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An extensively valid and stable method for derivation of all parameters of a solar cell from a single current-voltage characteristic

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It is important to precisely estimate values of all solar cell parameters in order to analyze the performance of a solar cell. We have proposed a stable method for the derivation of solar cell parameters from a single *I-V* curve under only the valid assumption for a general solar cell. Our method was applied to experimental *I-V* curves of a selection of solar cells, comprising of silicon, organic, and dye-sensitized solar cells, which were previously reported. Each *I-V* curve was calculated using the values of parameters derived from our method, and was found to be in good agreement with the experimental *I-V* curves, compared to previous work. Our extensively valid and stable method can be applied to for the analysis of all kinds of solar cells showing various characteristics that follow the single diode model and is very useful for improving the performance of developing solar cells. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2895396]

I. INTRODUCTION

The solar cell is a very useful device for clean electric power generation, and its performance has been continuously improved through intensive research in this field. A solar cell is generally characterized using the equivalent circuit of the single diode model as shown in Fig. 1 and the relation between the current I and the voltage V is given by

$$I = I_{\mathrm{ph}} - I_0\left[\exp\left\{\frac{q(V+R_sI)}{nk_BT}\right\} - 1\right] - \frac{V+R_sI}{R_{\mathrm{sh}}}, \qquad (1)$$

where I_{ph}, I_0, R_s, R_{sh}, q, n, k_B, and T are the photocurrent, the saturation current of the diode, the series resistance, the shunt resistance, the electron charge, the ideality factor, the Boltzmann constant, and the temperature, respectively. The equivalent circuit includes many parameters. In the case of analysis of mature silicon solar cells, the shunt resistance R_{sh} is generally neglected.[1,2] However, the shunt resistance R_{sh} of even one silicon solar cell, although large, cannot be neglected as it strongly influences estimation of the ideality factor n and the series resistance R_s.[3–5] Therefore, it is crucially important to precisely estimate all the parameters of a solar cell for improvement of its performance. Many methods for determining the parameters of a solar cell have been already reported.[2] However, most of the previously reported methods neglect a shunt resistance R_{sh} or requires an assumption that is valid in only a special condition. For example, it is well known that the series resistance R_s can be estimated from *I-V* plots obtained at two different illumination levels.[1,6] This method is the "different illumination level method." However, it was pointed out that a difference in temperature between two measured points causes a large error.[2] Therefore, it is important to estimate all the parameters from a single current-voltage characteristic using the "constant illumination level method." In the constant illumination level method, it is well known that the series resistance R_s and the ideality factor n are often obtained from the slope and the y-intercept of the $-dV/dI$ plot which is a function of $(I_{\mathrm{sc}}-I)^{-1}$, where I_{sc} is the short circuit current. This graphical method is very simple. However, it requires the assumption that R_{sh} is large enough to neglect. In this method, n and R_s are estimated with large errors when the assumption is invalid. Some computational approaches such as the curve fitting method have been also proposed.[3,4,7–9] However, the least mean square method, which is often used in the curve fitting method, cannot be directly applied to the analysis of the electrical characteristics of a solar cell,[3,4,7–9] as the fitting method depends strongly on initial values of the parameters. The main cause for the initial value dependence and instability of the fitting method has not been resolved yet.[4,7,8] Therefore, it is important to develop a widely applicable method for determining all the parameters of a solar cell from its single *I-V* curve, in order to improve its performance. This step is essential in the effective development of a new type of solar cells such as an organic solar cell[10] or a dye-sensitized solar cell.[8,9] Here, we have proposed an extensively valid and stable method for estimating all the parameters of a solar cell by combining an improved constant illumination level method and the fitting method.

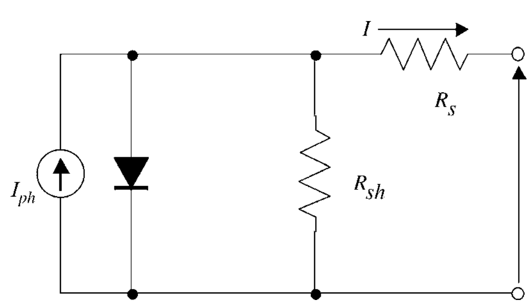

FIG. 1. The equivalent circuit (single diode model) of a solar cell.

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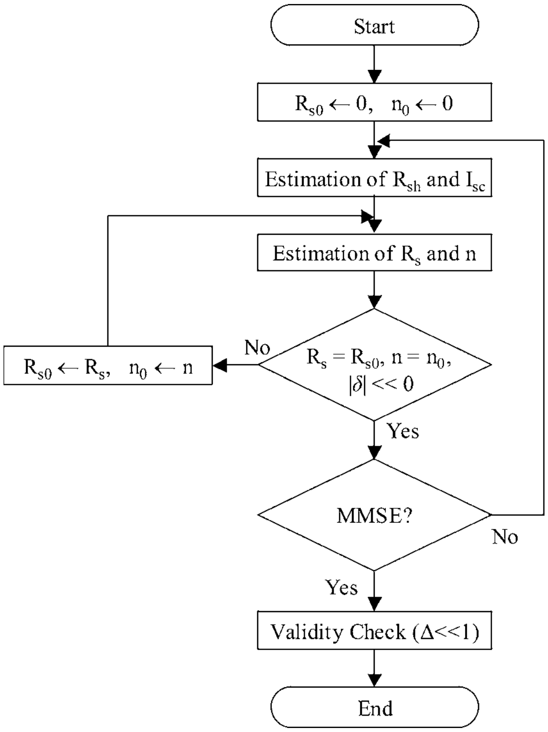

FIG. 2. The flow chart of our method employed to derive the values of solar cell parameters.

II. THEORY

The flow chart for determining all the parameters of a solar cell is shown in Fig. 2. The method we proposed requires only one assumption, which follows

$$\Delta \equiv \exp\left\{-\frac{q(V_{oc}-R_sI_{sc})}{nk_BT}\right\} \ll 1, \tag{2}$$

where V_{oc} is the open circuit potential. The assumption (2) is generally valid enough for various solar cells.

From Eq. (1), $-dV/dI$ is given by

$$-\frac{dV}{dI} = \frac{nk_BT/q}{I_{ph}+I_0-I-(V+R_sI-nk_BT/q)/R_{sh}} + R_s. \tag{3}$$

Substituting $V=0$ and $I=I_{sc}$ into Eq. (1) and substituting $V=V_{oc}$ and $I=0$ give

$$I_{sc} = I_{ph} - I_0\left\{\exp\left(\frac{qR_sI_{sc}}{nk_BT}\right)-1\right\} - \frac{R_sI_{sc}}{R_{sh}} \tag{4a}$$

and

$$0 = I_{ph} - I_0\left\{\exp\left(\frac{qV_{oc}}{nk_BT}\right)-1\right\} - \frac{V_{oc}}{R_{sh}}, \tag{4b}$$

respectively. As mentioned in the Appendix, applying the assumption (2) to Eqs. (4a) and (4b) gives

$$I_{ph} + I_0 \sim I_{sc} + \frac{R_sI_{sc}}{R_{sh}}. \tag{5}$$

Substituting Eq. (5) into Eq. (3) derives

$$-\frac{dV}{dI} = \frac{nk_BT/q}{I_{sc}-I-\{V-R_s(I_{sc}-I)-nk_BT/q\}/R_{sh}} + R_s. \tag{6}$$

In the case of $I=I_{sc}$ ($V=0$), Eq. (6) is modified to

$$-\left.\frac{dV}{dI}\right|_{I=I_{sc}} = R_{sh}+R_s \sim R_{sh}, \tag{7}$$

where $R_{sh} \gg R_s$, which is generally valid. Equation (7) indicates that R_{sh} can be independently derived from the slope of the I-V curve of a solar cell at $I=I_{sc}$ ($V=0$). On the other hand, in order to determine the ideality factor n and the series resistance R_s, temporary parameters R_{s0} and n_0 are introduced to Eq. (6), which yields

$$-\frac{dV}{dI} = \frac{nk_BT/q}{I_{sc}-I-\dfrac{V-R_{s0}(I_{sc}-I)-n_0k_BT/q}{R_{sh}}(1-\delta)} + R_s, \tag{8}$$

where δ is defined by

$$\delta \equiv \frac{(R_s-R_{s0})(I_{sc}-I)+(n-n_0)k_BT/q}{V-R_{s0}(I_{sc}-I)-n_0k_BT/q}. \tag{9}$$

When $|\delta| \ll 1$, Eq. (8) is given by

$$-\frac{dV}{dI} = \frac{nk_BT/q}{I_{sc}-I-\{V-R_{s0}(I_{sc}-I)-n_0k_BT/q\}/R_{sh}} + R_s. \tag{10}$$

The condition $|\delta| \ll 1$ can be valid when the parameters R_{s0} and n_0 are close in value to the series resistance R_s and the ideality factor n, respectively. Note that the series resistance R_s and the ideality factor n disappear from the denominator in the right-hand side of Eq. (10) by introducing the temporary parameters R_{s0} and n. Then, the series resistance R_s and the ideality factor n are derived by the y-intercept and the slope of the plot of $-dV/dI$ as a function of $\{I_{sc}-I-[V-R_{s0}(I_{sc}-I)-n_0k_BT/q/R_{sh}]\}^{-1}$ under the condition $|\delta| \ll 1$, respectively. In order to find the appropriate values of the temporary parameters, the estimation of series resistance R_s and the ideality factor n according to Eq. (10) is iterated using 0 as the initial value of each temporary parameter, as shown in Fig. 2. At the first iteration count of the loop, the values of the series resistance R_s and the ideality factor n include large errors because $|\delta| \ll 1$ is not generally satisfied when $R_{s0}=0$ and $n_0=0$. However, they are close to the appropriate values as compared with the initial ones, respectively. More accurate values are obtained by using the values of the series resistance R_s and the ideality factor n given at this step as those of R_{s0} and n_0. The most appropriate values are finally yielded by repeating this procedure until $R_s=R_{s0}$, $n=n_0$, and $|\delta|=0$, as shown in Fig. 2.

Equations (A3) and (4b) give

$$I_0 \sim \left(I_{sc} - \frac{V_{oc}-R_sI_{sc}}{R_{sh}}\right)\exp\left(-\frac{qV_{oc}}{nk_BT}\right) \tag{11}$$

and

$$I_{ph} = I_0\left\{\exp\left(\frac{qV_{oc}}{nk_BT}\right) - 1\right\} + \frac{V_{oc}}{R_{sh}}, \tag{12}$$

respectively. After determination of the series resistance R_s, the shunt resistance R_{sh} and the ideality factor n, substitution of the derived parameters into Eq. (11) yields the saturation current I_0. Finally, applying the value of I_0 to Eq. (12) gives the photocurrent I_{ph}.

In this stage, candidates for all the parameters are obtained and the *I-V* curve can be calculated. Optimal parameters are derived by repeating their estimation for various values of the shunt resistance R_{sh} and the short circuit current I_{sc} until the mean squared error (MSE) between the calculated *I-V* curve and experimental data are minimized (MMSE). Finally, the validity of the derived values of the parameters is checked by confirming the assumption (2).

III. APPLICATION TO ANALYSIS OF SOLAR CELLS

We applied our method to analysis of various solar cells, of which the experimental *I-V* curves were obtained from literature. Some are silicon solar cells,[3,5] another is an organic solar cell with triple heterojunctions,[10] and the other is a dye-sensitized solar cell (DSSC), which was called "C4" in the literature.[8]

Our method requires deriving the value of dV/dI from discrete data points of the experimental *I-V* curve. We estimated it from the following expression:

$$\left.\frac{dV}{dI}\right|_{V=(V_{m+1}+V_m)/2} = \left.\frac{dV}{dI}\right|_{I=(I_{m+1}+I_m)/2} = \frac{V_{m+1} - V_m}{I_{m+1} - I_m} \quad (m = 1, \ldots, M-1), \tag{13}$$

where V_m and I_m are values of the voltage and the current of the mth point in the *I-V* curve, respectively, and M is the number of the data points. When the current I is close in value to the short circuit current I_{sc}, the estimation of the right-hand side of Eq. (10) includes a large error because the ratio of the error in I to $(I_{sc}-I)$ is very large even though the error in I is small. Here, the value of the right-hand side of Eq. (10) was estimated from data satisfying the following condition:

$$\frac{I_{sc} - I - V/R_{sh}}{I_{sc}} > 0.1. \tag{14}$$

The analysis of the differential requires smoothness of data because the insufficiency of it causes a large error. Therefore, the value of the right-hand side of Eq. (10) was estimated from the *I-V* curve approximated by a polynomial expression under the condition (14) when data were deficient in smoothness. In this study, the ninth-degree polynomial approximation was used in the case of analysis of DSSC(C4).[8]

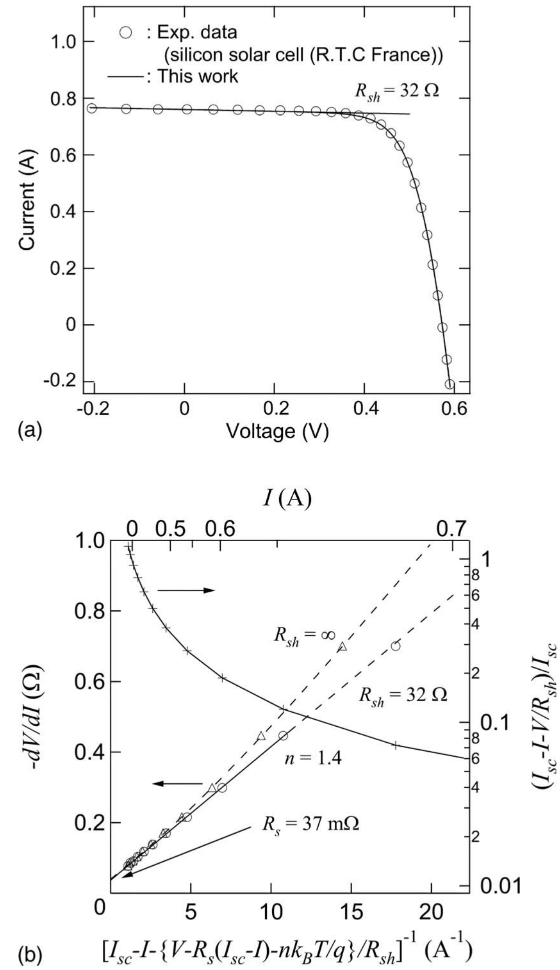

FIG. 3. (a) The experimental *I-V* curve of a silicon solar cell (R.T.C France) (open circles) (Ref. 3) and the *I-V* curve calculated using the value of the parameters derived by our method. (b) The plots of $-dV/dI$ and $(I_{sc}-I-V/R_{sh})/I_{sc}$ as a function of $[I_{sc}-I-\{V-R_s(I_{sc}-I)-nk_BT/q\}/R_{sh}]^{-1}$ in the case of R_{sh}=32 Ω when $R_{so}=R_s$, $n_0=n$, and $\Delta=0$.

IV. RESULTS AND DISCUSSION

A. Application to silicon solar cells

We applied our method to an *I-V* curve of a silicon solar cell (R.T.C France).[3] Figure 3(a) shows the experimental data of the *I-V* curve (open circles) and the result of calculation by our method (solid line). It is clearly seen that the calculation by our method is in good agreement with experimental data. Table I shows the values of parameters derived by our method, those of previous reports and the values of Δ. The values of the parameters derived by our method were very close to those obtained by another method and $\Delta \ll 1$, which showed that the assumption (2) was satisfactory enough in this case. This result supports the adequacy of our method. Figure 3(b) shows the plots of $-dV/dI$ and $(I_{sc}-I-V/R_{sh})/I_{sc}$ as a function of $\{I_{sc}-I-[V-R_s(I_{sc}-I)-nk_BT/q]/R_{sh}\}^{-1}$ in the case of R_{sh}=32 Ω when $R_{so}=R_s$, $n_0=n$, and $\Delta=0$. The plot of $-dV/dI$ has good linearity in the case of using our method, although it was nonlinear in the case of neglecting the shunt resistance R_{sh}. This indicates that the shunt resistance R_{sh} cannot be neglected. Under the condition (14), the ideality factor n and the series resistance R_s were estimated at 1.4 and 37 mΩ, respectively. Figure 4

TABLE I. The values of solar cell parameters derived by our method and the previous work and Δ.

Cell or module	V_{oc} (V)	I_{sc} (A)	I_{ph} (A)	I_0 (μA)	R_s (Ω)	R_{sh} (kΩ)	n	T (°C)	Δ
Si (R.T.C France)[a]	0.5728	0.760 8	0.7608	0.3223	0.0364	0.0538	1.484		8.9×10^{-7}
A previous work[b]	⋯	⋯	0.7609	0.4039	0.0364	0.0495	1.504	33	⋯
This work	0.57	0.76	0.77	0.20	0.037	0.032	1.4		5.6×10^{-7}
Si (Photowatt-PWP-201)[a]	16.778	1.030	1.0318	3.2876	1.2057	0.549	48.450		8.3×10^{-6}
A previous work[b]	⋯	⋯	1.0359	6.77	1.146	0.2	51.32	45	⋯
This work	17	1.0	1.0	2.3	1.3	0.83	47		6.3×10^{-6}
Silicon[c]	⋯	0.050 0	⋯	⋯	3.3	⋯	2.5	17	3.4×10^{-3}
This work	0.52	0.050	0.050	0.99	0.73	0.16	1.9		4.5×10^{-5}
Organic solar cell[d]	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
This work	1.2	0.004 6	0.0047	0.92	48	1.4	5.8	20[e]	1.1×10^{-3}
DSSC(C4)[f]	0.704	0.002 06	⋯	0.035	43.8	3.736	2.5	⋯	⋯
This work	0.70	0.002 1	0.0021	0.023	42	3.2	2.5	20[e]	4.9×10^{-5}

[a]See Ref. 3.
[b]See Ref. 4.
[c]See Ref. 5.
[d]See Ref. 10.
[e]No information on the temperature T in the reference.
[f]See Ref. 8.

shows convergence properties of (a) the ideality factor n and the series resistance R_s and (b) the dependence of MSE on a shunt resistance R_{sh}. As shown in Fig. 4(a), an ideality factor n and a series resistance R_s rapidly converged by iterating the loop several times in spite of using 0 as each initial value. The MSE drastically decreased for the MMSE, as shown in Fig. 4(b). These indicate that our method has a rapid convergence property.

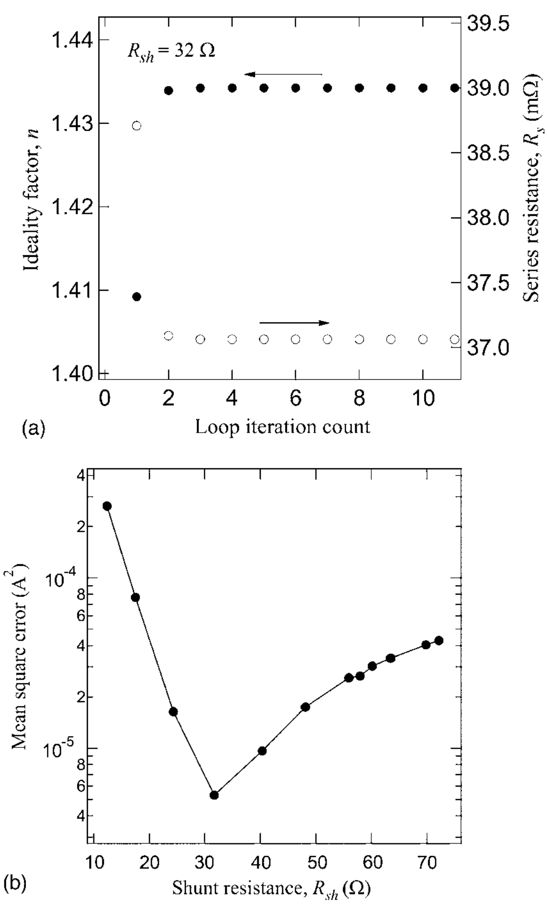

FIG. 4. The convergence property of (a) the loop for deriving the ideality factor n and the series resistance R_s, and (b) the dependence of MSE on a shunt resistance R_{sh} .

A silicon solar module (Photowatt-PWP 201) (Ref. 3) was also analyzed using our method. The values of the parameters we obtained were close to those in Ref. 3, and another previously reported method[4] gave a smaller value of the shunt resistance R_{sh} and a larger value of the ideality factor n, as shown in Table I. The failure in the estimation of the parameters indicates the difficulty and instability of the parameter estimation using a general fitting method, which depends strongly on the initial values of parameters.

Figure 5 shows an I-V curve of another silicon solar cell

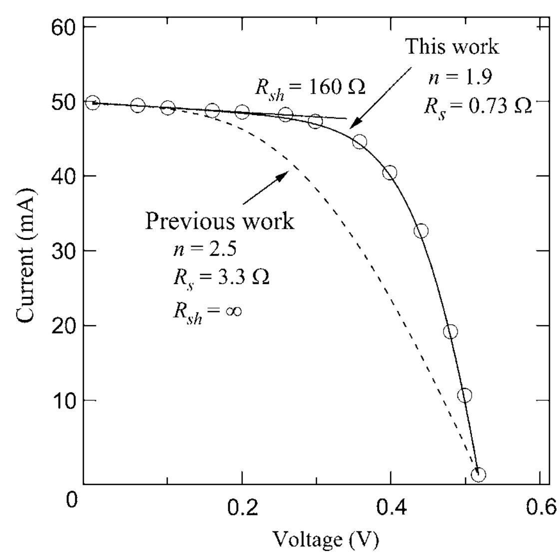

FIG. 5. The experimental I-V curve of a silicon solar cell (open circles) (Ref. 5) and the I-V curves calculated using the value of the parameters derived by our method (solid line) and those of the previous work (broken line) (Ref. 5).

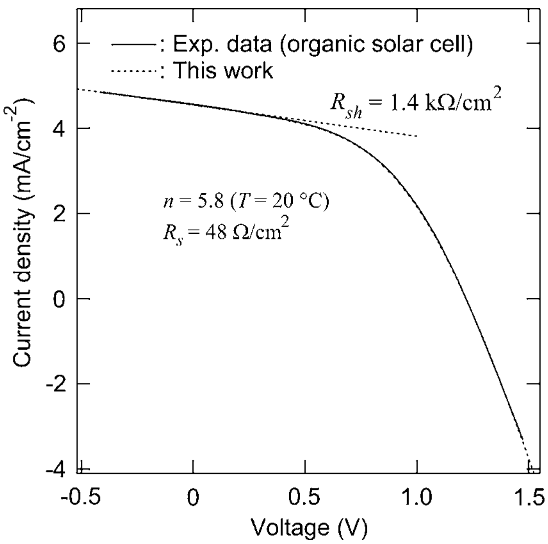

FIG. 6. The experimental *I-V* curve of an organic solar cell with triple heterojunctions (solid line) (Ref. 10) and the *I-V* curves calculated using the value of the parameters derived by our method (broken line).

(open circles)[5] and the curves calculated using the values of parameters derived by our method (solid line) and those in Ref. 5 (broken line). The broken line did not express the *I-V* curve of the solar cell at all. In Ref. 5, the series resistance R_s of 3.3 Ω was estimated from values of only one point of the *I-V* curve, where the power was maximum, under an assumption that the ideality factor n was 2.5 and that the shunt resistance R_{sh} was large enough to neglect, leading to the large deviation of the calculation from the experimental *I-V* curve. On the other hand, the solid line perfectly fitted with the experimental data. This indicates that the parameters yielded by our method reflect the actual performance of the solar cell well, and that fixing the ideality factor n and neglecting the shunt resistance R_{sh} cause the estimated parameter values to include large errors.

B. Application to an organic solar cell

Our method is adaptable enough for analysis of an organic solar cell. Figure 6 shows the experimental data of the *I-V* curve of an organic solar cell (solid line)[10] and the curve calculated using the values given by our method (broken line). The calculated *I-V* curve was in excellent agreement with the experimental data, to the extent that they could not be distinguished from each other, and assumption (2) was satisfactory enough, as shown in Table I. The agreement of the calculation results with the experimental data indicates that our method is available not only for a silicon solar cell but also for another solar cell showing various performances.

C. Application to a DSSC

Figure 7(a) shows the experimental *I-V* curve of a DSSC called C4 in Ref. 8 (open circles) and the *I-V* curves calculated using values of parameters yielded by the previous method (broken line)[8] and our method (solid line), respectively. In this case, the smoothness of the experimental data was insufficient to directly apply our method to them, as shown in Fig. 7(b). Here, the experimental data were approximated by a ninth-degree polynomial expression under the condition (14). The insertion in Fig. 7(a) shows the experimental *I-V* curve (open circle) and the approximation by the polynomial expression at the region of the condition (14). The polynomial expression reflected the experimental data well. In the case of using the polynomial approximation, the plot of $-dV/dI$ as a function of $\{I_{sc}-I-[V-R_s(I_{sc}-I)-nk_BT/q]/R_{sh}\}^{-1}$ had good linearity although that of the raw data was scattered, which prevented from drawing the accurate line, as shown in Fig. 7(b). The ideality factor n and the series resistance R_s were determined by the $-dV/dI$ plot using the polynomial approximation under the condition (14). The solid line in Fig. 7(a) was calculated using the values of the parameters estimated by the polynomial approximation. The solid line was in better agreement with the experimental data than the broken line that was obtained by the fitting method with fixing the ideality factor n to 2.5 in Ref. 8. As mentioned before, the method that requires fixing an unknown parameter can never yield accurate values of the parameters.

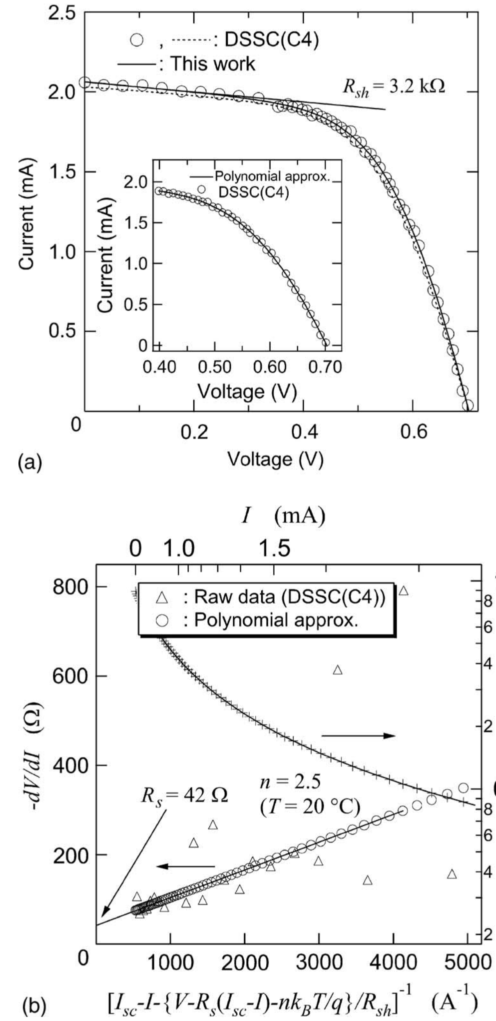

FIG. 7. (a) The experimental *I-V* curve of DSSC(C4) (open circles) (Ref. 8) and the *I-V* curves calculated using the value of the parameters derived by our method (solid line). The insertion shows the experimental *I-V* curve (open circle) and the approximation of the polynomial expression. (b) The plots of $-dV/dI$ and $(I_{sc}-I-V/R_{sh})/I_{sc}$ as a function of $[I_{sc}-I-\{V-R_s(I_{sc}-I)-nk_BT/q\}/R_{sh}]^{-1}$ when $R_{so}=R_s$, $n_0=n$, and $\Delta=0$.

V. CONCLUSION

We have proposed a stable method for deriving all the parameters from a single *I-V* curve, which requires only the valid assumption for a general solar cell. We have demonstrated that our method is very stable and is adaptable for analyzing various solar cells, compared to previous efforts which could not derive the accurate values of the parameters. Our method also has a rapid convergence property. Our extensively valid and stable method is very useful for analyzing various solar cells, leading to an improvement of their performance.

APPENDIX

Subtracting Eq. (4b) from Eq. (4a) gives

$$I_{sc} = I_0 \exp\left(\frac{qV_{oc}}{nk_BT}\right)\left[1 - \exp\left\{-\frac{q(V_{oc}-R_sI_{sc})}{nk_BT}\right\}\right] + \frac{V_{oc}-R_sI_{sc}}{R_{sh}}. \quad \text{(A1)}$$

By applying approximation (2), we obtained

$$I_{sc} \sim I_0 \exp\left(\frac{qV_{oc}}{nk_BT}\right) + \frac{V_{oc}-R_sI_{sc}}{R_{sh}},$$

$$I_0 \exp\left(\frac{qV_{oc}}{nk_BT}\right) + \frac{V_{oc}}{R_{sh}} \sim I_{sc} + \frac{R_sI_{sc}}{R_{sh}}. \quad \text{(A2)}$$

From Eq. (4b), $I_{ph}+I_0$ was given by

$$I_{ph} + I_0 = I_0 \exp\left(\frac{qV_{oc}}{nk_BT}\right) + \frac{V_{oc}}{R_{sh}}. \quad \text{(A3)}$$

Substituting Eq. (A2) into Eq. (A3) gives Eq. (5).

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