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A Game-theoretic Approach to Cost Allocation for Infrastructure Arrangement in An Urban Renewal Project

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The public sector and landowners, including private entities, need to cooperate in urban renewal projects to develop the infrastructure. It is important to allocate fairly the joint costs to each agent. In this paper we present a discussion on the rationality of the cost allocation of infrastructure arrangement among landowners and the public sector in the urban renewal project. A mathematical model based on cooperative game theory is formulated and then applied to an assumed three-player situation. We show that the model well explains an appropriate way of allocating costs, and also extra cooperative benefits.

KEYWORDS: cost allocation, cooperative game theory, infrastructure planning, urban renewal project

1. Introduction

A city continually changes under social and economic circumstances. New activities emerge which demand suitable sites within the city. On the other hand, some activities dwindle and make less use of their land. The bulk of such low-use land appears in the district where the "old industry" agglomerated but then started to decline. Such typical examples of declining districts may be commonly formed within the center of a city; i.e., idling areas due to the closing of a plant or relocation of a railway car shed to a suburban district. It may also be the case that such an area finds itself in the so-called "waterfront" or "bay" districts where factories, warehouses, loading and unloading facilities, stock yards, etc. have become too obsolete to meet industrial and technological changes.

We will focus on such problem areas and discuss the renewal projects with a view to resolving the problem. Specifically we will highlight the following types of urban renewal projects. The scale ranges from 50,000 [m^2] (5 [ha]) to 500,000 [m^2] (50 [ha]) in area. The structure of land ownership is relatively simple in the sense that lands are owned by a small number of landowners (mostly industries). Many facilities are planned for construction in the area. These facilities are classified into two kinds: (i) privately owned buildings available for office, retail, dwelling and other urban activities, and (ii) public facilities including roads, energy supply facilities, and open spaces. Privately owned buildings are constructed within private land. Sometimes public facilities are also located over the private lands, to be utilized by the private land uses. Accordingly the area demands to be reconfigured as a whole. Provision of public facilities and reconfiguration of land are collectively called infrastructure arrangement. Figure 1 shows the relation between architecture of privately owned buildings and infrastructure arrangement in an urban renewal project.

The urban renewal project is expected to contribute to the creation of a high-quality urban core. However, it is very difficult to complete the project successfully, especially due to the difficulty in coordinating the agents related to infrastructure arrangement. A critical question to be asked is, "what factors are involved in making this type of urban renewal project feasible?". To answer this question, the following assumptions are made:
1) The project can be executed by at least three types of agents. They are the landowners, a developer, and the public sector. Each of their concerns in the project varies remarkably.
2) The primary interest of a landowner is whether the project could produce benefits from making the best use of his land. If he is able to obtain some benefits, he may have intention to invest in the project corresponding to the expected benefit.
3) Commonly the landowners commission a developer to undertake the planning of the project. We can assume that the developer virtually works on their behalf.
4) The essential concern of the public sector is to improve public welfare of the area. The public sector necessarily invests in the infrastructure arrangement at the standard level of the region.
5) The public facilities are shared mostly by the users who work or live in the project area. That is to say, the users range very restrictedly in the case of renewal project area. For this reason, the private sector should become responsible for a portion of the investment in infrastructure arrangement. The landowners will be expected to support the land for siting.

6) As a result of 5), the private sector and the public sector may possibly conflict in payment of infrastructure arrangement costs, even if any of them has the intention to arrange some infrastructure in the area. There is also the possibility of conflict among the landowners, who may consider the necessity of infrastructure arrangement differently.
7) We note that there is basically no hierarchical relation among the landowners and the public sector. Each may work independently or cooperate; each can decide autonomously.

In order to make an urban renewal project feasible, the cost of infrastructure arrangement should be allocated fairly among them. To the authors' knowledge, there has yet to be an established procedure of cost allocation for the practice of urban renewal projects in Japan. Currently, landowners form an association (such as in the case of the Osaka Business Park Project, 1992) and they negotiate with the public sector over the cost allocation problem, as well as the scheduling of the project, and so on. The allocation of costs sometimes undergoes an unstable bargaining process.

With the above assumptions stated to set up our analytical framework, we present a discussion on the rationality of the cost allocation of infrastructure arrangement among the different types of landowners and the public sector in the urban renewal project. Infrastructure arrangement is often planned over a further extensive area. Thus it brings the regional benefits as well as the benefits for the problem area. We do not take the regional effects of the arrangement into consideration, in the situation that the landowners and the public body would allocate the shares of the cost for themselves.

In the next section, we show that application of cooperative game theory explains very well the structure of this problem. In the third section we treat a particular game situation of three-players: two landowners and the road administrator (the public sector). In the fourth section we consider the allocation of the extra benefit due to cooperation. The final section concludes this paper.

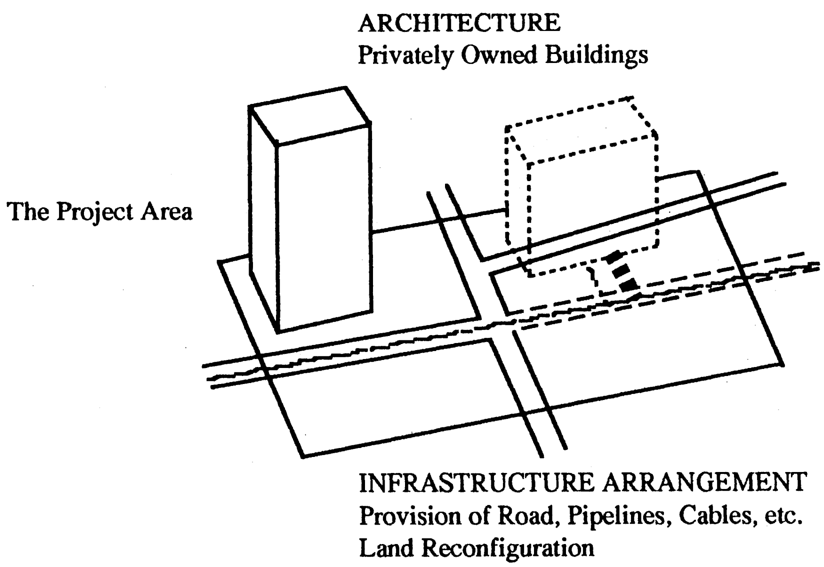

Fig. 1 Architecture and Infrastructure Arrangement in a Renewal Project.

2. Cost Allocation Problem in An Urban Renewal Project

2.1 The Possibility of Cooperation

We focus on the arrangement of the road as a part of infrastructure arrangement. A road handles the traffic of automobiles and pedestrians. It furthermore provides the room for utilities, sunlight and wind; it can also protect from the spread of fire. Since it has multiple functions as such, the road serves an important part of infrastructure. Therefore all public roads are commonly arranged by the road administrator (city government, national administrator, etc., as one agent). The road administrator invests in the arrangement of these roads for the unspecified users of the district.

Particularly in the project area, the landowners who supply the new buildings will expect roads for the private uses. A large amount of specific traffic flow will originate from the newly constructed buildings. As a result, the roads in the project area will serve both publicly and privately. Thus the investment should be shared by both the road administrator and the landowners. If a partnership is established among them, it would better aid in the arrangement of the road. Indeed such partnership often realizes the economies of scale and the economies of scope in the cost of construction and the scale of demanded land.

Nevertheless, an unfavorable situation may possibly occur when the intent to cooperate is deficient for these related agents. Consequently, the road administrator might even choose another alternative route and be less motivated by the project. Also the landowners might limit the traffic not related to their private lands. In short, any of the agents has the possibility of deviating from the partnership.

In so far as the conflict in payment is concerned, with the potential of deviation, each agent attempts to lessen his share of payment. The acceptability of the cost allocation may well depend on all the agents' durability against any deviation from the partnership. Cooperation can be attained by considering an appropriate way of cost allocation from such a standpoint.

Actually, the problem of cost allocation is one of the most common practices of examining such merits and demerits of cooperation by measuring them in cost terms. The rationale for such a practice is that the costs are relatively easy and straightforward to estimate and that the allocation of direct costs, such as those for constructing related facilities, becomes a crucial question for those who contemplate participation in the grand project.

Naturally, every agent would obtain some benefits corresponding to their investment in road arrangement. We assume any agent is guaranteed to acquire the benefits accrued to himself to the fullest extent, by the road arrangement whether the agent acts independently or cooperatively. Then it follows that the benefits of this type do not have to be controlled or redistributed if the payment of the cost is appropriately allocated.

2.2 Model Formulation

Given the framework as set up in the Introduction, cooperative game theory gives us a proper way of understanding the rationality of cooperation. Although a landowner and the road administrator have different interests in the project, both are regarded as one of the players of the game, with the point that both will attempt to lessen their share of payment. A player is concerned with the incentive of cooperation with the other players. A player deviates with other players, or by himself, from the group of all members when he doesn't have incentive to belong to the group of all members.

A cooperative group including the case of a single player is called a coalition. The group of all members is called the grand coalition. This set is denoted by the symbol N ($=\{1, 2, .., i, .., n\}$). The symbol S, is the set of members of a coalition which has deviated from the grand coalition N. The symbol $\{i\}$, is the set of a single player. We can estimate a coalition's potential of deviation by assuming the cost value which the coalition requires if it acts independently. Many assumed cost values exist corresponding to the possible forms of coalition. The cost values are called characteristic functions of the game. Let the characteristic function of cost, $C(\{i\})$, $C(S)$, and $C(N)$, denote the assumed cost values of coalition.

The cost value of the grand coalition $C(N)$, should be allocated amongst all the players. The quotas of the costs are derived from the solution of the game $Xc = (Xc1, Xc2, \ldots, Xcn)$. Many concepts of solution have been considered in cooperative game theory (for example, see Myerson 1991). In order for the allocation to be acceptable by all players, the solution should satisfy the following conditions:

The condition of grand rationality: $\sum_{N} Xci = C(N)$. (1)

The condition of individual rationality for each player: $Xci \leqq C(\{i\}) \quad (\forall i \in N)$. (2)

This equation describes the condition of the individual's incentive to belong to the grand coalition N. Furthermore, for the purpose of ensuring the dominance of the grand coalition from any other coalition, the solution should satisfy the following conditions:

The condition of group rationality: $\sum_{S} Xci \leqq C(S) \quad (\forall S \subset N)$. (3)

The "Core" is defined as follows: an allocation Xc, is in the Core if Xc is feasible and no coalition can improve on Xc with respect to equations (2) and (3). The Nucleolus (Schmeidler 1973) is a concept of fair allocation based on the Core. We firstly consider the "Excess" $\varepsilon(S)$, for the definition of the Nucleolus.

$$\varepsilon(S) = \sum_{S} Xci - C(S) \quad (i \in S \subset N, i \in N). \tag{4}$$

We obtain the Nucleolus so as to minimize the maximum value of the Excess. The principle of the Nucleolus as a type of solution of the game is to attend to the largest difference between or among the quotas and to lessen this difference to the fullest extent. The Nucleolus is sought by means of linear programming.

The Nucleolus can still be calculated regardless of the existence of the Core. Then, how should we interpret this result on the cost allocation in the renewal project? There are three ways of interpretation. One way is that they give way to the road arrangement by the grand coalition. Another way is that they should obey the Nucleolus regardless of the nonexistence of the Core, neglecting their own excess in the investment. The third way is that they should re-allocate the quotas to accommodate some of the deviating coalitions. This paper treats the situation all of the players are eager for the infrastructure arrangement. In other situation the public body may accomodate such deviating players unavoidably so as to attain a plan for a further extensive area. It is treated in

Hideshima, et al. (1995).

The game-theoretic approach is significant for the situation when the players are identified and the relevant information on the incentive of player to participate in the coalition is well expressed in the form of characteristic function rationally. It may be interpreted that the value of the alternate coalitions are the opportunity benefits of the agents (players) who are able to deviate from the grand coalition. In most cases, we have the difficulty in acceding plainly to the benefit, as the value, revealed *ex ante*, of the participation. The cost value can properly be substituted for this unidentified benefit value. In fact, the cost allocation game as formulated in equations (1) through (4), precisely fits in with this analytical framework. Okada, et al. (1982) have dealt with the cost allocation problem in water resource management by way of cooperative game theory. They have also shown that any cost allocation game is identical to its corresponding cost saving game (see Section 3.2). In other words, the cost allocation game reduces to the allocation of a particular type of "cooperative benefits", as explained later in Section 4.

In the urban renewal project the players including the private entities are those who wish to arrange the urban infrastructure cooperatively. We assume the private entity as well as the public agent acts rationally. Rationality means that one is to choose the most reasonable alternative among those available. The next section illustrates a three-player game although more players may exist in practical situations. There are the two reasons for dealing with only three players. One reason is that it essentially serves our purpose: analysis of the two types of relations between the agents; the relation between the private sector and the public sector, and the one between the two landowners (in the private sector) whose physical features of their own land differ. The other reason is that a three-player game is more tractable mathematically and is more instrumental in deriving formal policy implications fundamental enough to the practical situation than the use of several players.

3. Analysis of A Three-Player Game

3.1 Problem

In this section we present a cost allocation problem of three players in an assumed urban renewal project. The project (Case I, mentioned in the following analysis) is to be undertaken between point A and point B in Figure 2. It covers an area of 60,000 square meters occupied by two landowners. The area shapes rectangularly and its longer edge lies from east to west. The road administrator is the first player of the game. It has a policy to connect a road as short as possible between the two points A and B. One landowner is the second player and the other is the third player. Let the players be denoted P1, P2 and P3. The route of road is determined by agreement with the players in coalition. There are seven coalitions in a three-player game: {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, and {1, 2, 3}. We show the route of each coalition in Figures 2 to 8. Figure 2 shows the planned route by the grand coalition {1, 2, 3}.

Coalition {1, 2, 3} (Figure 2)

In the case of the grand coalition, the road administrator (P1) and the two landowners (P2, P3) agree on where to locate the road. The route lies in a straight line between A and B. It functions well both publicly and privately.

Coalition {1} (Figure 3), Coalition {2} (Figure 4), Coalition {3} (Figure 5)

As illustrated above, each player plans a road independently. In addition we assume that the player is unable to locate the road on the other players' land (Figure 3, 4 and 5). This forces the road to detour out of one or two

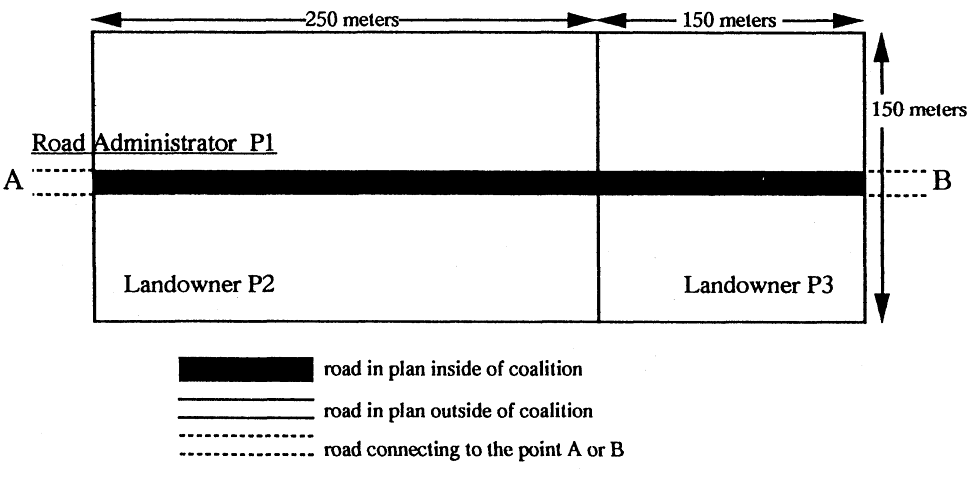

Fig. 2 Road Arrangement by All the Members {1,2,3} in Case I.

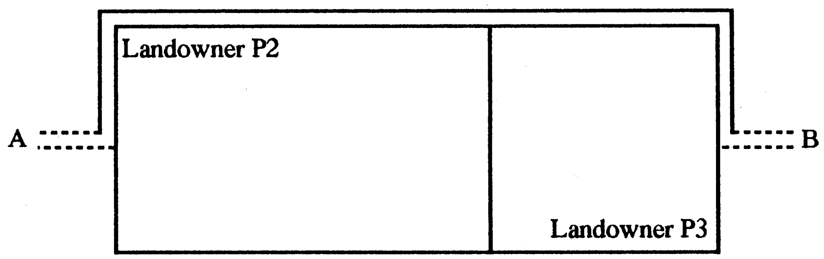

Fig. 3 Road Arrangement by {1} in Case I.

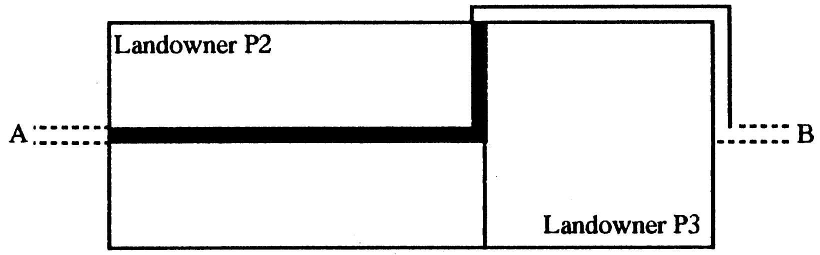

Fig. 4 Road Arrangement by {2} in Case I.

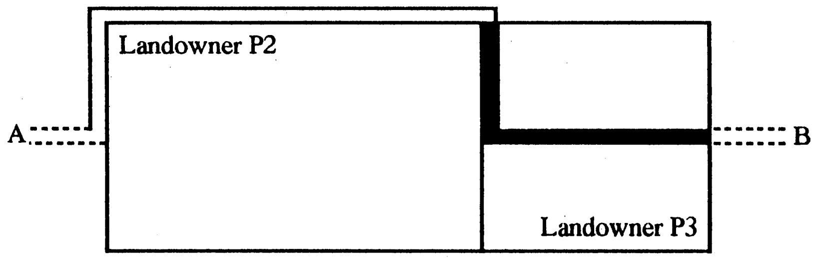

Fig. 5 Road Arrangement by {3} in Case I.

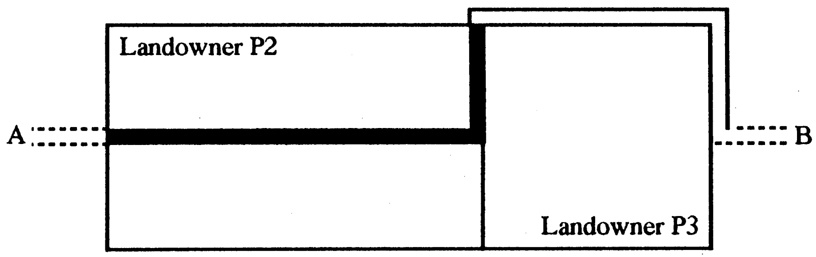

Fig. 6 Road Arrangement by {1,2} in Case I.

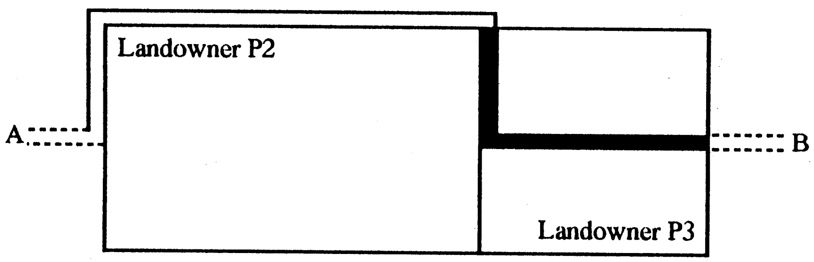

Fig. 7 Road Arrangement by {1,3} in Case I.

of the rejecting landowner's land. We assume that such a route touches the outside perimeter of the rejecting landowner's land. Of course every player must purchase the land for the road by himself which is on the outer perimeter of the project area. It is natural that the land price per unit inside the project area (r) should be lower than the price outside the project area (r_0), due to the relatively low quality of the road infrastructure at present

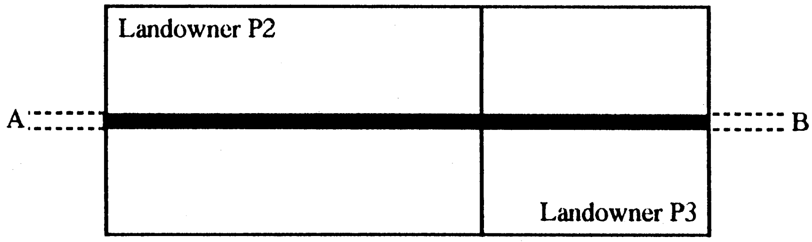

Figure 8. Road Arrangement by {2,3} in Case I

Fig. 8 Road Arrangement by {2,3} in Case I.

as compared to its outside; also, the landowners inside would not have enough incentive to cooperate in the project.

Coalition {1, 2} (Figure 6), Coalition {1, 3} (Figure 7)

In the above-illustrated cases, one landowner does not accept the proposal of the grand coalition. We assume that the route of such a coalition is forced to detour in the same way as stated above.

Coalition {2, 3} (Figure 8)

The two landowners arrange a private road inside their own land. They might prohibit entry of traffic not related to their area.

Cost functions of a coalition are identified with some assumptions in equations (5) to (8). Here the symbol S represents any set of coalition such as a single player {i} and the grand coalition N. The cost of coalition S denoted as C(S) [yen], is the sum of land cost $C_L(S)$, and construction cost $C_C(S)$.

$$C(S) = C_L(S) + C_C(S). \tag{5}$$

In regards to the cost of land $C_L(S)$ in equation (5), the following is noted. It is possible to divide up the land and then calculate its price by a qualified evaluator accordingly. The price per unit area differs inside and outside the project area. The parameter r is used inside the project area and the parameter r_0 is used outside the project area where, by assumption, $r_0 > r$ holds. Thus the cost of land acquisition by a coalition S, $C_L(S)$ [yen] is expressed as:

$$C_L(S) = rA(S)\text{inside} + r_0A(S)\text{outside}. \tag{6}$$

The unit prices of land which are used in this paper are: (inside) r = 300,000 [yen/m^2] and (outside) r_0 = 350,000 [yen/m^2].

A(S) [m^2] denotes the area of road for coalition S. The width is given as 20 meters throughout this paper and the length depends on the determined route. L(S) denotes the length of coalition S's route.

$$A(S) = 20L(S). \tag{7}$$

There is, however, a problem in the evaluation of the real estate. The specifications of land, for instance, the shape and the spatial relations of the lands, have not been paid attention to in this paper. It is very difficult to generalize the method of this evaluation. Most of this problem may well be left to other studies (for example, Asami 1993). Later, we will merely deal with a special feature of the effects of land specifications such as those which come out when the agents cooperate in the grand coalition.

Construction cost $C_C(S)$ [yen] is dependent upon the area of the road.

$$C_C(S) = gA(S). \tag{8}$$

The average construction cost per area g [yen/m^2], is calculated from the data of road construction reports (Osaka Business Park Project, 1992, etc.). We found that g = 500,000 [yen/m^2].

3.2 The Solutions

Although there are a variety of other solution concepts based on the core, we use the Nucleolus as the solution of the allocation problem in this analysis. This is because the Nucleolus is the most basic solution among them, and because it is not over analytically intent to compare the different core-based solution concepts.

The induction process of the Nucleolus is not analytically tractable in an n-player game. In a three-player game (N = {1, 2, 3}), the solution is introduced analytically in the following procedure. The cost game is replaced by the cost-saving game. In the cost-saving game the players allocate the amounts of saving of the grand coalition to themselves. Eventually, each player should pay his alternate cost minus the quota of the saving cost. Let V(N), V(S) and V({i}) be the characteristic functions of the cost-saving game for N, S, and {i} in equations (9) to (11) succeedingly.

$$V(N) = \sum_{N} C(\{i\}) - C(N) \tag{9}$$

$$V(S) = \sum_{S} C(\{i\}) - C(S) \quad (S \subset N) \tag{10}$$

$$V(\{i\}) = C(\{i\}) - C(\{i\}) = 0 \quad (i \in N). \tag{11}$$

We seek the solution Xv = (Xv1, Xv2, Xv3), which is shown by equation (12).

$$\sum_{N} Xvi = V(N). \tag{12}$$

A quota of cost Xci, is reproducible from a quota of saving Xvi, as follows:

$$Xci = C(\{i\}) - Xvi. \tag{13}$$

We note that Xvi is regarded as the allocation of the savings from the grand coalition; i.e., a special type of "cooperative benefits". Without loss of generality we may assume the following conditions to hold:

$$V(N) \geqq V(\{\mathit{1}, \mathit{2}\}) \geqq V(\{\mathit{1}, \mathit{3}\}) \geqq V(\{\mathit{2}, \mathit{3}\}). \tag{14}$$

We sometimes need to reorder the players according to condition (14) in the calculation. Italic numbers *1*, *2* and *3* below represent the numbers of reordered players.

The different forms of solution are derived according to the following five types i) to v) of the relationships among the cost-saving values.

i) $V(N) \geqq 3V(\{\mathit{1}, \mathit{2}\})$ (15)

$Xv\mathit{1} = Xv\mathit{2} = Xv\mathit{3} = V(N)/3$

ii) $V(N) \leqq 3V(\{\mathit{1}, \mathit{2}\})$, (16)

$[V(N) + V(\{\mathit{1}, \mathit{2}\})]/2 \geqq V(\{\mathit{1}, \mathit{3}\}) + V(\{\mathit{2}, \mathit{3}\})$ (17)

and $V(N) \geqq (\{\mathit{1}, \mathit{2}\})] + 2V(\{\mathit{1}, \mathit{3}\})$ (18)

$Xv\mathit{1} = V(N)/4 + V(\{\mathit{1}, \mathit{2}\})/4$

$Xv\mathit{2} = V(N)/4 + V(\{\mathit{1}, \mathit{2}\})/4$

$Xv\mathit{3} = V(N)/2 - V(\{\mathit{1}, \mathit{2}\})/2$

iii) $V(N) \leqq 3V(\{\mathit{1}, \mathit{2}\})$, (19)

$[V(N) + V(\{\mathit{1}, \mathit{2}\})]/2 \geqq V(\{\mathit{1}, \mathit{3}\}) + V(\{\mathit{2}, \mathit{3}\})$ (20)

and $V(\{\mathit{1}, \mathit{2}\}) + 2V(\{\mathit{1}, \mathit{3}\}) \geqq V(N) \geqq V(\{\mathit{1}, \mathit{2}\}) + 2V(\{\mathit{2}, \mathit{3}\})$ (21)

$Xv\mathit{1} = V(\{\mathit{1}, \mathit{2}\})/2 + V(\{\mathit{1}, \mathit{3}\})/2$

$Xv\mathit{2} = V(N)/2 - V(\{\mathit{1}, \mathit{3}\})/2$

$Xv\mathit{3} = V(N)/2 - V(\{\mathit{1}, \mathit{2}\})/2$

iv) $V(N) \leqq 3V(\{\mathit{1}, \mathit{2}\})$, (22)

$[V(N) + V(\{\mathit{1}, \mathit{2}\})/2 \geqq V(\{\mathit{1}, \mathit{3}\}) + V(\{\mathit{2}, \mathit{3}\})$ (23)

and $V(\{\mathit{1}, \mathit{2}\}) + 2V(\{\mathit{2}, \mathit{3}\}) \geqq V(N)$ (24)

$Xv\mathit{1} = V(N)/4 + V(\{\mathit{1}, \mathit{2}\})/4 + V(\{\mathit{1}, \mathit{3}\})/2 - V(\{\mathit{2}, \mathit{3}\})/2$

$Xv\mathit{2} = V(N)/4 + V(\{\mathit{1}, \mathit{2}\})/4 - V(\{\mathit{1}, \mathit{3}\})/2 + V(\{\mathit{2}, \mathit{3}\})/2$

$Xv\mathit{3} = V(N)/2 - V(\{\mathit{1}, \mathit{2}\})/2$

v) $V(N) \leqq 3V(\{\mathit{1}, \mathit{2}\})$, (25)

and $[V(N) + V(\{\mathit{1}, \mathit{2}\})/2 \leqq V(\{\mathit{1}, \mathit{3}\}) + V(\{\mathit{2}, \mathit{3}\})$ (26)

$Xv\mathit{1} = [V(N) + V(\{\mathit{1}, \mathit{2}\}) + V(\{\mathit{1}, \mathit{3}\}) + V(\{\mathit{2}, \mathit{3}\})/3 - V(\{\mathit{2}, \mathit{3}\})$ (27)

$Xv\mathit{2} = [V(N) + V(\{\mathit{1}, \mathit{2}\}) + V(\{\mathit{1}, \mathit{3}\}) + V(\{\mathit{2}, \mathit{3}\})/3 - V(\{\mathit{1}, \mathit{3}\})$ (28)

$Xv\mathit{3} = [V(N) + V(\{\mathit{1}, \mathit{2}\}) + V(\{\mathit{1}, \mathit{3}\}) + V(\{\mathit{2}, \mathit{3}\})/3 - V(\{\mathit{1}, \mathit{2}\})$. (29)

Here we attempt to understand the feature of the Nucleolus. The Nucleolus belongs to any type, i) to v). In type i), when the cost-saving value of the grand coalition V(N) is extremely large as compared to the values of the other coalitions $V(\{\mathit{1}, \mathit{2}\})$, $V(\{\mathit{1}, \mathit{3}\})$ and $V(\{\mathit{2}, \mathit{3}\})$, every player receives the equal quota of saving V(N)/3. As we move from type i) through type v), the cost-saving value of the grand coalition V(N) diminishes. Eventually we come to type v) where the solution becomes symmetrical and proportional to the advantage of each player. In type v), the first term on the right-hand side of equations (27) to (29) takes on the common values, whereas the second term takes on a negative value of the cost-saving amounts if the remaining party excluding the particular player forms a coalition by themselves. That is to say that particular player may claim a commonly allocated amount minus the potential benefit of the remaining party.

3.3 Analysis of Spatial Relation Patterns

Let us apply our allocation method to the different examples of renewal projects (Case I–Case III), which are illustrated in Figures 2, 9 and 10. Case I has been already explained in Section 4.2. The other two cases contrast in spatial relation patterns between the two landowners in the renewal project area. The characteristic functions and the cost allocation of each case are shown in Table 1, under the "standard condition" when the land price per unit inside the project area (r) is 300,000 [yen/m^2], the price per unit outside (r_0) is 350,000 [yen/m^2] and the average construction cost per area (g) is 500,000 [yen/m^2].

The solution of Case I and that of Case II are true of type v) (Case I requires reordering the players: $1 \rightarrow 3$, $2 \rightarrow 2$, $3 \rightarrow 1$ in the analytical solution). Inequalities (25) and (26) of type v) suggest that the cost-saving effect of the grand coalition is stronger, but to a limited extent, than those of the other coalitions. The reason for this

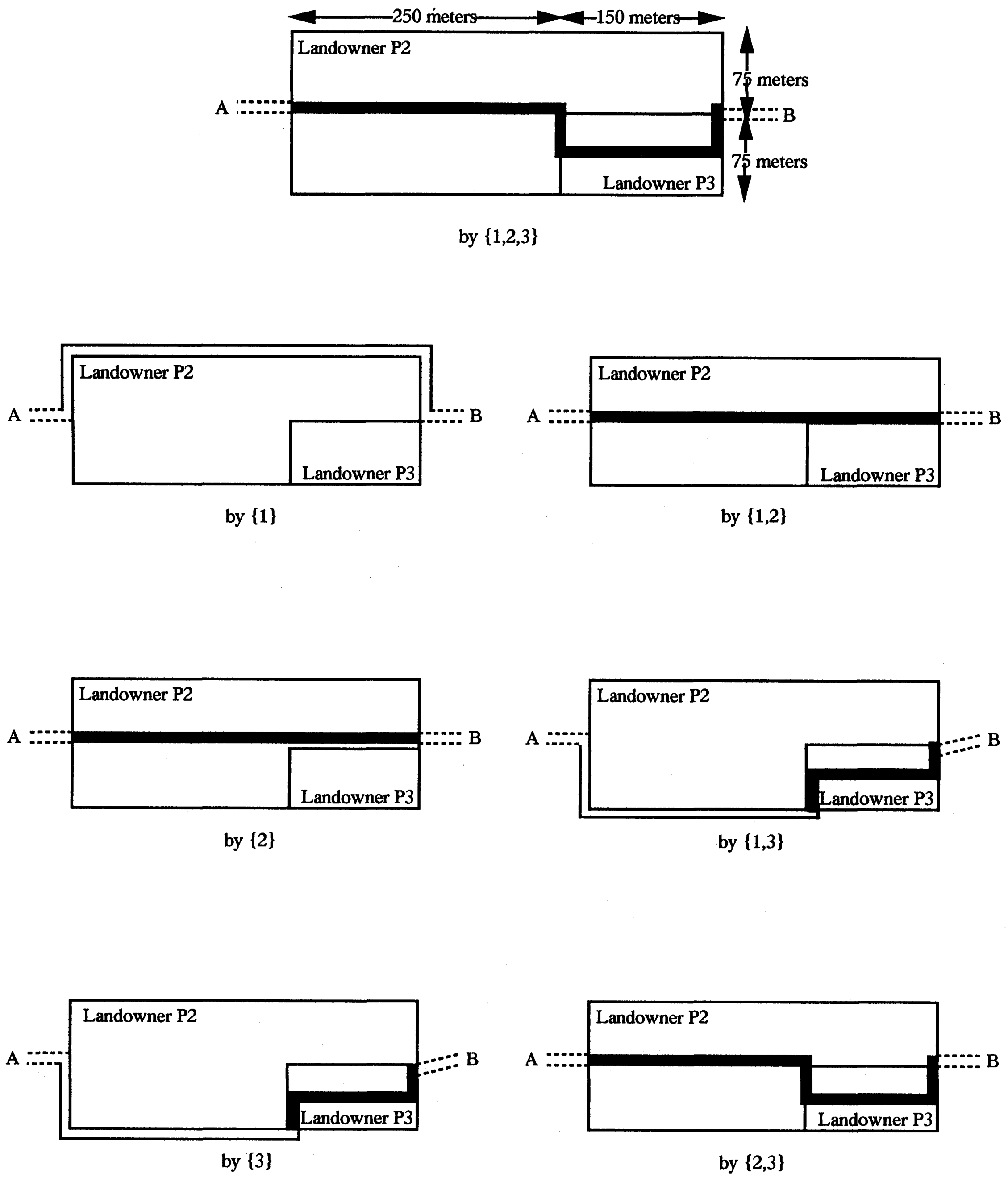

Fig. 9 Road Arrangement in CaseII.

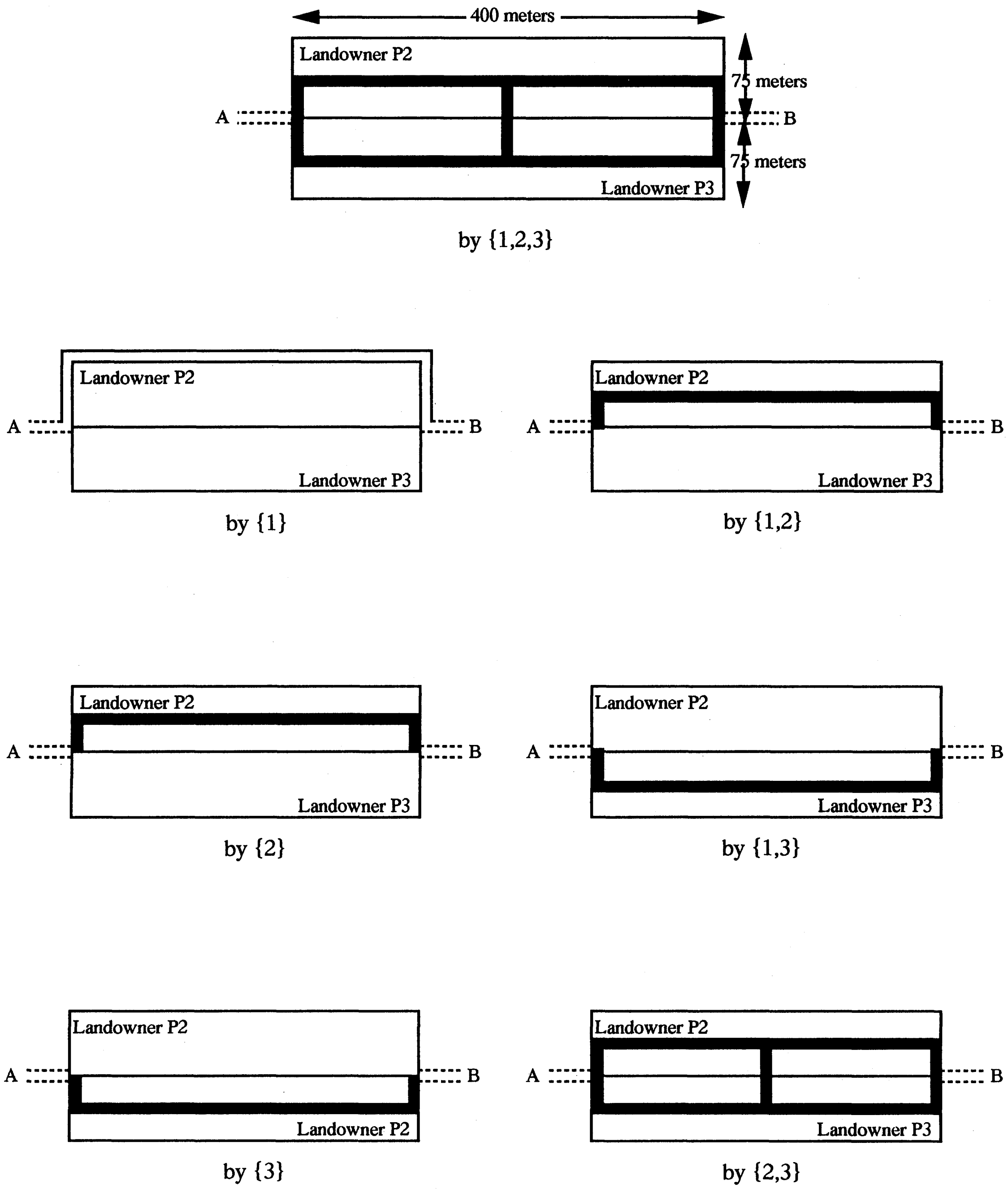

Fig. 10 Road Arrangement in CaseIII.

fact is discussed in the following parametric analysis (in Section 3.4).

From the viewpoint of the Core of the game, we observe the following fact. In the analytical solution, the Core diminishes as the number of solution type increases. Although the Core always exists in types i) and ii), it sometimes disappears in types iii) to v). Case I and Case II possess the Core, but Case III does not. This is because in Case III the grand coalition is irrational in cost efficiency, due to the fact that the two landowners would arrange the additional road connecting their lands if they cooperate (see Figure 10).

3.4 Parametric Analysis of the Relationships among the Cost-saving Values

The classification of the analytical solution in types i) to v) depends on the relationships among the characteristic functions of cost-saving value, V(N), V({*1*, *2*}), V({*1*, *3*}) and V({*2*, *3*}). We observe these relationships as we change the values of the land prices r and r_0, and the average construction cost per area g, as shown in conditions (A) to (D) in Table 2. The results of this parametric analysis suggest that no matter how much we change the parameters, the cost-saving values satisfy inequality (16), which is equivalent to condition (19), (22), and (25). Thus any solution may not belong to type i).

In Table 2 we find that V(N) is at most, approximately two times as large as the second largest cost-saving

Table 1. Cost Allocation—Project Cases I, II, III—.

	Case I	Case II	Case III
Size of Land [m^2]			
P1	0	0	0
P2	37,500	48,750	30,000
P3	22,500	11,250	30,000
$C_L(\{1, 2, 3\})$	24,000	28,500	61,500
$C_L(\{1\})$	38,500	38,500	38,500
$C_L(\{2\})$	35,250	24,000	28,500
$C_L(\{3\})$	36,250	36,250	28,500
$C_L(\{1, 2\})$	35,250	24,000	28,500
$C_L(\{1, 3\})$	36,250	36,250	28,500
$C_L(\{2, 3\})$	24,000	28,500	61,500
$C_C(\{1, 2, 3\})$	40,000	47,500	102,500
$C_C(\{1\})$	55,000	55,000	55,000
$C_C(\{2\})$	55,000	40,000	47,500
$C_C(\{3\})$	55,000	55,000	47,500
$C_C(\{1, 2\})$	55,000	40,000	47,500
$C_C(\{1, 3\})$	55,000	55,000	47,500
$C_C(\{2, 3\})$	40,000	47,500	102,500
Characteristic Functions [10^6 yen]; $C(S) = C_L(S) + C_C(S) = r$ and/or $r_0A(S) + gA(S)$			
$C(\{1, 2, 3\})$	64,000	76,000	164,000
$C(\{1\})$	93,500	93,500	93,500
$C(\{2\})$	90,250	64,000	76,000
$C(\{3\})$	91,250	91,250	76,000
$C(\{1, 2\})$	90,250	64,000	76,000
$C(\{1, 3\})$	91,250	91,250	76,000
$C(\{2, 3\})$	64,000	76,000	164,000
Characteristic Functions of Cost-saving Value [10^6 yen]; $V(S) = \Sigma C(\{i\}) - C(S)$			
$V(N)$	211,000	172,750	81,500
$V(\{1, 2\})$	93,500	93,500	93,500
$V(\{1, 3\})$	93,500	93,500	93,500
$V(\{2, 3\})$	117,500	79,250	−12.000
Nucleolus of Cost-saving Game [10^6 yen]			
Xv1	54,334	67,084	93,500
Xv2	78,333	52,833	−6,000
Xv3	78,333	52,833	−6.000
Ouotas of Cost [10^6 yen]; $Xci = C(\{i\}) - Xvi$			
Xc1	39,166	26,417	0
Xc2	11,917	11,167	82,000
Xc3	12,917	38,416	82,000

$r = 0.30$ [10^6 yen/m^2] $r_0 = 0.35$ [10^6 yen/m^2] $g=0.50$ [10^6 yen/m^2]

value $V(\{1, 2\})$. Case I in condition (B) has the largest value of $V(N)/V(\{1, 2 = 1, 3\})$. We shall pay particular attention to the composition of $V(N)$ and $V(\{1, 2\})$ of Case I in condition (B).

$$V(N) = C(\{1\}) + C(\{2\}) + C(\{3\}) - C(\{N\}) \tag{30}$$

$$V(\{1, 3\}) = C(\{1\}) + C(\{3\}) - C(\{1, 3\}) \tag{31}$$

$C(N)$ and $C(\{1, 3\})$ are not so different from $C(\{1\})$, $C(\{2\})$ and $C(\{3\})$ of Case I in condition (B). Therefore the value of $V(N)/V(\{1, 2 = 1, 3\})$ results:

$$\begin{aligned} V(N)/V(\{1, 3\}) &\fallingdotseq 2C(\{1\})/C(\{1\}) \\ &= 2. \end{aligned} \tag{32}$$

From equation (32) it can be said that our cost allocation method will satisfy condition (25) in type v), which is equivalent to conditions (16) in type ii), (19) in type iii) and (22) in type iv). It may be concluded that the solution of our allocation problem would be true of any type, ii) to v).

In this paper, we have assumed the situations where the road administrator does not have any land in the

Table 2. Parametric Analysis of the Cost-saving Values

		Case I	Case II	Case III
(A) (standard)	V(N) [yen]	211,000	172,750	81,500
$r = 0.30$ [10^6 yen/m^2]	V({1, 2}) [yen]	93,500	93,500*	93,500*
$r_0 = 0.35$ [10^6 yen/m^2]	V({1, 3}) [yen]	93,500	93,500*	93,500*
$g = 0.50$ [10^6 yen/m^2]	V({2,3}) [yen]	117,500*	79,250	−12,000
V(N)/V({*1*, *2*})		1.796	1.848	0.872
Solution Type		v)	v)	iii)
(B) (r_0 larger)	V(N)	90,400	72,400	42,800
$r = 0.30$	V({1, 2})	44,000	44,000*	44,000*
$r_0 = 3.50$	V({1, 3})	44,000	44,000*	44,000*
$g = 0.50$	V({2, 3})	46,400*	28,400	−1,200
V(N)/V({*1*, *2*})		1.948	1.645	0.973
Solution Type		v)	v)	iii)
(C) (g larger)	V(N)	133,600	109,525	50,900
$r = 0.30$	V({1, 2})	58,850	58,850*	58,850*
$r_0 = 0.35$	V({1, 3})	58,850	58,850*	58,850*
$g = 5.00$	V({2, 3})	74,750*	50,675	−7,950
V(N)/V({*1*, *2*})		1.787	1.861	0.865
Solution Type		v)	v)	iii)
(D) (r and r_0 larger)	V(N)	872,500	712,750	344,750
$r = 30.00$	V({1, 2})	390,500	390,500*	390,500*
$r_0 = 35.00$	V({1, 3}) 390,500	390,500*	390,500*	
$g = 0.50$	V({2, 3}) 482,000*	322,250	−45.750	
V(N)/V({*1*, *2*})		1.810	1.825	0.883
Solution Type	v) v)	iii)		

*The second largest cost-saving value, V({*1*, *2*}): *after reordering*

renewal project area. If the road administrator has some land in the project area, the cost function of {1: the road administrator by himself} C({1}) diminishes and the value V(N)/V({1, 3}) in equation (32) becomes less than 2. The stated conclusion still holds.

4. The Allocation of Cooperative Benefits

4.1 Cost Allocation Associated with Redistribution of Cooperative Benefits

In the former sections we discussed application of cooperative game theory to the cost allocation problem in the urban renewal project. As stated above, the game-theoretic approach is available to the situation that the characteristic functions are measured rationally. Thereby, we used the cost values of coalition as characteristic functions. It is assumed that a player has the incentive to participate in the larger coalition when his payment is saved due to scale and/or scope economies. That is to say, the incentive of a player to participate in infrastructure arrangement is estimated through the cost account.

As referred to in Section 2.2, the cost allocation game itself, reduced to a special type of the allocation of "cooperative benefits", i.e., the game of allocating the savings from the grand coalition. From the viewpoint of the benefit from the project, a player may possibly lose some other extra benefits, due to cooperation and he may become less motivated. This fact is exemplified by a situation such as the one shown in Figure 11. Two landowners, P1 and P2 are concerned with road arrangement. They attempt to locate a new road through the area. P1 occupies the left side of the area and P2 occupies the right side in Figure 11. P1's land is faced to the point which is to be connected to the area by road. If they cooperate, they arrange the road through both P1's land and P2's, as shown in Figure 11. If P1 arranges it independently, he will choose the shorter route within his land as shown in Figure 12. Although the cooperation saves each player's payment, the route planned by the cooperation may make the land-use efficiency of P1 worse by dividing it too small. This case shows that cooperation possibly brings some demerits (negative externalities) to a player.

In other cases, cooperation may bring some merits (positive externalities). An example is the land replotting, when the landowners cooperatively exchange some parts of their land to reconfigure the area. It can improve the utility of their lands totally.

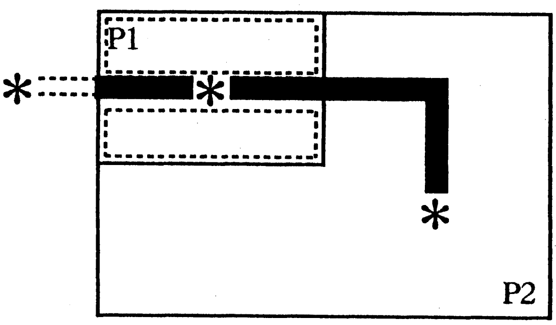

Fig. 11 Road Arrangement by {1,2}; P1 is less motivated.

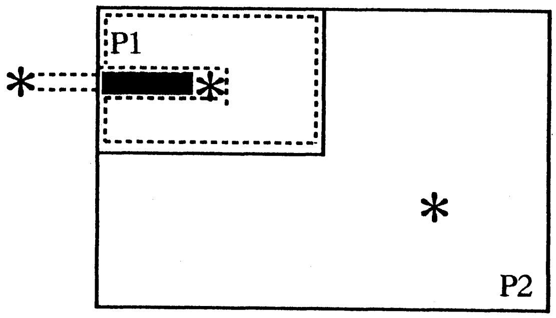

Fig. 12 Road Arrangement by {1}.

These examples leave the problem of how they should share proportionally such extra benefits (either positive or negative) which appears due to cooperation. They should consider these merits and demerits due to cooperation, and the redistribution of the extra benefits through the side payment. The ordinary cost allocation method mentioned in the former sections requires only assumed cost functions to discuss the possibility of cooperation. If the cooperative benefits conspicuously vary according to the formation of coalition, as compared to the cost, the difference of the extra cooperative benefits must not be neglected.

The cooperative benefits can be simply summed up into a value and then allocated to all the related agents. The cost saving effect (V(S)) is also included in the cooperative benefits. These matters are explained mathematically, with the cost saving value (a special type of cooperative benefit) V(N) and V(S), and the other extra cooperative benefit B(N) and B(S), in equations (33) and (34) succeedingly.

$$TB(N) = V(N) + B(N) \tag{33}$$

$$TB(S) = V(S) + B(S) \quad (S \subset N). \tag{34}$$

A player cannot obtain these cooperative benefits by himself.

$$TB(\{i\}) = V(\{i\}) + B(\{i\}) = 0 \quad (i \in N). \tag{35}$$

We call TB the total cooperative benefit. It can be allocated collectively. The merits of cooperation are evaluated as positive and the demerits are evaluated as negative. When the merits exceed the demerits, such collective allocation enables the conditions for individual rationality and for group rationality (cf. the condition (1) and (3) of cost allocation game) to be achieved in some cases.

We seek the solution $X_{TB} = (X_{TB}1, X_{TB}2, X_{TB}3)$ by cooperative game theory.

$$\sum_N X_{TB}i = TB(N). \tag{36}$$

The total cooperative benefit should be redistributed for the players in this way, otherwise some players would obtain more benefits unfairly. Then we attempt to associate the redistribution of the total cooperative benefit with the cost allocation problem. We suppose that each player should owe an appropriately allocated portion of the cost and the excess of the cooperative benefits. We define the adjusted cost quota of player i, X_Ci' as follows:

$$X_Ci' = C(\{i\}) + Bi(N) - X_{TB}i. \tag{37}$$

It would be confirmed that the amount of the cost quotas are equal to the cost of the grand coalition.

$$\begin{aligned} \sum_N X_Ci' &= \sum_N C(\{i\}) + B(N) - TB(N) \\ &= \sum_N C(\{i\}) + B(N) - B(N) - V(N) \\ &= C(N). \end{aligned} \tag{38}$$

4.2 Model Analysis

We can allocate the cooperative benefits to the related agents in the same way as the ordinary cost allocation. This problem is also dealt with by means of cooperative game theory. The remaining problem is how to evaluate the benefits, besides the cost saving benefit. One useful measure is by the land price estimation. For example, we assume that the more squared the land is, the higher the land price becomes. We adopt a very simple method of evaluating the benefit, considering the land price estimation, and apply it to the three cases (Case I, II, and III) of the three-player game situation mentioned in Section 3.

Each landowner loses a certain part of his own land, but the remaining land increases in price, due to road construction. For the sake of simplification of analysis, let us assume that the effect of road construction is measured in terms of the number of major points connected by the road. The major points around the area are remarked (*) in Figures 13 to 15. In consequence, the land price (per unit) is assumed to increase as they improve the road in the following way.

$$r(m) = q(m)r(0) \tag{39}$$

m; the number of the major points connected by the road
r = r(m); the land price per area [yen/m^2]
r(0); the land price before the project; no roads were located in the area. r(0) is 300,000 [yen/m^2] in this paper.
q = q(m); the multiplier concerning the effect of road arrangement as the function of m.

The function q(m) takes the form in equation (40).

$$q(m) = 1 + 0.5m. \tag{40}$$

The expected land price per area after the road arrangement by the coalition S (or N) is denoted by ri(S (or N)). Since the landowner loses his own land due to the road arrangement, the land loss of player i in the coalition S (or N) is denoted by ai(S (or N)). The cooperative benefit of player i in the coalition S, evaluated by the land estimation, Bi(S) is formulated in equation (41).

$$Bi(S) = ri(S)(Ai - ai(S)) - ri(\{i\})(Ai(\{i\}) - ai(\{i\})) \quad (i \in S). \tag{41}$$

The benefit of each player who belongs to the coalition S is summed up into B(S).

$$B(S) = \sum_S Bi(S). \tag{42}$$

We add the cost saving value to the cooperative extra benefit measured by the land estimation for each coalition. Let TB(N), TB(S) be the characteristic functions of the cooperative benefits game for N, S in equations (43) and (44) succeedingly.

$$TB(N) = B(N) + V(N) \tag{43}$$

$$TB(S) = B(S) + V(S) \quad (S \subset N). \tag{44}$$

Then we calculate the quotas of the total cooperative benefit for the three cases of the project, Case I, II, and III. We compare the cost quotas of ordinary cost allocation and the cost quotas after side payment of the total

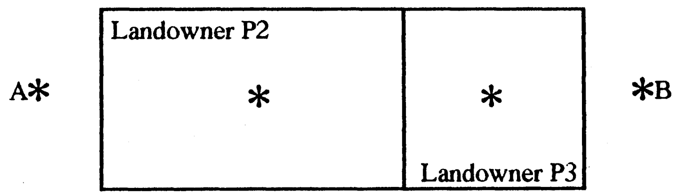

Fig. 13 Major Points in Case I.

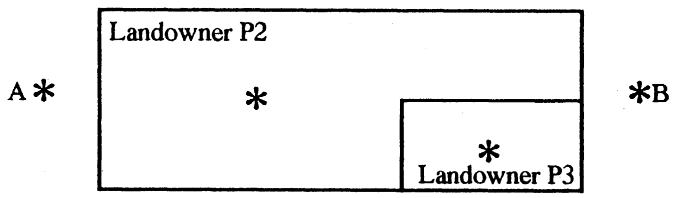

Fig. 14 Major Points in Case II.

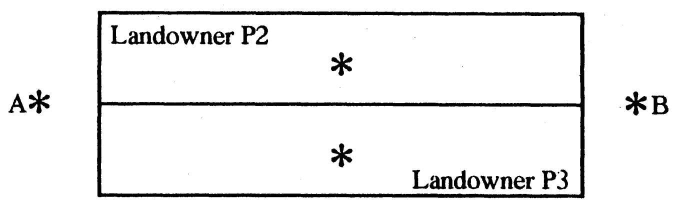

Fig. 15 Major Points in Case III.

cooperative benefit, both shown in Table 3. The results are as follows: in Case III, the cost allocation associated with the redistribution of the cooperative benefits achieves all the rationalities although the ordinary cost allocation is irrational in some of the group rationalities.

5. Conclusions

We have pursued a method of fair cost allocation for infrastructure arrangement in the urban renewal project. A game situation is apparent among the parties concerned with the infrastructure arrangement. We then applied cooperative game theory to this real game situation and formulated a mathematical model which appropriately describes the economic characteristics of the parties.

In this paper two landowners and one public agent are the players of the game. Hideshima, et al. (1993) treated a different situation, amongst three private landowners. However, that paper did not use the analytical solution of the cost-saving game which can reproduce the solution of the original cost game. By way of the analytical solution, we observed that particular patterns of solution are suitable for our problem. It can be concluded that commonly in the renewal project, each party may make claims comparatively proportional to his cost-saving advantages in the cost allocation for infrastructure arrangement. Furthermore we considered the cooperative merits. If cooperation drastically varies the benefit value of infrastructure arrangement, we should pay attention to this matter, in order to keep the grand coalition away from dissociating.

We pay no attention, in this paper, to the costs encountered after completion of the project, such as maintenance costs of infrastructure facility. We suppose the allocation of such costs are to be dealt with after establishment of the partnership in the project.

As newer types of infrastructure and newer styles of towns are developed, infrastructure planning becomes oriented towards a more complex situation. A widely general set of rules for the coordination of the planning process is required especially under such circumstances. We are convinced that the proposed game-theoretic approach should be applied to cover even such problems.

Table 3. Cooperative Benefit Allocation and Adjusted Cost Allocation

		Case I	Case II	Case III
Characteristic Functions [10^6 yen]				
B(N)		32,100	31,200	23,250
B({1, 2})		9,300	23,550	9,450
B({1, 3})		5,400	2,025	0
B({2, 3})		32,100	31,200	23,250
V(N)	(from Table 1)	211,000	172,750	81,500
V({1, 2})	(from Table 1)	93,500	93,500	93,500
V({1, 3})	(from Table 1)	93,500	93,500	93,500
V({2, 3})	(from Table 1)	117,500	79,250	−12.000**
Characteristic Functions of Total Cooperative Benefit [10^6 yen]; TB = B + V				
TB(N)		243,100	203,950	104,750
TB({1, 2})		102,800	117,050	102,950
TB({1, 3})		98,900	95,525	93,500
TB({2, 3})		149,600	110,450	11,250
Nucleolus of Total Cooperative Benefit Game [10^6 yen]				
$X_{TB}1$		48,534	65,208	92,900
$X_{TB}2$		99,233	80,134	10,650
$X_{TB}3$		95,333	58,608	1,200
Cost Ouotas by Ordinary Allocation [10^6 yen]; $X_{C}i' = C(\{i\}) - X_{V}i$				
$X_{C}1$	(from Table 1)	39,166	26,417	0
$X_{C}2$	(from Table 1)	11,917	11,167	82,000
$X_{C}3$	(from Table 1)	12,917	38,416	82,000
Cost Ouotas adjusted by Total Cooperative Benefit Allocation [10^6 yen]; $X_{C}i' = C(\{i\}) + Bi(N) - X_{TB}i$				
$X_{C}1'$		44,966	28,292	600
$X_{C}2'$		10,967	11,017	76,975
$X_{C}3'$		8,067	36,691	86,425

N.B.) **indicates rationality does not hold for this coalition ({2, 3}).

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