Sci. Rep. RITU A42 (1996) pp. 359-364
No. 2, May 1996

Thermal Conductivity and Scattering Mechanisms in High-T_c Oxide Superconductors *

K.Noto[a], M.Matsukawa[a], K.Iwasaki[a], K.Watanabe[b], T.Sasaki[b], and N.Kobayashi[b]

[a] *Faculty of Engineering, Iwate University, 4-3-5 Ueda, Morioka 020*
[b] *Institute for Materials Research, Tohoku University, Sendai 980-77, Japan*

(Received February 26, 1996)

Scattering mechanisms of a c-axis aligned $(Bi,Pb)_2Sr_2Ca_2Cu_3O_y$ have been studied by a numerical analysis based on a model in which two scattering mechanisms, phonon-electron scattering and electron-electron scattering, are coexisting as the main cause of the thermal conductivity enhancement below the superconducting transition temperature, T_c. We found a fairly good agreement between experimental results of temperature dependence and magnetic field dependence of the thermal conductivity and the coexisting model, as well as the model with electron-electron scattering (Proc.US-Japan WS on J_c in high T_c Oxide Superconductors (Oct.1995,Tsukuba): to be published), suggesting the coexistence of above mentioned two scattering mechanisms.

KEYWORDS: thermal conductivity, BPSCCO, scattering mechanism, BRT theory, quasiparticle damping

1. Introduction

Thermal transport studies on metals and alloys give important informations about thermal carriers such as phonons and electrons, and their scattering processes.[1,2] Electronic transport properties of superconductors, i.e. dc conductivity and Seebeck coefficient, vanish in the superconducting state, while the thermal conductivity has been observed even in such a state. Thus, the thermal conductivity measurement is a significant probe to examine scattering mechanisms not only in the normal state but also in the superconducting state.

Recently, a number of studies on the thermal conductivity κ of high-T_c oxide superconductors have been reported.[3~5] A common feature in their studies except for the 2-1-4 systems such as $La_{2-x}Sr_xCuO_4$ and $Nd_{2-x}Ce_xCuO_4$ crystals is that high-T_c oxide superconductors show an anomaly in the thermal conductivity associated with the superconducting transition. The origin of such an anomaly in κ is not made clear yet at present and two-types of scenarios have been mainly presented to interpret the observed anomaly. One scenario is essentially concerned with the Bardeen, Rickayzen and Tewordt (BRT) theory proposed firstly for the conventional superconductors, which attributes the anomaly in thermal conductivity to the phonon thermal conductivity.[6,7] In the superconducting state, quasiparticles as thermal carriers are condensed into Cooper pairs and the quasiparticle number rapidly decrease, so that the electron thermal conductivity decreases below T_c.[8] When the electron-phonon interaction is operative and the phonon component in κ is dominant, a rapid decrease in the quasiparticle number gives rise to a large reduction in the scattering cross section of phonons and the an observed thermal conductivity shows an enhancement in the superconducting state. On the other hand, an alternative scenario is based on an unconventional idea firstly proposed by Yu *et al*[9] to explain the anomaly in the *ab*-plane κ of the untwinned $YBa_2Cu_3O_{7-x}$ (YBCO) single crystal. According to their idea, the damping rate in quasiparticle is strongly suppressed in the superconducting state, in other words, the scattering cross section of quasi-particles decreases more remarkably in comparison with a decrease in the quasiparticle number. As a result, the electronic thermal conductivity shows a peak below T_c. On the background of this idea, there is a fact of experimental results in which *ac* conductivity of the YBCO and $Bi_2Sr_2CaCu_2O_y$ (BSCCO or Bi-2212) systems also show a rapid increase below T_c similar to the thermal conductivity.[10,11] In addition, the marginal Fermi-liquid phenomenology proposed by Varma *et al*.[12] has previously predicted such behaviours both in the ac conductivity and in the electronic thermal conductivity. The former approach focuses the origin of the anomaly on the phonon component, while the latter ascribes it to the electronic component.

For type-II superconductors in a magnetic field higher than their lower critical fields, thermal carriers interact with quasiparticles within the normal region produced by an external field and are scattered by quasiparticles in the normal cores.[13,14] Thus, the thermal conductivity study in magnetic fields gives further information about the vortex state of the superconductor and scattering mechanisms. The thermal conductivity of high-T_c oxide superconductors in a magnetic field has been investigated

* IMR Report No. 2046

to clarify extraordinary properties in the vortex-state of such materials.[15,16] In particular, a precise study on the angular dependence of κ in a field strongly suggests that the BSCCO system is a highly anisotropic or a quasi-2D superconductor which is consistent with the electronic transport study or the superconducting fluctuation study in the specific heat.[17~19] Moreover, the vortex-state thermal conductivity is considered to be a powerful tool to approach the origin of a peak in κ since the interactions of thermal carriers with quasiparticles in the vortex cores gives information about electron-phonon interaction or electron-electron interaction.

In this paper, scattering mechanisms in the thermal conductivity of a c-axis aligned $(Bi,Pb)_2Sr_2Ca_2Cu_3O_y$ crystal has been reexamined by a numerical analysis based on a model in which two scattering mechanisms, phonon-electron scattering and electron-electron scattering, are coexisting for the main cause of the thermal conductivity enhancement below the superconducting transition temperature, T_c, succeeding to the pervious report.[20]

2. Theoretical background

A. Analysis based on the conventional superconductor model

The measured thermal conductivity is written as a sum of the electronic and phonon components, $\kappa = \kappa_e + \kappa_{ph}$. The normal-state electronic thermal conductivity κ_e^n is estimated from the electric conductivity data using the Wiedemann-Franz (W-F) law, $\kappa_e^n = L_0 \sigma T$, where L_0 and σ are Lorenz number and the electrical conductivity. The value of κ_e^n shows almost no dependence on temperature because the resistivity of the BPSCCO sample varies linearly with temperature from room temperature down to $T_{c\ onset}$. The estimated value of κ_e^n becomes ~7 mW/cmK at 150 K. This indicates that 25 % of the total thermal conductivity is made by electrons at this temperature. The superconducting electronic thermal conductivity κ_e^s is calculated on the basis of the Kadanoff-Martin (K-M) theory ;

$$\bar{\kappa}^s_e = \frac{3}{2\pi^2} t \int_0^\infty d\varepsilon\, \varepsilon^2 \operatorname{sech}^2(E/2) \frac{1+a}{\varepsilon/E + at^n} \tag{1}$$

with $E=\sqrt{\varepsilon^2+\Delta^2}$, where $\bar{\kappa}_e^s$ is normalized value by the electronic thermal conductivity at T_c.[8] The variables of ε and $\Delta(T)$ normalized by kT denote quasiparticle energy measured from the Fermi-level and the superconducting energy gap according to the Bardeen, Cooper and Schrieffer (BCS) theory. The denominator in integrand of eq.(1) represents the scattering rate of quasiparticles. The first term in the denominator means the residual scattering rate of quasiparticles due to impurities. In the second term, the parameter a represents the strength of a power-law scattering rate to the residual scattering rate. If $n = 3$, a power law of t^3 in the second term of eq.(1) is reduced to the scattering rate of quasiparticles due to phonons and then the parameter of a describes thermal resistivity ratio of phonons to impurities, as discussed in the original paper by Kadanoff and Martin. If a approaches to zero, eq. (1) is reduced to the Bardeen, Rickayzen and Tewordt (BRT) expression, which describes the superconducting-state electronic thermal conductivity in the predominant impurity scattering.[6]

In high-T_c superconductors such as the BPSCCO system, the T-linear dependence of the electrical resistivity yields that the inverse relaxation time of quasiparticles in the normal state is proportional to the temperature. In the assumption of the W-F law, the inverse relaxation time in the thermal transport also becomes a linear function of T. Accordingly, the power exponent in the second term of eq.(1) is taken as $n = 1$ if the relaxation time of quasiparticles in the superconducting state is equivalent to that in the normal state.

In calculation for the s-wave and d-wave pairing states, the wave functions are taken as $\psi_s = 1$ and $\psi_d = \sqrt{2}\cos(2\phi)$, where ϕ is the azimuthal angle of wave number k in the ab-plane.[21] The electronic thermal conductivity of the superconductor for the d-wave pairing is calculated such as

$$\bar{\kappa}^s_e = \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{3}{2\pi^2} t \int_0^\infty d\varepsilon\, \varepsilon^2 \operatorname{sech}^2(E/2) \frac{1+a}{\varepsilon/E + at^n} \tag{2}$$

with $\Delta(t,\phi) = \chi \Delta_{BCS}(t)\psi_d(\phi)$ and $E=\sqrt{\varepsilon^2+\Delta^2}$, where χ is a scaling parameter for the superconducting gap to the value of the BCS gap i.e., $\chi = \Delta(0)/\Delta_{BCS}(0)$.

In order to separate the measured thermal conductivity into the phonon and electronic components, the electronic thermal conductivity is calculated using the W-F law and Kadanoff-Martin expressions. The value of κ_e^s rapidly decreases with lowering temperatures since the thermal carrier is condensed into Cooper pairs, carrying no heat flux. It should be noted that within the framework of the electronic thermal conductivity theory for the conventional superconductors, the origin of anomaly below T_c is attributed to the phonon component. Subtracting the value of κ_e^s from the measured value, the phonon thermal conductivity κ_{ph} is obtained.

Next, the phonon thermal conductivity κ_{ph} is discussed in terms of the relaxation time approach by Tewordt and Wölkhausen (TW)[7] which was originally proposed to explain the behavior of $\kappa(T)$ in YBCO system. The expression in κ_{ph} in a magnetic field is given in the following form,

$$\kappa_{ph} = AT^3 \int_0^{\Theta/T} \frac{x^4 e^x}{(e^x - 1)^2} \tau_{\text{ph}}(x) dx \tag{3}$$

and

$$\frac{1}{\tau_{\text{ph}}} = \frac{1}{\tau_{\text{ph-b}}} + \frac{1}{\tau_{\text{ph-sh}}} + \frac{1}{\tau_{\text{ph-p}}} + \frac{1}{\tau_{\text{ph-e}}^{s}} + \frac{1}{\tau_{\text{ph-f}}}, \tag{4}$$

where Θ and $\tau_{ph}(x)$ are the Debye temperature and the total relaxation time of phonons, [7,22] respectively. In eq.(4), $\tau_{ph\text{-}b}$, $\tau_{ph\text{-}sh}$, $\tau_{ph\text{-}p}$, $\tau_{ph\text{-}e}{}^{s}$ and $\tau_{ph\text{-}f}$ represent the relaxation time of phonon scattered by boundaries, sheetlike-faults, point-defects, quasi-particles in the superconducting state and vortex cores or quasiparticles within normal cores , respectively. For convenience, eq. (3) is rewritten such as,

$$\kappa_{ph} = At^3 \int_0^{\Theta/T} \frac{x^4 e^x}{(e^x - 1)^2} \times \frac{1}{1 + B(tx)^2 + C(tx)^4 + D[(1-h)g(t,x,y)(tx) + h(tx)]} dx\,, \tag{5}$$

where the variables of t , x, y and h denote reduced temperature (T / T_c), reduced phonon energy ($\hbar\omega/kT$), reduced gap energy (Δ/kT) and reduced field (H/H_{c2}), respectively. The parameters of A,B,C and D correspond to the strength of boundary scattering, sheetlike-fault scattering, point-defect scattering and electron-phonon scattering, respectively. An exact expression in the function of $g(x,y,t)$ ($= \tau_{\text{ph-e}}{}^{n} / \tau_{\text{ph-e}}{}^{s}$) is given in the BRT and TW papers as follows,

$$\begin{aligned}\frac{\tau_{\text{ph-e}}^{n}}{\tau_{\text{ph-e}}^{s}} &= g(x,y,t) = x[1-\exp(-x)]\{2\int_0^\infty dx'[N(x')N(x+x') \\ &- M(x')M(x+x')]f(x')f(-x'-x) + \int_0^\infty dx'[N(-x')N(x-x') \\ &+ M(x')M(x-x')]f(-x')f(x'-x)\}.\end{aligned} \tag{6}$$

Here, functions of $N(x)$ and $M(x)$ are given as

$$N(x) = \frac{x}{\sqrt{x^2 - y^2}},\ M(x) = \frac{y}{\sqrt{x^2 - y^2}},\ \text{for } s\text{-wave pairing} \tag{7}$$

$$N(x) = \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{x}{\sqrt{x^2 - y^2}},\ M(x) = 0 \ \text{ for } d\text{-wave pairing.} \tag{8}$$

In addition to the various scattering centers of phonons in zero-field, the phonon scatterer due to quasiparticles within normal cores is newly introduced in a thermal resistance in a magnetic field. The inverse relaxation time ($1/\tau_{ph\text{-}f}$) is proportional to the product of the fraction h of normal region produced by the applied field and the inverse relaxation time scattered by normal electrons ($1/\tau_{ph\text{-}e}{}^{n}$) i.e.,

$$\frac{1}{\tau_{\text{ph-f}}} = \pi\xi^2 n \frac{1}{\tau_{\text{ph-e}}^{n}} = \pi\xi^2 \frac{\mu_0 H}{\phi_0} \frac{1}{\tau_{\text{ph-e}}^{n}} = \frac{H}{H_{c2}} \frac{1}{\tau_{\text{ph-e}}^{n}} = h \frac{1}{\tau_{\text{ph-e}}^{n}}, \tag{9}$$

where ξ, $\pi\xi^2$, n and ϕ_0 denote the radius of normal core, the cross section of normal core, the number of the normal core per the unit cross section and a flux quantum, respectively. Moreover, it is assumed that the upper critical field is taken as a parabolic function of temperature,

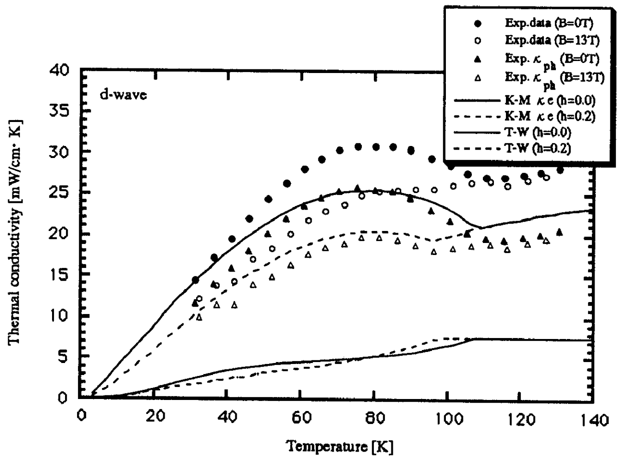

Figure.1 Comparison of the temperature dependence of the thermal conductivity with the calculated one on the basis of the conventional superconductor model.

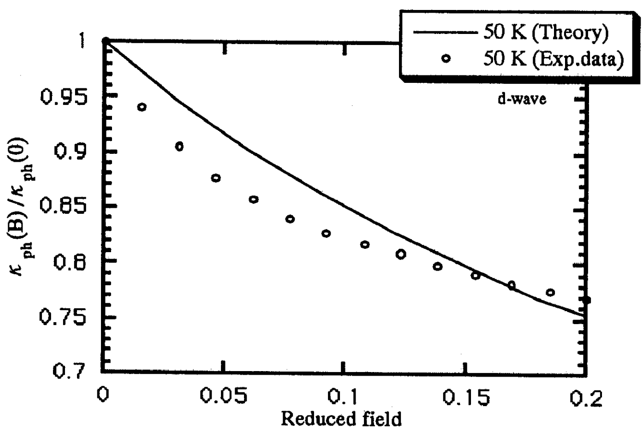

Figure.2 Comparison of the field dependence of the thermal conductivity with the calculated one on the basis of the conventional superconductor model.

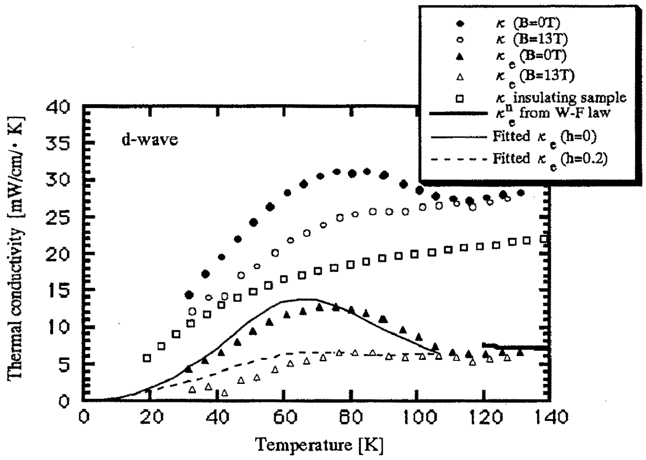

Figure.3 Comparison of the temperature dependence of the thermal conductivity with the fitted curve using the modified K-M expression.

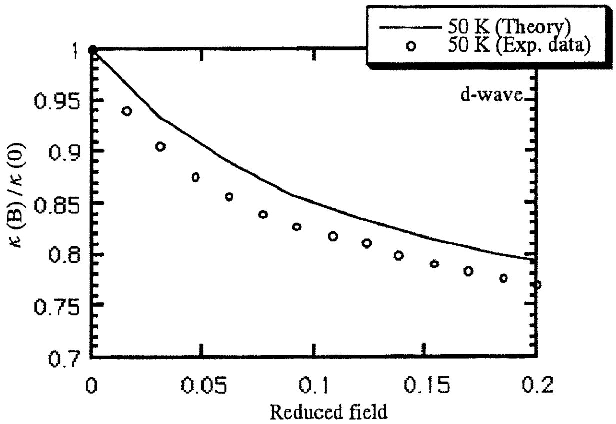

Figure.4 Comparison of the field dependence of the thermal conductivity with the calculated one on the basis of the modified K-M expression.

$H_{c2}(T) = H_{c2}(0)[1-(T/T_c)^2]$ and the dependence of the superconducting energy gap on the field is assumed as $\Delta(T)=\Delta(0)[1-(H/H_{c2})]^{1/2}$.

Following this model, the origin of a peak in κ below T_c is explained by a rapid decrease of the phonon scattering cross section due to the condensation of quasiparticles into Cooper pairs in the superconducting state. The slope in κ_{ph} just below T_c for *d*-wave state becomes less steep in comparison with that for *s*-wave state because the density of state of quasiparticles inside the superconducting energy gap remains finite in the former state.[22]

B. Analysis based on the strong suppress model of quasiparticle scattering rate.

Next, we describe the procedure of analysis on the basis of the strong suppression model of quasiparticle scattering rate.[9] Firstly, the measured thermal conductivity in the absence of a magnetic field is separated into κ_e and κ_{ph} using the value of κ of an insulating sample in which the charge carrier was removed by an annealing in a vacuum from the metallic sample cut out in the same batch as the superconducting sample. Here, it is assumed that the electron-phonon interaction strength is very small compared with the electron-electron interaction strength. On the basis of this assumption, the value of κ_{ph} is not sensitive to the existence of charge carriers, so that the value of κ of the insulating sample is almost corresponding to the value of κ_{ph} of the superconducting BPSCCO sample. Subtracting the value of κ of the insulating sample from that of the superconducting BPSCCO sample, the electronic thermal conductivity is obtained. The origin of the peak in κ is attributed not to phonon component but to the electronic component. The value of κ_e is in agreement with that estimated from the W-F law within an accuracy of 25 %.[20] The experimental value of the normalized κ_e for zero-magnetic field is fitted using the modified K-M expression varying the power-law value in the quasiparticle scattering rate in eq.(2).

Next, the thermal conductivity of the BPSCCO sample in a field is studied using the electron-electron interaction model. Subtracting the value of κ of the insulating sample from measured value of κ in the vortex-state, one can obtain the electronic thermal conductivity in a field. It should be emphasized that not the κ_{ph} value but the κ_e value is dramatically suppressed with increasing the field. The data in κ_e in a field is fitted using modified K-M expression as follows,

$$\bar{\kappa}_e^s = \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{3}{2\pi^2} t \int_0^{\infty} d\varepsilon\, \varepsilon^2 \operatorname{sech}^2\left(\frac{E}{2}\right) \frac{1+a}{\varepsilon / E + a[(1-h)t^n + ht]}, \quad (10)$$

where the quasiparticle scattering rate due to the normal region h is taken into account.

C. Analysis based on the coexistence model

In this model we assume the coexistence of two scattering mechanisms mentioned above for the main cause of the thermal conductivity enhancement below T_c in the c-axis oriented BPSCCO sample.[20] First, electronic thermal conductivity κ_e^n in the normal state was estimated by using the W-F law in terms of measured $\rho(T)$ in the normal state. Then, superconducting state thermal conductivity κ_e^s is estimated using eq.(2) by assuming an appropriate value of n. The phonon thermal conductivity κ_{ph} is obtained by subtracting the estimated κ_e^n for the temperature region above T_c and κ_e^s for the temperature region below T_c, respectively, from the measured total thermal conductivity κ. A numerical parameter fitting analysis is examined for thus obtained phonon thermal conductivity κ_{ph} by using the expression of eq.(5).

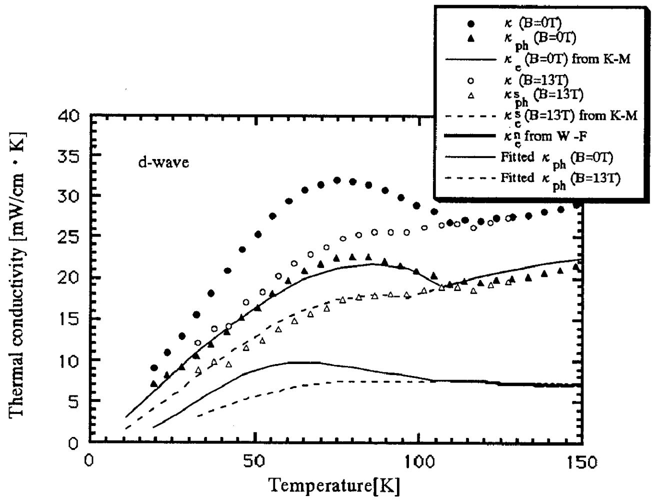

Figure.5 Comparison of the temperature dependence of the thermal conductivity with the calculated one on the basis of the coexisting model.

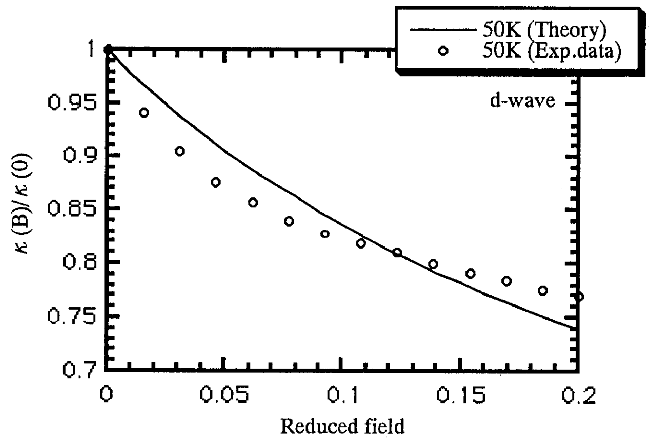

Figure.6 Comparison of the field dependence of the thermal conductivity with the calculated one on the basis of the coexisting model.

3. Results of numerical analyses and discussion

Analysis based on the conventional superconductor model and on the strong suppression model on the quasiparticle scattering rate are reported in the previous paper.[20] Figure 1 shows the measured temperature dependence of thermal conductivity and analyzed results for B=0T and 13T. The best fit is obtained for parameters of a=10, A=2284, B=150, C=20, D=150, Θ=400K, and χ=1.7 for the d-wave state . Figure 2 shows the comparison in the reduced magnetic field dependence of the reduced phonon thermal conductivity calculated taking into account the phonon - normal core scattering process with measured results for T=50K. There are a fairly good agreement in the temperature dependence between measured results and analyzed values of the thermal conductivity . However, the phonon - normal core scattering model based on the electron - phonon interaction does not give a satisfactory explanation on the field effect of the thermal conductivity behavior for the BPSCCO sample as can be seen in Fig.2.

Figure 3 shows the measured and derived temperature dependence of the thermal conductivity and the best fitted curve of $\kappa_e{}^S$ for d-wave pairing with a=10, n=6 , and χ=1.2. In Fig.4, reduced value of $\kappa(B)/\kappa(0)$ $(\cong[\kappa_{ph}+\kappa_e(B)]/[\kappa_{ph}+\kappa_e(0)])$, where κ_e(B) is calculated on the basic of K-M expression , is shown up to a reduced field of 0.2 at the temperature of 50K. As already pointed out in the previous paper, we can see a fairly good agreement between experimental results and theoretical results based on the electron - electron scattering model.

A numerical analysis based on the coexisting model is firstly examined in this paper. We found a better agreement for the d-wave pairing in the numerical analysis based on the strong suppress model . Therefore, we also use d-wave pairing expression in this numerical analysis based on the coexisting model. The best fitting is obtained with parameters of a=10, n=4, A=2300, B=150, C=2, D=150, Θ=400K, and χ=1.2. Figure 5 shows a comparison of measured temperature dependence with the fitted curve with this model. Figure 6 shows the comparison in the magnetic field dependence. As can be seen in these figures, we can see fairly good agreement in the temperature dependence , while intermediate magnetic field dependence between conventional model and strong suppression model. One of the reason of this intermediate agreement in the magnetic field dependence might be that the assumed upper critical field $B_{c2}(0)\approx$ 65T is too much smaller than actual value in this material.

Since the thermal conductivity enhancement was also observed even in the samples in which phonon part are very dominant and hence electronic part is very small, as well as in the high quality samples like single crystals in which electronic conduction makes a significant part, it is very plausible that both scattering mechanisms coexist in this sample with intermediate quality.

4. Summary

Succeeding to the previous study [20], we examined numerical analysis based on a coexisting model in order to explain the observed thermal conductivity enhancement in a highly c-axis oriented Bi(2223) sintered sample. As a result, we can see fairly good agreement in the temperature dependence , while intermediate agreement in the magnetic field dependence between conventional model and the strong suppression model.

Acknowledgments

The authors would like to thank Mr.S.Fujinuma at Iwate University for his technical support. A part of this work was carried out under the inter-university cooperative research program of the Institute for Materials Research, Tohoku University.

1) P. G. Klemens: Solid State Phys. 7 (Academic Press, New York 1958) p1.
2) J. M. Ziman: Electrons and Phonons (Clarendon, Oxford 1960) .
3) See for example a review paper, C. Uher : J. Supercond. 3 (1990) 337.
4) S. Hagen, Z. Z. Wang and N. P. Ong: Phys.Rev. B40, (1989) 9389
5) M. F. Crommie and A. Zettle: Phy. Rev. B41, (1990) 10978.
6) J. Bardeen, G. Richayzen and L. Tewordt: Phys. Rev. 113 (1959) 1014
7) L. Tewordt and T. H. Wölkhausen : Solid State Commun. 70 (1989) 839.
8) L. P. Kadanoff and P. C. Martin: Phys. Rev. 128 (1961) 2390.
9) R. C. Yu, M. B. Salamon, Jian Ping Lu and W. C. Lee: Phys. Rev.Lett. 69 (1992) 1431
10) Martin C. Nuss, P. M. Mankiewich, M. L. O'malley and E. H. Westerwick :Phys.Rev.Lett. 66 (1991) 3305.
11) D. B. Romero, C. D. Porter, D. B. Tanner, L. Forro, L. Mihaly, G. L. Carr and G. P. Williams : Phys. Rev. Lett. 68 (1992) 4980
12) P. B. Littlewood and C. M. Varma : J.Appl.Phys. 69 (1991) 4980
13) J. B. Sousa: Physica 55 (1971) 507.
14) N. A. Red'ko and B. K. Chakal'skii : Sov.Phys. JETP 65 (1987) 619.
15) R. A. Richardson, S. D. Peacor, Franco Nori and C. Uher : Phys. Rev. Lett. 67 (1991) 3856.
16) Da-Ming Zhu, A. C. Anderson, T. A. Friedmann and D. M. Ginsberg : Phys. Rev. B41 (1990) 6605.
17) A. V. Inyushkin, A. N. Taldenkov, S. Yu. Shabanov, L. N. Dem'yanets and T. G. Uvarova : J. Supercond. 7 (1994) 331.
18) Y. Iye, I. Oguro, T. Tamegai and W. R. Datars : Physica C 199 (1992) 154.
19) N. Kobayashi, K. Egawa, K. Miyoshi, H. Iwasaki, H. Ikeda and R. Yoshizaki : Physica C 219 (1994) 265.
20) K. Noto, M. Matsukawa and K. Iwasaki : Proc. US-Japan Workshop on Critical Current in High-T_c Oxide Superconductors (Oct.1995, Tsukuba) to be published.
21) C. T. Rieck, D. Fay and L. Tewordt : Phys. Rev. B 41 (1990) 7289.
22) L. Tewordt and T. H. Wölkhausen : Solid State Commun. 75 (1990) 515.