigma}$, reflecting that market size effectively rises. Then, using the price index above, the number of agents is expressed as (12) where $\Xi \equiv \sigma f\left(\delta + \alpha^{\sigma-1}\mu^F\right)$ is the same as that in autarky (for a given ratio θ). Finally, welfare expression in (13) is expressed as

$$W = \frac{\sigma-1}{\sigma}\left(\frac{(1+\tau_x^{1-\sigma})L}{\sigma f}\right)^{\frac{1}{\sigma-1}}.$$

This completes the characterization of X-integration equilibrium where all firms export.

If search frictions are prohibitively large, equilibrium properties of X-integration are identical to that derived by Krugman (1980). As the number of matched firms is zero ($n=0$) and there is no fixed export cost ($f_x=0$) in that case, we have $f = f_e^F$ by setting $\phi^F = 0$ in the first equality of (A.10). Since the number of agents is still expressed as (12) where $\Xi = \delta\sigma f_e^F$ (from $\mu^F = 0$), the number of firms is given by $N^F = L/\sigma f_e^F$ in both autarky and X-integration. Moreover, noting $f = f_e^F$ in both autarky and X-integration, the above welfare expression reveals that welfare is higher in X-integration than in autarky due solely to increased product variety.

A.5 Free Entry Condition in M-Integration

A.5.1 Baseline Case

Consider the Bellman conditions of agents under the assumption that the rate at which unmatched agents meet unmatched partners is the same between domestic and cross-border matches. In this baseline case, we have

$$\begin{aligned}
\gamma V^F &= \frac{r}{\sigma} + \frac{\mu^F}{2}\left(V_d^F(\alpha) - V^F\right) + \frac{\mu^F}{2}\left(V_m^F(\alpha) - V^F\right) - \delta V^F + \dot{V}^F,\\
\gamma V_d^F(\alpha) &= \frac{r_d^F(\alpha)}{\sigma} - \delta V_d^F(\alpha) + \dot{V}_d^F(\alpha),\\
\gamma V_m^F(\alpha) &= \frac{r_m^F(\alpha)}{\sigma} - \delta V_m^F(\alpha) + \dot{V}_m^F(\alpha),\\
\gamma V^S &= \frac{\mu^S}{2}\left(V_d^S(\alpha) - F_d - V^S\right) + \frac{\mu^S}{2}\left(V_m^S(\alpha) - F_m - V^S\right) - \delta V^S + \dot{V}^S,\\
\gamma V_d^S(\alpha) &= \frac{r_d^S(\alpha)}{\sigma} - \delta V_d^S(\alpha) + \dot{V}_d^S(\alpha),\\
\gamma V_m^S(\alpha) &= \frac{r_m^S(\alpha)}{\sigma} - \delta V_m^S(\alpha) + \dot{V}_m^S(\alpha).
\end{aligned}$$

Setting $\gamma = 0$ and $\dot{V}^F = \dot{V}_d^F(\alpha) = \dot{V}_m^F(\alpha) = \dot{V}^S = \dot{V}_d^S(\alpha) = \dot{V}_m^S(\alpha) = 0$ in the Bellman equations, the value functions of agents corresponding to (6) are

$$
\begin{aligned}
V^F &= \frac{r}{\delta\sigma} + \left(\frac{\mu^F}{2(\delta+\mu^F)}\right)\left(\frac{r_d^F(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma}\right) + \left(\frac{\mu^F}{2(\delta+\mu^F)}\right)\left(\frac{r_m^F(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma}\right), \\
V_d^F(\alpha) &= \frac{r_d^F(\alpha)}{\delta\sigma}, \\
V_m^F(\alpha) &= \frac{r_m^F(\alpha)}{\delta\sigma}, \\
V^S &= \left(\frac{\mu^S}{2(\delta+\mu^S)}\right)\left(\frac{r_d^S(\alpha)}{\delta\sigma} - F_d\right) + \left(\frac{\mu^S}{2(\delta+\mu^S)}\right)\left(\frac{r_m^S(\alpha)}{\delta\sigma} - F_m\right), \\
V_d^S(\alpha) - F_d &= \frac{r_d^S(\alpha)}{\delta\sigma} - F_d, \\
V_m^S(\alpha) - F_m &= \frac{r_m^S(\alpha)}{\delta\sigma} - F_m.
\end{aligned}
\tag{A.11}
$$

We assume $r_d^F(\alpha)/\delta\sigma - r/\delta\sigma > 0,\ r_m^F(\alpha)/\delta\sigma - r/\delta\sigma > 0$ in the first equation of (A.11), which ensures $\left(V_d^F(\alpha) - V^F\right) + \left(V_m^F(\alpha) - V^F\right) > 0$ and the net value of matched agents is higher than that of unmatched agents. In a similar vein, we assume $r_d^S(\alpha)/\delta\sigma - F_d > 0,\ r_m^S(\alpha)/\delta\sigma - F_m > 0$ in the fourth equation of (A.11), which ensures $\left(V_d^S(\alpha) - F_d - V^S\right) + \left(V_m^S(\alpha) - F_m - V^S\right) > 0$.

Agents matched with home and foreign partners determine profit sharing by symmetric Nash bargaining. Similarly to (3), this sharing imposes the following conditions for each type of matched agents:

$$
\begin{aligned}
V_d^F(\alpha) - V^F &= \frac{1}{2}\Big(V_d^F(\alpha) - V^F + V_d^S(\alpha) - F_d - V^S\Big), \\
V_m^F(\alpha) - V^F &= \frac{1}{2}\Big(V_m^F(\alpha) - V^F + V_m^S(\alpha) - F_m - V^S\Big).
\end{aligned}
$$

Adding up these two equalities and rearranging, we get

$$
\Big(V_d^F(\alpha) - V^F\Big) + \Big(V_m^F(\alpha) - V^F\Big) = \Big(V_d^S(\alpha) - F_d - V^S\Big) + \Big(V_m^S(\alpha) - F_m - V^S\Big).
$$

Substituting the value functions (A.11) into the equality above and rearranging, we get the following profit sharing rule:

$$
\begin{aligned}
\left(\frac{r_d^F(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma}\right) + \left(\frac{r_m^F(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma}\right) &= \beta\left[\left(\frac{r_d(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d\right) + \left(\frac{r_m(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_m\right)\right], \\
\left(\frac{r_d^S(\alpha)}{\delta\sigma} - F_d\right) + \left(\frac{r_m^S(\alpha)}{\delta\sigma} - F_m\right) &= (1-\beta)\left[\left(\frac{r_d(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d\right) + \left(\frac{r_m(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_m\right)\right],
\end{aligned}
\tag{A.12}
$$

where β is the same as (8). Thus, (A.12) shows that agents matched with either home or foreign partners split the total economic rent weighted by effective bargaining power. Using (9), (A.11) and (A.12) for the free entry condition in (5) and rearranging, we get

$$
\begin{aligned}
\frac{r}{\sigma} + \frac{n}{2N^F}\beta\left[\left(\frac{r_d(\alpha)}{\sigma} - \frac{r}{\sigma} - f_d\right) + \left(\frac{r_m(\alpha)}{\sigma} - \frac{r}{\sigma} - f_m\right)\right] - f_e^F &= 0, \\
\frac{n}{2N^S}(1-\beta)\left[\left(\frac{r_d(\alpha)}{\sigma} - \frac{r}{\sigma} - f_d\right) + \left(\frac{r_m(\alpha)}{\sigma} - \frac{r}{\sigma} - f_m\right)\right] - f_e^S &= 0,
\end{aligned}
\tag{A.13}
$$

where $r_d(\alpha)/\sigma - r/\sigma - f_d$ and $r_m(\alpha)/\sigma - r/\sigma - f_m$ are the economic rent earned by agents matched with home and foreign partners respectively, and the terms in square brackets represent the total economic rent earned by matched agents. The expression in (22) follows from noting that $r_d(\alpha)/\sigma + r_m(\alpha)/\sigma = r(\alpha)/\sigma$ in (A.13).

A.5.2 Extended Case

Consider the Bellman conditions of agents under the assumption that the rate at which unmatched agents meet unmatched partners differs between domestic and cross-border matches. Let μ_d^F, μ_d^S denote the rate at which unmatched agents meet unmatched domestic partners, while let μ_m^F, μ_m^S denote the rate at which unmatched agents meet unmatched partners abroad. In this extended case, we have

$$\begin{aligned}
\gamma V^F &= \frac{r}{\sigma} + \mu_d^F\left(V_d^F(\alpha) - V^F\right) + \mu_m^F\left(V_m^F(\alpha) - V^F\right) - \delta V^F + \dot{V}^F,\\
\gamma V_d^F(\alpha) &= \frac{r_d^F(\alpha)}{\sigma} - \delta V_d^F(\alpha) + \dot{V}_d^F(\alpha),\\
\gamma V_m^F(\alpha) &= \frac{r_m^F(\alpha)}{\sigma} - \delta V_m^F(\alpha) + \dot{V}_m^F(\alpha),\\
\gamma V^S &= \mu_d^S\left(V_d^S(\alpha) - F_d - V^S\right) + \mu_m^S\left(V_m^S(\alpha) - F_m - V^S\right) - \delta V^S + \dot{V}^S,\\
\gamma V_d^S(\alpha) &= \frac{r_d^S(\alpha)}{\sigma} - \delta V_d^S(\alpha) + \dot{V}_d^S(\alpha),\\
\gamma V_m^S(\alpha) &= \frac{r_m^S(\alpha)}{\sigma} - \delta V_m^S(\alpha) + \dot{V}_m^S(\alpha).
\end{aligned}$$

Suppose that the matching function of domestic matches is $m_d(u^F, u^S) = m(u^F, u^S)/(1+\kappa)$ while that for cross-border matches is $m_m(m^F, u^S) = \kappa m(u^F, u^S)/(1+\kappa)$ where $m(u^F, u^S)$ is given in Section 2.3. Using μ^F, μ^S defined in Section 2.3, the probability of matches for each type of agents is defined as $\mu_d^F = \mu^F/(1+\kappa)$, $\mu_d^S = \mu^S/(1+\kappa)$ for domestic matches while $\mu_m^F = \kappa\mu^F/(1+\kappa)$, $\mu_m^S = \kappa\mu^S/(1+\kappa)$ for cross-border matches. Substituting these probabilities and rearranging, the value functions of agents corresponding to (A.11) are

$$\begin{aligned}
V^F &= \frac{r}{\delta\sigma} + \left(\frac{\mu^F}{(1+\kappa)(\delta+\mu^F)}\right)\left(\frac{r_d^F(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma}\right) + \left(\frac{\kappa\mu^F}{(1+\kappa)(\delta+\mu^F)}\right)\left(\frac{r_m^F(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma}\right),\\
V_d^F(\alpha) &= \frac{r_d^F(\alpha)}{\delta\sigma},\\
V_m^F(\alpha) &= \frac{r_m^F(\alpha)}{\delta\sigma},\\
V^S &= \left(\frac{\mu^S}{(1+\kappa)(\delta+\mu^S)}\right)\left(\frac{r_d^S(\alpha)}{\delta\sigma} - F_d\right) + \left(\frac{\kappa\mu^S}{(1+\kappa)(\delta+\mu^F)}\right)\left(\frac{r_m^S(\alpha)}{\delta\sigma} - F_m\right),\\
V_d^S(\alpha) - F_d &= \frac{r_d^S(\alpha)}{\delta\sigma} - F_d,\\
V_m^S(\alpha) - F_m &= \frac{r_m^S(\alpha)}{\delta\sigma} - F_m.
\end{aligned}$$

The profit sharing rule corresponding to (A.12) is now given by

$$\begin{aligned}
&\left(\frac{r_d^F(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma}\right) + \kappa\left(\frac{r_m^F(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma}\right) = \beta\left[\left(\frac{r_d(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d\right) + \kappa\left(\frac{r_m(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_m\right)\right],\\
&\left(\frac{r_d^S(\alpha)}{\delta\sigma} - F_d\right) + \kappa\left(\frac{r_m^S(\alpha)}{\delta\sigma} - F_m\right) = (1-\beta)\left[\left(\frac{r_d(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d\right) + \kappa\left(\frac{r_m(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_m\right)\right].
\end{aligned}$$

Relative to the baseline case, the economic rent earned by agents matched with foreign partners is discounted by $\kappa < 1$ in the extended case. This, of course, reflects that search frictions are relatively larger for cross-border matches than for domestic matches.

Finally, noting that the steady-state number of agents (9) holds and using the above equilibrium relationships for the free entry condition in (5), we obtain the following expression of free entry corresponding to (A.13):

$$\begin{aligned} &\frac{r}{\sigma} + \frac{n}{(1+\kappa)N^F}\beta\left[\left(\frac{r_d(\alpha)}{\sigma} - \frac{r}{\sigma} - f_d\right) + \kappa\left(\frac{r_m(\alpha)}{\sigma} - \frac{r}{\sigma} - f_m\right)\right] - f_e^F = 0, \\ &\frac{n}{(1+\kappa)N^S}(1-\beta)\left[\left(\frac{r_d(\alpha)}{\sigma} - \frac{r}{\sigma} - f_d\right) + \kappa\left(\frac{r_m(\alpha)}{\sigma} - \frac{r}{\sigma} - f_m\right)\right] - f_e^S = 0. \end{aligned} \tag{A.14}$$

Obviously, when $\kappa = 1$, (A.14) collapses to (A.13) in the baseline case. The expression in Section 5.4 follows from noting that $r_d(\alpha)/\sigma + \kappa r_m(\alpha)/\sigma = r(\alpha)/\sigma$ in (A.14). Comparing the economic rent in (10) and (A.14), the condition under which M-integration improves welfare given in (25) is exactly the same even in this case, which implies that the expected profit is always lower in M-integration than in autarky.

Using the free entry condition (A.14), we can prove that the equilibrium characterization and the impact of M-integration in the extended case is qualitatively similar to those in the baseline case. Solving (A.14) for the variable profit of unmatched firms, we get the following equalities:

$$\begin{aligned} \frac{r}{\sigma} &= \frac{f_e^F + (f_d + \kappa f_m)\frac{\phi^F}{1+\kappa}}{1 + \left[\left(1 + \kappa\tau_m^{(1-\sigma)(1-\eta)}\right)\alpha^{\sigma-1} - (1+\kappa)\right]\frac{\phi^F}{1+\kappa}}, \\ \frac{r}{\sigma} &= \frac{f_e^S + (f_d + \kappa f_m)\frac{\phi^S}{1+\kappa}}{\left[\left(1 + \kappa\tau_m^{(1-\sigma)(1-\eta)}\right)\alpha^{\sigma-1} - (1+\kappa)\right]\frac{\phi^S}{1+\kappa}}. \end{aligned} \tag{A.15}$$

Since ϕ^F and ϕ^S are decreasing and increasing in θ, the FF and SS curves derived from the first and second equalities in (A.15) are downward- and upward-sloping in the $(\theta, r/\sigma)$ space respectively, which establishes the existence and uniqueness of the M-integration equilibrium in the extended case. Furthermore, differentiating (A.15) reveals that the expected fixed cost in the right-hand side is increasing not only in trade costs τ_m, f_m but also in search frictions of cross-border matches κ. Thus, a reduction in trade costs shift down two curves while a reduction in search frictions abroad shift up two curves, suggesting that these two measures have an opposite impact on two key endogenous variables of the model. As in X-integration, a reduction in trade costs leads to an increase in the economic rent of matched firms, which in turn induces further entry and shifts two curves downwards in M-integration. In contrast, a reduction in search frictions abroad (an increase in κ) leads to an increase of the share of cross-border matches in the overall expected profit. Indeed, inspection of (A.14) shows that an increase in κ decreases the expected profit of domestic matches but increases the expected profit of cross-border matches. Since agents matched with foreign partners are less efficient, this in turn leads to the weaker degree of competition of the industry and hence shifts two curves upwards.