Orientation Determination of Cubic Crystal Plates by the Light-Figure Method*

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Synopsis

A procedure of the perfect, accurate, and rapid orientation determination by the light-figure method of cubic crystal plates, in particular, strips has been worked out. Full accounts are given of the procedure for the orientation determination, of the kind of applicable light figures or of accurately determinable orientation angles, of the stereographic representation of orientations, and of the accuracy in orientation determination with reference to actual examples of iron and silicon iron strip crystals. It is shown that the kind of applicable light figures or of accurately determinable orientation angles depends on the crystal orientation to be determined, of which the relation indicates that the perfect orientation determination can be made for most crystals of which the orientations of the plate normal lie close to the [100], [110], or [111] direction. Orientations of single crystal strips of pure iron and silicon iron can be determined with an accuracy well within $\pm 0.5°$ by this meteod.

I. Introduction

We have so far studied on the rapid, accurate, and perfect orientation determination of various metal crystals by the light-figure method and applied this method with a great success to single crystal rods of cubic[1,2] (nickel, copper, iron, aluminium, silver, nickel-iron, nickel-copper, nickel-cobalt, copper-zinc iron-silicon, iron-aluminium, iron-silicon-aluminium), hexagonal[3] (zinc), trigonal[4] (bismuth), and tetragonal[5] (tin) metals. The light-figure method is based on such a property of the light figure that its symmetry characteristics may remain always the same as that of a reflecting crystal plane, even though its shape changes diversely with etching condition. Accordingly, when a light figure is reflected symmetrically with respect to a pin-hole of a screen through which an incident light beam passes perpendicularly, the normal of the reflecting crystal plane coin-

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(1) M. Yamamoto, Nippon Kinzoku Gakkai-shi, **5** (1941), 214; Sci. Rep. Tôhoku Univ., **31** (1943), 121; Butsurigaku Kôenshû, **3** (1943), 193.

(2) M. Yamamoto and J. Watanabé, Nippon Kinzoku Gakkai-shi, **17** (1953), 5 and 9; Sci. Rep. RITU, **A7** (1955), 173.

(3) M. Yamamoto and J. Watanabé, Nippon Kinzoku Gakkai-shi, **13** (1949), No. 4; Sci. Rep. RITU, **A2** (1950), 270.

(4) M. Yamamoto and J. Watanabé, Nippon Kinzoku Gakkai-shi, **B15** (1951), 572; Sci. Rep. RITU, **A5** (1953), 135.

(5) M. Yamamoto and J. Watanabé, Nippon Kinzoku Gakkai-shi, **17** (1953), 113; Sci. Rep. RITU, **A7** (1955), 161.

cides with the incident light beam (Fig. 1). Then, the orientation of a rod crystal can be determined readily and perfectly by measuring the angle between the rod axis and the incident light beam in this configuration with two important crystal planes.

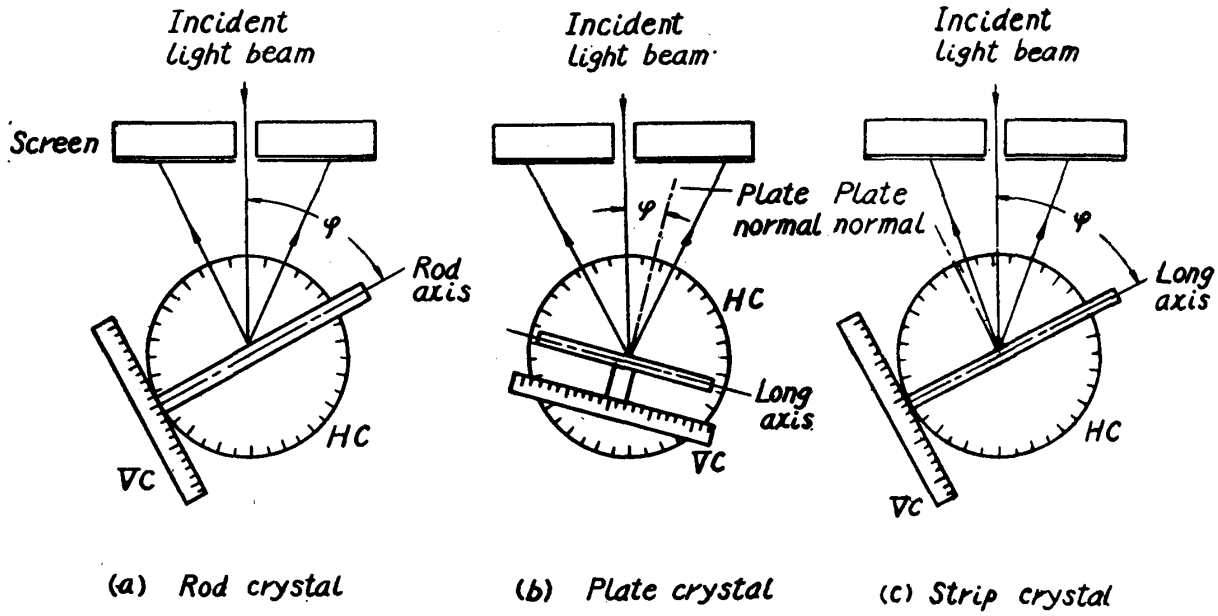


Fig. 1. Modes of mounting the specimen crystals of the rod, plate, and strip forms on the goniometer for the orientation determination by the light-figure method. *HC* and *VC* are the horizontal and vertical circles of the goniometer, respectively.

In the experimental arrangement, a two-circle goniometer[(6)] is so arranged that the incident light beam may coincide with the rotation axis of the vertical circle of the goniometer when the vertical circle is faced perpendicularly to the incident beam. The specimen crystal is mounted at the center of the vertical circle in such a way that its rod axis may coincide with the rotation axis of the vertical circle (Fig. 1 (a)), and a light figure produced by a principal crystal plane is catched on the screen by rotating the specimen crystal about its rod axis as well as by rotating the horizontal circle of the goniometer. Then, positions of the two circles of the goniometer are so adjusted that the symmetric center of the light figure may coincide with the pin-hole of the screen, when the reading of angle on the horizontal circle, φ_1, is recorded. After the specimen crystal has been rotated by 180° about its rod axis and then by so much angle about the rotation axis of the horizontal circle that the symmetric center of the same light figure may coincide again with the pin-hole of the screen, the reading on the horizontal circle, φ_2, is taken. The angle, φ, between the rod axis and the normal of the crystal plane concerned is, then, given by

$$\varphi = (\varphi_1 \sim \varphi_2)/2\,. \qquad (1)$$

This is the *fundamental procedure* for the orientation determination by the light-figure method. It is needless to say that the fundamental procedure can be applied

(6) In a two-circle goniometer, the rotation axis of its horizontal circle is fixed, around which the rotation axis of its vertical circle can be rotated.

for determining the orientation of rod crystal of any cross-section since it determines orientation angles of the rod axis alone.

Now, let us consider the procedure for the orientation determination of a plate crystal, namely, for the the determination of angles which the plate normal makes with the principal crystal axes, which may be more troublesome than that for a rod crystal. We take a rectangular coordinate system XYZ, of which the X axis coincides with an incident light beam fixed in space and the Y axis is the rotation axis of the horizontal circle of the two-circle goniometer (Fig. 2). For a practical convenience, let the XYZ coordinate system rotate around the Y axis by an angle $\pi/2-\varphi_0$, then we have a new coordinate system $X'YZ'$ of which the Z' axis is the rotation axis of the vertical circle. Further, a rectangular coordinate system $X''Y''Z'$ is obtained by rotating the $X'YZ'$ system around the Z' axis by an angle $-\theta_0$. Let the polar coordinates of two orientations $\overrightarrow{OC}$ and $\overrightarrow{ON}$, referred to the $X''Y''Z'$ coordinate system, be denoted as (φ_C, θ_C) and (φ_N, θ_N), respectively. Then, the angle, δ, between $\overrightarrow{OC}$ and $\overrightarrow{ON}$ is given by

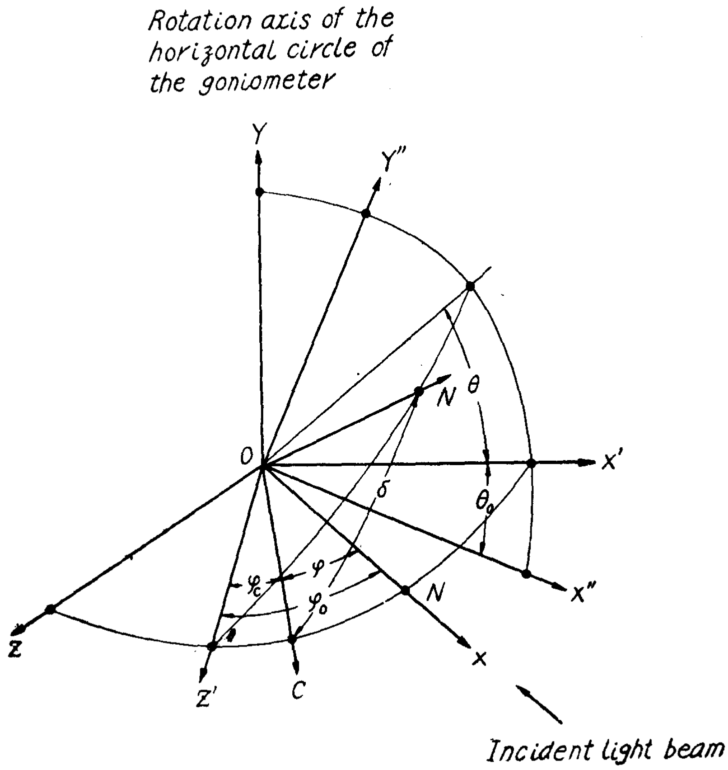


Fig. 2. To illustrate the subsidiary procedure for the orientation determination of a strip crystal by the light-figure method.

$$\cos\delta = \cos\varphi_C\cos\varphi_N + \sin\varphi_C\sin\varphi_N\cos(\theta_C \sim \theta_N)\,. \tag{2}$$

In the orientation determination of a rod crystal, $\overrightarrow{OC}$ is the rod axis and $\overrightarrow{ON}$ is the normal of a certain crystal plane. Both orientations lie in the $Z'OX'$ plane $(\theta_C = \theta_N = \theta_0)$, if we neglect a possible small deviation of the rod axis from the $Z'OX'$ plane, $\overrightarrow{ON}$ coinciding with $\overrightarrow{OX}$ $(\varphi_N = \varphi_0)$, and thus Eq. (2) is reduced to

$$\delta = \varphi_0 - \varphi_C. \tag{3}$$

Further, since the goniometer is so arranged that, when the rod axis set so as to coincide with the rotation axis of the vertical circle lies perpendicularly to the incident beam, $\varphi_0 = 90°$, namely, the Z' and X' axes coincide with the Z and X axes, respectively, Eq. (3) eventually becomes

$$\delta = 90° - \varphi_C = \varphi\,, \tag{4}$$

indicating that this single angle, namely, the angle between the directions of the rod axis and of the incident light beam can be determined by means of the above-mentioned fundamental procedure. In general cases, however, Eq. (2) indicates that four angles θ_N, θ_C, φ_N and φ_C, are required to determine the orientation.

If a plate crystal is set on the vertical circle of the goniometer in such a way that the plate normal may coincide with the rotation axis of the vertical circle and an extension of the plate surface may pass through the center of the horizontal circle (Fig. 1 (b)), the orientation angles of the plate normal can also be determined by the fundamental procedure since such a situation of the plate normal corresponds just to the axis of a rod crystal. It is to be noted, however, that several difficulties are encountered in setting a plate crystal in this way on the goniometer. First, the size of the specimen crystal is limited by the diameter of the vertical circle (about 6 cm in our Fuess goniometer), and moreover the goniometer must be provided with an attachment for moving the specimen crystal parallel to its plate surface in order to determine the orientation of the plate normal at an arbitrary place on the plate surface.

II. Procedure for the orientation determination of a single crystal strip by the light-figure method

For a long plate or strip crystal, the orientation can be determined by setting the crystal at the center of the vertical circle of the goniometer in such a way that its long axis may coincide with the rotation axis of the vertical circle, as in the above-mentioned case of rod crystal (Fig. 1 (c)). It is evident, however, that, in such an arrangement, angles which can be determined by the fundamental procedure are only those which the long axis makes with the crystal axes, and the orientation of the plate normal must be determined by a different procedure based on Eq. (2). We suppose, in this case, that $\overrightarrow{ON}$ and $\overrightarrow{OC}$ in Fig. 2 are the plate normal of the crystal and the normal of a certain crystal plane, respectively. Let us consider that $\overrightarrow{ON}$ is made to coincide with the incident light beam, $\overrightarrow{OX}$, $(\varphi_N = \varphi_0,\ \theta_N = \theta_0)$, and then the vertical circle of the goniometer is rotated by such an angle, θ, that $\overrightarrow{OC}$ is brought in the $Z'OX'$ plane when $\overrightarrow{ON}$ reaches to $\overrightarrow{ON'}$ and the horizontal circle is rotated by such an angle, φ, that $\overrightarrow{OC}$ coincides with $\overrightarrow{OX}$. Then, the angle, δ, between the plate normal $\overrightarrow{ON'}$ $(\varphi_N = \varphi_0,\ \theta_N = \theta_0 + \theta)$ and the normal of the crystal plane concerned $\overrightarrow{OC}$ $(\varphi_C = \varphi_0 - \varphi,\ \theta_C = \theta_0)$ is given, from Eq. (2), by the following expression: —

$$\cos\delta = \cos\varphi_0\cos(\varphi_0 - \varphi) + \sin\varphi_0\sin(\varphi_0 - \varphi)\cos\theta\,. \qquad (5)$$

If the long axis of the crystal is set so as to coincide exactly with the rotation axis of the vertical circle of the goniometer ($\varphi_0 = 90°$, $\theta_0 = 0°$), Eq. (5) may be reduced simply to

$$\cos\delta = \cos\varphi\cdot\cos\theta\,. \qquad (6)$$

This procedure will be called the *subsidiary procedure* for the orientation determination by the light-figure method in the following.

As readily seen from the above description, in the subsidiary procedure, it is first required to determine the orientational angles, φ_0 and θ_0, of the plate normal $\overrightarrow{ON}$ when it coincides with the direction of incident light beam $\overrightarrow{OX}$, referred to the goniometer or to the $X''Y''Z'$ coordinate system in Fig. 2. For this purpose, we tried two methods. The first is a procedure which determines θ_0 after the specimen crystal has been set on the goniometer in such a way that its long axis coincides with the rotation axis of the vertical circle, namely, after φ_0 has been made to become 90° as seen from the mentioned above. First, we let the long axis and plate surface of the specimen crystal be, respectively, normal and parallel to the incident light beam. We rotate, then, the vertical circle of the goniometer about its axis gradually, so that a light reflected from the plate surface may go away from the filament image of the light source thrown on the wall standing perpendicularly behind the screen and the specimen crystal (Fig. 3). We take a reading of the vertical circle, θ_1, when the reflected light just leaves from either the upper or the lower end of the filament image on the wall, and a similar reading, θ_2, after the vertical circle has been rotated by 180° about its axis. Then, the angle θ_0 is given by

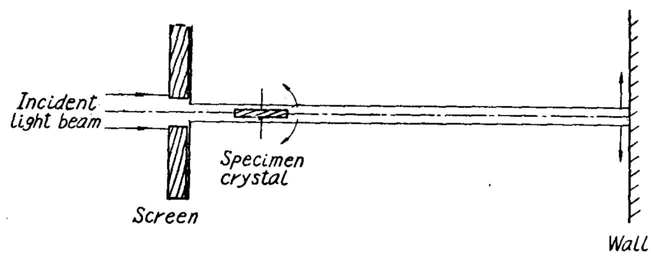


Fig. 3. To illustrate the first procedure for determining the orientation relative to the goniometer of the plate normal of a strip crystal.

$$\theta_0 = (\theta_1 + \theta_2)/2. \qquad (7)$$

It is to be noted, however, that this procedure can only be applied for a strip crystal of which the width is less than that of the incident light beam and the plate surface is so bright that the incident light beam can be reflected.

The second procedure for determining φ_0 and θ_0 utilizes a light image on the screen reflected from a deck-glass sticked to the surface of the strip crystal. Readings of the horizontal and vertical circles of the goniometer, φ_1 and θ_1, are recorded when the goniometer is so adjusted that the light spot reflected from the deck-glass may coincide with the pin-hole of the screen, namely, the plate normal of the crystal specimen may coincide with the incident light beam, and then similar readings φ_2 and θ_2 after both horizontal and vertical circles have been rotated by 180° are recorded. Then, φ_0 and θ_0 are given, respectively, by

$$\left.\begin{aligned} \varphi_0 &= (\varphi_1 \sim \varphi_2)/2 \\ \text{and} \qquad \theta_0 &= (\theta_1 + \theta_2 - 180°)/2. \end{aligned}\right\} \qquad (8)$$

As mentioned before, φ_0 must be equal to 90°, if the long axis of the strip coincides with the rotation axis of the vertical circle. Therefore, the deviation of φ_0 from 90° indicates the degree of correctness of setting the specimen crystal on the goniometer.

Next, the orientation angles, φ_C and θ_C, with respect to the goniometer, of the normal of a given crystal plane when it coincides with the direction of incident light beam may be determined by a procedure similar to the second procedure for determining φ_0 and θ_0. It is to be noted, however, that, in this case, the deck glass is not required and the symmetric center of the light figure corresponding to the crystal plane concerned is made to coincide with the pin-hole of the screen.

We get, then, the angle δ by substituting

$$\left.\begin{aligned}\varphi &= \varphi_0 - \varphi_C \\ \text{and} \quad \theta &= \theta_0 \sim \theta_C\end{aligned}\right\} \tag{9}$$

into Eq. (5) or (6). It is to be noted that the angle between the long axis of the strip and the normal of the crystal plane concerned is given by the angle φ_C itself. Thus, the procedure for determining the orinetation of single crystal strip with the light-figure method has been established.

III. Relationship between the orientation to be determined and the kind of light figures applicable to the accurate determination

For an actual application of the present procedure, orientation angles should be measured accurately using light figures corresponding to the principal crystal planes. It is to be noticed, however, that the kind of light figures applicable to the accurate orientation determination is limited by the inclination of the normal of the crystal plane producing the light figure to the geometrical surface of crystal (plate and side surfaces of a plate crystal)[(2)] and thus the kind of applicable light figures varies with the crystal orientation to be determited.

According to our experience, the ranges of angle δ as may be determined accurately (within $\pm 0.1°$) using the {100}, {110} and {111} light figures of cubic crystal are approximately $0° \sim 30°$, $0° \sim 20°$ and $0° \sim 10°$, respectively. Then, if the angles between the plate normal and the [100], [110] and [111] axes are denoted by α_1, β_1, and γ_1, respectively, the accurate determinable ranges of these principal orientation angles are

$$0° < \alpha_1 < 30°, \quad 0° < \beta_1 < 20° \quad \text{and} \quad 0° < \gamma_1 < 10°. \tag{10}$$

Accordingly, if the orientation of the plate normal is represented by a point in a (100)-(110)-(111) stereographic triangle, this triangle is divided into five regions I-V, as shown in Fig. 4. Two angles, α_1 and β_1, can be determined accurately for crystals with orientations in region II which lies intermediately on the (100)-(110) side, while only one angle, α_1 or β_1 or γ_1, for crystals with orientations in regions I, III and IV which lie closely to one of the (100), (110) and (111) corners and nothing for crystals with orientations in region V which locates intermediately in the triangle.

Since crystal orientation may be fixed perfectly, at least, by two orientation angles, it follows, then, that the perfect orientation determination can not be made for most crystals except ones in the region II, so far as light figures revealed by the plate surface are used. But, if we employ also the light figures revealed by the side surfaces of the strip crystal, the range of orientation where the perfect determination is possible can be extended. Light figures revealed by the side

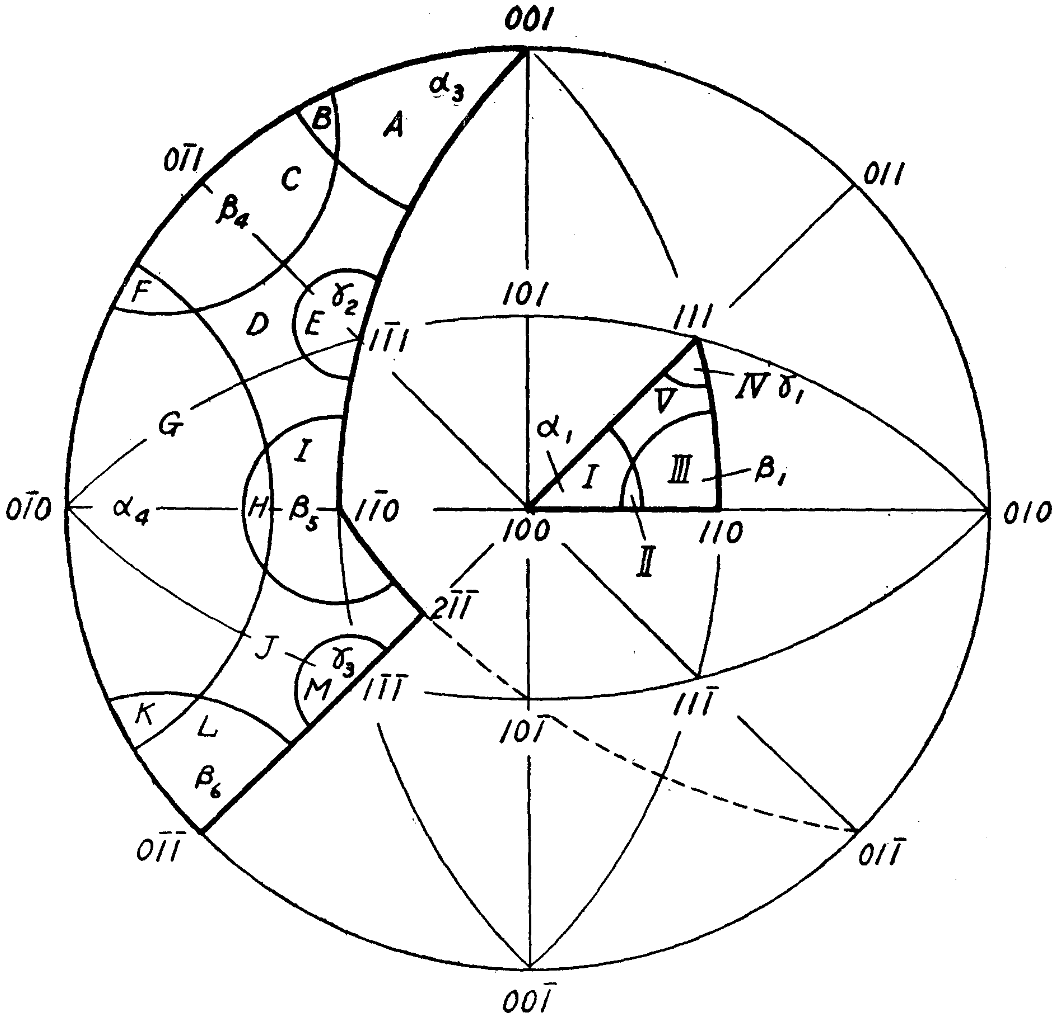


Fig. 4. Stereographic illustration of the relationship between light figures applicable to the accurate orientation determination or orientation angles accurately determinable by light figures and orientations to be determined in a cubic crystal strip.

surfaces, applicable for this purpose, are the (001), $(0\bar{1}0)$, $(0\bar{1}1)$, $(1\bar{1}0)$, $(0\bar{1}\bar{1})$, $(1\bar{1}1)$ and $(1\bar{1}\bar{1})$ light figures, seven in all, all of which are contained in an outer half-crescent region outlined with thick lines in Fig. 4. It is to be noted, however, that all of these light figures are not always revealed by the side surfaces and, moreover, even all of the observed light figures can not always be applied to the accurate orientation determination but only those for which the angle between the normals of the corresponding crystal planes and the normal of the side surface lie in the above-mentioned limits expressed by Eq. (10) may be used. Further, since the orientation of the normal of the side surface may vary for the same orientation of the normal of the plate surface, the kind of light figures actually applicable to the accurate orientation determination or of accurately determinable orientation angles vary with orientations of the normals of the plate and side surfaces.

The orientations of the side-surface normal corresponding to orientations of the plate normal contained in the (100)-(110)-(111) triangle are in a crescent region connecting (001), $(0\bar{1}1)$, $(0\bar{1}0)$, $(0\bar{1}\bar{1})$, $(00\bar{1})$, $(01\bar{1})$, $(10\bar{1})$, $(2\bar{1}\bar{1})$, $(1\bar{1}0)$, $(1\bar{1}1)$ and $(00\bar{1})$ poles in Fig. 4, but it is sufficient, in view of the cubic symmetry, to take the above-mentioned half-crescent (001)-$(0\bar{1}1)$-$(0\bar{1}0)$-$(0\bar{1}\bar{1})$-$(1\bar{1}\bar{1})$-$(2\bar{1}\bar{1})$-$(1\bar{1}0)$-$(1\bar{1}1)$-(001) region. Let angles between the plate-surface normal and the [001], $[0\bar{1}0]$, $[0\bar{1}1]$, $[1\bar{1}0]$, $[0\bar{1}\bar{1}]$, $[1\bar{1}1]$ and $[1\bar{1}\bar{1}]$ directions, which may be determined by using the (001), $(0\bar{1}0)$, $(0\bar{1}1)$, $(1\bar{1}0)$ $(0\bar{1}\bar{1})$, $(1\bar{1}1)$ and $(1\bar{1}\bar{1})$ light figures, be denoted as α_3, α_4, β_4, β_5, β_6, γ_2, and γ_3, respectively. Then, since accurately determinable range of these angles are limited by the conditions similar to Eq. (10), the half-crescent orientation region of the side-surface normal may be divided into thirteen regions A∼M, as shown in Fig. 4. For crystals of which the side-surface normals lie in narrow regions B, F. H and K, two angles α_3 and β_4, α_4 and β_4, α_4 and β_5, or α_4 and β_6 can be determined accurately, for crystals in regions A, C, G, E, I, L and M, only one of α_3, β_4, α_4, γ_2, β_5, β_6 and γ_3 can be determined accurately, while for crystals in regions D and J, none can be determined accurately. Although individual orientation regions of the side-surface normal corresponding to each of orientation regions of I∼V of the plate-surface normal are not shown in Fig. 4, it may be seen that, for crystals of which the plate-surface normals lie in I, III and IV regions and the side-surface normals lie in regions other than D and J, at least two orientation angles can always be determined accurately and so the perfect orientation determination can be made, while, for crystals of which the plate-surface normals lie in V region and the side-surface normals lie in regions other than B and H the orientation can not be determined perfectly.

It is to be noted that the perfectly determinable orientation range can be enlarged further if light figures revealed by the end surfaces of the strip crystal are used together. Further, the perfect orientation determination may be conducted using only one light figure revealed by the plate surface of the strip crystal, but with a somewhat lower accuracy, if the indices of crystal planes or zone axes corresponding to special spots or streaks of the light figure are known.(6)

IV. Geometrical relationships required for the calculation of the three principal orientation angles from any two measured orientation angles

Since the orientation of a cubic crystal is usually represented by the three angles, α_1, α_2 and α_3, referred to the tetragonal axes [100], [010] and [001], α_1, α_2 and α_3 have to be calculated from the measured data of any two of angles α_1, α_3, α_4, β_1, β_4, β_5, β_6, γ_1, γ_2 and γ_3. Expressions for the geometrical relations required for this purpose are not given here, because most of them are the same as those required for the case of a cubic crystal rod(2) in which notations and signs of the angles are appropriately altered.

(6) M. Yamamoto and J. Watanabé, Nippon Kinzoku Gakkai-shi, **20** (1956), 85; Sci. Rep. RITU, **A9** (1957), 410.

It is to be noted, however, that, among angles $(\alpha_1)_n$, $(\alpha_2)_n$ and $(\alpha_3)_n$, and $(\alpha_1)_l$, $(\alpha_2)_l$ and $(\alpha_3)_l$ which the plate-surface normal and long axis of a strip crystal make with the tetragonal axes, the following relationship holds : —

$$\cos(\alpha_1)_n \cos(\alpha_1)_l + \cos(\alpha_2)_n \cos(\alpha_2)_l + \cos(\alpha_3)_n \cos(\alpha_3)_l = \cos\omega = 0, \qquad (11)$$

from which whether or not the orientation determination has been made accurately is seen (cf. Table 3) and the check of determination is also possible.

V. Stereographic representation of the orientations of the plate normal and long axis of a strip crystal

There are two ways for representing stereographically thus determined orientations of the plate normal and long axis of a strip crystal. In the first method, the orientations are plotted, with an aid of Wulff's net, on the (100) standard stereographic projection as points apart from the [010] and [001] axes by angular distances α_2 and α_3. As seen from Section III, if the orientation of the plate normal, N, is plotted as a point in the (100)-(110)-(111) triangle, that of the long axis, L, is necessarily represented as a point in the outer crescent region mentioned above (Fig. 5 (a)). The second method plots, by the use of the polar and Wulff's nets, the cube poles (100), (010) and (001) on a stereographic projection in which the plate normal of the strip crystal is at the center and the long axis corresponds to the north or south pole (Fig. 5 (b)). Of these two, the first is more convinient for mutual comparison since more orientations can be plotted on the same projection.

VI. Actual examples and accuracy of the orientation determination: the orientation determination of strip crystals of iron and silicon iron

As examples the above-described procedure was applied to the orientation determination of iron and 3.2 percent silicon-iron strip crystals. Iron crystal specimens, which were produced by the recrystallisation (strain-anneal) method(7) from Blogen iron, were 8 mm in width, 3.5 mm in thickness, and 10 cm in length and silicon-iron crystal specimens grown by the same method, were 4 mm in width, 1 mm in thickness, and 6~7 cm in length, the latter being the same as used previously for the light-figure experiments(8). It is to be noted that the specimens used were actually not monocrystalline but consist of several coarse grains connected along their length.

Both iron and silicon-iron crystal specimens were etched for two and half hours with concentrated hydrochloric acid. The reason why we dared such a long time etching is the necessity of getting equally distinct light figures with definite symmetric centers on all of the three principal crystal planes {100}, {110} and {111}.

(7) M. Yamamoto and R. Miyasawa, Nippon Kinzoku Gakkai-shi, **16** (1952), 300 ; Sci. Rep. RITU, **A5** (1953), 493.

(8) M. Yamamoto and J. Watanabé, Nippon Kinzoku Gakkai-shi, **16** (1952), 136.

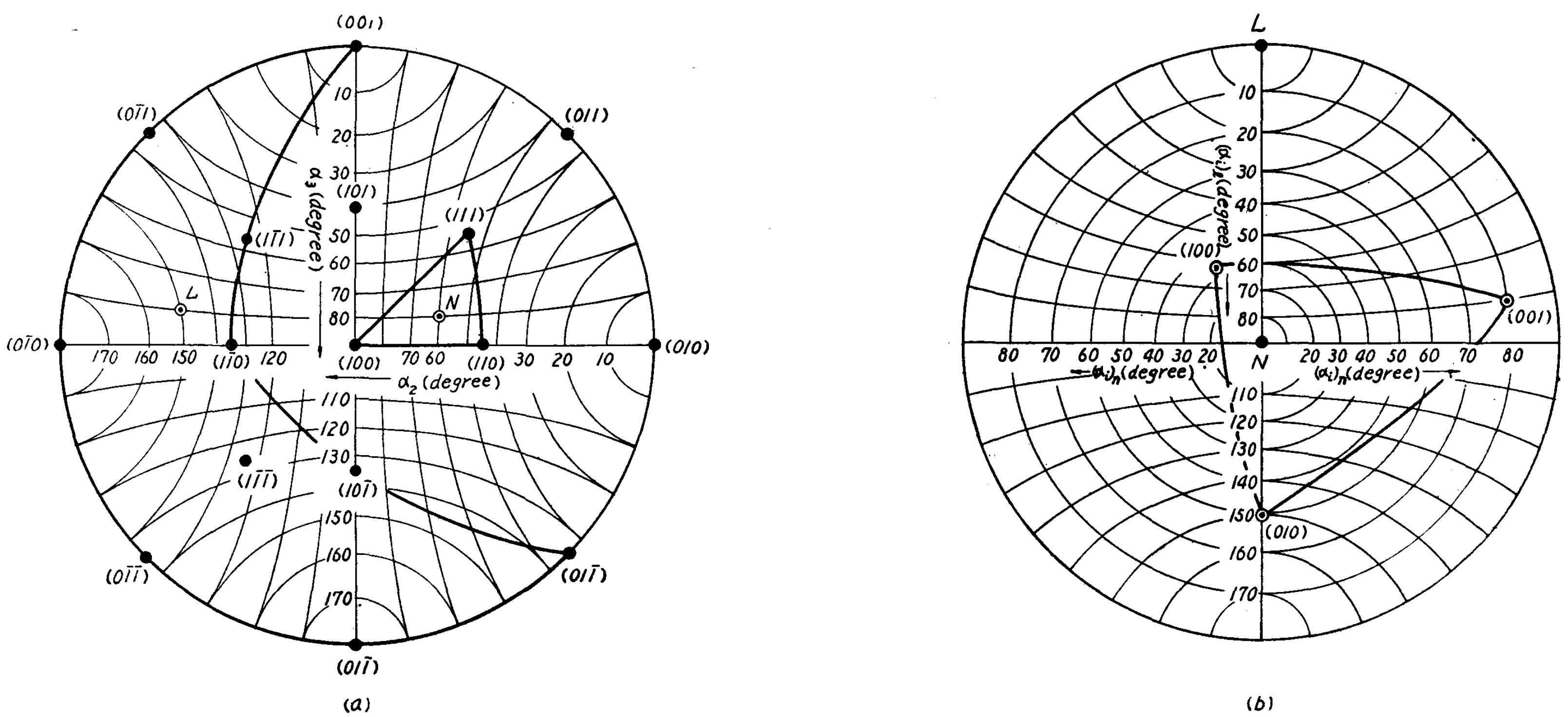
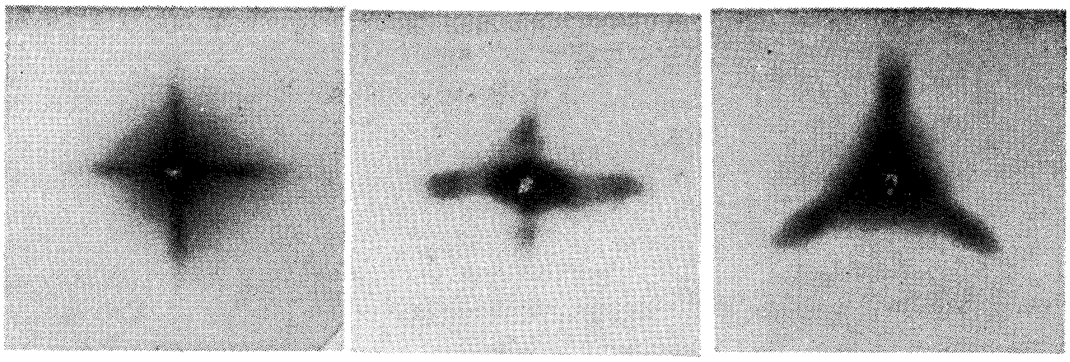


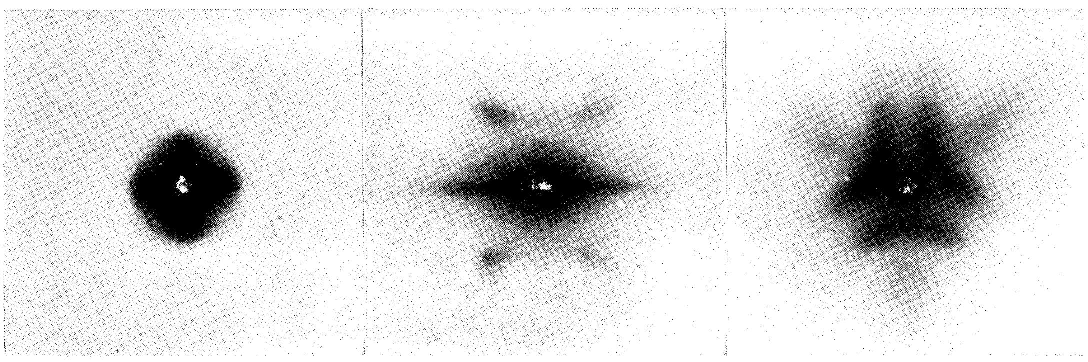
Fig. 5. Stereographic representation of the orientations of the plate normal (N) and long axis (L) of a cubic crystal strip. (a) using the standard stereographic projection on the (001) plane, and (b) using the stereographic projection on the plate surface of the crystal strip. Orientations N and L represent $(\alpha_1)_n = 32.0°$, $(\alpha_2)_n = 60°$ and $(\alpha_3)_n = 80°$, and $(\alpha_1)_l = 62.2°$, $(\alpha_2)_l = 150°$ and $(\alpha_3)_l = 80°$, respectively.

The light figures revealed by this etching are shown in Photos. 1 and 2. Thus etched specimen crystal was, then, set on the center of the vertical circle of a two-circle goniometer in such a way that its long axis may coincide with the rotation axis of the vertical circle (cf. Fig. 1 (c)).



(a) {100} (b) {110} (c) {111}

Photo. 1. Light figures produced by iron crystals etched with concentrated hydrochloric acid for two and half hours.



(a) {100} (b) {110} (c) {111}

Photo. 2. Light figures produced by silicon-iron crystals etched with concentrated hydrochloric acid for two and half hours.

First, the orientation angles, φ_0 and θ_0, of the plate normal with respect to the goniometer were determined. Actual examples as determined by the first and second procedures are shown in Table 1, from which it may be seen that the accuracy in the determination of φ_0 and θ_0 is within $\pm 0.1°$. The values measured after the deck-glass was re-sticked to the crystal surface is given in 1′ of Table 1(b), which indicates that the sticking of the deck-glass to the crystal surface does not introduce an error in the determination. Next, the orientation angles, φ_C and θ_C, of the normals of the principal crystal planes $\{hkl\}$ with respect to the goniometer were determined by using the $\{hkl\}$ light figures. The measurements were made thrice at one or two positions along the length of a specimen crystal and the mean values were taken as the final ones. As an example, the measured values with crystal grains contained in the same specimen crystals as those of Table 1 are shown in Table 2, which indicates that the determinations of φ_C and θ_C can be made with an accuracy within $\pm 0.5°$. Finally, the orientation angles $(\alpha_1)_n$, $(\alpha_2)_n$ and $(\alpha_3)_n$ of

Table 1. Two examples of the determination of orientation angles, φ_0 and θ_0, of the plate normal relative to the goniometer.

(a) Determination with the first procedure.*
Silicon iron, No. 2 (Length 6.0 cm, width, 3.9 mm, thickness 1 mm)

Measured point	θ_{00} (degree)			$\theta_{0\pi}$ (degree)			θ_0 degree
	Measured value	Mean Value	Final mean	Measured value	Mean value	Final mean	
1	189.6 .8 .4	189.6	189.6	9.8 .7 .5	9.7	9.6	99.6
2	189.4 .6 .6	189.5		9.6 .4 .6	9.5		

* φ_0 is essentially equal to 90°

(b) Determination with the second procedure.
Iron, No. 2 (Length 10.0 cm, width 7 mm, thickness 3.5 mm)

Measured point	φ_0 (degree)			θ_0 (degree)		
	Measured value	Mean value	Final mean	Measured value	Mean value	Final mean
1	90.6 .5 .7	90.6	90.6	282.9 .9 .8	282.9	283.0
1′	90.6 .6 .6	90.6		282.9 283.0 .0	283.0	

Table 2. Examples of the determination of orientation angles, φ_C anc θ_C, of the normals of the principal crystal planes $\{hkl\}$ relative to the goniometer.

(a) Silicon iron No. 2C (Length 0.8 cm)

(hkl)	φ_C (degree)		θ_C (degree)		Remark
	Measured value	Mean value	Measured value	Mean value	
(110)	93.4 .4 .7	93.5	101.6 .4 .4	101.5	$\varphi_0=90°$ $\theta_0=99.6°$ (in Table 1(a))
$(1\bar{1}0)$	80.2 .1 .2	80.2	190.0 189.6 .6	189.7	

(b) Iron No. 2B (Length 2.5 cm)

(hkl)	φ_C (degree)		θ_C (degree)		Remark
	Measured value	Mean value	Measured value	Mean value	
(110)	71.6 .5 .2	71.4	290.5 .6 .6	290.6	$\varphi_0=90.6°$ $\theta_0=283.0°$ (in Table 1(b))
$(1\bar{1}0)$	77.4 .5 .8	77.6	206.2 .2 .8	206.4	

Table 3. Various examples of orientations of single crystal strips of silicon iron and iron as determined by the light-figure method.

Crystal mark	φ_0 degree	θ_0 degree	Light figures used	Kind of measured angles	φ_C degree	θ_C degree	Orientation of the plate normal $(\alpha_1)_n$ degree	$(\alpha_2)_n$ degree	$(\alpha_3)_n$ degree	Orientation of the long axis $(\alpha_1)_l$ degree	$(\alpha_2)_l$ degree	$(\alpha_3)_l$ degree	ω degree
Fe-Si,2B	90*	181.4*	(100)	α_1	76.2	160.6	24.8	65.7	85.5	76.2	132.9	46.1	90.5
			(110)	β_1	108.2	192.2							
" ,2C	90*	181.4*	(110)	β_1	86.8	183.1	44.2	46.0	86.5	80.7	94.8	169.5	90.1
			$(1\bar{1}0)$	β_5	80.0	92.3							
" ,2D	90*	11.2*	(110)	β_1	82.1	14.1	41.8	49.2	82.5	52.0	114.9	131.7	90.0
			$(1\bar{1}1)$	γ_2	77.6	282.5							
Fe ,1C	89.8**	175.4**	(110)	β_1	73.4	173.1	38.4	55.1	76.0	86.8	117.2	27.6	90.2
			$(1\bar{1}\bar{1})$	γ_3	77.6	266.5							
" , 2B	90.6**	283.0**	(110)	β_1	71.4	290.6	34.6	60.0	74.4	86.3	67.2	156.9	90.0
			$(1\bar{1}0)$	β_5	103.2	206.4							
" ,4	90.2**	280.6**	(100)	α_1	80.0	290.6	10.2	79.9	88.9	80.0	169.6	92.7	90.6
			$(0\bar{1}0)$	α_4	87.3	189.5							

* Determined by the first procedure. ** Determined by the second procedure.

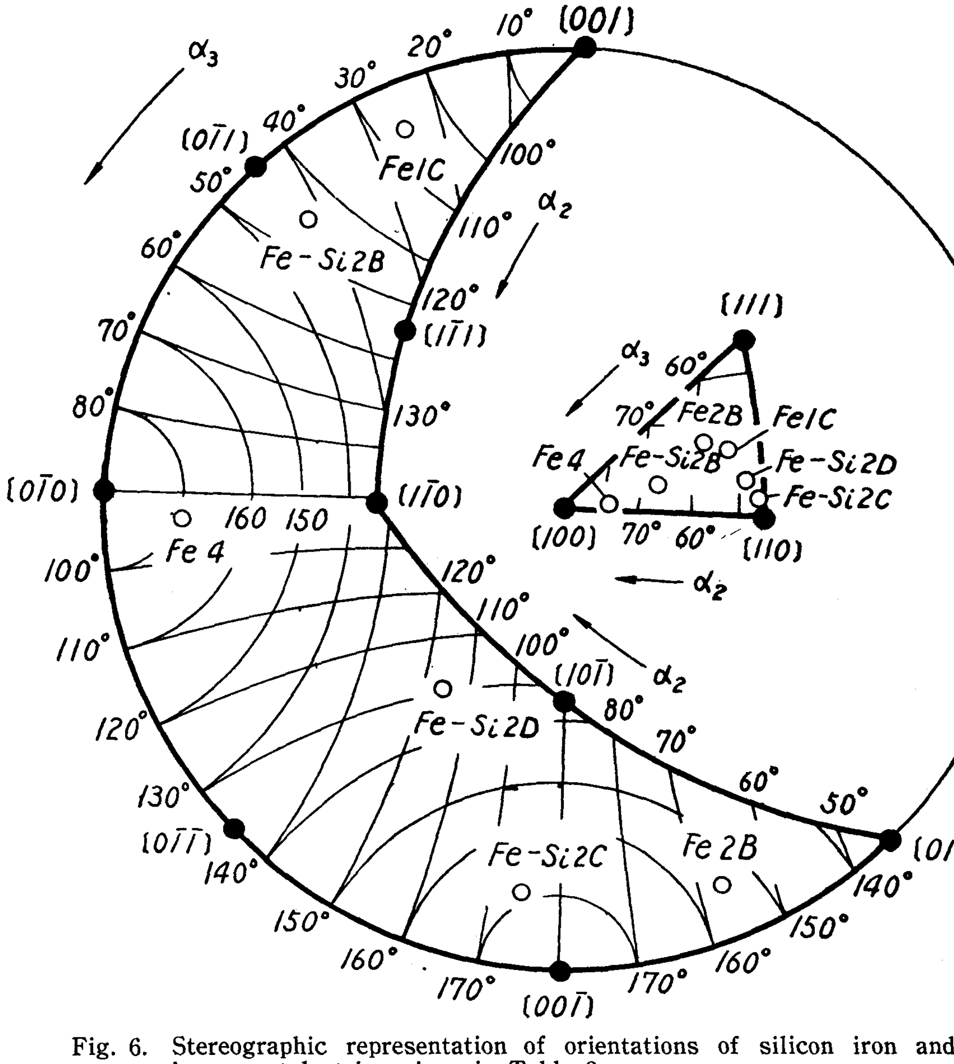


Fig. 6. Stereographic representation of orientations of silicon iron and iron crystal strips given in Table 3.

the plate normal and $(\alpha_1)_l$, $(\alpha_2)_l$ and $(\alpha_3)_l$ of the long axis referred to the tetragonal axes were calculated from the data of θ_0, φ_0, θ_C and φ_C thus obtained.

Various examples of orientation determinations are given in Table 3. For example, for Fe-Si, No. 2B crystal, the orientation angles of the plate normal, φ_0 and θ_0, were determined by the first procedure and the orientation angles, φ_C and θ_C, of the normals of (100) and (110) planes were measured using (100) and (110) light figures, respectively. Then, orientations of the plate normal and long axis, $(\alpha_i)_n$ and $(\alpha_i)_l$ $(i = 1, 2, 3)$, were computed from geometrical relations using values of $(\alpha_1)_n$ and $(\beta_1)_n$ as calculated by means of Eq. (6) and those of $(\alpha_1)_l$ and $(\beta_1)_l$ which were the measured values of φ_C themselves. The value of ω calculated from Eq. (11) is 90.5°, being nearly equal to the theoretical value 90°. Fig. 6 shows the stereographic representation of crystal orientations given in Table 3.

Summary

The application of the light-figure method to the orientation determination of cubic metal single crystal plates, particularly strips, has been investigated. It has been shown that when a cubic crystal strip is mounted on the two-circle goniometer in such a manner that its long axis may coincide with the rotation axis of the vertical circle of the goniometer, the angle δ which the plate normal of the crystal strip makes with the normal of a certain crystal plane is given by

$$\cos\delta = \cos\varphi_0\cos\varphi_C + \sin\varphi_0\sin\varphi_C\cos(\theta_0\sim\theta_C)\,,$$

where φ_0 and θ_0 or φ_C and θ_C are the readings of the horizontal and vertical circles of the goniometer when the plate normal or the crystal plane concerned coincides with the direction of the incident light beam, the angle between the long axis of the crystal strip and the crystal plane concerned being given by φ_C. For the determination of φ_0 and θ_0 we have devised two procedures with the accuracy within $\pm 0\,1°$.

Since the accuracy of the determination of φ_C and θ_C by a light figure is influenced by an inclination of a crystal plane corresponding to the light figure to the crystal surface, the kind of light figures applicable to the accurate orientation determination or of accurately determinable orientation angles varies with orientation to be determined. For cubic strip crystals, the ranges of orientation angles of the plate normal of the crystal strip which can be determined accurately by means of the {100}, {110}, and {111} light figures are, in practice, approximately 0°~30°, 0°~20° and 0°~10°, respectively. It follows, then, that if the (100), (110) and (111) light figures revealed by the plate surface are used, the perfect orientation determination of the plate normal can not be made for most orientations except for ones lying in a narrow region lying intermediately on the (100)-(110) side of the (100)-(110)-(111) stereographic triangle. But, the perfectly determinable range of orientation of the plate normal can be extended to nearly the entire orientation range, if we use also light figures revealed by the side surface of the crystal strip, namely the (001), $(0\bar{1}0)$, $(0\bar{1}1)$, $(1\bar{1}0)$, $(0\bar{1}\bar{1})$, $(1\bar{1}1)$ and $(1\bar{1}\bar{1})$ light figures, although the ac-

curately determinable ranges of orientation angles corresponding to these light figures are also limited similarly to the {100}, {110} and {111} light figures.

The procedure of orientation determination has been illustrated with reference to actual examples of crystal strips of iron and 3 percent silicon-iron, and it has been shown that the orientation determination can be conducted with a precision within $\pm 0.5°$. An account has also been given of the geometrical relations necessary for the normal representation and of the methods of stereographic represntation, of the orientations of the plate normal and of the long axis of the crystal strip.