On the Positions of the Equivalent Poles in a Magnetized Bar Tested by the Magnetometer Method*

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Synopsis

According as the two coils used in the magnetometer method are in the same axis or in parallel, two equations showing the relations of the equivalent distance between two poles to the distance between the magnetometer and the magnetized bar were deduced, and using the values of free magnetism in a soft iron bar determined by T. Yoshida and H. Kadooka, the positions of the equivalent poles were calculated. It was found that the positions of the equivalent poles were remarkably influenced by the distance between the magnetometer and the magnetized bar, and by the intensity of external field.

I. Introduction

The term "magnetic poles" in a magnetized bar usually means respectively the positions of the center of mass of positive and of negative free magnetism of the bar. On the other hand, when the magnetized bar exerts force upon the outside magnetic charge, the term "magnetic poles" is also used. The latter, however, is to be called "equivalent poles" and is used in the case of a magnetized bar exerting force on a magnetic needle of the magnetometer. Riecke(1), for the first time, investigated the positions of the equivalent poles in a bar magnet, but did not determine them numerically, showing only a general formula. Hallock and Kohlrausch(2) experimentally determined the distance between the positive and negative equivalent poles in a magnetized bar, showing that it was practically equal to $5/6 \fallingdotseq 0.833$ of the total length of the bar. This value, however, should not generally be used, since it was obtained when the bar was far from the outside acting point. In Graetz's Handbuch(3), we find two formulas showing how the equivalent poles vary with the distance between a magnetized bar and an outside accepting point, when the point is on the longitudinal axis, or on the perpendicular bisector of the bar, assuming that the bar is linear and that the density of the distribution of free magnetism on it is given by $\delta = \pm cx^2$, in which the longitudinal axis of the bar is taken as X-axis, the center of the bar as the origin, and c is a constant. These formulas, however, are applicable only when the distance is sufficiently large.

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(1) E. Riecke, Ann. d. Phys. und Chem., **8** (1879), 299.

(2) W. Hallock and F. Kohlrausch, Wied. Ann., **22** (1884), 411.

(3) Graetz, Handbuch der Elektrizität und des Magnetismus IV (Magnetismus und Elektromagnetismus) (1920), 98.

In the present investigation, the positions of the equivalent poles were calculated on a soft iron bar, using the values obtained by Yoshida and Kadooka[4] of the density of the distribution of free magnetism on the bar, provided that the magnetizing coil and the compensating coil in the magnetometer method are in the same axis or in parallel, and that the outside accepting point is a point magnetic charge.

II. Calculations

In the first place, the so-called magnetic poles, or the centers of mass of positive and negative magnetism in a magnetized bar will be considered.

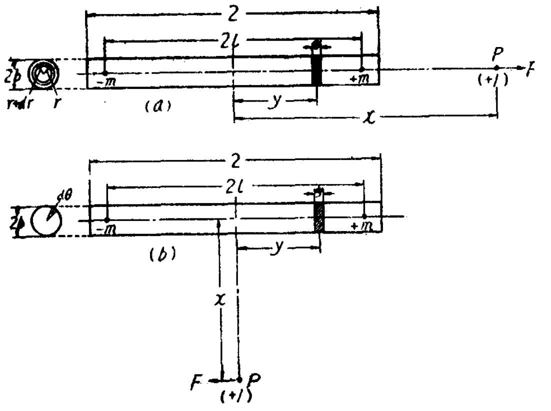

Fig. 1

Let us take a magnetized cylindrical bar, $2b$ in diameter and 2 in length, in which the distributions of positive and negative free magnetism are symmetrical, the densities of free magnetism on the cylindrical surface and on the end surface being respectively σ and σ_1. Taking the bar-axis as y-axis and the center of the bar as the origin(cf. Fig. 1), the magnetic moment of the bar will be expressed as follows:

$$M = \int_{-1}^{1} 2\pi b\sigma y dy + 2\int_{0}^{b} 2\pi\sigma_1 r dr , \tag{1}$$

and the sum of positive free magnetism may be expressed as follows:

$$m = \int_{0}^{1} 2\pi b\sigma dy + \int_{0}^{b} 2\pi\sigma_1 r dr . \tag{2}$$

Now, we replace the magnetized bar with a mathematical magnet with the intensity of magnetic poles of $\pm m$ and with the pole distance of $2l$. Since the magnetic moment of this magnet is $2ml$, l is given by

$$l = \frac{\int_{-1}^{1} 2\pi b\sigma y dy + 2\int_{0}^{b} 2\pi\sigma_1 r dr}{2\left(\int_{0}^{1} 2\pi b\sigma dy + \int_{0}^{1} 2\pi\sigma_1 r dr\right)} . \tag{3}$$

Next, the equivalent poles will be considered in two cases in which the free magnetism in the magnetized bar acts upon a point magnetic charge existing in the following two principal positions:

(1) The case in which a point charge lies on the longitudinal axis of the bar. (cf. Fig. 1, a)

First, we will obtain the force F acting along the axis upon a point charge at a distance x from the center of the bar. F is the integrated sum of two forces of the free magnetism in the ranges of $2\pi b dy$ on the bar and $2\pi r dr$ on the end surfaces, that is,

(4) T. Yoshida, H. Kadooka, Proc. Tokyo Mathematico-Physical Soc., 3 (1906), 150.

$$F=\int_{-1}^{1}2\pi b\sigma\frac{x-y}{\sqrt{(x-y)^2+b^2}}dy+\int_0^b 2\pi\sigma_1\left(\frac{x-1}{\sqrt{(x-1)^2+r^2}^3}-\frac{x+1}{\sqrt{(x+1)^2+r^2}^3}\right)rdr. \quad (4)$$

By replacing the magnetized bar with the above-mentioned mathematical magnet, the force F will be expressed as follows:

$$F=m\frac{4xl}{(x^2-l^2)^2}\ . \quad (5)$$

Eliminating m and F from Eqs. (2), (4) and (5),

$$\frac{4lx}{(x^2-l^2)^2}=\frac{\int_{-1}^{1}2\pi b\sigma\frac{x-y}{\sqrt{(x-y)^2+b^2}^3}dy+\int_0^b 2\pi\sigma_1\left(\frac{x-1}{\sqrt{(x-1)^2+r^2}^3}-\frac{x+1}{\sqrt{(x+1)^2+r^2}^3}rdr\right.}{\int_0^1 2\pi b\sigma dy+\int_0^b 2\pi\sigma_1 rdr}. \quad (6)$$

Thus, the relation between l and x can be obtained from this equation with the experimental values of σ and σ_1.

(2) The case in which a point charge lies on the perpendicular bisector of the bar. (cf. Fig. 1, b)

First, we will obtain the force acting along the axis upon a point charge on a perpendicular bisector at distance x from the center of the bar. F is the integrated sum of two forces of the free magnetism in the ranges of $bd\theta dy$ on the bar and $rd\theta dr$ on the end surfaces, that is,

$$F=\int_{-1}^{1}\int_0^{2\pi}\frac{\sigma byd\theta dy}{\sqrt{x^2+y^2+b^2-2bx\cos\theta}^3}+2\int_0^b\int_0^{2\pi}\frac{\sigma_1 rd\theta dr}{\sqrt{x^2+1+r^2-2rx\cos\theta}^3}, \quad (7)$$

which becomes

$$F=m\frac{2l}{\sqrt{x^2+l^2}^3}, \quad (8)$$

by utilizing the mathematical magnet as in the above case. Eliminating m and F from Eqs. (2), (7) and (8),

$$\frac{2l}{\sqrt{x^2+l^2}^3}=\frac{\int_{-1}^{1}\int_0^{2\pi}\frac{\sigma byd\theta dy}{\sqrt{x^2+y^2+b^2-2bx\cos\theta}^3}+2\int_0^b\int_0^{2\pi}\frac{\sigma_1 rd\theta dr}{\sqrt{x^2+1+r^2-2rx\cos\theta}^3}}{\int_0^1 2\pi b\sigma dy+\int_0^b 2\pi\sigma_1 rdr}, \quad (9)$$

from which the relation between l and x can be obtained with the experimental values of σ and σ_1 as before.

(3) Finally, we will obtain the equations applicable in practical use with experimental values.

Since the values obtained practically are the leakage flux $-\frac{d\Phi}{dy}$ through the cylindrical surface of unit length and Φ_1 through the end surfaces of the magnetized bar, the density on the cylindrical surface σ and the density on the end surfaces σ_1 may be expressed as follows:

$$\sigma=\frac{-\frac{d\Phi}{dy}}{2\pi b\cdot 4\pi}, \quad (10)$$

$$\sigma_1=\frac{\Phi_1}{\pi b^2\cdot 4\pi}, \quad (11)$$

provided that the distribution of free magnetism on the end surfaces is uniform.

Substituting them in Eqs. (3), (6) and (9), and expanding a part of Eq. (9), Eqs. (3), (6) and (9) may be expressed respectively as follows:

$$l = \frac{\int_0^1 \Phi dy}{\Phi_0}, \tag{3'}$$

$$\frac{4lx}{(x^2-l^2)^2} = \frac{-\int_{-1}^1 \frac{d\Phi}{dy}\frac{x-y}{\sqrt{(x-y)^2+b^2}^3}dy + \frac{2\Phi_1}{b^2}\left(\frac{x+1}{\sqrt{(x+1)^2+b^2}} - \frac{x-1}{\sqrt{(x-1)^2+b^2}}\right)}{\Phi_0}, \tag{6'}$$

$$\frac{2l}{\sqrt{x^2+l^2}^3}$$

$$= \frac{-2\int_0^1 \frac{d\Phi}{dy}\frac{y}{\sqrt{x^2+y^2+b^2}^3}\left\{1+\frac{15}{16}\left(\frac{2bx}{x^2+y^2+b^2}\right)^2+\cdots\right\}dy + \frac{4\Phi_1}{b^2}\int_0^b \frac{1+\frac{15}{16}\left(\frac{2rx}{x^2+1+r^2}\right)^2+\cdots}{\sqrt{x^2+1+r^2}^3}rdr}{\Phi_0}, \tag{9'}$$

where Φ_0 is the total flux through the central section of the specimen.

III. Numerical calculations

As an example, the positions of equivalent poles will be shown in a magnetized soft iron bar utilizing the value of $-\frac{d\Phi}{dy}$ measured by Yoshida and Kadooka[4], by drawing out a differential coil, 6.1 mm in diameter and 10 cm in length, along the bar and with a ballistic galvanometer. The values are denoted only by the deflection of galvanometer $-c\frac{d\Phi}{dy}$, c being a constant.

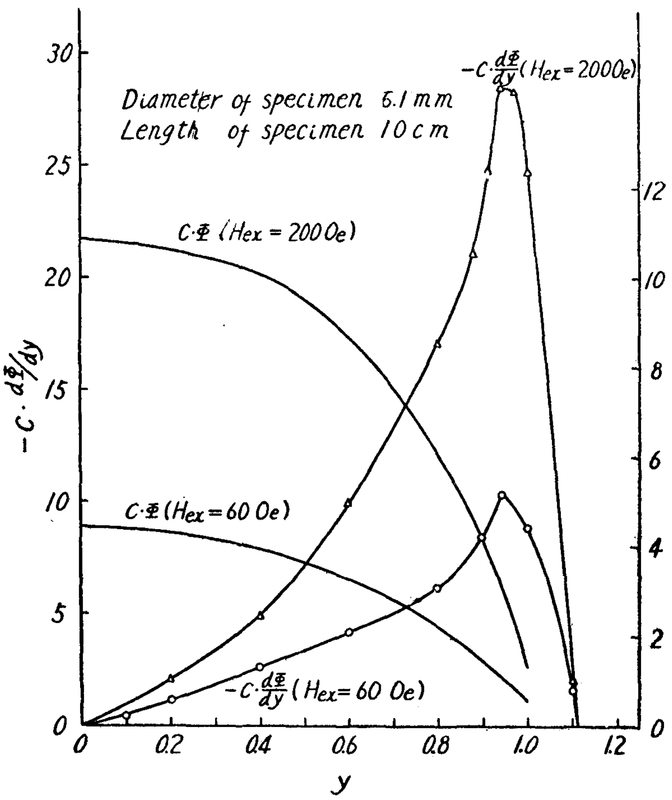

Fig. 2. The relations of magnetic flux Φ and flux leakage $-d\Phi/dy$ to the distance y from the center of the magnetized bar.

Taking the mean of the absolute values on both sides of specimen, the distribution curves of $-c\frac{d\Phi}{dy}$ and $c\cdot\Phi$ at the longitudinal field of 60 and 200 Oe are shown in Fig. 2.

First, from Eq. (3′) the distance between the positions of centers of mass of positive and negative free magnetism, l, is expressed as follows:

$$l = 0.743, \text{ when } H_{ex} \text{ is } 60\,\text{Oe}, \tag{12}$$

$$l = 0.772\,, \text{ when } H_{ex} \text{ is } 200 \text{ Oe}\,. \tag{13}$$

Next, the variation of l with the distance between the specimen and an outside accepting point at the fields of 60 and 200 Oe obtained from Eqs. (6′) and (9′) are respectively shown in Figs. 3 and 4. As seen in Fig. 3, when an outside point charge is on the longitudinal axis of the bar, l decreases first rapidly from the initial value of about 0.9, then slowly and finally approaches the values at $y=\infty$, namely, 0.743 and 0.772 in respective cases. As seen in Fig. 4, when an outside point charge is on the perpendicular bisector of the bar, l, which at first is almost equal to zero, increases rapidly at first and then slowly approaches 0.743 and 0.772 in respective fields. These values coincide with those of (12) and (13), that is, when $y=\infty$, the positions of the equivalent poles coincide with those of centers of mass of positive and negative free magnetism.

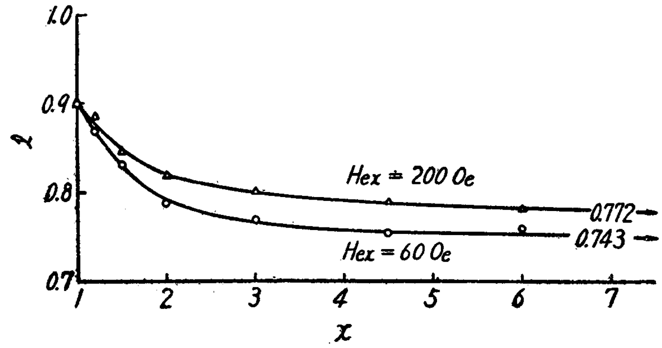

Fig. 3. The relations of the equivalent pole distance l to the distance x between the magnetometer and the center of the magnetized bar when the two coils are on the same axis.

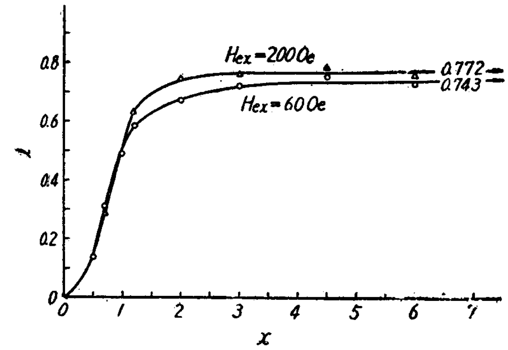

Fig. 4. The relations of the equivalent pole distance l to the distance x between the magnetometer and the center of the magnetized bar when the two coils are in parallel.

IV. Consideration

As mentioned in I, when the magnetization of a ferromagnetic material is measured by the magnetometer method, the distance between the equivalent magnetic poles of the specimen is considered to be nearly 5/6 of the total length of the bar. However, as seen in the above calculations, when the magnetizing coil and the compensating coil are in the same axis, the dsitance of the equivalent magnetic poles varied with the distance between the specimen and the magnetometer from about 0.9 to 0.743 in the case where $H_{ex} = 60$ Oe and from about 0.9 to 0.743 at the field where $H_{ex} = 200$ Oe. Further, when the two coils are in parallel, it varied from about 0 to 0.743 in the case where $H_{ex} = 60$ Oe and from about 0 to 0.772 in the case where $H_{ex} = 200$ Oe. Thus, it may be said that the distance between the equivalent poles is remarkably influenced by the distance between the specimen and the magnetometer and by the intensity of external field.

Summary

According as the two coils in the magnetometer method are in the same axis or in parallel, two equations were deduced, from which the positions of the equivalent poles were calculated with the distance between the magnetized bar and a point charge taken in place of a magnetic needle as an outside accepting point and with the distribution density of free magnetism on the bar. Using the values of free magnetism of a soft iron bar determined by Yoshida and Kadooka, the positions of the equivalent poles were calculated by means of the above equations, and the following results were obtained:

(1) In both cases of external fields of 60 Oe and 200 Oe, when the magnetizing coil and the compensating coil are on the same axis, the equivalent magnetic poles of almost 0.9 decreases at first rapidly and then slowly with the distance of the specimen from the magnetometer, and finally approaches the values 0.743 and 0.772 respectively.

(2) In both cases of external fields of 60 Oe and 200 Oe, when the magnetizing coil and the compensating coil are in parallel, the equivalent poles of almost zero increases at first rapidly and then slowly with the distance between the specimen and the magnetometer, and finally approaches the values 0.743 and 0.772 respectively.

(3) When the specimen is infinitely away from the magnetometer, the distance between the equivalent magnetic poles coincides with that between the centers of mass of positive and negative free magnetism.

(4) It is erroneous to use 5/6 ≒ 0.833 of the total length of the specimen as the distance between the equivalent poles, when the magnetization is measured by the magnetometer method, especially when the specimen approaches the magnetometer.

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