Proceedings of the 35th
Conference on Decision and Control
Kobe, Japan • December 1996

Reaction Null-Space Based Control of Flexible Structure Mounted Manipulator Systems

D.N. Nenchev K. Yoshida M. Uchiyama
Department of Aeronautics and Space Engineering, Tohoku University
Aramaki-aza-Aoba, Aoba-ku, Sendai 980-77, Japan
nenchev@space.mech.tohoku.ac.jp
yoshida@space.mech.tohoku.ac.jp
uchiyama@space.mech.tohoku.ac.jp

Abstract

The control of a dexterous manipulator mounted on a flexible structure is discussed. Using a concept called *Reaction Null Space*, the manipulator dynamics is totally decoupled from the base dynamics. As a consequence of the decoupling, feedback control gains can be determined in a straightforward manner. We examine the simultaneous performance of the following two tasks: (1) base vibration suppression and (2) end-point path tracking without base disturbance.

1 Introduction

The interest toward complex robot systems is expanding for new application areas. A class of such robot systems are so-called underactuated systems, characterized by the number of control actuators being less than the number of state variables. Typical examples are free-flying space manipulators, serial robots with passive joints, flexible-link manipulators, multiarm manipulation systems with loose grasp contacts, space vehicles with fuel sloshing models [1].

Another example of an underactuated system, which will be discussed in this paper, is a dexterous manipulator mounted on a flexible base. We shall refer to such a system as *flexible structure mounted manipulator system (FSMS)*. We assume that the flexible base can be regarded as a passive structure. The motivation behind that is that even if the base represents another manipulator (i.e. an "active" base), it will be used just for relocation of the dexterous manipulator. Once located at the work site, the dexterous manipulator only is controlled. This approach has been suggested by Book and Lee [2]. They proposed a controller that stabilizes the fast (base vibration control) and the slow (manipulator control) submodels. Torres and Dubowsky [3], [4], proposed several concepts for path planning and control of a FSMS. Both groups pointed out the possibility for utilizing kinematic redundancy for base motion control. A more detailed treatment of the latter topic has been done by Hanson and Tolson [5]. Control inputs from the manipulator Jacobian null space have been applied, as in conventional redundancy resolution techniques. This approach yields, however, kinematic instabilities due to conflicts between the manipulator motion subtask and the base motion subtask.

In this paper we propose a control decomposition scheme based on the null space of the so-called inertia coupling matrix. We call the new framework "Reaction Null Space." The reaction null-space concept has its roots in our earlier work on free-floating space robots, where the Fixed-Attitude-Restricted (FAR) Jacobian has been proposed as a means to plan [6] and control [7] manipulator motion that does not disturb the attitude of the free-floating base. Application of the reaction null space with relation to impact dynamics can be found in [8]. In a recent study [9] we emphasized the fact that the reaction null space concept is general, and can be applied to a broad class of moving base robots. The main advantage of this approach is total decoupling of the interaction dynamics.

2 Dynamics Model

We consider a dexterous rigid n-link manipulator attached to an elastic base. The *system dynamics* of the FSMS is represented in the following form [2], [4]:

$$\begin{bmatrix} \boldsymbol{H}_b & \boldsymbol{H}_{bm} \\ \boldsymbol{H}_{bm}^T & \boldsymbol{H}_m \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{x}}_b \\ \ddot{\boldsymbol{\phi}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{D}_b\dot{\boldsymbol{x}}_b + \boldsymbol{K}_b\boldsymbol{x}_b \\ \boldsymbol{0} \end{bmatrix} + \begin{bmatrix} \mathbf{c}_b \\ \mathbf{c}_m \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{\tau} \end{bmatrix}, \tag{1}$$

where $\boldsymbol{x}_b \in \Re^m$ denotes the positional and orientational deflection of the base with respect to the inertial frame[1], $\boldsymbol{\phi} \in \Re^n$ stands for the manipulator generalized coordinates, $\boldsymbol{H}_b, \boldsymbol{D}_b, \boldsymbol{K}_b \in \Re^{m\times m}$ denote inertia, damping and stiffness matrices of the base, respectively, $\boldsymbol{H}_m \in R^{n\times n}$ is the manipulator inertia matrix,

[1] Generally, $m = 6$.

0-7803-3590-2/96 $5.00 © 1996 IEEE

$\boldsymbol{H}_{bm} \in \Re^{m\times n}$ denotes an inertia matrix for manipulator/base coupling which will be referred to as the *inertia coupling matrix*, $\boldsymbol{c}_b$ and $\boldsymbol{c}_m$ denote velocity-dependent terms, and $\boldsymbol{\tau} \in \Re^n$ are the joint torques. As it is seen, no external forces are present, and this assumption holds throughout the paper. Both zero and nonzero initial conditions will be examined, however.

We shall give some details for the derivation of the system dynamics. First, we note that the *base dynamics* is

$$\boldsymbol{H}_b\ddot{\boldsymbol{x}}_b + \boldsymbol{D}_b\dot{\boldsymbol{x}}_b + \boldsymbol{K}_b\boldsymbol{x}_b = -\mathcal{F}, \qquad (2)$$

where $\mathcal{F} = \begin{pmatrix} \boldsymbol{f}^T & \boldsymbol{t}^T \end{pmatrix}^T$ is the disturbance reaction force/torque induced from the manipulator motion. The *reaction dynamics,* defined with respect to the base coordinate frame, is:

$$\mathcal{F} = \begin{pmatrix} m\ddot{\boldsymbol{r}}_{cm} \\ \frac{d}{dt}[m\boldsymbol{r}_{cm} \times \dot{\boldsymbol{r}}_{cm} + \sum_{j=1}^{n}(\boldsymbol{I}_j\boldsymbol{\omega}_j + m_j\boldsymbol{r}_j \times \dot{\boldsymbol{r}}_j)] \end{pmatrix}, \qquad (3)$$

where $\boldsymbol{r}_{cm}$ denotes the manipulator center of mass position, $\boldsymbol{I}_j$, $\boldsymbol{\omega}_j$, m_j, $\boldsymbol{r}_j$ stand for the inertia matrix, angular velocity, mass and center-of-mass position for link j, respectively, and $m = \sum m_j$. The expression $\sum_{j=1}^{n}(\boldsymbol{I}_j\boldsymbol{\omega}_j + m_j\boldsymbol{r}_j \times \dot{\boldsymbol{r}}_j)$ denotes angular momentum and imposes a nonholonomic constraint. On the other hand, the upper part of Equation (3) denoting the reaction force, represents a holonomic constraint. In terms of manipulator generalized coordinates, Equation (3) can be rewritten as

$$\mathcal{F} = \boldsymbol{H}_{bm}\ddot{\boldsymbol{\phi}} + \dot{\boldsymbol{H}}_{bm}\dot{\boldsymbol{\phi}}. \qquad (4)$$

Equation (4) can be integrated.

Definition 1: The integral of the reaction dynamics (Equation (4)), denoted as

$$\mathcal{L} = \boldsymbol{H}_{bm}\dot{\boldsymbol{\phi}} \qquad (5)$$

is called *the coupling momentum of a FSMS.*

It must be emphasized that the coupling momentum thus defined is different from the generalized system momentum:

$$\boldsymbol{H}_b\dot{\boldsymbol{x}}_b + \boldsymbol{H}_{bm}\dot{\boldsymbol{\phi}}. \qquad (6)$$

3 Reaction Null-Space and Reactionless Motion of FSMS

In this section we shall introduce a special case of manipulator motion that conserves the coupling momentum. In this case, $\mathcal{L}$ =**const**.

Proposition 1: (Zero reaction) The manipulator does not induce any reactions to the base if and only if the coupling momentum is conserved ($\mathcal{L}$ =**const** $\Leftrightarrow \mathcal{F} = \mathbf{0}$).

The proof follows from the direct examination of Equations (4) and (5).

Let us assume that $n > m$ holds, which denotes a kinematic redundancy condition with respect to the base motion task [6].

Proposition 2: At a manipulator configuration $\tilde{\boldsymbol{\phi}}$ such that rank $\boldsymbol{H}_{bm}(\tilde{\boldsymbol{\phi}}) = \max_{\boldsymbol{\phi}}$ rank $\boldsymbol{H}_{bm}(\boldsymbol{\phi})$, zero reaction/coupling momentum conservation is achieved with:

1. the joint acceleration

$$\ddot{\boldsymbol{\phi}}_c = -\boldsymbol{H}_{bm}^{+}\dot{\boldsymbol{H}}_{bm}\dot{\boldsymbol{\phi}} + (\boldsymbol{E} - \boldsymbol{H}_{bm}^{+}\boldsymbol{H}_{bm})\boldsymbol{\zeta} \qquad (7)$$

   where $\boldsymbol{H}_{bm}^{+} \in \Re^{n\times m}$ denotes the Moore-Penrose generalized inverse of the inertia coupling matrix, $\boldsymbol{E} \in \Re^{n\times n}$ stands for the unit matrix, $\dot{\boldsymbol{\phi}}$ and $\boldsymbol{\zeta} \in \Re^n$ are arbitrary;

2. the joint velocity

$$\dot{\boldsymbol{\phi}}_c = \boldsymbol{H}_{bm}^{+}\bar{\mathcal{L}} + (\boldsymbol{E} - \boldsymbol{H}_{bm}^{+}\boldsymbol{H}_{bm})\boldsymbol{\zeta} \qquad (8)$$

   where $\boldsymbol{\zeta}$ denotes again an arbitrary vector, and $\bar{\mathcal{L}}$ =**const**.

Proof: Substituting $\ddot{\boldsymbol{\phi}}_c$ and $\dot{\boldsymbol{\phi}}_c$ into Equations (4) and (5), respectively, and taking into account that under the above rank condition $\boldsymbol{H}_{bm}\boldsymbol{H}_{bm}^{+} = \boldsymbol{E}$, one obtains $\mathcal{F} = \mathbf{0}$ and $\mathcal{L} = \bar{\mathcal{L}}$, respectively. □

The condition for maximum rank of the inertia coupling matrix is in fact a controllability condition, as discussed by Spong with regard to passive-joint manipulators [11]. Spong coined the term "strong inertially coupled system" to address this condition.

The expression $\boldsymbol{P}_{RNS} \equiv (\boldsymbol{E} - \boldsymbol{H}_{bm}^{+}\boldsymbol{H}_{bm})$ which appears in both Equations (7) and (8), stands for the projector onto the null space of the inertia coupling matrix.

Definition 2: The null space of the inertia coupling matrix is called *the reaction null-space of a FSMS.*

From Equation (8) it is apparent that the joint velocity comprises two components: one from the reaction null space, and the other from its orthogonal complement. The reaction null-space component does not contribute to the coupling momentum, and hence, it would yield zero reaction.

Corollary: With zero initial coupling momentum, zero reaction is obtained with the velocity

$$\dot{\boldsymbol{\phi}}_{RNS} = \boldsymbol{P}_{RNS}\boldsymbol{\zeta}. \qquad (9)$$

We are interested in the component $\dot{\phi}_{RNS}$ especially from the standpoint of integrability. At each manipulator configuration ϕ the columns of the null space projector $\boldsymbol{P}_{RNS}(\phi)$ induce a smooth co-distribution [12] in joint space, which, for a strong inertially coupled system, is nonsingular. According to Frobenius' theorem, a distribution is completely integrable, if and only if it is involutive. Involutivity can be examined via Lie brackets on the columns of $\boldsymbol{P}_{RNS}$. If such involutivity can be established, then the reaction null space component of the joint velocity will be integrable.

Definition 3: The integral of Equation (9), if it exists, is called *the set of reactionless paths of a FSMS.*

The reactionless paths guarantee total decoupling between the base dynamics and the manipulator dynamics. We note that these paths differ from the minimum-disturbance paths of the Coupling Map [3]. The main advantage of the reactionless paths is that they can be generated on-line, without using a graphical tool. Unfortunately, integrability cannot be always guaranteed. The only case when integrability is guaranteed, is that of a one-dimensional distribution (*i.e.*, $n-m=1$). Nevertheless, in some important practical cases the system can be recast to fit into this category.

4 Existence of the Reaction Null-Space

A necessary condition for the existence of the reaction null-space is the availability of any of the following features:

- kinematic redundancy;
- dynamic redundancy;
- selective reaction null-space;
- singularity of the inertia coupling matrix.

We utilized *kinematic redundancy* when deriving the solution in the previous section. As an example we point out the Space Station RMS/SPDM system [13].

The concept of *dynamic redundancy* has been introduced by Arai et al [14]. We applied the concept to the general problem of moving base robotics and reaction management control [9] in assuming that special devices, called reaction compensators, are present. These devices are used just to control the reaction on the base, similarly to the usage of reaction wheels for satellite attitude control.

There are some applications when the stiffness of the elastic base along the generalized coordinates can be sufficiently characterized as high-stiffness and low-stiffness. Reactions along the high-stiffness directions do not disturb the base at all. In this case we introduce the *selective reaction null space.* Denote by $\boldsymbol{S}=\mathrm{diag}(s_1, s_2, ..., s_6)$ a selection matrix, where $s_i=1$ specifies a Cartesian-space low-stiffness direction, requiring zero base reaction, while $s_i=0$ otherwise. Then, we denote the selective reaction null space as $\aleph(\boldsymbol{S}\boldsymbol{H}_{bm})$. Obviously, $\dim\aleph(\boldsymbol{S}\boldsymbol{H}_{bm}) \geq \dim\aleph\,(\boldsymbol{H}_{bm})$. Generally, a reaction null space of higher dimension is desirable, since it yields more DOF when planning the reactionless motion.

Finally, the reaction null space will also exist when the inertia coupling matrix $\boldsymbol{H}_{bm}$ is singular. There will be, however, singular directions in which no coupling would be possible at all. Further analysis is needed, which goes beyond the scope of the present work.

5 Energy Exchange between the Base and the Manipulator

We have seen that the reaction null space projector guarantees total inertial decoupling between the base and the manipulator. A reasonable question to ask is: which is the most efficient inertial coupling manipulator motion? Efficient coupling is related to the transfer of base strain energy to manipulator kinetic energy or vice versa.

Proposition 3: (Efficient inertial coupling) The most efficient (in a least-squares sense) inertial coupling between the base and the manipulator is provided through the projector $\boldsymbol{H}_{bm}^T\boldsymbol{H}_{bm}$.

Proof: Follows from the orthogonality between $\boldsymbol{H}_{bm}^T\boldsymbol{H}_{bm}$ and the null space projector. □

The energy exchanged between the manipulator and the base at each instant of time is

$$dE = d\mathcal{L}^T\dot{\phi} = \mathcal{F}^T d\boldsymbol{x}_b. \tag{10}$$

Note that the general solution of Equation (4) is

$$\ddot{\phi} = \boldsymbol{H}_{bm}^{+}(\mathcal{F} - \dot{\boldsymbol{H}}_{bm}\dot{\phi}) + \boldsymbol{P}_{RNS}\boldsymbol{\zeta}. \tag{11}$$

The only component that contributes to the reaction (cf. also Equation (7)) is:

$$\ddot{\phi}_d = \boldsymbol{H}_{bm}^{+}\mathcal{F}. \tag{12}$$

Since the coupling momentum $\mathcal{L}$ is the integral of the reaction $\mathcal{F}$, we can write

$$d\dot{\phi}_d = \boldsymbol{H}_{bm}^{+}d\mathcal{L}. \tag{13}$$

The instantaneous change of joint velocity $d\dot{\phi}_d$ ensures the most efficient (in a least-squares sense) decrease/increase of coupling momentum, which is related via Equation (10) to the exchange of energy. Note that with strong inertial coupling $\boldsymbol{H}_{bm}^{+} = \boldsymbol{H}_{bm}^T(\boldsymbol{H}_{bm}\boldsymbol{H}_{bm}^T)^{-1}$, which clearly shows that $d\dot{\phi}_d$ comes from the orthogonal complement of the reaction null space.

6 Control of FSMS

In the sequel we shall consider base vibration control and end-effector motion control.

6.1 Base vibration control

The main concern here is deviation of the base from its equilibrium, which results in vibration. Such deviation may be caused by the manipulator motion itself, or by an external force which was applied to the base in some previous time.

Proposition 4: (Base vibration suppression control) Under the condition of strong inertial coupling, the control law

$$\boldsymbol{\tau} = \tilde{\boldsymbol{G}}_f \dot{\boldsymbol{x}}_b + \tilde{\boldsymbol{c}} + \boldsymbol{H}_m \boldsymbol{n} \tag{14}$$

where

$$\tilde{\boldsymbol{G}}_f = -\boldsymbol{H}_m \boldsymbol{H}_{bm}^{+} \boldsymbol{G}_f,$$
$$\tilde{\boldsymbol{c}} = \boldsymbol{c}_m - \boldsymbol{H}_m \boldsymbol{H}_{bm}^{+} \boldsymbol{c}_b,$$

$\boldsymbol{G}_f$ denoting a constant feedback gain matrix and $\boldsymbol{n} \in \Re^n$ is an arbitrary vector from the reaction null space, guarantees that system damping is achieved, resulting in base vibration suppression.

Proof: From the system dynamics equation (1) we eliminate the joint acceleration:

$$\boldsymbol{\tau} = \tilde{\boldsymbol{H}} \ddot{\boldsymbol{x}}_b + \tilde{\boldsymbol{D}} \dot{\boldsymbol{x}}_b + \tilde{\boldsymbol{K}} \boldsymbol{x}_b + \tilde{\boldsymbol{c}} + \boldsymbol{H}_m \boldsymbol{n} \tag{15}$$

where,

$$\tilde{\boldsymbol{H}} = \boldsymbol{H}_{bm}^T - \boldsymbol{H}_m \boldsymbol{H}_{bm}^{+} \boldsymbol{H}_b,$$
$$\tilde{\boldsymbol{D}} = -\boldsymbol{H}_m \boldsymbol{H}_{bm}^{+} \boldsymbol{D}, \ \tilde{\boldsymbol{K}} = -\boldsymbol{H}_m \boldsymbol{H}_{bm}^{+} \boldsymbol{K}$$

The closed-loop system is given by

$$\begin{aligned} & \tilde{\boldsymbol{H}} \ddot{\boldsymbol{x}}_b + \tilde{\boldsymbol{D}} \dot{\boldsymbol{x}}_b + \tilde{\boldsymbol{K}} \boldsymbol{x}_b + \tilde{\boldsymbol{c}} + \boldsymbol{H}_m \boldsymbol{n} \\ = \ & \tilde{\boldsymbol{G}}_f \dot{\boldsymbol{x}}_b + \tilde{\boldsymbol{c}} + \boldsymbol{H}_m \boldsymbol{n}. \end{aligned} \tag{16}$$

Under the assumption of strong inertial coupling, matrix $\tilde{\boldsymbol{H}}$ can be shown to be of full rank m [11]. Then, the closed-loop dynamics becomes

$$\ddot{\boldsymbol{x}}_b + \tilde{\boldsymbol{H}}^{+} (\tilde{\boldsymbol{D}} - \tilde{\boldsymbol{G}}_f) \dot{\boldsymbol{x}}_b + \tilde{\boldsymbol{H}}^{+} \tilde{\boldsymbol{K}} \boldsymbol{x}_b = \boldsymbol{0}.$$

With proper choice of the constant gain $\boldsymbol{G}_f$ system damping is achieved, and hence, base vibration will be suppressed. □

The vibration suppression is done effectively, since the control law is based on the pseudoinverse of the inertia coupling matrix, and the solution (in terms of joint acceleration) is orthogonal to the reaction null space. We note that the vibration suppression control law is generally of the same form as that proposed by Book and Lee for FSMS [2], and by Uchiyama and Konno for a flexible link manipulator[2] [16]. The main difference is the appearing of the term $\mathbf{H}_m \boldsymbol{n}$ in Equation (14). The role of this term will be highlighted next.

[2]In their case, the left pseudoinverse had to be used, however.

6.2 End-effector motion control

The control law (14) can be superimposed to a manipulator joint-space nonlinear control law [2], [16], provided the gains are selected with special care [2]. We note that the superposition of just a joint damping term would be appropriate. A low-gain joint stiffness term can be also included to obtain a desired final manipulator configuration. Without superposition, the vibration suppression will result in nonzero coupling momentum conservation, and hence, constant "floating" of the manipulator.

The term $\boldsymbol{H}_m \boldsymbol{n}$ in Equation (14) yields yet another possibility: that of controlling the end-effector motion. In order to determine the null space vector $\boldsymbol{n}$ we shall make use of the general solution (11). The arbitrary vector $\boldsymbol{\zeta}$ is determined by substituting the joint acceleration into the end-effector kinematics

$$\ddot{\boldsymbol{x}}_e = \boldsymbol{J} \ddot{\boldsymbol{\phi}} + \dot{\boldsymbol{J}} \dot{\boldsymbol{\phi}}, \tag{17}$$

where $\boldsymbol{x}_e \in \Re^p$ denotes task coordinates and $\boldsymbol{J}(\boldsymbol{\phi}) \in \Re^{p \times n}$ is the end-effector Jacobian. Note that the reference frame is at the base. After some formula manipulation, one obtains

$$\begin{aligned} \ddot{\boldsymbol{\phi}} \ = \ & \boldsymbol{H}_{bm}^{+} (\mathcal{F} - \dot{\boldsymbol{H}}_{bm} \dot{\boldsymbol{\phi}}) \\ & + \bar{\boldsymbol{J}}^{+} [\ddot{\boldsymbol{x}}_e - \dot{\boldsymbol{J}} \dot{\boldsymbol{\phi}} - \boldsymbol{J} \boldsymbol{H}_{bm}^{+} (\mathcal{F} - \dot{\boldsymbol{H}}_{bm} \dot{\boldsymbol{\phi}})] \end{aligned} \tag{18}$$

where $\bar{\boldsymbol{J}} \stackrel{def}{=} \boldsymbol{J} \boldsymbol{P}_{RNS}$ is the Jacobian, restricted by the reaction null space. It can be shown that $\bar{\boldsymbol{J}}^{+} \in \aleph(\boldsymbol{H}_{bm})$, and hence, the second term on the right hand side of the last equation is indeed a reaction null space vector.

Proposition 5: (Reactionless end-effector motion control) Let the control law be given by (14) with

$$\boldsymbol{n} = \bar{\boldsymbol{J}}^{+} [\ddot{\boldsymbol{x}}_e^d + \boldsymbol{G}_v \dot{\boldsymbol{e}}_e + \boldsymbol{G}_p \boldsymbol{E}_e - \dot{\boldsymbol{J}} \dot{\boldsymbol{\phi}} - \boldsymbol{J} \boldsymbol{H}_{bm}^{+} (\boldsymbol{G}_f \dot{\boldsymbol{x}}_b - \dot{\boldsymbol{H}}_{bm} \dot{\boldsymbol{\phi}})] \tag{19}$$

where $\boldsymbol{x}_e^d$ is the desired end-effector path, $\boldsymbol{e}_e = \boldsymbol{x}_e^d - \boldsymbol{x}_e$ is the path tracking error and $\boldsymbol{G}_v$ and $\boldsymbol{G}_p$ denote proper gain matrices. Under the condition of strong inertial coupling *and* full rankness of the restricted Jacobian $\bar{\boldsymbol{J}}$, as well as when $n \geq m + p$, the base deflection $\boldsymbol{x}_b \rightarrow 0$, and the end-effector error converges to zero asymptotically.

Proof: The base vibration suppression ability is not influenced from the additional control (19) since it is taken from the reaction null space. Combining Equations (2), (14), (15), (18) and (19), the closed loop system is written as

$$\begin{aligned} \boldsymbol{0} \ = \ & \tilde{\boldsymbol{H}} \ddot{\boldsymbol{x}}_b + (\tilde{\boldsymbol{D}} - \tilde{\boldsymbol{G}}_f) \dot{\boldsymbol{x}}_b + \tilde{\boldsymbol{K}} \boldsymbol{x}_b \\ & + \boldsymbol{H}_m \bar{\boldsymbol{J}}^{+} [\ddot{e}_e + \boldsymbol{G}_v \dot{e}_e + \boldsymbol{G}_p \boldsymbol{e}_e \\ & + \boldsymbol{J} \boldsymbol{H}_{bm}^{+} (\ddot{\boldsymbol{x}}_b + (\boldsymbol{D} - \boldsymbol{G}_f) \dot{\boldsymbol{x}}_b + \boldsymbol{K} \boldsymbol{x}_b)] \end{aligned} \tag{20}$$

which shows that also the end-effector error must go to zero, asymptotically. □

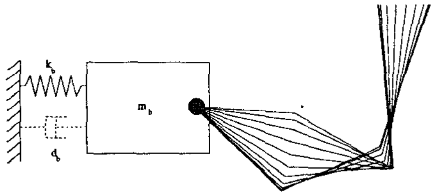

Figure 1: Model of an FSMS tracking a reactionless path.

It is seen that the control law (14) in combination with the reaction null space component (19) is capable of controlling reactionless end-effector motion and suppressing base disturbances in the same time, without imposing any restrictions on the type of manipulator, or without the need of graphical path planning tools.

7 An Example

We shall illustrate our approach with a planar 3R manipulator mounted on a base translating horizontally, which is attached to the inertial frame through a linear spring and a damper. Zero gravity environment is assumed. Figure 1 shows the system, tracking with its end-point a path without inducing any disturbances to the base. Since the reaction null space is 2-dimensional, it is possible to track any path in task space which complies with the condition of strong inertial coupling and non-singularity of matrix $\bar{\boldsymbol{J}}$. Because of the total decoupling property of the reaction null space, the selection of the feedback gains is not critical: for example, for the end-point control high gains are used ($\boldsymbol{G}_p$ =diag{200}, $\boldsymbol{G}_v$ =diag{100}). The gain for base vibration suppression control is G_f =10.

First, base vibration suppression is demonstrated. We assume that base vibration is excited due to some external force at $t = 1$ s (see Figure 2). The base vibrates, with decreasing amplitude because of the natural damping. At $t = 3$ s the vibration control is activated. We have included a joint damping term into the control law for vibration suppression, which guarantees that joint velocity decreases to zero. As already mentioned, if joint damping would be not present, the manipulator would be loaded with a nonzero coupling momentum which is conserved, and which would result in constant drift of the manipulator.

Next, end-effector control is demonstrated. There is no initial deflection of the base. The path is similar to that depicted in Figure 1, and was planned through a fifth order spline. Other planning can be also used; there is no requirement for zero boundary conditions. Figure 3 shows the results. The path is tracked perfectly, with zero base disturbance.

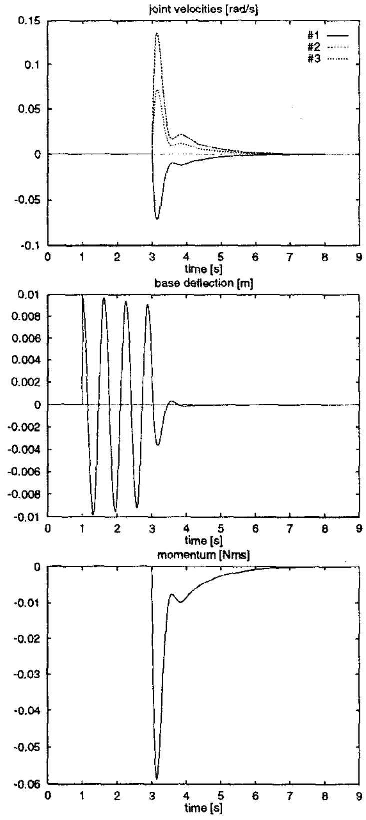

Figure 2: Base vibration control.

8 Discussion and Conclusion

The reaction null space control approach proposed here ensures total decoupling of the dynamics of two interacting mechanical systems. This approach is especially useful, but not only limited to so-called underactuated systems. We focused on an application to a flexible structure mounted manipulator system. The approach could be very useful for teleoperation of the manipulator, when the desired path is not known in advance, or no off-line path planning techniques such as non-holonomic path planning are applicable. Also, in case of emergency, the reactionless paths obtained through the reaction null space provide a fast escaping route, without exciting undesirable base vibration. We pointed out several possibilities for the existence of the reaction null space. In future it would be appropriate to discuss any combination of these possibilities. For systems with abundant DOF such as the SPDM, there is the possibility to use some of the DOF under kinematic redundancy, and some of them under dynamic redundancy. In this combination one could also include the selective reaction null space, provided the configuration of the elastic base (i.e. SSRMS) is such that high/low stiffness directions can be distinguished.

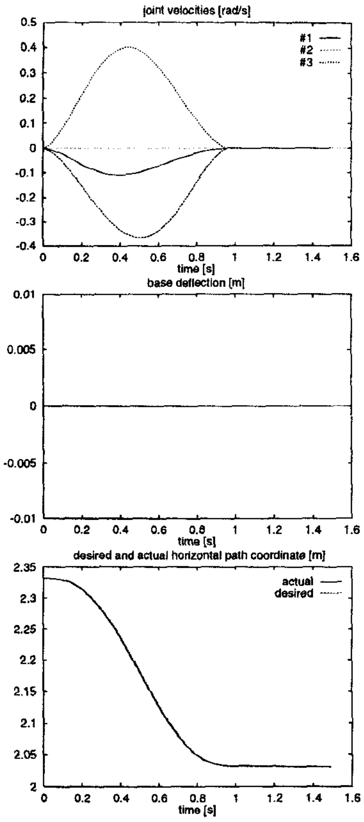

Figure 3: End-point path tracking.

The theory presented in this paper has been meanwhile experimentally validated [10].

Acknowledgments

This work was supported through the Ippan C Research Project 07805027 of the Grand-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan.

References

[1] A. Jain and G. Rodriguez, "An analysis of the kinematics and dynamics of underactuated manipulators," *IEEE Trans. Robotics and Automation*, Vol. 9, No. 4, pp. 411–422, 1993.

[2] W.J. Book and S.H. Lee, "Vibration control of a large flexible manipulator by a small robotic arm, " in *Proc. American Control Conference*, Pittsburgh, Pennsylvania, 1989, pp. 1377–1380.

[3] M.A. Torres and S. Dubowsky, "Path-planning in elastically constrained space manipulator systems," in *Proc. 1993 IEEE Int. Conf. on Robotics and Automation*, Atlanta, Georgia, May 2-6, 1993, pp. 812–817.

[4] M.A. Torres, "Modelling, path-planning and control of space manipulators: the coupling map concept," *Ph.D. Thesis*, MIT, 1993.

[5] M.L. Hanson and R.H. Tolson, "Reducing flexible base vibrations through local redundancy resolution," *J. of Robotic Syst.*, Vol. 12, No. 11, pp. 767–779, 1995.

[6] D.N. Nenchev, Y. Umetani, and K. Yoshida, "Analysis of a redundant free-flying spacecraft/manipulator system," *IEEE Trans. on Robotics and Automation*, Vol. 8, No. 1, pp. 1–6, Febr. 1992.

[7] D.N. Nenchev, "A controller for a redundant free flying space robot with spacecraft attitude/manipulator motion coordination," in *Proc. 1993 IEEE/RSJ Int. Conf. on Intelligent Robots and Syst. (IROS'93)*, Yokohama, Japan, July 1993, pp. 2108–2114.

[8] K. Yoshida and D.N. Nenchev, "Space robot impact analysis and satellite-base impulse minimization using reaction null space," in *Proc. 1995 IEEE Int. Conf. on Robotics and Automation*, Nagoya, Japan, May 21-27, 1995, pp. 1271–1277.

[9] K. Yoshida, D.N. Nenchev and M. Uchiyama, "Moving base robotics and reaction management control," *Robotics Research: The Seventh International Symposium*, Ed. by G. Giralt and G. Hirzinger, Springer Verlag, 1996, pp. 101–109.

[10] K. Yoshida *et al*, "Experiments on the PTP operations of a flexible structure mounted manipulator system," to be presented at *IEEE/RSJ Int. Conf. on Intelligent Robots and Syst. (IROS'96)*, Osaka, Japan, Nov. 4-8, 1996.

[11] M.W. Spong, "Partial feedback linearization of underactuated mechanical systems," in *Proc. 1994 IEEE/RSJ Int. Conf. on Intelligent Robots and Syst. (IROS'94)*, Munich, Germany, Sept. 1994, pp. 314–321.

[12] A. Isidori. Nonlinear Control Systems. Third Edition. *Springer Verlag*, 1995.

[13] S.B. Skaar and C.F. Ruoff (eds). Teleoperation and Robotics in Space. *AIAA Series*, Vol. 161, 1994.

[14] T. Arai et al., "Proposal of dynamic redundancy in robot control," in *Proc. IEEE/RSJ Int. Workshop on Intelligent Robots and Syst. (IROS'92)*, Raleigh, North Carolina, July 7-10, 1992, pp.1921–1926.

[15] D.N. Nenchev, K. Yoshida, and Y. Umetani, "Analysis, design and control of free-flying space robots using fixed-attitude-restricted Jacobian matrix," *Robotics Research: The Fifth International Symposium*, Ed. by H. Miura and S. Arimoto, The MIT Press, 1990, pp. 251–258.

[16] M. Uchiyama, and A. Konno, "Modeling, controllability and vibration suppression of 3D flexible robots," *Robotics Research: The Seventh International Symposium*, Ed. by G. Giralt and G. Hirzinger, Springer Verlag, 1996, pp. 90–99.