

Measurement of Dielectric Constant of Germanium at Microwave Frequencies*

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Synopsis

Dielectric constants and electrical conductivity of germanium were measured at microwave frequencies in the temperature range from 1.6° to 300°K by the resonant cavity method. The observed values of dielectric constants showed the size effect, and were corrected by taking depolarizing field into account. The imaginary or reactive part of the conductivity was shown apparently as the reduction of the effective dielectric constant. The temperature dependence of the relaxation time of charge carriers was estimated by using the simple classical relaxation formula.

I. Introduction

The temperature dependence of dielectric constant of germanium at microwave frequencies has been reported by several workers^{(1)~(5)}. So far as known, however, the measurement at low temperatures has not been reported. As the temperature dependence would give us many informations concerning (1) the relaxation time of charge carriers, (2) the plasma effects and (3) the impurity band conduction, its measurements would be of value for the study of the conduction mechanism in semiconductors. The reason why there had been no such experiment was due to the difficulty of temperature regulation of waveguide systems. In the present experiment, the difficulties were overcome by the use of a resonant cavity instead of the waveguide, which was an extension of the work on the galvanomagnetic effect of germanium by Fukuroi and Date⁽⁶⁾. As is well known, some improvements in the experimental technique are necessary to obtain the precise values of relatively large dielectric constants by the cavity method.

II. Experimental method

1. Principles of the measurement

According to the theory of microwave circuits⁽⁷⁾ by Slater, the equation

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(1) T. S. Benedict and W. Shockley, *Phys. Rev.*, **89** (1953), 1152.

(2) T. S. Benedict, *Phys. Rev.*, **91** (1953), 1565.

(3) F. D'Altroy and H. Y. Fan, *Phys. Rev.*, **94** (1954), 1415.

(4) F. D'Altroy and H. Y. Fan, *Phys. Rev.*, **98** (1955), 1561.

(5) J. M. Gordy and S. C. Brown, *ibid.*, 1761.

(6) T. Fukuroi and M. Date. *Sci. Rep. RITU*, **A9** (1957), 190; hereafter referred to as F.D.

(7) J. C. Slater, *Rev. Mod. Phys.* **18** (1946), 441.

showing the effects of dielectrics and conductors on the character of a resonant cavity is as follows:

$$\frac{1}{Q_s} - 2j \frac{\Delta\omega_a}{\omega_a} = \frac{1}{\varepsilon_0\omega_a} \frac{\int \mathbf{J} \cdot \mathbf{E}_a d\mathbf{v}}{\int \mathbf{E} \cdot \mathbf{E}_a d\mathbf{v}}, \quad (1)$$

where Q_s is the Q -value due to the sample, ω_a one of the proper angular frequencies of the cavity, ε_0 the ε of the vacuum ($(1/36\pi) \times 10^{-9} \text{ Fm}^{-1}$ in the M. K. S. rationalized units), \mathbf{E} the electric field in the cavity, \mathbf{J} the current density, and \mathbf{E}_a the normalized function belonging to ω_a ; the integral should be carried out in the whole volume inside the cavity.

If a mode having an angular frequency ω is excited in the resonator, the current density will be given by the next equation:

$$\mathbf{J} = \sigma \mathbf{E} + j\omega(\varepsilon - \varepsilon_0)\mathbf{E}. \quad (2)$$

Slater took the electric field in the sample as

$$\mathbf{E} \approx \mathbf{E}_a \int \mathbf{E} \cdot \mathbf{E}_a d\mathbf{v} \quad (3)$$

to a first approximation.

Inserting Eqs. (2) and (3) into (1), one obtains

$$\frac{1}{Q_s} - 2j \frac{\Delta\omega_a}{\omega_a} = \frac{1}{\varepsilon_0\omega_a} \int (\sigma + j\omega(\varepsilon - \varepsilon_0)) \mathbf{E}_a^2 d\mathbf{v}. \quad (4)$$

Provided ε and σ have constant values throughout the sample, Eq. (4) becomes

$$\frac{1}{Q_s} - 2j \frac{\Delta\omega_a}{\omega_a} = \frac{1}{\varepsilon_0\omega_a} (\sigma + j\omega(\varepsilon - \varepsilon_0)) \int_{\text{Sample}} \mathbf{E}_a^2 d\mathbf{v}. \quad (4')$$

Okamura, Fujimura and Date⁽⁸⁾ performed more detailed calculation of the relation (4') and gave the relations which connect ε to the variation in the length of a cavity, Δl , and Q_s to the variation of Q -value of the cavity respectively, when a rectangular cavity was excited in its TE_{10n} mode. Their results were used in the present work to a first approximation.

2. Experimental setups

The arrangement of experimental apparatus was similar to those by O.F.D.⁽⁸⁾ and by F.D.⁽⁶⁾, and the block diagram is shown in Fig. 1. The microwave power having a frequency of about 9400 Mc/sec was fed from a klystron into the cavity through a coaxial cable made of stainless steel. The resonator was excited in its TE_{104} mode and its unloaded Q -value was near 900. The transmitted power detected by a crystal detector and amplified by an audio amplifier was fed into the Y-axis of an oscilloscope. The X-axis was swept by saw-tooth waves which were also supplied to the repeller of a klystron 2K25. Thus, the Q -curve could be observed directly, which was used as an indicator of shift of the characteristic

(8) T. Okamura, T. Fujimura and M. Date, Sci. Rep. RITU, A4 (1952), 191; hereafter referred to as O. F. D.

frequency of the cavity caused by the sample.

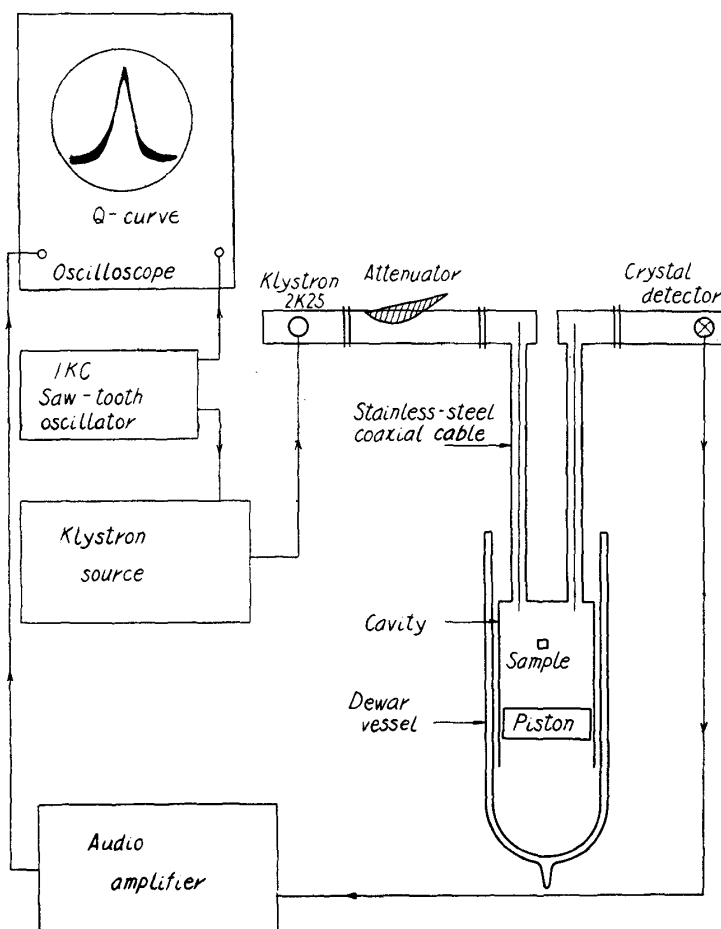


Fig. 1. Block diagram of the apparatus.

The cavity and the system relating to it are shown in Fig. 2. Two heaters were used to produce intermediate temperatures. The temperature of the sample was measured with a carbon resistor below 77°K. The uniformity of the temperature over the specimen was secured by using a can covering the cavity, the cover being made of copper sheet 0.5 mm in thickness. The variation in the length of the cavity could be read with the accuracy of ± 0.005 mm, and the frequency shift caused by germanium samples could be compensated by displacing the piston by 0.5 mm. So, the errors in the measurement of Δl were less than ± 1 per cent.

III. Depolarization corrections

As a preliminary measurement, the dielectric constants of polystyrol rods, were measured and calculated using the formula given by O.F.D. As shown in Table 1, the size effect was observed, i.e., the thicker the sample the smaller the measured value. There are two possible corrections with which the same value seems to be able to be obtained for various specimens: one comes from the approximation made by O.F.D. and the other by Slater. O.F.D.'s approximation is

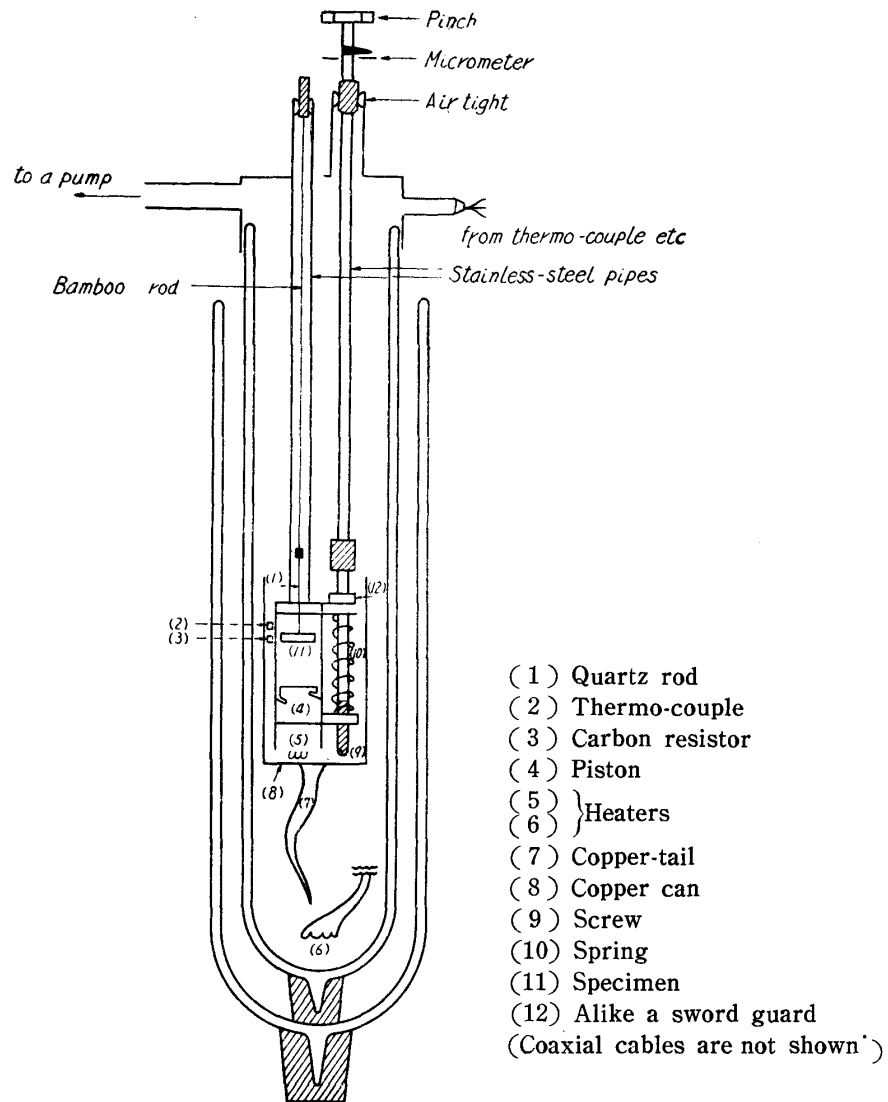


Fig. 2. Experimental setups inside the cryostat.

Table 1. Uncorrected values of polystyrol rods.

No.	Size	Order of Δl in TE_{1013}	Calculated after O.F.D.
	mm	mm	
1	9.2 × 3.1 × 2.8 (thicker)	2.0	2.51 ± 0.02*
2	7.3 × 1.8 × 1.0 (middle)	0.3	2.63 ± 0.05
3	7.2 × 1.6 × 0.5 (thinner)	0.1	2.67 ± 0.05

* "±0.02" shows the probable error.

as follows.

In calculating the integral

$$\int_{\text{Sample}} E_a^2 dv \quad (5)$$

in Eq. (4'), they assumed that throughout the sample E_a is given approximately by the maximum value of E_a . It is clear that this assumption leads to the smaller

values of ϵ for the thicker samples. The correct calculation of the quantity (5) is an elementary integration, and the result shows

$$\epsilon_{(\text{corrected})}^{-1} = \{\epsilon_{(\text{uncorrected})}^{-1}\} \frac{2}{1 + \frac{\lambda_g}{2\pi C} \sin \frac{2\pi C}{\lambda_g}} \cdot \frac{2}{1 + \frac{a}{\pi A} \sin \frac{\pi A}{a}}, \quad (6)$$

where A and C denote the sample size. Corrections by the above equation are shown in the third column of Table 2. The size effect is not completely cor-

Table 2. Corrected values of polystyrol rods.

No.	Uncorrected	Using the correct values of $\int E_a^2 dv$	After depolarization corrections
1 (thicker)	2.51	2.55	2.68 ± 0.02
2 (middle)	2.63	2.63	2.68 ± 0.05
3 (thinner)	2.67	2.67	2.70 ± 0.05

rected by it. However, for the reason that the correction is less important as germanium samples are thinner than the polystyrol ones, it will not be touched hereafter. Slater's approximation is expressed by Eq.(3), in which he assumed that the electric field in the sample was the same as that without specimen. In other words, the effect of depolarizing field is ignored together with the effects of higher modes. Taking the depolarizing field into account, the field inside the sample should be written as follows: ⁽⁹⁾

$$E = \frac{E_a}{1 + A(\epsilon/\epsilon_0 - 1)} \int E \cdot E_a dv. \quad (7)$$

Then, equating the electric polarization represented by corrected dielectric constants to that by uncorrected ones (they are written as ϵ and ϵ^* respectively), we obtain

$$\left(\frac{\epsilon^*}{\epsilon_0} - 1\right) E_a \int E \cdot E_a dv = \frac{(\epsilon/\epsilon_0 - 1) E_a \int E \cdot E_a dv}{1 + A(\epsilon/\epsilon_0 - 1)}. \quad (8)$$

Accordingly, the following relation will be obtained:

$$\frac{\epsilon}{\epsilon_0} - 1 = \frac{\frac{\epsilon^*}{\epsilon_0} - 1}{1 - A\left(\frac{\epsilon^*}{\epsilon_0} - 1\right)}, \quad (9)$$

in which A is the effective depolarizing factor. To a first approximation, the A of cylinder having the same cross section⁽¹⁰⁾ was adopted as that of the sample. The corrected values show no longer the size effect, and are listed in the fourth column of Table 2. It is quite natural to conclude that no further correction depending on the sample size is needed within an accuracy of $\pm 0.2 \sim \pm 0.5$. In the case of germanium the size effect due to the depolarizing

(9) J. A. Stratton, *Electromagnetic Theory*, New York (1941), 212.

(10) R. M. Bozorth, *Ferromagnetism*, New York (1951), 846.

field was more pronounced and the depolarization correction was essential in order to obtain a plausible value from the experimental results. Some examples are shown in Table 3, and the temperature dependence of some specimens in Table 3 are shown in Fig. 3. The samples were cut from an intrinsic-grade (53.8 Ω cm at 19.8°C) single crystal of p-type, and dielectric constants in $\langle 100 \rangle$ directions were measured in the temperature range from 1.7 to 30°K, in which the dielectric constants of the samples are constant.

Table 3. Size effect in germanium samples.

No.	Size	Uncorrected	Corrected
1	mm 6.8 × (0.4) ² thinner	13.7 ± 0.3	14.2 ± 0.3
2	8.8 × (0.6) ² middle	13.3	14.1 ± 0.3
3	7.3 × 1.2 × 0.7 thicker	11.8	14.0 ± 0.3

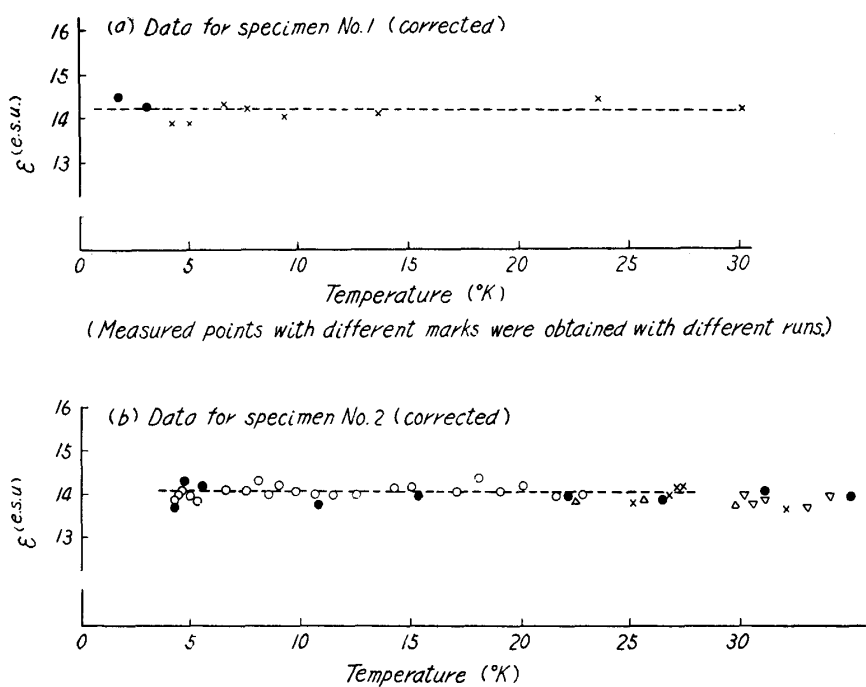


Fig. 3. Examples of the data listed in Table 3.

IV. Experimental results

Before describing the temperature dependence of the dielectric constant of a germanium specimen, the results of a few preliminary measurements will be shown. First, the measured temperature dependence of the dielectric constant of liquid helium showed a quite good agreement with that of Grebenkemper and Hagen⁽¹¹⁾ as shown in Fig. 4. The molar polarizability calculated by using the values of the density obtained by Kerr⁽¹²⁾ was $0.1247 \pm 0.0015 \text{ cm}^3 \text{ mol}^{-1}$,

(11) C. J. Grebenkemper and J. P. Hagen, Phys. Rev., **80** (1950), 89

(12) E. G. Kerr, Jour. Chem. Phys., **26** (1957), 511.

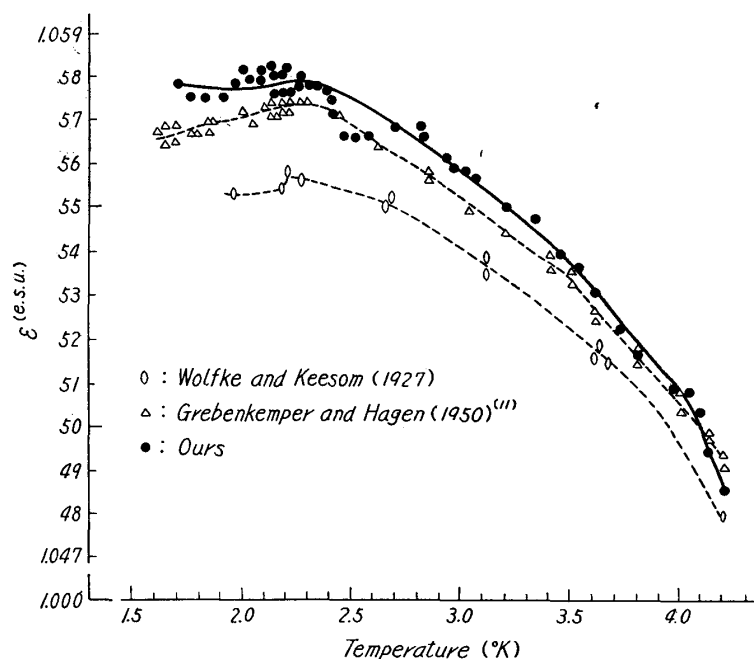


Fig. 4. Temperature dependence of dielectric constant of liquid helium.

when converted into optical frequencies, and it was also in good agreement with that reported by Edwards⁽¹³⁾ (the error was less than 1 per cent). It shows that there was no essential error in the present measurements. Second, the dielectric constants of various samples of germanium and silicon were measured at very

Table 4. ϵ of various Ge and a Si sample.

No.	Material	Characteristics	Temperature	ϵ
1	Ge	single crystal, p-type, $53.8 \Omega \text{cm}$ at 19.8°C , $\langle 100 \rangle$ direction, used to observe the size effect	$1.7 \sim 35^\circ \text{K}$	14.2 ± 0.3
2	Ge	polycrystals, p-type, $61.8 \Omega \text{cm}$ at 19.9°C , temperature dependence will be reported	$1.6 \sim 5^\circ \text{K}$	14.6 ± 0.3
3	Ge	Sb and Ni doped single crystal, p-type $0.99 \Omega \text{cm}$ at 20.7°K	$4.3 \sim 32^\circ \text{K}$	15.1 ± 0.2
4	Ge	Ga doped single crystal, $0.11 \Omega \text{cm}$ at 18.5°C	$4.3 \sim 4.6^\circ \text{K}$	20.4 ± 0.3
5	Si	single crystal, $0.12 \Omega \text{cm}$ at 20.7°C	$4.3 \sim 19^\circ \text{K}$	11.2 ± 0.2

low temperatures. The results are shown in Table 4, and for example, the data for silicon are shown in Fig. 5. The polarizability of gallium impurity calculated from the value of the sample No. 4 is $1.1 \times 10^{17} \text{cm}^3/\text{atom}$, which agrees with that reported by D'Altroy and Fan⁽¹⁴⁾.

(13) M. H. Edwards, *Canad. Jour. Phys.*, **36** (1958), 884.

(14) F. D'Altroy and H. Y. Fan, *Phys. Rev.*, **103** (1956), 1671.

Now, the temperature dependence of the effective dielectric constant of a germanium sample will be stated. The variation of effective dielectric constant

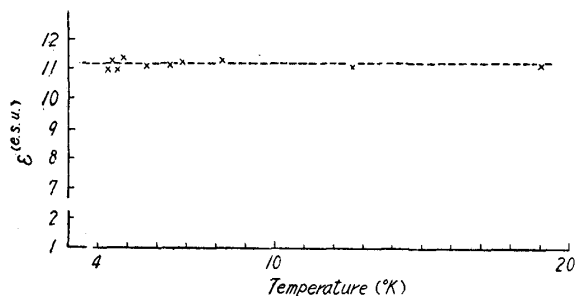


Fig. 5. Dielectric constant of Si.

is plotted as a function of temperature in Fig. 6. The reason why the notation K is adopted for the dielectric constant in place of ϵ is as follows: In the rf electric field, the polarization due to bound electrons and the current due to free carriers in the specimen have imaginary or out-of-phase parts, respectively.

They are represented by the imaginary parts of ϵ and σ respectively;

$$\epsilon = \epsilon' - j\epsilon'' \quad (10)$$

and

$$\sigma = \sigma' - j\sigma'' \quad (11)$$

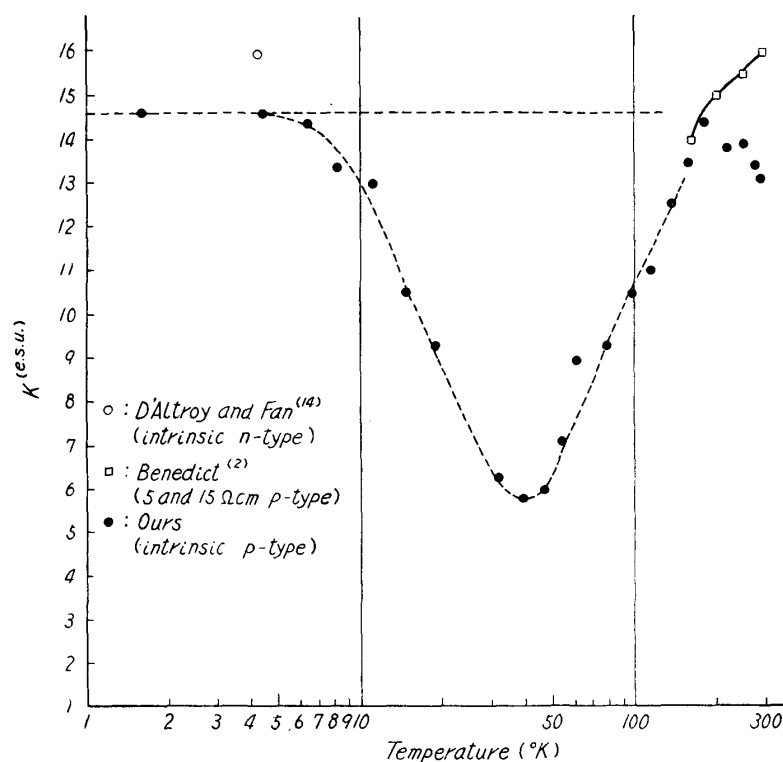


Fig. 6. Temperature dependence of dielectric constant K of a Ge sample.

It is clear from Eq. (4') that, by the present experiment,

$$\sigma' + \omega\epsilon'' \quad (12)$$

or

$$\epsilon' - \frac{\sigma''}{\omega} \quad (13)$$

cannot be separated into two parts. Physically, it signifies that the decrease in

Q -value is caused by a dielectric loss as well as by a current loss, and that the frequency shift is caused by reactive motion of carriers as well as the dielectric polarization. It should be noticed that the reactive part of the steady motion of the free electrons is opposite to that of the bound electrons. So, the quantity (13) may be called the "effective dielectric constant" K . The decrease appearing in Fig. 6 will be discussed below.

V. Discussion

Let us consider the physical nature of the temperature dependence of the effective dielectric constant shown in Fig. 6. As stated in the previous section, the dielectric constant ϵ and the electric conductivity σ of the semiconductors owe their origins to the bound electrons and the free carriers respectively. There are in the semiconductor the bound electrons which can be specified into three groups: (1) the core electrons, (2) the valence electrons, and (3) the electrons or the holes bound to the impurities. ϵ of the core- and the valence-electrons may have no imaginary part in ϵ as their excitation needs a frequency in the infra-red region. Their temperature dependence is small at low temperatures, at which the thermal expansion can be neglected. Electrons trapped by impurity vary in number with temperature, and its effect would change the dielectric constant in the case of the specimen having a considerable amount of impurities. However, in the present specimen, being cut from a block of germanium polycrystals which shows the intrinsic conductivity at room temperature, the active impurities are estimated to be less than $10^{14}/\text{cc}$. Therefore, it can easily be shown that the above-mentioned effect may be small. The effect of conduction electrons or holes is as follows: If one assumes a simple relaxation time τ and a simple effective mass m^* , the out-of-phase motion of carriers will be specified by the following equations⁽¹⁵⁾

$$\sigma' = \frac{\sigma_0}{1 + (\omega\tau)^2} \quad (13)$$

and

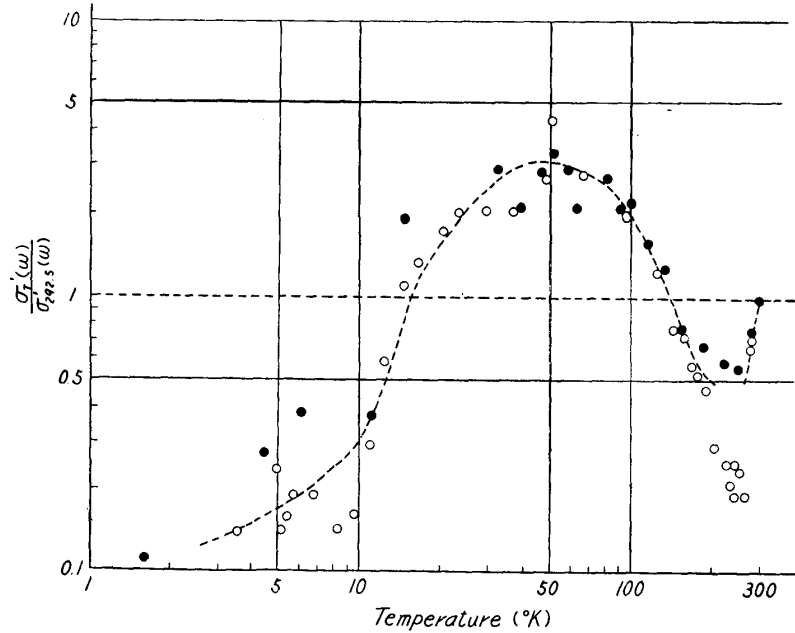
$$\sigma'' = \frac{\omega\tau\sigma_0}{1 + (\omega\tau)^2}, \quad (14)$$

where

$$\sigma_0 = \frac{ne^2\tau}{m^*}. \quad (15)$$

The decrease in K seems to come from Eq. (14). In order to confirm this view, τ was calculated from K and σ' which was measured by using the ratio of transmitted power at various temperatures to that of 292.5°K. The calculated values of σ' are shown in Fig. 7. The calculation of τ is as follows: ϵ' is

(15) F. Seitz and D. Turnbull, *Solid State Physics*, 1, New York, (1955), 283 (H. Y. Fan, Valence Semiconductors, Germanium and Silicon)



(Measured points with different marks were obtained at different runs.)

Fig. 7. Temperature dependence of σ' of the Ge sample.

assumed to be of constant value 14.6 at low temperatures, that is,

$$14.6 - K = \frac{\sigma''}{\omega \epsilon_0}. \quad (16)$$

As easily be seen from Eqs. (13) and (14), the following relation holds

$$\frac{\sigma''}{\omega} = \tau \sigma'. \quad (17)$$

Then, τ will be obtained by the next calculation

$$\begin{aligned} \tau &= \frac{1}{\sigma'} \frac{\sigma''}{\omega} \\ &= \frac{1}{\sigma'} \cdot \epsilon_0 \left(\frac{\sigma''}{\omega \epsilon_0} \right) \\ &= 0.62 \times \frac{\sigma''}{\sigma' T' \sigma'_{292.5}}, \end{aligned} \quad (18)$$

where the following relations are used

$$\sigma'_{T'}(\omega) / \sigma'_{292.5}(\omega) \doteq \sigma'_{T'}(\omega) / \sigma'_{292.5}(0) \quad (19)$$

and

$$\begin{aligned} \sigma'_{292.5}(0) &= \frac{1}{70} \text{ } \bar{\sigma} \text{ cm}^{-1} \\ &= \frac{10}{7} \bar{\sigma} \text{ m}^{-1}. \end{aligned} \quad (20)$$

The calculated values of τ by using the smooth curve in Fig. 7 are shown in Fig. 8, in which the temperature dependence is reasonable at low temperatures.

Accordingly, it is plausible to conclude that the decrease in K is the effect of free carriers.

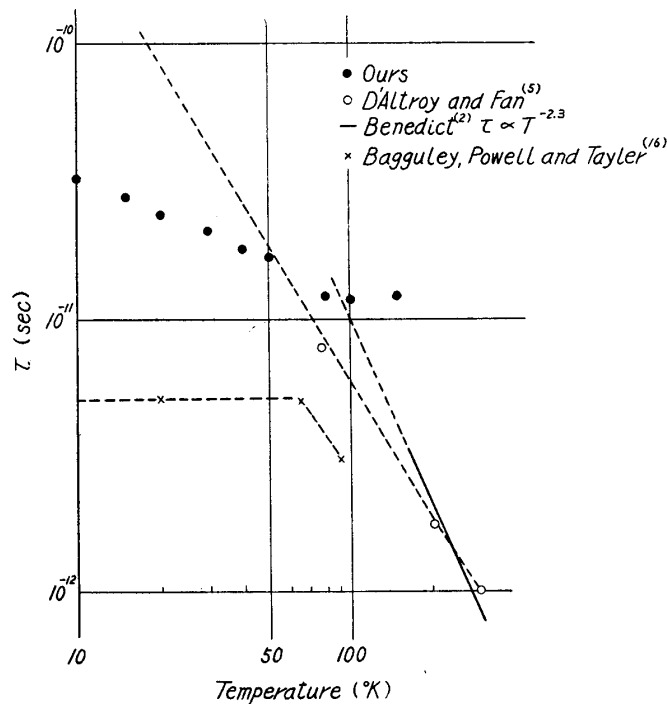


Fig. 8. Calculated temperature dependence of relaxation time τ .

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(16) D. M. S. Bagguley, J. A. Powell and D. J. Tayler, Proc. Phys. Soc., **70A** (1957), 759.