A New Variable Step Size LMS Algorithm-Based Method for Improved Online Secondary Path Modeling in Active Noise Control Systems

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***Abstract*—This paper proposes a new method for online secondary path modeling in active noise control systems. The existing methods for active noise control systems with online secondary path modeling consist of three adaptive filters. The main feature of the proposed method is that it uses only two adaptive filters. In the proposed method, the modified-FxLMS (MFxLMS) algorithm is used in adapting the noise control filter and a new variable step size (VSS) least mean square (LMS) algorithm is proposed for adaptation of the secondary path modeling filter. This VSS LMS algorithm is different from the normalized-LMS (NLMS) algorithm, where the step size is varied in accordance with the power of the reference signal. Here, on the other hand, the step size is varied in accordance with the power of the disturbance signal in the desired response of the modeling filter. The basic idea of the proposed VSS algorithm stems from the fact that the disturbance signal in the desired response of the modeling filter is decreasing in nature, (ideally) converging to zero. Hence, a small step size is used initially and later its value is increased accordingly. The disturbance signal, however, is not available directly, and we propose an indirect method to track its variations. Computer simulations show that the proposed method gives better performance than the existing methods. This improved performance is achieved at the cost of a slightly increased computational complexity.**

***Index Terms*—Active noise control, FxLMS algorithm, modified FxLMS algorithm, online secondary path modeling, variable step size least mean square (VSS LMS) algorithm.**

I. Introduction

A FEEDFORWARD active noise control (ANC) system using FxLMS algorithm [1], [2] comprises two filters; a noise control filter, and a secondary path modeling filter. As shown in Fig. 1, the control filter $W(z)$ is adaptive. It generates signal $y(n)$, to drive the secondary source to produce the secondary canceling signal $y'(n)$. The objective of the modeling filter $\hat{S}(z)$ is to compensate for the secondary path $S(z)$ present between the output of the control filter and that of the error microphone. The FxLMS algorithm is fairly robust to the modeling errors between the secondary path and the modeling filter; however, in general, the noise reduction performance is inferior to that under ideal conditions. Accordingly, we should use online identification of the secondary path characteristics to ensure the stability and to maintain the noise reduction performance when the secondary path is time varying [3].

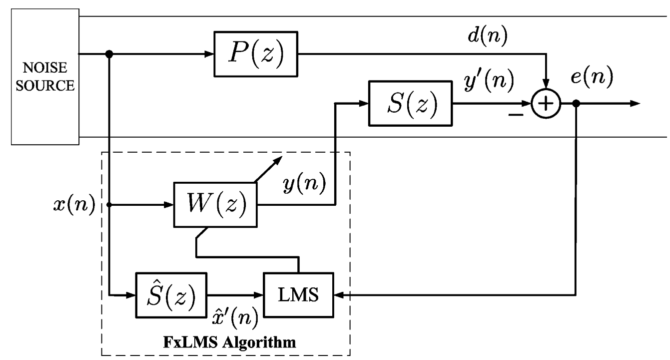

Fig. 1. FxLMS algorithm-based feedforward ANC system.

In ANC systems, there are two different approaches for online secondary path modeling. The first approach, involving the injection of additional random noise into the ANC system, utilizes a system identification method to model the secondary path. The second approach attempts to model it from the output of the ANC controller, thus avoiding the injection of additional random noise into the ANC system. A detailed comparison of these two online modeling approaches can be found in [4], which concludes that the first approach is superior to the second approach on convergence rate, speed of response to changes of primary noise, updating duration, computational complexities, etc. Consequently, only the design of the first approach is examined further.

The basic additive random noise technique for online secondary path modeling in ANC systems is proposed by Eriksson *et al.* [5]. As shown in Fig. 2, this ANC system comprises two adaptive filters: FxLMS algorithm-based noise control filter $W(z)$, and the least mean square (LMS) algorithm-based secondary path modeling filter $\hat{S}(z)$. The main problem with this system is that the white random noise, $v(n)$, injected into the ANC system for the modeling process, appears in the residual error signal $e(n)$. This constraints $v(n)$ to be a low-level signal, which results in slow convergence of the modeling filter. Furthermore, now $e(n)$ comprises two parts, a part required for the control process and a part required for the modeling process. Since $e(n)$ is used in both the control process and modeling process, the part required for one acts as a disturbance for the

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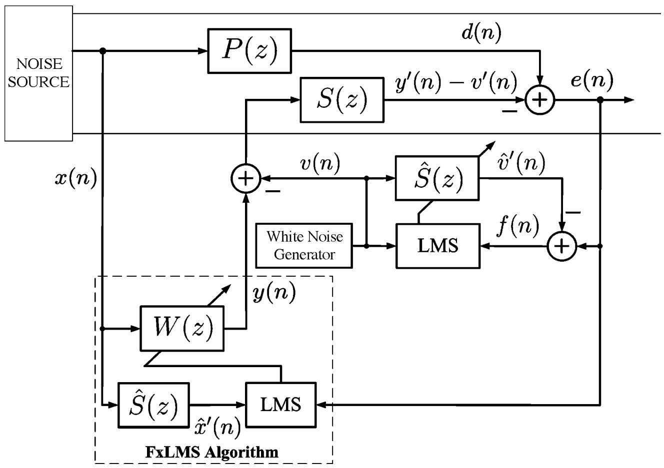

Fig. 2. FxLMS algorithm-based ANC system of Fig. 1 with online secondary path modeling (Eriksson's method).

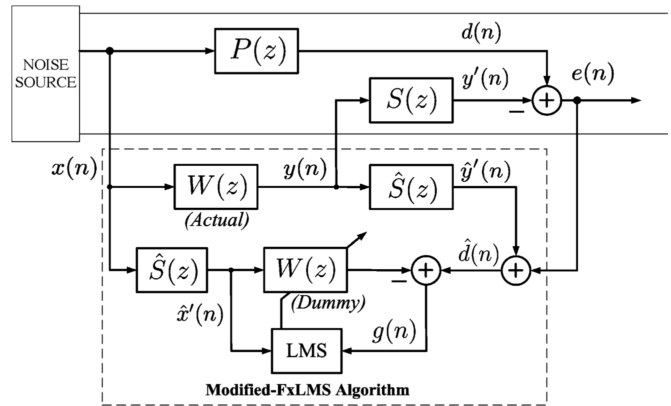

Fig. 3. Modified-FxLMS algorithm-based feedforward ANC system.

other. Due to this intrusion between the control process and modeling process, the overall performance of the ANC system is further degraded. Improvements in the Eriksson's method have been proposed in [6]–[8]. These improved methods introduce another adaptive filter into the ANC system of Fig. 2. Simulation results presented in [8] show that their proposed method, in which the control filter, the modeling filter, and the third filter are *cross-updated*, gives the best performance for ANC systems with online secondary path modeling.

The main drawback of the existing improved methods is the introduction of the third adaptive filter, which increases the design complexity. In this paper we propose a new method for active noise control system with online secondary path modeling. The main features of the proposed method are summarized below.

- In contrast to existing improved methods, it comprises two adaptive filters.
- *Adaptation of $W(z)$ using MFxLMS algorithm:* In FxLMS algorithm, one cause of the slow convergence speed is the delay introduced by the secondary path. Due to this delay the upper bound for the step size is reduced from $2/[LP_{\hat{x}'}]$ to $2/[(L+\Delta)P_{\hat{x}'}]$ [9], where L is the order of the control filter $W(z)$, Δ is the delay introduced by the secondary path and $P_{\hat{x}'}$ is the power of the filtered reference signal $\hat{x}'(n)$. In modified-FxLMS (MFxLMS) algorithm two extra (fixed) filters are used for the secondary path modeling filter and the control filter [9]. As shown in Fig. 3, the extra secondary-path-modeling filter is used to generate modified error signal for the control filter. The extra control filter is used to avoid adaptation using FxLMS algorithm. The control filter is adapted using simple LMS algorithm and hence upper bound for the step size parameter is larger than that for FxLMS algorithm. Since larger step size can be selected, fast convergence can be achieved. In the proposed method, therefore, we use MFxLMS algorithm in adapting $W(z)$.
- *New variable step size algorithm for $\hat{S}(z)$*: In the modeling filter, the step size is varied in accordance with the power of the disturbance signal in the desired response of the modeling filter. This variable step size (VSS) LMS algorithm is different from the normalized-LMS (NLMS) algorithm, where the step size is varied with the power of reference signal power. It is also different from the other VSS algorithms [10]–[14] where initially a large step size is selected for fast convergence and finally a small value is used for small misadjustment. The proposed VSS LMS algorithm stems from the fact that the desired response for the modeling filter is corrupted by a disturbance signal which is decreasing in nature, (ideally) converging to zero. Infact, initially this interference may be so large that the online secondary path modeling is much slower as compared with offline modeling. The proposed VSS LMS algorithm, in contrast to the existing VSS algorithms, uses a small step size initially and later its value is increased accordingly.

The organization of this paper is as follows. Section II describes the proposed method. Section III presents computational complexity analysis, Section IV details the computer simulations, and Section V gives some concluding remarks.

A short version of this paper was presented at a conference [15].

II. Proposed Method for ANC Systems With Online Secondary Path Modeling

A. Proposed Method

Consider Fig. 4 which shows the block diagram of the proposed method. Assuming that $W(z)$ is an FIR filter of tap-weight length L, the output signal $y(n)$ is computed as

$$y(n) = \boldsymbol{w}^T(n)\boldsymbol{x}_L(n) \tag{1}$$

where $\boldsymbol{w} = [w_0(n), w_1(n), \ldots, w_{L-1}(n)]^T$ is the tap-weight vector, and $\boldsymbol{x}_L(n) = [x(n), x(n-1), \ldots, x(n-L+1)]^T$ is the L sample reference signal vector. An internally generated zero mean white Gaussian noise signal, $v(n)$, uncorrelated with the reference signal $x(n)$, is injected at the output $y(n)$ of the control filter. Thus, the residual error signal $e(n)$ is given as

$$e(n) = [d(n) - y'(n)] + v'(n) \tag{2}$$

where $d(n) = p(n) * x(n)$ is the primary disturbance signal, $y'(n) = s(n) * y(n)$ is the canceling signal, $v'(n) = s(n) * v(n)$ is the modeling signal, $*$ denotes linear convolution, and $p(n)$ and $s(n)$ are the impulse responses of the $P(z)$ and $S(z)$, respectively.

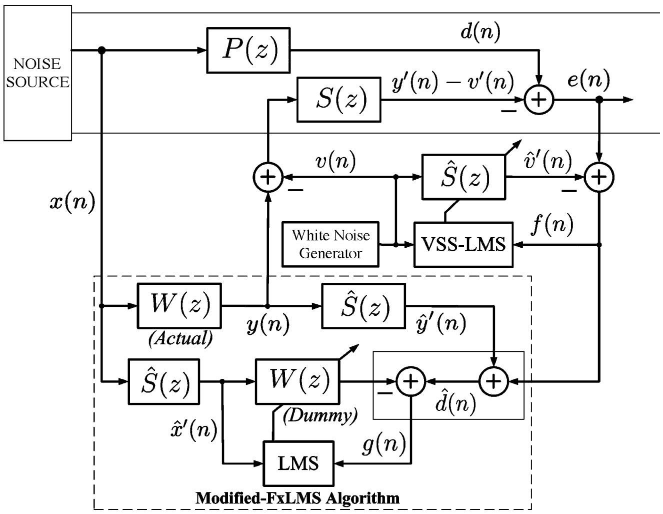

Fig. 4. Proposed method for ANC systems with online secondary path modeling.

Assuming that the modeling filter $\hat{S}(z)$ is an FIR filter of tap-weight length M, its output $\hat{v}'(n)$ is given as

$$\hat{v}'(n) = \hat{\boldsymbol{s}}^T(n)\boldsymbol{v}(n) \tag{3}$$

where $\hat{\boldsymbol{s}}(n) = [\hat{s}_0(n), \hat{s}_1(n), \ldots, \hat{s}_{M-1}(n)]^T$ is the impulse response of $\hat{S}(z)$, and $\boldsymbol{v}(n) = [v(n), v(n-1), \ldots, v(n-M+1)]^T$.

The output of $\hat{S}(z)$, $\hat{v}'(n)$, is subtracted from $e(n)$ to get the error signal for the modeling filter $\hat{S}(z)$

$$f(n) = [d(n) - y'(n)] + [v'(n) - \hat{v}'(n)]. \tag{4}$$

The tap-weights of the modeling filter $\hat{S}(z)$ are updated using LMS algorithm

$$\hat{\boldsymbol{s}}(n+1) = \hat{\boldsymbol{s}}(n) + \mu_s(n) f(n)\boldsymbol{v}(n) \tag{5}$$

where $\mu_s(n)$ is the step size parameter. Note the time dependence of the step size, it will be explained later.

Here output of the (actual) control filter $y(n)$ is filtered through another modeling filter $\hat{S}(z)$ to get the desired response for the (dummy) control filter

$$\hat{d}(n) = f(n) + \hat{\boldsymbol{s}}^T(n)\boldsymbol{y}(n) \tag{6}$$

where $\boldsymbol{y}(n) = [y(n), y(n-1), \ldots, y(n-M+1)]^T$ is a vector containing M samples of the controller output $y(n)$.

The input to the (dummy) control filter $W(z)$ is derived by filtering the reference signal through $\hat{S}(z)$

$$\hat{x}'(n) = \hat{\boldsymbol{s}}^T(n)\boldsymbol{x}_M(n) \tag{7}$$

where $\boldsymbol{x}_M(n) = [x(n), x(n-1), \ldots, x(n-M+1)]^T$ is an M sample reference signal vector.

The noise control filter $W(z)$ is updated using LMS algorithm

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu_w g(n)\hat{\boldsymbol{x}}'(n) \tag{8}$$

where μ_w is the step size parameters for the control filter, $g(n)$ is the error signal for $W(z)$ given as $g(n) = \hat{d}(n) - \boldsymbol{w}^T(n)\hat{\boldsymbol{x}}'(n)$, and $\hat{\boldsymbol{x}}'(n) = [\hat{x}'(n), \hat{x}'(n-1), \ldots, \hat{x}'(n-L+1),]^T$ is the filtered reference signal vector. After updating, the tap weights are copied to the (actual) control filter $W(z)$.

Consider the structure of the MFxLMS algorithm in the proposed method, as shown by dashed box in Fig. 4. In the upper path, the reference signal $x(n)$ is first filtered through $W(z)$ and then through $\hat{S}(z)$. In the lower path this situation is exactly reversed. Thus, the adder generating the error signal $g(n)$, is taking two similar inputs with opposite signs. The third input being driven from the modeling error signal $f(n)$, we find that both the modeling filter and the control filter are effectively updated by using a same error signal, i.e., $g(n) = f(n) = e(n) - \hat{v}'(n) = e'(n)$(say). Taking the z-transform and making necessary substitutions, we get the following expression for this error signal:

$$E'(z) = [P(z) - S(z)W(z)]\,X(z) + \left[S(z) - \hat{S}(z)\right] V(z). \tag{9}$$

By convergence of $W(z)$, we mean that the error signal is minimized to (ideally) zero. This requires $W(z)$ to adapt to the following transfer function:

$$W^{\circ}(z) = \frac{P(z)}{S(z)} + \left(\frac{S(z) - \hat{S}(z)}{S(z)}\right)\frac{V(z)}{X(z)}. \tag{10}$$

This equation shows that $W(z)$ will converge to the optimal solution $P(z)/S(z)$, if and only if, modeling error reduces to zero, i.e., $\hat{S}(z) \to S(z)$. Converse is also true, that modeling error reduces to zero, if and only if, $W(z)$ converges to the optimal solution. Thus, in the proposed method the convergence of the control filter and the modeling filter is mutually dependent.

B. New Variable Step Size LMS Algorithm

Equation (4) shows that $f(n)$, the error signal for modeling filter, comprises two parts; $[d(n) - y'(n)]$ and $[v'(n) - \hat{v}'(n)]$. Here $[d(n) - y'(n)]$ acts as disturbance to the modeling process. We observe the following.

- Due to large disturbance (note that initially canceling signal $y'(n)$ is zero) the convergence of the modeling filter is degraded, and in worst case it may be unstable.
- As $n \to \infty$, $y'(n)$ would converge to $d(n)$ and thus (ideally) $[d(n) - y'(n)]$ would converge to zero.

These observations show that initially we should use a small value for the step size parameter μ_s, and later, when the disturbance signal $[d(n) - y'(n)]$ is reduced, the step size can be increased accordingly. This disturbance signal, however, is not available directly. We have access to residual error signal $e(n)$ [as given in (2)] being picked up by the error microphone. The error signal for $\hat{S}(z)$, $f(n)$ [as given in (4)], also contains this disturbance signal. On the basis of $e(n)$ and $f(n)$, we propose an indirect procedure to vary the step size $\mu_s(n)$.

Define $\rho(n) = P_f(n)/P_e(n)$ where $P_e(n)$ is the power of the residual error signal $e(n)$ and $P_f(n)$ is the power of the modeling error signal $f(n)$. These powers can be estimated from the following low-pass estimators, respectively:

$$P_e(n) = \lambda P_e(n-1) + (1-\lambda)e^2(n) \tag{11}$$

$$P_f(n) = \lambda P_f(n-1) + (1-\lambda)f^2(n) \tag{12}$$

where λ is the forgetting factor $(0.9 < \lambda < 1)$. From (2) $P_e(n)$ can be written as $P_e(n) = P_{[d(n)-y'(n)]} + P_{v'(n)}$. Similarly

TABLE I
COMPUTATIONAL COMPLEXITY (NUMBER OF COMPUTATIONS PER ITERATION) COMPARISON OF THE PROPOSED METHOD WITH THE EXISTING METHODS

	Number of Computations per Iteration			
			$M = N = L$	
	Multiplications	Additions	Multiplications	Additions
Eriksson's method	$2L + 3M + 2$	$2L + 3M - 1$	$5L + 2$	$5L - 1$
Zhang's method	$2L + 3M + 2N + 3$	$2L + 3M + 2N + 1$	$7L + 3$	$7L + 1$
Proposed method	$3L + 4M + 10$	$3L + 4M + 5$	$7L + 10$	$7L + 5$

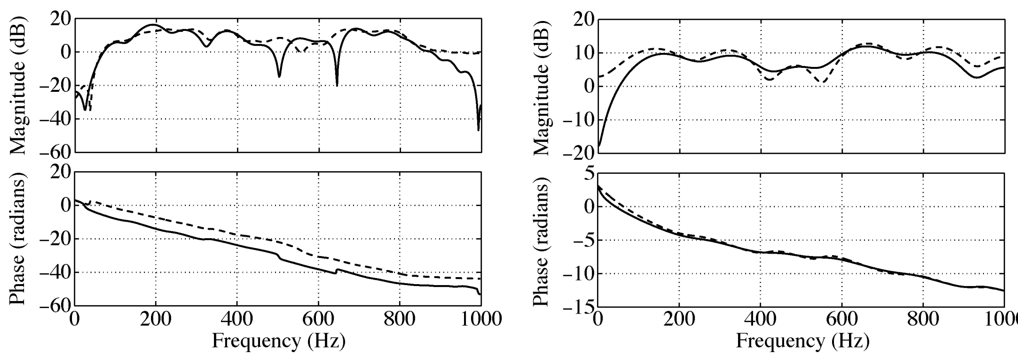

Fig. 5. Frequency response of the acoustic paths. (a) Primary path $P(z)$ and (b) secondary path $S(z)$.

from (4) $P_f(n)$ can be expressed as $P_f(n) = P_{[d(n)-y'(n)]} + P_{[v'(n)-\hat{v}'(n)]}$. Thus, $\rho(n)$ can be expressed as

$$\rho(n) = \frac{P_f(n)}{P_e(n)} = \frac{P_{[d(n)-y'(n)]} + P_{[v'(n)-\hat{v}'(n)]}}{P_{[d(n)-y'(n)]} + P_{v'(n)}}. \qquad (13)$$

As stated in Section I, the injected random noise $v(n)$ is a low-level constant excitation signal as compared with the reference noise signal $x(n)$. Initially at $n = 0$, there is no canceling signal, i.e., $y'(n) = 0 \Rightarrow$, $P_{[d(n)-y'(n)]} \gg P_{v'(n)}$ and $P_{[d(n)-y'(n)]} \gg P_{[v'(n)-\hat{v}'(n)]}$, and hence $\rho(n) \approx 1$. In steady state, as $n \to \infty$, $y'(n) \to d(n)$ and $\hat{v}'(n) \to v'(n)$, thus the expression in the numerator in (13) converges to zero, whereas the denominator is nonzero due to $P_{v'(n)}$, and hence $\lim_{n\to\infty} \rho(n) \to 0$. This observation leads to the following mechanism for the step size selection in the modeling process:

$$\mu_s(n) = \rho(n)\mu_{s_{\min}} + (1 - \rho(n))\,\mu_{s_{\max}} \qquad (14)$$

where $\mu_{s_{\min}}$ and $\mu_{s_{\max}}$ are the experimentally determined values for lower and upper bounds of the step size. These values are selected so that neither adaptation is too slow nor it becomes unstable. To make sure that initially $\mu_s(n) = \mu_{s_{\min}}$, the estimators (11), (12) are initialized by the same value, preferably unity, i.e., $P_e(0) = P_f(0) = 1$. Also it is recommended that the same λ is used for the two estimators.

III. COMPUTATIONAL COMPLEXITY COMPARISON

The computationally complexity is normally determined by the number of computations required per iteration of the algorithm [16]. The computational complexity analysis of the proposed method, in comparison with the existing methods, is given in Table I. Here L and M, are tap-weight lengths of the control filter $W(z)$, and modeling filter $\hat{S}(z)$, respectively. The N is the tap-weight length of the third filter $H(z)$ in Zhang's method [8]. In the proposed method, although we have avoided using the third adaptive filter, however, it adds two extra (fixed FIR) filters for $W(z)$ and $\hat{S}(z)$. Furthermore, it uses VSS LMS algorithm in adapting the modeling filter. The computational complexity of the proposed method, therefore, is comparable to Zhang's method.

TABLE II
SIMULATION PARAMETERS FOR COMPUTER EXPERIMENTS

	Case 1	Case 2	Case 3	Case 4
	Zhang's Method			
μ_w	5×10^{-4}	5×10^{-4}	5×10^{-4}	5×10^{-4}
μ_s	1×10^{-2}	1×10^{-2}	7.5×10^{-3}	1×10^{-2}
μ_h	1×10^{-2}	2.5×10^{-2}	1×10^{-2}	2.5×10^{-2}
	Proposed Method			
	$P_e(0) = P_f(0) = 1$; $\lambda = 0.99$;			
μ_w	5×10^{-4}	5×10^{-4}	5×10^{-4}	5×10^{-4}
$\mu_{s_{\min}}$	7.5×10^{-3}	7.5×10^{-3}	7.5×10^{-3}	7.5×10^{-3}
$\mu_{s_{\max}}$	2.5×10^{-2}	2.5×10^{-2}	2.5×10^{-2}	2.5×10^{-2}

IV. SIMULATIONS

This section presents the simulation experiments performed to verify the effectiveness of the proposed method. The performance of the proposed method is compared with Zhang's method [8] which is known to be the best existing method for

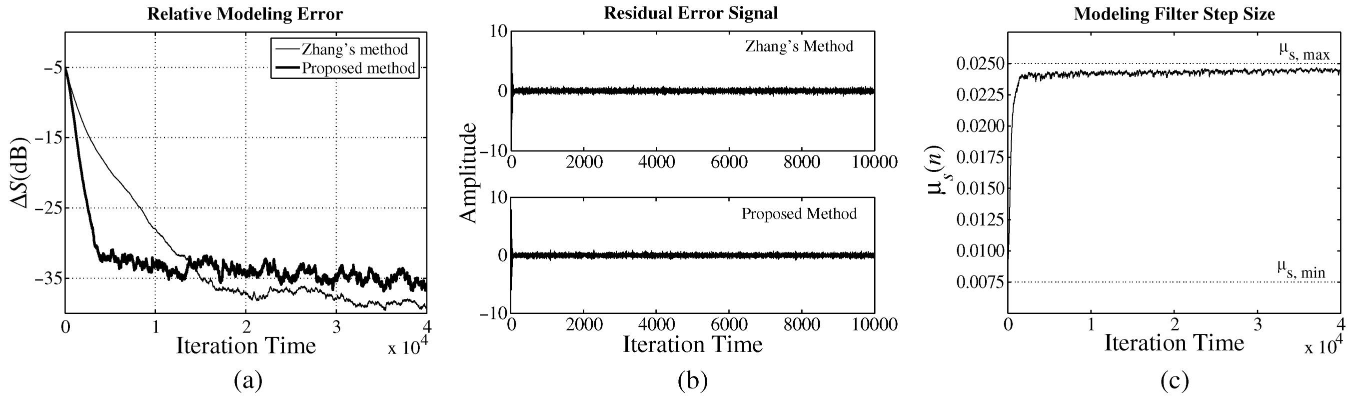

Fig. 6. Simulation results in Case 1. (a) Relative modeling error ΔS (decibels) versus iteration time n, (b) residual error signal $e(n)$ versus iteration time n, and (c) variation of step size $\mu_s(n)$ in the proposed method.

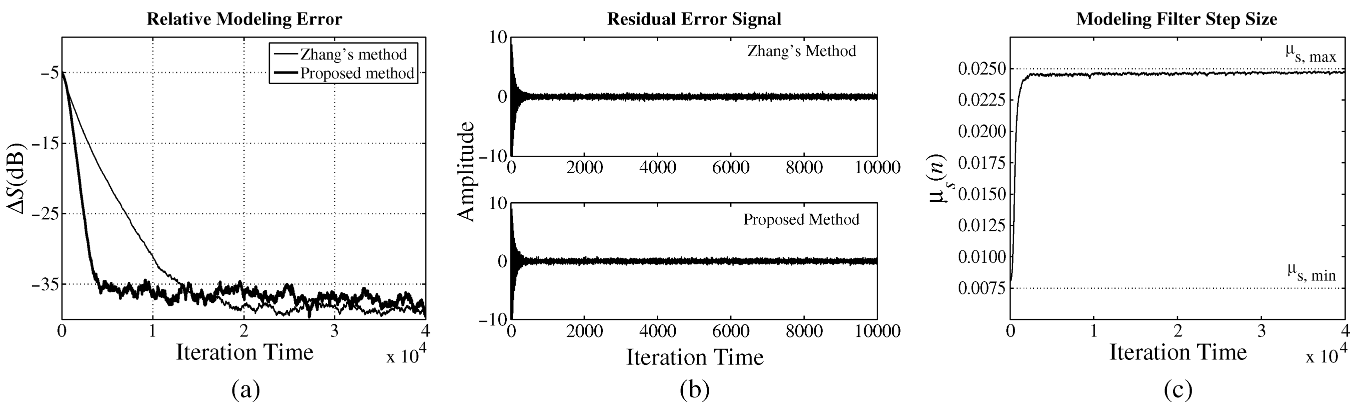

Fig. 7. Simulation results in Case 2. (a) Relative modeling error ΔS (decibels) versus iteration time n, (b) residual error signal $e(n)$ versus iteration time n, and (c) variation of step size $\mu_s(n)$ in the proposed method.

ANC systems with online secondary path modeling.[1] The performance comparison is done on the basis of two performance measures; 1) the residual error signal $e(n)$, and 2) the relative modeling error, ΔS(dB), being defined as

$$\Delta S(\mathrm{dB}) = 10\log_{10}\left\{\frac{\sum_{i=0}^{M-1}\left[s_i(n)-\hat{s}_i(n)\right]^2}{\sum_{i=0}^{M-1}\left[s_i(n)\right]^2}\right\}. \tag{15}$$

The primary path $P(z)$ and the secondary path $S(z)$ are FIR filters of tap-weight length 48 and 16, respectively (the data is taken from disk included with [1]). The frequency response of $P(z)$ and $S(z)$ are shown (by solid curves) in Fig. 5. The control filter $W(z)$ and modeling filter $\hat{S}(z)$ are FIR filters of tap-weight length $L = 32$, and $M = 16$, respectively. The third adaptive filter in Zhang's method [8], $H(z)$, is selected as an FIR filter of tap-weight length $N = 16$. The delay Δ in Zhang's method is 16. The control filter is initialized by a null vector $\boldsymbol{w}(0)$. The third filter $H(z)$ in the Zhang's method is also initialized by a null vector $\boldsymbol{h}(0)$.

We have seen in Section II, that: 1) the convergence of $W(z)$ and $\hat{S}(z)$ is mutually dependent and 2) the proposed VSS LMS algorithm uses a smaller step size at the beginning of the ANC operation and a larger step size when the residual noise is relatively small. With this kind of treatment, if $\hat{S}(z)$ is initialized by a null vector, then ANC system may be unstable. To avoid this, offline secondary path modeling is performed which is stopped when the modeling error [as defined in (15)] has been reduced to -5 dB. The resulting weights are used for $\hat{\boldsymbol{s}}(0)$ when the ANC system is started. It should be noted that this is not a serious problem as offline measurements are first step in any ANC system design [20].

A sampling frequency of 2 kHz is used and simulations are carried out with the signals having frequency below 500 Hz. The step size parameters are adjusted, by trial-and-error, for fast and stable convergence and are summarized in Table II. Extensive simulations are performed to show the effectiveness of the proposed method. All the results shown below are averaged over ten experiments.

A. Case 1

Here the reference signal $x(n)$ is a tonal of 300 Hz. The variance of this signal is adjusted to 2.0 and a zero-mean white Gaussian noise is added with SNR of 30 dB. In order to ensure low residual noise in the steady state, a zero-mean white Gaussian noise of variance 0.05 is used in the modeling process. Fig. 6(a) shows the curves of the relative modeling error, ΔS,

[1]The authors have proposed another two adaptive filter-based method [17] for ANC systems with online secondary path modeling. Here $W(z)$ is adapted by adaptive filtering with averaging (AFA) [18] based filtered-x, FxAFA, algorithm [19]. It gives better noise reduction and secondary path modeling performance than the Zhang's method, but has poor tracking properties, and hence not included in the simulations presented here.

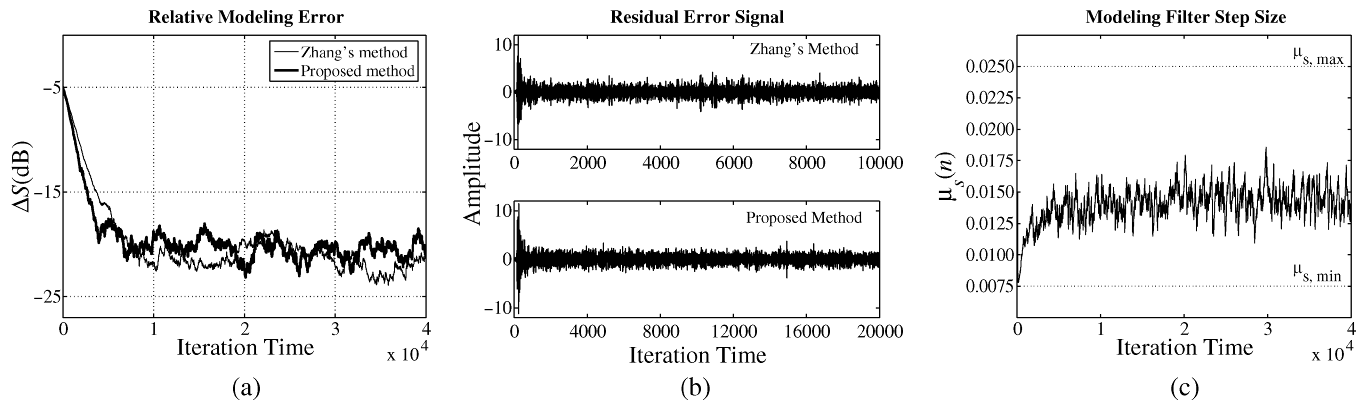

Fig. 8. Simulation results in Case 3. (a) Relative modeling error ΔS (decibels) versus iteration time n, (b) residual error signal $e(n)$ versus iteration time n, and (c) variation of step size $\mu_s(n)$ in the proposed method.

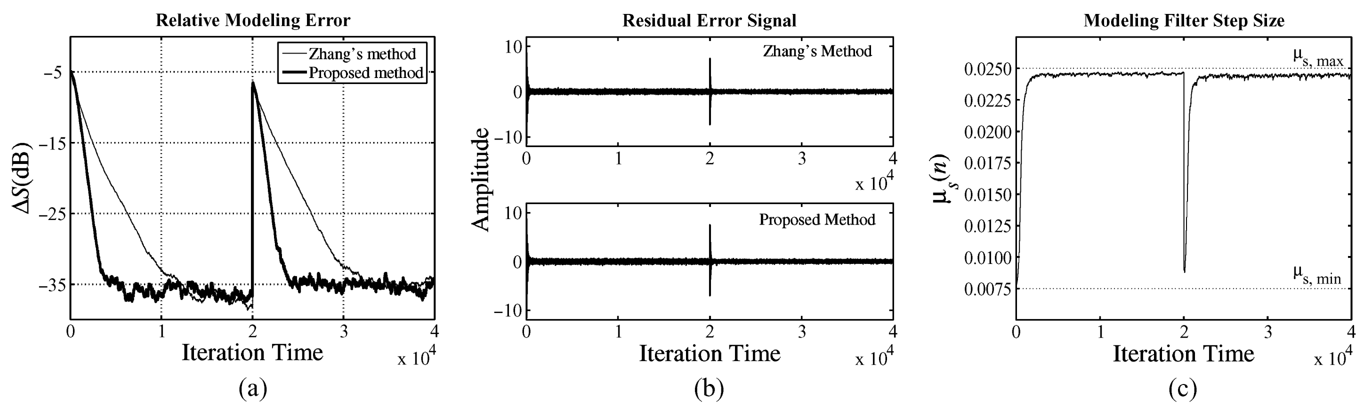

Fig. 9. Simulation results in Case 4. (a) Relative modeling error ΔS (decibels) versus iteration time n, (b) residual error signal $e(n)$ versus iteration time n, and (c) variation of step size $\mu_s(n)$ in the proposed method.

as defined in (15). We see that the proposed method can reduce the modeling error at a faster convergence rate as compared with Zhang's method. The corresponding curves for the residual error signal are shown in Fig. 6(b). The variation of step size $\mu_s(n)$ in the proposed method is shown in Fig. 6(c). Initially it selects small step size $\mu_{s_{\min}}$, and converges toward the maximum value $\mu_{s_{\max}}$.

B. Case 2

In this case the reference signal is a narrow-band signal comprising frequencies of 100, 200, 300, and 400 Hz. The variance of the resulting signal is adjusted to 2.0 and a zero-mean white Gaussian noise is added with SNR of 30 dB. As in Case 1, the modeling excitation signal $v(n)$, is a zero-mean white Gaussian noise of variance 0.05. The simulation results are shown in Fig. 7. We observe similar performance as in Case 1.

C. Case 3

Here we consider a broad-band reference noise signal. A zero mean white Gaussian noise of unit variance is filtered through a bandpass filter with the passand 100–400 Hz. The variance of the filtered signal is adjusted to 2, and is used as reference signal $x(n)$. As in the previous cases, zero-mean white Gaussian noise of variance 0.05 is used in the modeling process. The simulation results are given in Fig. 8. The performance of the proposed method is comparable to that of the Zhang's method.

D. Case 4

Here we consider a situation of varying acoustic paths. The reference signal is same as described in Case 2. The system is started with the same conditions as described for Case 2. At $n = 20\,000$ the acoustic path change to as shown by dotted curves in Fig. 5. The simulation results for two methods are shown in Fig. 9. We see that the proposed methods gives better performance before and after the change. Fig. 9(c) shows that step size $\mu_s(n)$ varies in accordance with the disturbance in the error signal. At $n = 20\,000$, when $P(z)$ and $S(z)$ change, the step size reduces to the minimum value. Later, it reconvenes toward the maximum value in accordance with the decrease in the disturbance signal.

V. Concluding Remarks

In this paper we have proposed a new method for ANC systems with online secondary path modeling. This method uses MFxLMS algorithm in adapting the control filter $W(z)$ and proposes a new VSS LMS algorithm for the modeling filter $\hat{S}(z)$. The proposed method comprises two adaptive filters only and yet can give better performance than the existing methods which comprise of three adaptive filters. This improved performance is achieved without any degradation in the noise control performance, which is the ultimate goal of ANC.

The proposed method avoids using the third adaptive filter, however, its computational complexity is slightly higher than the three adaptive filter-based methods. The increased computational complexity is due to the structure of MFxLMS algorithm [9]. To realize the improved performance at a reduced computational cost is a task of future work.

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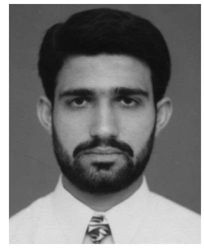

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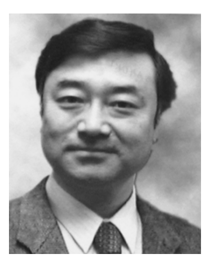

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