# Dielectric Constant and Permeability of Various Ferrites in the Microwave Region

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**Synopsis**

The dielectric constant $\varepsilon$ and permeability $\mu$ of ferrites have already been studied at audio and high fiequency region, but they have scarcely been measured in the microwave region.

We have determined $\varepsilon$ and $\mu$ on various simple polycrystalline specimens of magnesium, copper, cobalt, nickel and manganese ferrites quenched or slow-cooled from high temperatures at the wave-length of 6.6 cm by observing changes in $Q$-value and frequency shift when a thin disk formed specimen, whose diameter is 20 mm and thickness is 1 mm or 1.5 mm, attached on a supporter, was inserted in the rectangular cavity ($TE_{104}$ mode) at the positions of the electric and magnetic field maxima, respectively.

From these measurements, both real and imaginary part of $\varepsilon$ and $\mu$ were determined from the formula which were derived by expanding the field in the cavity by using orthogonal function introduced by Slater.

## I. Introduction

Till quite recently, not a few reports were published concerning the method to measure the dielectric constant in the microwave region and the loss factor $\tan\delta$ obtainable in treating gases, liquid and solid bodies.(1) Among them are found the plane wave method, the transmission line method, the resonant cavity method and so on. In the remarkable and representative method adopted by Surber(2) and others, the dielectric constant was obtained from the relation between the standing wave ratio and the change of the site of the voltage minimum. Jen(3) and other researchers used the resonant cavity and found the dielectric constant from the change of $Q$ and resonance frequency. As for the measuring method of permeability, though few papers have appeared, Bloembergen's work(4), for example, who measured the permeability of Ni by substituting a part of resonant cavity for Ni, is worth noting. But no one has ever tried to measure the dielectric constant $\varepsilon$ and the permeability $\mu$ with a single resonant cavity at a time. Of late the studies

(1) For example:
W. D. Hershberger, J. App. Phys. **17** (1946), 495.
W. R. Mclean, J. App. Phys. **17** (1946), 588.
S. Roberts and A. V. Hippel, J. App. Phys. **17** (1946), 610.
T. W. Dakin and C. N. Works, J. App. Phys. **18** (1947), 789.
ETC.

(2) W. H. Surber, Jr, J. App. Phys. **19** (1948), 514.

(3) C. K. Jen, J. App. Phys. **19** (1948), 649.
″ , A method for measuring the complex dielectric constant of gases at micro-wave frequencies by using a resonant cavity.

(4) N. Bloembergen, Phys. Rev. **78** (1950), 572.

of ferrite both theoretical and experimental has attracted general attention and extensive studies of ferrite as the materials for electric communication have been carried out extensively, so the property of ferrite for high frequency material is now known with certainty as well as its nature near several mega cycles. The present authors, too, have deeply been interested in the study of dielectric and magnetic properties of ferrite in the microwave region and tried to measure $\varepsilon$ and $\mu$, separating the real and the imaginary parts with a single resonant cavity at a time. First, supposing that some substance (which has $\varepsilon$ and $\mu$) had been put in the resonant cavity, the relation between $\varepsilon$ or $\mu$ and the change of $Q$ and the relation between $\varepsilon$ or $\mu$ and the shift of resonance frequency could be deduced analogically from the theoretical results of Slater[5]. Next, with a resonant cavity of transmission type, the measurement was carried out at normal temperature on various ferrite specimens (e. g. Mg. Cu. Co. Ni, Mn &. c) of discoid form which were about 1 mm and 1.5 mm in thickness. The method applied here was so called "resonant cavity method" and the wave-length was 6.6 cm. But, strictly speaking, the method we adopted was different from these ordinary resonant cavity methods. In our experiments, $\varepsilon$ and $\mu$ were to be measured at a time. For this purpose, the position of the maximum of electric and magnetic fields in the resonant cavity was used and the real parts of complex $\varepsilon$ and $\mu$ were obtained from the change of resonance frequency and the imaginary ones from the value of $Q$ of the cavity. Both theoretically and experimentally, the specimen must be thin and so we used those having a thickness of about 1 mm. The result of measurement shows the order $\varepsilon'\sim 10$ and $\mu'\sim 1.5$ in each case.

## II. Theory

Applying analogically the method of calculation which Slater[5] introduced, first we shall consider how the change in $Q$ of resonant cavity, in which some material has been provided, and the change of resonance frequency related to the dielectric constant $\varepsilon$ and to the permiability $\mu$. Next, with a cavity of transmission type, a discussion will be given on the transmissioned energy spectrum (i. e. one type of $Q$-curve) in order that we may prove that the separation of real and imaginary parts of $\varepsilon$ and $\mu$ can really be done.

1) Derivation of the formula for finding $\varepsilon$ and $\mu$ separately.

Under the condition that comparatively small material was provided in the resonant cavity, the change of resonance condition was calculated.

Generally, the equations of electro-magnetic field are given by

$$\left.\begin{aligned} \text{Curl } E + \frac{\partial B}{\partial t} &= 0 \\ \text{Curl } H - \frac{\partial D}{\partial t} &= J. \end{aligned}\right\} \tag{1}$$

When the electro-magnetic field in the cavity is developed with orthogonal function,

(5) J. C. Slater, Rev. Mod. Phys. 18 (1946), 441.

the following equations are obtained according to Slater's notation: (Slater's equations III 34 and 35 of reference (5))

$$\left.\begin{aligned} k_a\int E\cdot E_a dv + \mu_0\frac{d}{dt}\int H\cdot H_a dv &= -\int_S (n\times H)H_a da \\ k_a\int H\cdot H_a dv - \varepsilon_0\frac{d}{dt}\int E\cdot E_a dv &= \int JE_a dv - \int_{S'}(n\times H)\,da . \end{aligned}\right\} \quad (2)$$

But these equations can be used only when $\varepsilon$ and $\mu$ are the same as those in the vacuum state, that is, $\varepsilon=\varepsilon_0$, $\mu=\mu_0$.

Now suppose that some part of the cavity would be filled with a certain material having different $\varepsilon$ and $\mu$, that this material would give pretty little influence on the whole cavity and that the same orthogonal function as in the case of vacuum would be applicable, and the Maxwell equations are

$$\left.\begin{aligned} k_a\int E\cdot E_a dv + \mu_0\frac{d}{dt}\int_{V'} H\cdot H_a dv + \mu\frac{d}{dt}\int_{V''} H\cdot H_a dv \\ = -\int_S (n\times E)H_a da \\ k_a\int H\cdot H_a dv - \varepsilon_0\frac{d}{dt}\int_{V'} E\cdot E_a dv - \varepsilon\frac{d}{dt}\int_{V''} E\cdot E_a dv \\ = \int JE_a dv - \int_{S'}(n\times H)E_a da . \end{aligned}\right\} \quad (3)$$

Here, by the sign $V$ the whole volume of the cavity is denoted. $V''$ is the volume of the specimen and $V'$ is that of the cavity containing no specimen. As generally understood, we introduced $P_E=(\varepsilon-\varepsilon_0)E$ and $P_M=(\mu-\mu_0)H$. The signs $P_E$ and $P_M$ respectively denotes electric and magnetic polarization. So, from eq. (3), the following equations are obtained:

$$\left.\begin{aligned} k_a\int E\cdot E_a dv + \mu_0\frac{d}{dt}\int H\cdot H_a dv &= -\frac{d}{dt}\int_{V''} P_M H_a dv - \int_S (n\times E)H_a da \\ k_a\int H\cdot H_a dv - \varepsilon_0\frac{d}{dt}\int E\cdot E_a dv &= \frac{d}{at}\int_{V''} P_E E_a dv - \int_{S'}(n\times H)E_a da + \int_{V''} JE_a dv . \end{aligned}\right\} \quad (4)$$

This makes the fundamental equation for the following discussion. The first term in the right hand side is the newly added one to the Slater's equation. Here, $J$ in the space other than that of the specimen itself was considered to be zero. Now, separating $E$ and $H$ from eq. (4),

$$\begin{aligned} \varepsilon_0\mu_0\frac{d^2}{dt^2}\int E\cdot E_a dv + k_a^2\int E\cdot E_a dv = -\mu_0\frac{d}{dt}\left\{\int_{V''} JE_a dv - \int_{S'}(n\times H)E_a dv\right\} \\ - k_a\int_S (n\times E)H_a da - k_a\frac{d}{dt}\int_{V''} P_M H_a dv - \mu_0\frac{d^2}{dt^2}\int_{V''} P_E E_a dv . \end{aligned} \quad (5)$$

Then we applied here the same procedure which had been adopted by Slater when he introduced his equation III, 55, considering the terms in the right hand side to be perturbation terms. Thus the following expression was obtained:

$$\frac{1}{Q}-2j\frac{\Delta\omega_a}{\omega_a}=\frac{1}{\varepsilon_0\omega_a}\left\{\frac{\int_{V''}JE_a dv}{\int E\cdot E_a dv}-\frac{\int_{S'}(n\times H)E_a da}{\int E\cdot E_a dv}\right\}-\frac{j}{\omega_a\sqrt{\varepsilon_0\mu_0}}\frac{\int_S(n\times E)H_a da}{\int E\cdot E_a dv}$$
$$+\frac{1}{\sqrt{\varepsilon_0\mu_0}}\frac{\int_{V''}P_M H_a dv}{\int E\cdot E_a dv}+\frac{j\omega}{\omega_a\varepsilon_0}\frac{\int_{V''}P_E E_a dv}{\int E\cdot E_a dv}, \qquad (6)$$

where the surface integral over $S$ was cavity wall loss and therefore Slater's equation III, 59 can be used. As for $S'$, we intended to apply characteristic impedance matrix obtained by Tomonaga and Miyajima and so it is left out of consideration.

Next, the relation $J=\sigma E$ and Slater's equation III 46 were applied to the above expression.

$$\frac{1}{Q}-2j\frac{\Delta\omega_a}{\omega_a}=(1+j)\int_S\frac{1}{2}\delta H_a^2 da+\frac{\sigma}{\varepsilon_0\omega_a}\int_{V''}E_a^2 dv+j\frac{\mu-\mu_0}{\mu_0}\int_{V''}H_a^2 dv$$
$$+j\frac{\varepsilon-\varepsilon_0}{\varepsilon_0}\int_{V''}E_a^2 dv\,. \qquad (7)$$

The first term in the right is cavity wall loss and it will be denoted $1/Q_0$ hereafter. Then the other terms can be evaluated, as $E_a$ and $H_a$ are normalized.

2) Measurement of $\varepsilon$ and $\mu$.

As a practical method of experiment, we used the resonant cavity of transmission type (Fig. 2). As the method of matching wave guide and cavity, the method of characteristic impedance matrix is available.

According to Tomonaga-Miyajima calculation, the matrix elements of microwave transmission is

$$C_{MN}=\left\{\frac{\dfrac{\omega_0}{\sqrt{Q_M Q_N}}}{\omega-\omega_0-\dfrac{j\omega_0}{2}\left(\dfrac{1}{Q_0}+\dfrac{1}{Q_1}+\cdots\cdots\right)}-j\delta_{MN}\right\}e^{j(\theta_M+\theta_N)}, \qquad (8)$$

where $\omega_0$ is proper frequency, $1/Q_0$ loss factor of cavity and $Q_N$ the coupling $Q$ of $N$th window of this cavity.

In our case, as shown in Fig. 2, $|C_{12}|^2$ is needed, that is,

$$|C_{12}|^2=\frac{\dfrac{1}{Q_1Q_2}}{\left\{\left(\dfrac{\omega}{\omega_0}-1\right)^2+\dfrac{1}{4Q^2}\right\}}, \quad \frac{1}{Q}=\frac{1}{Q_0}+\frac{1}{Q_1}+\frac{1}{Q_2}\,. \qquad (9)$$

When measurement is carried out practically, oscillation frequency $\omega$ must be fixed and the detector should be connected to the galvanometer. The volume of the cavity, in which no specimen is provided, is changed by moving its piston and then the resonance curve can be obtained. (Fig. 1, left) Next, in the same manner, but this time a specimen is put in the cavity, another curve is obtained (Fig. 1. right). From these curves, the difference between two maximum points, namely, the difference of resonance frequency, was measured. We also obtained here $f_i$ and

$f$ half line width of each curve. With these results, $Q$ and $\Delta\omega/\omega$ can be known easily in the following procedure. In our experiment, the change of micrometer means the change of $\omega_0$ in Eq. (9). Therefore, when $|C_{12}|^2$ at the maximum is substituted for A,

$$\frac{\frac{1}{Q_1Q_2}}{\frac{1}{4Q^2}}=A\,,\quad \frac{\frac{1}{Q_1Q_2}}{\frac{\delta\omega^2}{\omega_0^2}+\frac{1}{4Q_0^2}}=\frac{A}{2}\,.$$

On the other hand, half line width is $2\delta\omega$, so denoting it by $f$,

$$f=\frac{\omega_0}{Q}\,.$$

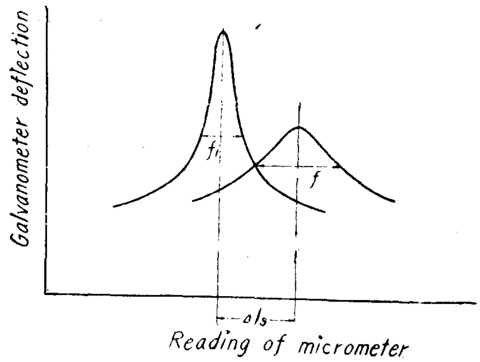

Fig. 1.

Accordingly, if we denote $Q$ of the specimen as $Q'$, the following expression can be obtained:

$$\frac{1}{\omega_0}(f-f_i)=\frac{1}{Q'}\,. \tag{10}$$

The term of shift $\Delta\omega/\omega$ is calculated from $\Delta l_S$ (Fig. 1), and so applying this, the part of $H$ (1) or that of $E$ (2) of the specimen, $\varepsilon$ and $\mu$ can be determined.

## III. Method of experiment

In the preceeding chapter, the relation between dielectric constant $\varepsilon$ of a certain substance in the resonant cavity and the shift of resonance frequency was obtained in relation to the change of $Q$ of the resonant cavity. Under the same condition, the relation between permeability $\mu$ of the substance and the shift of frequency was also obtained. As already described, if the change of $Q$ and the shift of resonance frequency are measured, $\varepsilon$ and $\mu$ could be calculated. We shall give some more explanations on the method of experiments below.

To measure the shift of resonance frequency, a piston was set up on the cavity as shown in Fig. 2. The frequency was then recorded by changing the volume of the resonant cavity and the shift was read in the micrometer attached to the piston.

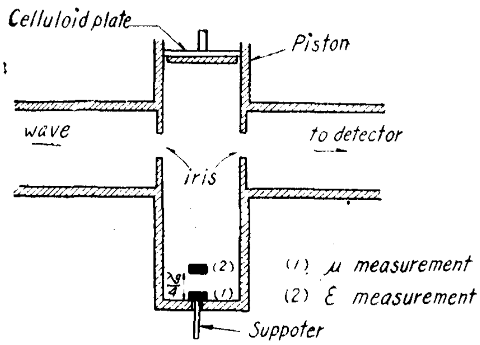

Fig. 2. Rectangular resonant cavity; $TE_{104}$-mode.

Various methods can be adopted for obtaining $Q$ of the resonant cavity. If the cavity is that of transmission type, as in the case of measuring the shift of resonance frequency, $Q$ can be measured from $Q$-curve and its half width, by changing the volume of the resonant cavity with the piston. In this case, an oscillation frequency of the oscillator must be fixed. When such a cavity is not used, $Q$ is obtained from the standing wave ratio. In the third method, $Q$ is calculated from the resonance curve which is drawn from the change of the oscillation frequency of the oscillator. Here the volume of the resonant cavity is always fixed.

For convenience sake, we tried first so called transmission method with a Braun tube. The oscillator was modulated with the saw-tooth wave and, at the same time, we made the saw-tooth wave sweep over the transverse axis of the Braun tube, then rectified with a crystal rectifier the wave which had been transmitted through the resonant cavity and obtained the output power. On the other hand, the wave coming through the oscillator was separated in $T$ section, and for the purpose of graduating the pips of frequency, we transmitted the separated wave through a cylindrical form wavemeter and rectified the output power again with a crystal rectifier. These output powers thus obtained were mixed and led through an amplifier and the amplified output power was brought in along the longitudinal axis of the Braun tube and then half width was measured from the resonance curve on the tube. Indeed we can obtain $Q$ in the above-mentioned procedure, but what we want to know is the change of $Q$ in both cases when some substance is provided in the resonant cavity and when it is not provided. The above-described method was found to be quite unsuitable for this purpose. Both theoretically and experimentally, the specimen must be thin and naturally the difference between the half widths of $Q$-curves may be considered to be small. If the specimen is pretty thick, the width of the line of $Q$-curve on the Braun tube would make a notable error. Besides, when the transverse axis is not straight against frequency, there may be an error. For these reasons mentioned above, we gave up this method and chose another one. We change the volume of the resonant cavity, using a piston, rectified the output power transmitted through the cavity, read the deflection of galvanometer and drew $Q$-curve. The most difficult point in this method is how to treat the piston attached to the resonant cavity. A further description of this piston will be given in the following section.

We described above how to measure $Q$ of the resonant cavity and the shift of resonance frequency. Now we shall explain the practical treatment of measurement below. First, the positions of the intensity of electric and magnetic field maxima were determined, respectively; as $TE_{104}$ mode was used in our experiments, the electric field did not exist on the wall of the cavity (See Fig. 2) and there the intensity of magnetic field reached the maximum; at the interval of one fourth of a wave-length, the intensity of electric field shows its maximum but magnetic field does not exist there. Practically we can measure the dielectric constant $\varepsilon$ and the permeability $\mu$ separately in this manner, but to do so, some support for the specimen is needed. As such a support, we provided a polystyrene rod whose diameter was about 2.5 mm. It was naturally expected that this support might change $Q$ of the resonant cavity and produce shift of resonancy frequency. So the rod had been graduated before it was put in the cavity, and we examined how the shift of resonance frequency and the half width would change. The results obtained are given in Fig. 3 where the transverse axis shows the length of the support while the longitudinal one indicates the graduation of micrometer. As shown in this figure, the polystyrene rod does not give much magnetic effect and, accordingly, near the maximum-point of the intensity in the magnetic field, the shift of resonance frequency is little influenced, while, in the case of electric field, it is influenced considerably and especially the slope of the curve showing the shift reaches its maximum. The half width is always near its average value in the limit of error and there is not much difference as in the case of resonant cavity itself.

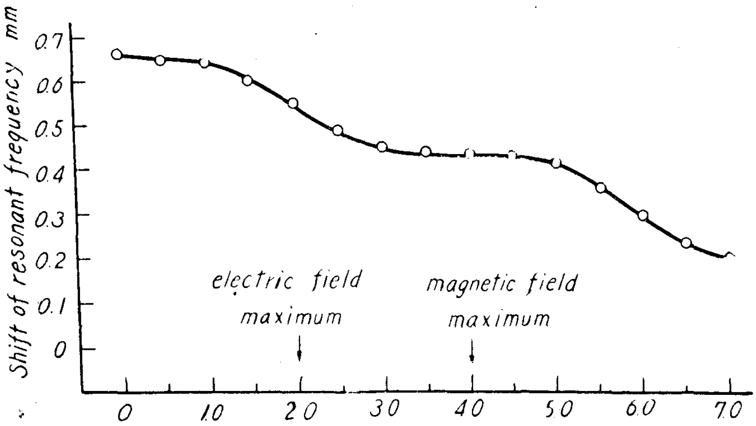


Fig. 3. Shift of resonant frequency by inserting the specimen-supporter made of polystyrol.

As described above, the change of resonance frequency and of half width caused by the polystyrene rod was known. Now we provided the specimen in the places where the intensity of both magnetic and electric fields were at the maximum and obtained the value of $Q$ and resonance frequency. From this resonance frequency and the value of $Q$, we deducted the values which had previously been obtained. Thus the shift of resonance frequency and the change of $Q$ caused only by the specimen are determined, and so $\varepsilon$ and $\mu$ can be calculated theoretically.

In Fig. 4, $Q$-curve which was actually measured is shown. The longitudinal axis shows the readings of the galvanometer while the graduations of the micrometer are given on the transverse axis. In the figure, "cavity" means $Q$-curve obtained without specimen, (E) means one concerning electric field and (H) denotes one relating to magnetic field.

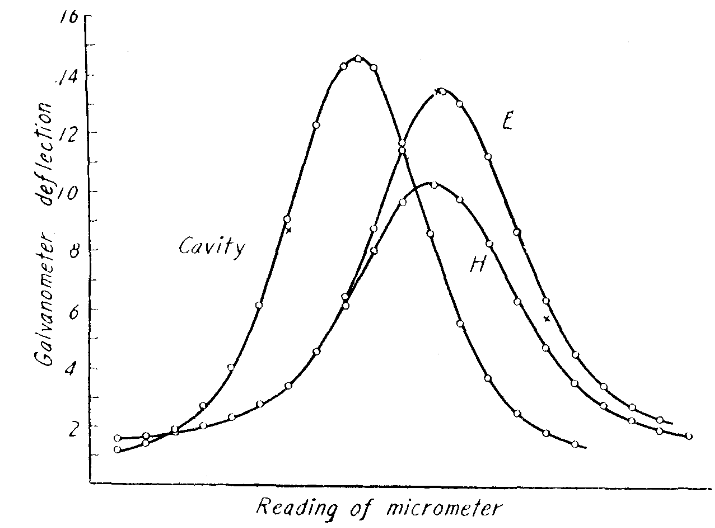


Fig. 4. An example of Q-curve.

## IV. Experimental apparatus

The experimental apparatus is shown in Fig. 5. The wave coming out of Klystron oscillator passes through the attenuator then goes into the resonant cavity which is rectangular in form and is joined to the wave guide by an iris of 15 mm × 6 mm. The distance between the oscillator and the resonant cavity is about 100 cm but these two parts are connected rather loosely and their mutual action is out of question. Batteries are employed for the oscillator so that oscillation is quite stable. A piston with a micrometer, which is used for measurement of the shift of resonance frequency, is attached to one end of the resonant cavity, while the other end has a hole whose diameter is about 2.5 mm and in which a polystyrene support is put. In consideration of the fact that the field may be disturbed near the iris, the resonant cavity is attached at the the wave-length of $3\lambda$ on the side where the specimen is provided and of $1\lambda$ on the other side where the piston is attached.

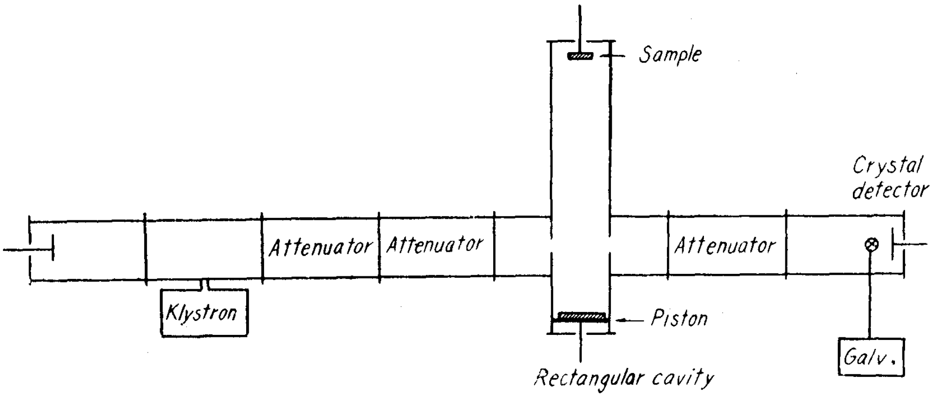


Fig. 5. Block diagram of apparatus for the measurement of dielectric constant and peameability at a wave length of 6.6 cm.

The transmission wave, passing through the resonant cavity, is led in the attenuator, rectified with the crystal rectifier—whose maximum output power is about $3\mu$ A and the crystal is used for square law detection—then it is brought in a galvanometer. When a piston of contact type is used here, rather strong contact resistance will result, so it is hard to obtain the state of resonance and in this case $Q$-curve is not reproducible, and symmetry of the curve, too, is not good. For these reasons, a piston of floating type was chosen for our experiment. If the resonant cavity were cylindrical in form, a discoid piston could be used but as it was rectangular, we used a rectangular brass plate coated with silver as piston (3 mm in thickness) reinforced by celluloid plate. (2 mm in thickness) Of course a polystyrene plate is also applicable. This plate could be moved, always keeping contact with the resonant cavity. To prevent resonance causable in the space at the back of the piston, the celluloid plate and the lid of the resonant cavity were painted with colloidal carbon. With this piston we could observe resonance curves and found the symmetry of $Q$-curve quite satisfactory and also gained good reproducibility in measurement. The wave-guide and the resonant cavity here used are made of brass plate coated with silver, of which thickness is 3 mm. The micrometer is an ordinary one with which we can measure up to 25 mm.

## V. Calculation and experimental results

In chapter II we gave the relations (Eq. 7) among $Q$ of the resonant cavity, $\Delta\omega$ the shift of resonance frequency, $\varepsilon$ and $\mu$. Now the complex $\varepsilon$ and $\mu$ are given by

$$\varepsilon=\varepsilon_0(\varepsilon'-j\varepsilon'') \ , \quad \mu=\mu_0(\mu'-j\mu'') \ .$$

Therefore, from Eq. (7), the following are given:

$$-2\frac{\Delta\omega_\varepsilon}{\omega}=\frac{\varepsilon'-\varepsilon_0}{\varepsilon_0}\int_{V''}E_a^2dv \ , \tag{7'a}$$

$$\frac{1}{Q_\varepsilon}=\varepsilon''\int_{V''}E_a{}^2dv+\frac{\sigma}{\varepsilon_0\omega}\int_{V''}E_a{}^2dv\,, \tag{7'b}$$

$$-2\frac{\Delta\omega_\mu}{\omega}=\frac{\mu'-\mu_0}{\mu_0}\int_{V''}H_a{}^2dv\,, \tag{7'c}$$

$$\frac{1}{Q_\mu}=\mu''\int_{V''}H_a{}^2dv\,. \tag{7'd}$$

What were measured in the experiments were the shift of resonance frequency shown in the micrometer and the half width determined from $Q$-curve. In other words, the left side of the above equation is determined, and each field is also determined from the resonant cavity. Thus, $\varepsilon'$, $\varepsilon''$, $\mu'$ and $\mu''$ can be obtained.

To rewrite the left side of the expression so that we may directely calculate by inserting the observed values, we must know the relations among the indication of micrometer, the change of $Q$ and of resonance frequency.

The indication of micrometer is denoted by the sign $\Delta l$, the shift by $\Delta l_s$ and half width by $\Delta l_H$.

As already known, from the relation

$$\lambda=\frac{\lambda_g}{\sqrt{1+\left(\frac{\lambda_g}{\lambda_c}\right)^2}}\,,$$

$$\Delta\lambda=\frac{\Delta\lambda_g}{\left\{1+\left(\frac{\lambda_g}{\lambda_c}\right)^2\right\}^{3/2}}$$

is obtained. On the other hand, from the eq. (10), the above expression beomes

$$\frac{1}{Q}=\frac{\Delta l_H}{\left\{1+\left(\frac{\lambda_g}{\lambda_c}\right)^2\right\}^{3/2}n\lambda}\,, \tag{11}$$

where $\Delta l_H$ denotes the difference of half width in two cases — with and without specimens — $n$ is the integer or the half integer showing the mode in the resonant cavity and $\lambda$, $\lambda_g$ and $\lambda_c$ denotes respectively the wave length in vacuum, in the cavity and the cut off wave length. Now the shift of resonance frequency is expressed by $\Delta l_s$, so the expression becomes

$$\frac{\Delta\omega}{\omega}=-\frac{\Delta l_s}{\left\{1+\left(\frac{\lambda_g}{\lambda_c}\right)^2\right\}^{3/2}n\lambda}\,. \tag{12}$$

Thus Eq. $(7)'$ was rewritten to be calculated. What we should obtain next are electric and magnetic fields; for this purpose $\int_{V''}E_a^2\,dv$, and $\int_{V''}H_a^2\,dv$ must be evaluated. Suppose the specimen is small and the field in $V''$ is uniform and further it is set in the place where intensity of electric and magnetic fields is respectively at maximum. Under this assumption, cosine and sine may be considered to be 1 or 0, respectively. Therefore, from the normalization constant $\int_V E_a^2\,dv=\int_V H_a^2\,dv=1$, the component of each field can be calculated as follows: we chose as a coordinate the base of the resonant cavity where $z=0$ and in the base axis $x$ was taken in the long side (a) while axis $y$ in the other. As described before in our experiment, $TE_{104}$-mode was used. Therefore,

$$\left.\begin{aligned} E_x &= 0 \\ E_y &= \frac{2}{\sqrt{V}} \sin\frac{\pi}{a} x \sin \alpha z \\ E_z &= 0 \\ H_x &= -\frac{2\lambda}{\lambda_g} \frac{1}{\sqrt{V}} \sin\frac{\pi}{a} x \cos \alpha z \\ H_y &= 0 \\ H_z &= \frac{\lambda}{a} \frac{1}{\sqrt{V}} \cos\frac{\pi}{a} x \sin \alpha z , \end{aligned}\right\} \quad (13)$$

where $\alpha = 2\pi/\lambda_g$. From Eq. (13) at each maximum of electric and magnetic field, the expressions become

$$\left.\begin{aligned} &(1);\ \int_{V''} E_a^2\,dv = 0, \qquad \int_{V''} H_a^2\,dv = \left(\frac{2\lambda}{\lambda_g}\right)^2 \frac{\Delta V}{V} \\ &(2);\ \int_{V''} E_a^2\,dv = 4\frac{\Delta V}{V}, \quad \int_{V''} H_a^2\,dv = 0 \end{aligned}\right\} \quad (14)$$

In the expressions, $\Delta V$ means volume $V''$ is small. Substituting Equations (11), (12) and (14) in Eq. (7′), the following is obtained:

$$\left.\begin{aligned} \varepsilon' &= \frac{\Delta l_{\varepsilon,S}}{\left\{1+\left(\frac{\lambda_g}{\lambda_c}\right)^2\right\}^{3/2} n\lambda} \frac{V}{2\Delta V} + 1 \\ \varepsilon'' &= \frac{\Delta l_{\varepsilon,H}}{\left\{1+\left(\frac{\lambda_g}{\lambda_c}\right)^2\right\}^{3/2} n\lambda} \frac{V}{4\Delta V} - \frac{\sigma}{\varepsilon_{0}\omega} \\ \mu' &= \frac{\Delta l_{\mu,S}}{\left\{1+\left(\frac{\lambda_g}{\lambda_c}\right)^2\right\}^{3/2} n\lambda} \frac{V}{2\Delta V} \left(\frac{\lambda_g}{\lambda}\right)^2 + 1 \\ \mu'' &= \frac{}{\left\{1+\left(\frac{\lambda_g}{\lambda_c}\right)^2\right\}^{3/2} n\lambda} \frac{V}{4\Delta V} \left(\frac{\lambda_g}{\lambda}\right)^2 . \end{aligned}\right\} \quad (15)$$

From Eq. (15), $\varepsilon'$, $\varepsilon''$, $\mu'$ and $\mu''$ can be calculated.

## Experimental results

Our experiments were carried out at the normal temperature, using $TE_{104}$-mode, and the wave length $\lambda$ was 6.6 cm. The section of the wave guide and the resonant cavity was measured to be 5.8 cm in the long side and 2.9 cm in other, and accordingly, cut off wave length was 11.6 cm. All these sections or both sides were coated with silver. When no specimen was provided, loaded $Q$ of the resonant cavity was about 1,300. The specimens were treated at 1,000°C for two hours and pressed into discoid form plates whose diameter was 20 mm, thickness 1 mm, 1.5 mm and so on, then these were sintered at 1,200°C for three hours, quenched in the air or cooled in the furnace slowely. Only Cu-ferrite was treated at 900°C for two hours and sintered at 1,000°C for three hours. The conductivity was measured of all the sintered specimens for the purpose of examining the increase of conductivity caused by the appearance of ferrous ion. All the specimens had resistance of more than $10^3 \Omega$ and so skin depth of such semiconducters must be above 2 cm near 4,500 M. c. and high frequency electric (or magnetic) field in the specimen can be considered to be uniform. The results obtained are shown in the Table 1.

Table 1. Dielectric constant, permeability and tan $\delta_\varepsilon$ of various ferrites at a wavelength of 6.6 cm; thickness of the specimen is 1.5 mm.

| Ferrite | $\varepsilon'$ | $\varepsilon''$ | tan $\delta_\varepsilon$ | $\mu'$ | $\mu''$ |
|---|---|---|---|---|---|
| $MgOFe_2O_3$ quenched from 1,200°C | 9.66 | 0.174 | 0.018 | 0.80 | 0.974 |
| $MgOFe_2O_3$ slow cooled | 8.53 | 0.132 | 0.016 | −0.84 | 0.341 |
| $CuOFe_2O_3$ quenched from 1,000°C | 9.29 | 0.520 | 0.056 | 0.06 | 1.240 |
| $CuOFe_2O_3$ slow cooled | 8.65 | 0.089 | 0.010 | 0.62 | 0.740 |
| $CoOFe_2O_3$ quenched from 1,200°C | 9.49 | 0.045 | 0.047 | 0.43 | 0.211 |
| $CoOFe_2O_3$ slow cooled | 9.00 | * | * | 0.10 | 0.116 |
| $NiOFe_2O_3$ quenched from 1,200°C | 13.40 | 3.520 | 0.260 | 0.26 | 3.460 |
| $NiOFe_2O_3$ slow cooled | 8.88 | 0.155 | 0.017 | 0.53 | 2.377 |
| $MnOFe_2O_3$ quenched from 1,200°C | 9.30 | 0.475 | 0.051 | −0.31 | 2.040 |

Generally speaking, the value of quenched specimen is large than that of slowly-cooled one. In the table of Co-ferrite, the blank columns means the half width did not change whether a specimen was provided in the resonant cavity or not and accordingly, we could not measure. When thickness is different, the results obtained are not precisely the same as shown in Table 2. This may be due to the fact that, in the expression from which $\varepsilon$ and $\mu$ were obtained, the effect of the specimen provided on the whole cavity, namely, the second order effect was neglected.

Table 2. Dielectric constant, permeability and tan $\delta_\varepsilon$ of various ferrites at a wavelength of 6.6 cm; thickness of the specimen is 1 mm.

| Ferrite | $\varepsilon'$ | $\varepsilon''$ | tan $\delta_\varepsilon$ | $\mu'$ | $\mu''$ |
|---|---|---|---|---|---|
| $CuOFe_2O_3$ quenched from 1,000°C | 10.79 | 0.530 | 0.049 | 0.38 | 1.150 |
| $CuOFe_2O_3$ slow cooled | 9.69 | 0.057 | 0.006 | 0.47 | 0.740 |
| $CoOFe_2O_3$ quenched from 1,200°C | 11.23 | 0.028 | 0.003 | 0.43 | 0.218 |
| $CoOFe_2O_3$ slow cooled | 9.59 | * | * | 0.47 | 0.097 |
| $NiOFe_2O_3$ quenched from 1,200°C | 18.93 | 5.17 | 0.270 | 0.69 | 1.590 |
| $NiOFe_2O_3$ slow cooled | 9.75 | 0.106 | 0.011 | 0.59 | 1.680 |
| $MnOFe_2O_3$ quenched from 1,200°C | 11.29 | 0.381 | 0.034 | 0.81 | 1.950 |

This problem can be solved to some extent if experiments are carried out on the specimens different in form and size, but, after all, the obtainable results may fairly agree with the every result in the Table.

## VI. Conclusion

From those experiments described above, single ferrite e. g. Mg, Cu, Co, Ni, Mn &c. has the values of order, $\varepsilon' \sim 10$, $\mu' \sim 0.5$. The value of $\varepsilon'$ of quenched Ni-ferrite is larger than others, and as for tan $\delta_\varepsilon$, that of slowly cooled Co-ferrite is extremely small. Generally speaking (except for a few speimens), the value of quenched specimen is larger than that of slowly-cooled one.

According to Hewitt(6), when he measured $MnOZnOFe_2O_3$ at a wavelength of 1.25 cm ; $\varepsilon'=18$, $\varepsilon''=25$, $\mu'=1.4$ and $\mu''=0.3$. But both the specimen and the wave length were different from ours. Recently Healy(7) puplished his experimental results that in the case of $NiOZnOFe_2O_3$, $\varepsilon'=11$, $\varepsilon''=0.13$. He studied at the wave-length of 3 cm, and though the different specimen he used, the results agreed satisfactorily in the order.

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(6) W. H. Hewitt, Phys. Rev., **73** (1948), 1118.

(7) Recently Healy published his paper in which he gave how to measure $\varepsilon$ and how to calculate. His results agree with our's. D. W. Healy, Jr, Contract N50RI-76, Task Order No. 1, NR-078-011, Cruft Laboratory, Harvard University, Cambridge Massachusetts, Technical Report No. 135, August 15, 1951.