Three-Directional FEM Analyses of Pre-Bending Effects for Nb₃Sn Composite Wires

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Abstract—Pre-bending effects of Nb₃Sn composite wires, enhancement in J_c at the as-cooled condition and J_c peak, J_{cm} , against applied tensile strain, are well known in especially Cu-Nb reinforced Cu stabilized Nb₃Sn wire. In an attempt to understand the effects, three directional strain analyses using FEM were studied by considering thermally-induced residual strain due to temperature difference between the reaction temperature and the cryogenic temperatures including the pre-bending process at room temperature for three types of Nb₃Sn composite wires. To evaluate effects of change in three directional strain, the von Mises strain was introduced. As a result, the minimum von Mises strain shows J_{cm} enhancement by the pre-bending for Cu-Nb reinforced Nb₃Sn wire. It was found that there is a direct correlation between the minimum von Mises strain and J_{cm} related to the pre-bending.

Index Terms—Composite superconductor, FEM, Nb₃Sn, prebending, three-directional strain.

I. INTRODUCTION

 Tb_3Sn superconductor has been widely used for not only N scientific applications, for instance high energy physics, fusion research, and the hybrid-magnet, but also for industrial applications of NMR. Generally the Nb₃Sn composite wire consists of superconducting filaments, a Cu stabilizer, Cu-Sn (bronze) and a diffusion barrier. To prevent the reduction of the superconducting properties, several types of reinforced Nb₃Sn composite wires have been developed using reinforced materials such as Cu-Nb [1], Ta [2], or copper-alumina [3] for the cryocooled superconducting magnets. Therefore, some Nb₃Sn composite wires have the reinforcement material. These component materials have different thermal expansion coefficients and mechanical properties. As the Nb₃Sn is formed at the reaction temperature of around 1,000 K and cooled at the cryogenic temperature of 4.2 K, the Nb_3Sn filament is subjected to the thermal strain, the residual strain, by the other component materials.

 Nb_3Sn is sensitive to stress/strain and its superconducting properties change under the influences of the residual strain and of the applied stress such as the electromagnetic force during the

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operation [4]. Especially, deterioration of critical current density, J_c , is a serious problem for the practical use. On the other hand, J_c enhancement is known under the strain-free state in the case of the loaded and unloaded tensile strain condition [5] and bending and unbending strain condition [6].

Recently, it is found that Cu-Nb reinforced Nb₃Sn wire shows increase of not only J_c but also upper critical field and critical temperature through the repeated bending strain at room temperature, so called the pre-bending effect [7]. So far the J_c enhancement by the pre-bending is observed in characteristics of J_c against the applied tensile stress/strain, for instance, J_c at as-cooled-state and J_c peak, J_{cm} , at some applied tensile stress/strain which corresponds to the axial residual strain-free state. The tendency is more remarkable for the Cu-Nb reinforced Nb₃Sn wire as compared with the ordinary Cu stabilized Nb₃Sn wires [8]. The above former case can be explained by considering only the axial (z-) strain, but in the latter case J_{cm} enhancement cannot be understood only using z-strain approach.

As it is suggested that the radial and tangential strains of the Nb₃Sn filament are not always zero even at the axial tensile strain-free state, three dimensional strain states for Cu/Nb₃Sn have been observed using the neutron diffraction [8]. The study brought the relaxation of the axial residual strain and unchanging of the radial (r) and tangential (θ) strains, in-plane strain, by the pre-bending, which explained the experimental results of J_c against applied strain for Cu/Nb₃Sn.

Furthermore, the analyses of the strain state for the pre-bending are necessary for understanding dependence of each of r- and θ - strains induced by thermal hysteresis on the axial (z) tensile stress/strain for the various kinds of Nb₃Sn composite wires. We have studied three directional strain states, radial, tangential and longitudinal strains, for various composite superconducting wires using FEM [9], [10]. In this paper, we report FEM analyses of three directional strain state with/without pre-bending strain for Cu – Nb/Nb₃Sn and two types of Cu/Nb₃Sn wires.

II. MODEL OF THE COMPOSITE SUPERCONDUCTOR

We analyzed a Cu-Nb reinforced Nb₃Sn model (CuNb) [6] and two types of Cu/Nb_3Sn models; a Cu matrix type (Cu-1) [8] and an external Cu type (Cu-2), having 1 mm in diameter. Those component volume fractions and schematic view of the cross-section for the models are shown in Tables I and II and Fig. 1, respectively.

Analysed steps are shown as follows. All strains are set to be free at the reaction temperature of 948 K and then the model

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 TABLE I

 COMPONENT MATERIALS AND THEIR VOLUME FRACTIONS

model	CuNb/Nb ₃ Sn (CuNb)	Cu/Nb ₃ Sn (Cu-1, Cu-2)
Cu (%)	24.0	43.9
Cu-Nb (%)	32.0	-
Barrier (%)	4.9 (Ta)	5.2 (Nb)
Cu-Sn (%)	29.9	36.5
Nb3Sn (%)	9.2	14.4

TABLE II PARAMETERS OF COMPONENT MATERIALS

Component materials	Thermal expansion coefficient (K)	Young's modulus (GPa) at R.T.	Poisson's ratio at R.T.
Nb ₃ Sn	7.64 x 10 ⁻⁶	165	0.3
Cu-Sn	17.3 x 10 ⁻⁶	124	0.345
Cu-Nb	15.1 x 10 ⁻⁶	109	0.346
Та	6.3 x 10 ⁻⁶	186	0.342
Nb	7.02 x 10 ⁻⁶	103	0.397
Cu	16.8 x 10 ⁻⁶	114	0.345



Fig. 1. Cross section of the Nb_3Sn wire model (a) $Cu - Nb/Nb_3Sn$ (CuNb), (b) Cu matrix Nb_3Sn (Cu-1), (c) external-Cu Nb_3Sn (Cu-2).

conductor is cooled at the room temperature of 300 K. The pre-bending of 0.5% strain is applied to the model at 300 K. As shown in Fig. 2, points A and B along the x-axis on the outermost surface are indicated on the cross-section of the model. The 0.5% tensile strain and 0.5% compressive strain are simultaneously applied to points A and point B, respectively. Bending strain is distributed linearly from A to B across the cross-section and still zero-strain is kept at the center of the model. Next the applied strains are put back to free and the strains are applied to the contrary; 0.5% tensile strain at point B and 0.5% compressive strain at point A (one-directional bending). Finally, the applied bending strains are returned to free. The bending procedure is repeated twice per one direction. The similar applied bending procedure is repeated along the y-axis in the case of two-directional bending which demonstrates the pulley bending process. After these pre-bending processes the model conductor is cooled at 4.2 K and then the z-axis tensile stress/strain is applied.



Fig. 2. Schematic drawing of applied bending strain to the model. Points A and B are located at the outermost surface of the in-plane (x - y) in the wire. If the 0.5% tensile strain is applied to point A, 0.5% compressive strain is done to point B simultaneously. The strain distribution is linear from A to B.

III. ANALYSIS METHODS

In the boundary condition, it is assumed that the ends of the composite superconductor keep a plain (x - y) plane during the thermal process and applying of tensile stress: all component materials unite to change during elongation and contraction along the longitudinal direction and each component displaces equally on z-axis on upper x - y plane. The other boundary conditions are used in the FEM analyses as follows; zero displacement of degree of freedom along z-axis on bottom on x - y plane, the symmetry boundary condition.

Each component is assumed to be equiaxial material. Approximate stress-strain curves of Cu, Cu-Sn and Cu-Nb are calculated elastic-plastically, taking temperature dependence into account, based on the previous paper [10] and inputted to the FEM analyses. Superconductors (Nb_3Sn) and Ta are analyzed elastically for the whole temperature range because they have high proof stress and their stress-strain curves do not depend strongly on the temperature. Constant thermal expansion coefficient and Poisson's ratio values against temperature are used in the analysis because their data are not presented. Temperature ranges from 948 K of Nb_3Sn reaction temperature to 4.2 K were performed in the strain analyses.

IV. RESULTS AND DISCUSSION

A. Characteristics of z-Strain State Under Applied Tensile Strain

Dependences of z-strain for the Nb₃Sn filament on applied tensile strain are shown in Fig. 3 for model CuNb. In this figure zero applied tensile strain means as-cooled condition; z-strain at the as-cooled corresponds to the residual strain. In Cu – Nb/Nb₃Sn wire the residual compressive strains were -0.43% and -0.40% for no pre-bending and for 0.5% pre-bending, respectively. The residual strain changes J_c value; lower residual strain for higher J_c in the case of the pre-bending condition. This analyzed results accorded to J_c in the experimental results.

As the applied tensile strain increases, the residual compressive strain relaxes and reaches to the residual strain free. The applied tensile strains at the residual strain free are 0.40% and



Fig. 3. Characteristics of z-strain versus applied tensile strain of model CuNb for no pre-bending and pre-bending.



Fig. 4. Characteristics of r- and θ -strains versus z-strain of model CuNb for no pre-bending and pre-bending.

0.43% for pre-bending and no pre-bending, respectively; they correspond to the peak in $J_{\rm c}$ against tensile strain. They were in almost good accord with each other.

In Cu/Nb₃Sn models (Cu-1 and Cu-2), the same results, the residual compressive strains of -0.28% for no pre-bending, were obtained, which were smaller than those of CuNb/Nb₃Sn. The residual strain relaxation values by the pre-bending in both Cu/Nb₃Sn models were 0.01\%, which were smaller than 0.03% of Cu - Nb/Nb₃Sn. It is considered that the large volume fraction of Cu-alloys (Cu, Cu-Sn and/or Cu-Nb) in Cu - Nb/Nb₃Sn, which is larger than that in Cu/Nb₃Sn, lead to larger residual compressive strain. The difference of the residual strain relaxation between Cu - Nb/Nb₃Sn and Cu/Nb₃Sn will be discussed later.

B. Characteristics of r- and θ -Strains versus z-Strain

One of the analyzed results of in-plane (r- and θ -) strains for model CuNb is shown in Fig. 4 where the horizontal axis shows z-strain and the vertical axis shows r-strain and θ -strain. The θ -strains are larger than r-strains at around z-strain-free. The in-plane strains are not relived even at z-strain-free, i.e., some strains remain at z-strain-free condition in the wire. Although in



Fig. 5. Characteristics of von Mises strain versus applied tensile strain for no pre-bending and pre-bending (a) CuNb, (b) Cu-1, (c) Cu-2.

 θ -stain relaxation by the prebending was observed, the r-strain increase by the pre-bending occurred, as shown in Fig. 4.

In model Cu-1 the in-plane strains remain even at z-strainfree and the θ -strains are larger than r-strains, which are similar to model CuNb. On the other hand, the strains of Cu-1 are compressive and the strains increase by pre-bending.

Even taking some factors that affect $J_{\rm CII}$ in the in-plane strains; the absolute value of the strain, difference between both in-plane strains and the effective strain into consideration, these behaviors cannot be explained. Therefore we led the main factor which is primary associated with changes in $J_{\rm CII}$, that is the von Mises strain which allows combination of three-dimension strains.

C. Characteristics of von-Mises Strain versus z-Strain

The von Mises strain, $\varepsilon_{\rm Y}$, that is widely used for evaluating the yield condition of three dimension strains is shown in (1),

$$\varepsilon_Y^2 = \frac{\left\{ (\varepsilon_r - \varepsilon_\theta)^2 + (\varepsilon_\theta - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_r)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right) \right\}}{2}$$
(1)

where τ is the shear strain. Characteristics of the von Mises strain versus applied tensile strain are shown for models CuNb, Cu-1 and Cu-2 in Fig. 5. The minimum values of von Mises strain against the applied tensile strain approach zero but do not reach zero. The minimum values of von Mises strain existed at around 0.4–0.5% of the applied tensile strain for CuNb and around 0.3% for Cu-1 and Cu-2. In the CuNb model the minimum values of von Mises strain are apparently separated between no pre-bending and pre-bending conditions; 0.018% at 0.41% tensile strain for pre-bending and 0.04% at 0.48% tensile strain for no pre-bending. The minimum von Mises strain of the pre-bending condition is smaller than that of no prebending. Enhancement of $J_{\rm Cm}$ by the pre-bending was in reasonable agreement with the results of von Mises strain analyses in model CuNb.

On the other hand in Cu/Nb₃Sn models the minimum values of von Mises strain are 0.032% for Cu-1 and 0.027% for Cu-2 at 0.28% tensile strain, which are almost the same for Cu-1 and Cu-2 and furthermore almost the same strain values are obtained for both of no pre-bending and pre-bending. No change in von Mises strain has been suggested to indicate no change in $J_{\rm cm}$ for Cu/Nb₃Sn wires.

D. Mechanism of the Pre-Bending Effects

Before and after the pre-bending process the elongation of the whole wire at room temperature, 0.076%, 0.047% and 0.046% for models CuNb, Cu-1 and Cu-2, respectively, was estimated in the analyses. Larger elongation for Cu – Nb/Nb₃Sn wire is assumed to bring the von Mises strain relief and $J_{\rm cm}$ enhancement. The calculated stress-strain curves showed improvement of the mechanical properties by the pre-bending for Cu – Nb/Nb₃Sn; large work-hardening was obtained for Cu – Nb/Nb₃Sn by pre-bending. On the other hand, improvement of mechanical properties of the Cu/Nb₃Sn was small by the pre-bending. Although it is well known that mechanical improvement by pre-bending, which is shown locally in the wire, gives the Jc enhancement at zero stress [6], the further study will be needed to clarify the effects of it on $J_{\rm cm}$ enhancement.

V. CONCLUSIONS

The pre-bending characteristics of three directional strains against applied tensile strain for CuNb/Nb₃Sn and two kinds

of Cu/Nb₃Sn wires were analyzed using FEM. Obtained r- and θ -strains decreased with increase in applied z-strain and various behaviors depending on the pre-bending and on the kind of the composite wires were shown. The minimum von Mises strain against applied tensile strain was introduced for evaluating the pre-bending effects and showed well the differences among no pre-bending and pre-bending, and CuNb/Nb₃Sn and Cu/Nb₃Sn; the minimum von Mises strain is 0.018% for pre-bending and 0.04% for no pre-bending in Cu – Nb/Nb₃Sn. Furthermore, low minimum von Mises strain corresponded to high J_{cm} enhancement. The pre-bending effects can be accounted for by von Mises strain in Nb₃Sn filaments. On the other hand, there were no apparent differences by the pre-bending in two types of Cu/Nb₃Sn; Cu stabilizer arrangement did not affect the von Mises strain.

REFERENCES

- [1] S. Murase, T. Murakami, T. Seto, S. Shimamoto, S. Awaji, K. Watanabe, T. Saito, G. Iwaki, and S. Meguro, "Normal zone propagation and quench characteristics of Nb₃Sn wires with jelly-roll and *in-situ* processed CuNb reinforcements," *IEEE Trans. Appl. Supercond.*, vol. 11, pp. 3627–3630, Mar. 2001.
- [2] T. Miyazaki, Y. Murakami, T. Hase, T. Miyatake, S. Hayashi, Y. Kawate, N. Matsukura, T. Kiyoshi, K. Itoh, T. Takeuchi, K. Inoue, and H. Wada, "Development of superconductor for 1 GHz class NMR magnets—High yield strength (Nb, Ti)₃Sn conductors," *Cryogenic Eng.*, vol. 35, pp. 126–131, 2000.
- [3] S. Murase, S. Nakayama, T. Masegi, K. Koyanagi, S. Nomura, N. Shiga, N. Kobayashi, and K. Watanabe, "Stress-strain effects in alumina-Cu reinforced Nb₃Sn wires fabricated by the tube method," *J. Jpn. Institute Metals*, vol. 61, pp. 801–806, 1997.
- [4] J. W. Ekin, "Strain scaling law and the prediction of uniaxial and bending strain effects in multifilamentary superconductors," in *Filamentary A15 Superconductors*, M. Suenaga and A. F. Clark, Eds. New York: Plenum Press, 1980, pp. 187–203.
- [5] S. Ochiai and K. Osamura, "Influence of cyclic loading at room temperature on critical current at 4.2 K of Nb₃Sn superconducting composite wire," *Cryogenics*, vol. 32, p. 584, 1992.
- [6] S. Awaji, K. Watanabe, and K. Katagiri, "Improvement of mechanical and superconducting properties in CuNb/(Nb, Ti)₃Sn wires by applying strain at room temperature," *Supercond. Sci. Technol.*, vol. 16, pp. 733–738, 2003.
- [7] S. Awaji, H. Oguro, G. Nishijima, and K. Watanabe, "Enhancement of upper critical field and critical temperature by prebending process for practical Nb₃Sn wires," *Jpn. J. Appl. Phys.*, vol. 43, pp. L709–L711, 2004.
- [8] S. Awaji, H. Oguro, G. Nishijima, P. Badica, K. Watanabe, S. Harjo, T. Kamiyama, and K. Katagiri, "Neutron diffraction study on prebending effects for bronze route Nb₃Sn wires without reinforcement," *IEEE. Trans. Appl. Supercond.*, vol. 16, pp. 1228–1231, 2006.
- [9] S. Murase, H. Okamoto, T. Wakasa, T. Tsukii, and S. Shimamoto, "Three-directional analysis of thermally-induced strains for Nb₃Sn and oxide composite superconductors," *IEEE Trans. Appl. Supercond.*, vol. 13, pp. 3386–3389, 2003.
- [10] S. Murase and H. Okamoto, "FEM analysis of three directional strain states under applied tensile stress for various composite superconductors," *IEEE Trans. Appl. Supercond.*, vol. 14, pp. 1130–1132, 2004.