Thermal fission rate around superfluid-normal phase transition

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Using Langer's Im*F* method, we discuss the temperature dependence of nuclear fission width in the presence of dissipative environments. We introduce a low frequency cutoff to the spectral density of the environmental oscillators in order to mimic the pairing gap. It is shown that the decay width rapidly decreases at the critical temperature, where the phase transition from superfluid to normal fluids takes place. A possible relation to the recently observed threshold for the dissipative fission is discussed.

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I. INTRODUCTION

Fission of a hot nucleus has attracted much interest of nuclear physicists in the past several years to study nuclear dissipation together with deep-inelastic heavy-ion collisions [1–3]. It is known that statistical codes to calculate the decay of a compound nucleus significantly underestimate the experimentally observed prefission neutron, charged particle, and γ-ray multiplicities at high excitation energies if the original Bohr-Wheeler formula for the fission width is used, though they work pretty well at low energies [4–9]. Two alternative interpretations of this fact have so far been proposed. The one attributes the large prefission neutron emission to the so-called transient effect [9]. In this case, one assumes that some amount of neutrons are emitted before the asymptotic fission rate given by the Bohr-Wheeler formula is achieved. The other is to consider that fission is hindered by nuclear dissipation. Based on the latter idea, Thoennessen and Bertsch have analyzed fission data on prefission neutron, charged-particle, and γ-ray multiplicities for various systems by using statistical codes, and obtained systematics of the threshold energy, where a dissipation starts to play a significant role in fission [4]. This systematics has been confirmed by experimentally studying the excitation energy dependence of the fission probability in ^{200}Pb compound nuclei [7].

On the other hand, the nuclear dissipation does not play any significant role in spontaneous fission because of the strong pairing correlation between nucleons [10,11]. When one discusses nuclear fission at moderate excitation energies, one has to take into account the temperature dependence of the pairing gap. The pairing gap decreases with temperature and the nucleus eventually undergoes a phase transition from a superfluid to a normal fluid [12–15]. The purpose of this paper is to investigate the effects of the superfluid-normal phase transition on the fission width at finite temperatures. Our study was partly motivated by that in [10], where the effect of pairing on the fission at zero temperature has been discussed.

We use Langer's Im *F* method, where the decay width of a metastable state is related to the imaginary part of the free energy [16–20]. In this method one can describe the decay process for a very wide range of temperature, i.e., from zero temperature, where the decay process is governed by the quantum tunneling, to high temperatures, where thermal decay dominates [21]. Also, the method can be applied to a system with many degrees of freedom [17].

The paper is organized as follows. In Sec. II, we briefly review Langer's Im *F* method for the decay of an unstable state at finite temperatures. In Sec. III, we apply this method to the fission of ^{248}Cf at temperatures in the region near the superfluid to normal phase transition. The summary and a discussion on the possible origin of the threshold phenomena discussed in [4] are given in Sec. IV.

II. LANGER'S Im *F* METHOD

We consider a system where a macroscopic degree of freedom q is coupled to environmental heat bath. In the problem of fission, q corresponds to the fission coordinate. We assume the following Lagrangian for this system [22]:

$$L=\frac{1}{2}M(q)\dot{q}^2-V(q)+\sum_i \frac{1}{2}m_i(\dot{x}_i^2-\omega_i^2x_i^2)-\sum_i c_ix_if(q)+\sum_i \frac{c_i^2f(q)^2}{2m_i\omega_i^2}, \tag{1}$$

where $\{x_i\}$ and $\{\omega_i\}$ are the coordinates of the environmental oscillators and the corresponding excitation energies, respectively. $V(q)$ is a potential for the macroscopic degree of freedom, which has a local minimum and a maximum at $q=q_0$ and $q=q_b$, respectively. $M(q)$ and $f(q)$ are the mass of the macroscopic motion and the coupling form factor, respectively. We assume general functions of q for them [18,20]. The last term is the so-called counter term which cancels the static potential renormalization due to the coupling between the macroscopic and the environmental degrees of freedom [22]. Takigawa and Abe have suggested that, in contrast to heavy-ion fusion reactions at sub-barrier energies where the static potential renormalization plays an important role in enhancing the fusion cross section over the predictions of a one-dimensional potential model [23,24], the static potential renormalization in the fission problem can lead to two opposite effects, i.e., it could either lower or increase the effective fission barrier compared with the bare potential barrier, thus leading to either hindrance or enhancement of the fission rate, depending on the properties of the coupling form factor

$f(q)$ [18]. Both cases lead to a temperature-dependent fission barrier height [5]. In this paper, we introduce the counterterm similarly to [22].

In order to obtain the free energy, we first express the partition function in the path-integral form. After integrating out the environmental degrees of freedom, the partition function at the temperature $k_BT=1/\beta$ takes the form [25]

$$Z(\beta)=\int \mathscr{D}[q(\tau)]e^{-S_{\rm eff}[q(\tau)]/\hbar}, \tag{2}$$

where the path integral is performed over all the periodic paths with the period $\beta\hbar$. the effective Euclidean action $S_{\rm eff}[q(\tau)]$ is given by

$$S_{\rm eff}[q(\tau)]=\int_0^{\beta\hbar} d\tau\left(\frac{1}{2}M[q(\tau)]\dot{q}^2+V[q(\tau)]\right)+\frac{1}{2}\int_0^{\beta\hbar}d\tau\int_0^{\beta\hbar}d\tau' k(\tau-\tau')f[q(\tau)]f[q(\tau')] \tag{3}$$

with the influence kernel $k(\tau)$ [18,25]

$$k(\tau)=\sum_i\left[\frac{c_i^2}{m_i\omega_i^2}:\delta(\tau):-\frac{c_i^2}{2m_i\omega_i}\frac{\cosh\{\omega_i[|\tau|-(1/2)\beta\hbar]\}}{\sinh[(1/2)\hbar\omega_i\beta]}\right], \tag{4}$$

where

$$:\delta(\tau):=\sum_{n=-\infty}^{\infty}\delta(\tau-n\beta\hbar) \tag{5}$$

is a generalized δ function with period $\beta\hbar$.

We consider now a high-temperature regime, where the decay of a metastable state is governed by the thermal hopping. Evaluating the path integral in Eq. (2) in the saddle-point approximation and using the relation between the decay width Γ and the imaginary part of the free energy [16]

$$\Gamma(T)=-\frac{2}{\hbar}\frac{T_c}{T}\,{\rm Im}F, \tag{6}$$

T_c being the crossover temperature where the transition between the thermal activated decay and the quantum tunneling occurs, we find that the decay width at temperature T can be expressed as [18]

$$\Gamma=\frac{\omega_0}{2\pi}\frac{\omega_R}{\omega_b}\sqrt{\frac{M(q_0)}{M(q_b)}}\,f_q e^{-\beta V_b}, \tag{7}$$

where ω_R is defined as $2\pi k_BT_c/\hbar$. ω_0, ω_b, and V_b are the curvature of the potential barrier $V(q)$ at the local minimum $q=q_0$, that at the barrier position q_b, and the height of the potential barrier, i.e., $V_b=V(q_b)-V(q_0)$, respectively. f_q is the quantum correction factor due to the quantum fluctuation of the paths around the classical paths $q_{\rm cl}(\tau)=q_b$, $q_{\rm cl}(\tau)=q_0$, and is given by

$$f_q=\prod_{n=1}^{\infty}\frac{\nu_n^2+\omega_0^2+(df/dq)_{q=q_0}^2\nu_n\hat{\gamma}_0(\nu_n)}{\nu_n^2-\omega_b^2+(df/dq)_{q=q_b}^2\nu_n\hat{\gamma}_b(\nu_n)}, \tag{8}$$

where $\nu_n=2\pi n/\beta\hbar$ are the Matsubara frequencies. $\hat{\gamma}$ is the Laplace transform of the retarded friction kernel [18], and is given by

$$\hat{\gamma}(z)=\frac{1}{M(q)}\sum_i\frac{c_i^2}{m_i\omega_i^2}\frac{z}{z^2+\omega_i^2}. \tag{9}$$

The subscripts 0 and b in Eq. (8) denote that the quantities with those indices should be evaluated at $q=q_0$ and $q=q_b$, respectively. The crossover temperature T_c is identified with the highest temperature at which the quantum correction factor f_q diverges [17]. This is the temperature where the so-called bounce path which describes a tunneling decay disappears as one increases the temperature from zero. At temperatures below T_c, the bounce solution dominates the decay and the decay rate has less temperature dependence [17]. In the absence of environments, this prescription assigns k_BT_c to be $\hbar\omega_b/2\pi$. This is consistent with the earlier observation by Affleck on the crossover temperature [16]. It should be noticed that Langer's Im F method implicitly assumes that the coupling of the macroscopic degree of freedom to the environmental degrees of freedom is strong enough to assure that the system is always in a thermal equilibrium.

III. FISSION OF A HOT NUCLEUS

We now apply Eq. (7) to the problem of the fission of a hot nucleus. Following [10] we introduce a low cutoff frequency ω_c to the distribution of the environmental oscillators in order to mimic that there is no nuclear levels below the two-quasiparticle state in even-even nuclei. Accordingly, we set the cutoff frequency to $2\Delta(T)/\hbar$, $\Delta(T)$ being the pairing gap at the temperature T, and take the spectrum density of the environmental oscillators as [10]

$$J(\omega)\equiv\frac{\pi}{2}\sum_i\frac{c_i^2}{m_i\omega_i}\delta(\omega-\omega_i)=\eta(\omega-\omega_c)\theta(\omega-\omega_c), \tag{10}$$

where η is the friction constant [22]. Note that $\omega_c=\infty$ and $\omega_c=0$ correspond to two extreme cases where there is no dissipation at all and where the spectrum density is given by the usual Ohmic dissipation, respectively. The former and the latter cases give the Bohr-Wheeler formula and the well-known Kramers's formula at moderate to strong friction for the decay rate, respectively, with a quantum correction factor [17]. For the spectrum density given by Eq. (10), Eq. (9) for the Laplace transform of the damping kernel reads

$$\hat{\gamma}(z)=\frac{\eta}{M(q)}+\frac{2}{\pi}\frac{\eta}{M(q)}\left(\frac{\omega_c}{z}\ln\frac{\omega_c}{\sqrt{\omega_c^2+z^2}}-\tan^{-1}\frac{\omega_c}{z}\right). \tag{11}$$

Note that the second term in this equation vanishes when the cutoff frequency ω_c is set zero.

We now apply the above arguments to the fission of ^{248}Cf. We take the reduced mass for the symmetric fission for $M(q)$ and the potential given in [26] for $V(q)$. $\hbar\omega_0$, $\hbar\omega_b$, q_b, and V_b then take the values of 1.18 MeV, 1.06 MeV, 3.4 fm, and 3.67 MeV, respectively. Though there are extensive experimental as well as theoretical studies on the dissipation coefficient for fission, its value is yet quite scattered [1]. In this paper, we assume 20×10^{21}/s. for the reduced dissipation coefficient $\beta\equiv\eta/M$. This is a typical value which one can find in the literature [1,2]. We checked that the results of the following part of this paper does not qualitatively change as long as one assumes a value for β, which is consistent with data. We assume a bilinear coupling form factor, i.e., $f(q)=q$. Since we are interested in the effects of pairing in the super to normal transition region, we use a simplified expression for the temperature-dependent pairing gap,

$$\Delta(T)=k_BT_c^{\rm pair}\sqrt{\frac{8\pi^2}{7\zeta(3)}\,[1-(T/T_c^{\rm pair})]}\quad (\text{for } T<T_c^{\rm pair}) \tag{12}$$

$$=0 \quad (\text{for } T>T_c^{\rm pair}), \tag{13}$$

which is valid near the transition temperature [15]. In Eq. (12) ζ is the zeta function and $T_c^{\rm pair}$ the critical temperature for the superfluid-normal phase transition. We assign the pairing gap at zero temperature to be $12/\sqrt{A}$, A being the mass number of a nucleus, and estimate the critical temperature $T_c^{\rm pair}$ using the relation $T_c^{\rm pair}\sim0.567\Delta_0$ [14,15].

Figure 1 shows the crossover temperature T_c as a function of the cutoff parameter $\hbar\omega_c$. This is given by the positive root of the equation

$$\omega_R^2+\omega_R\frac{\eta}{M}-\omega_b^2+\omega_R\frac{2}{\pi}\frac{\eta}{M}\left(\frac{\omega_c}{\omega_R}\ln\frac{\omega_c}{\sqrt{\omega_c^2+\omega_R^2}}-\tan^{-1}\frac{\omega_c}{\omega_R}\right)=0. \tag{14}$$

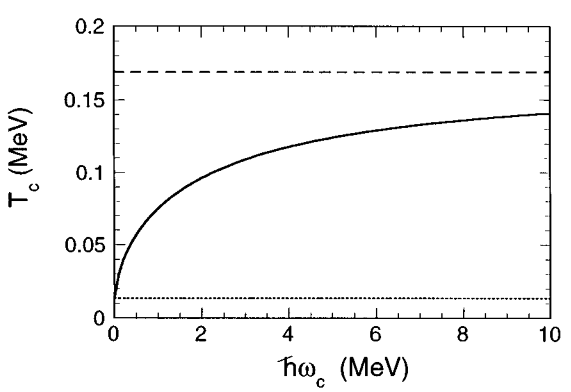

FIG. 1. The cutoff frequency dependence of the crossover temperature T_c between the quantum and the thermal regimes. The solid line was obtained by numerically solving Eq. (14). The dashed and the dotted lines are the crossover temperature in the absence of environments and that in the system with Ohmic dissipation without cutoff, respectively.

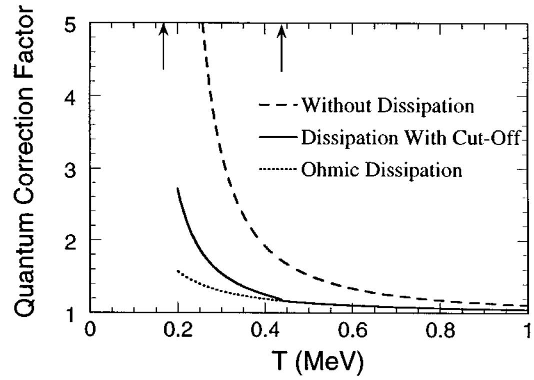

FIG. 2. Quantum correction factor as a function of temperature. The dashed and the dotted lines are the quantum correction factor in the absence of environment and that in the system with Ohmic dissipation without cutoff, respectively. The solid line is the quantum correction factor when a lower cutoff frequency has been introduced through the temperature dependence of the pairing gap. The left and the right arrows are the crossover temperature from a quantal to a thermal decay, and the critical temperature for the super to normal phase transition, respectively.

Notice that there is only one positive root for Eq. (14). It should be remarked that in calculating the decay rate based on Eq. (7) the crossover temperature ω_R has to be evaluated at each temperature T with corresponding cutoff frequency ω_c, i.e., one must solve Eq. (14) by treating ω_c as though it is independent of temperature. Otherwise, one cannot recover the decay rate formula of Kramers modified by the quantum correction factor at temperatures higher than $T_c^{\rm pair}$, where the pairing gap vanishes. The solid line in Fig. 1 is the solution of Eq. (14). The dashed line is the crossover temperature in the absence of environments, i.e., $\hbar\omega_b/2\pi$. If one sets ω_c to be zero, the crossover temperature is given by $(\sqrt{1+\alpha^2}-\alpha)\hbar\omega_b/2\pi$, α being $\eta/2M\omega_b$ [17]. This value is denoted by the dotted line in the figure. The crossover temperature gradually decreases as the cutoff frequency decreases reflecting the increasing dissipation [10].

Figure 2 shows the quantum correction factor given by Eq. (8) as a function of the temperature. In the limits of $\omega_c\to0$ and ∞, the infinite product in Eq. (8) can be simplified by using Γ function [17,19]. In the case of finite ω_c, one has to evaluate it directly until one gets convergence. In general cases, however, this is a fairly difficult numerical task because the ratio for each n in Eq. (8) never becomes sufficiently close to one even for very large n. Consequently, numerical errors accumulate as one performs the production many times. In our applications, where we used a constant mass and a bilinear coupling, the infinite product series converged. The dashed and the dotted lines are the quantum correction factor in the limit of $\omega_c\to0$ and ∞, respectively. The solid line is the quantum correction factor when the lower cutoff for each temperature has been introduced. The left and the right arrows in the figure show the crossover temperature from a quantal to a thermal decay, i.e., $T_c=0.169$ MeV, in the absence of environment and the transition temperature from super to normal fluids, i.e., $T_c^{\rm pair}=0.432$ MeV. The solid line coincides with the dotted line at temperatures higher than $T_c^{\rm pair}$, as is expected. Note

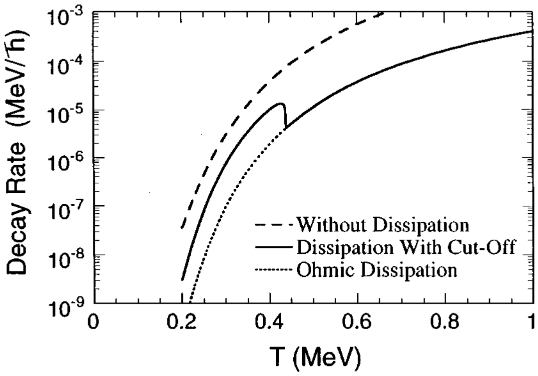

FIG. 3. Decay rate as a function of temperature. The dashed and the dotted lines are the decay rate in the absence of environment and in the Kramers limit, where there is no cutoff, respectively. The solid line takes the effects of cutoff into account.

that the quantum correction factor approaches one at high temperatures.

The decay rate for this system is shown in Fig. 3 as a function of the temperature. The meaning of each line is the same as that in Fig. 2. We observe a sudden decrease of the decay rate at the critical temperature $T_c^{\rm pair}$. This behavior agrees with that found in [27], where the diffusion of muons in metal was studied by taking a superconducting phase transition of the environmental electrons into account. Notice that the cusp behavior in the transitional region will be smeared out to some extent in actual cases, for example, by the gradual disappearance of the pairing gap with temperature.

IV. SUMMARY AND DISCUSSION

We made use of the Im F method of Langer to discuss the fission dynamics of hot nuclei in the presence of a dissipative environment. We modified the Caldeira-Leggett model by introducing a low cutoff frequency in order to mimic the effects of nuclear superfluidity due to pairing interaction. We took into account the temperature dependence of the pairing gap, and thus the phase transition from a superfluid to a normal liquids. The cutoff makes the dissipation weak. This accords with the fact that the nuclear dissipation plays less or no significant role in nuclear fission at low temperatures [28]. The pairing gap gets smaller as the temperature increases. We suggested that the decay rate suddenly decreases at the critical temperature, where the pairing gap disappears. This could be related to the sudden decay of superdeformed band at some critical angular momentum [29].

In this paper, we assumed the standard value for the pairing gap parameter. The critical temperature was then found to be much lower than the threshold temperature for the dissipative fission discussed in [4]. The nonmonotonic behavior of the decay rate shown in Fig. 3 in this paper might therefore indicate the existence of the second critical temperature other than the threshold temperature discussed in [4]. In this connection, we wish to add comments on the possible change of our critical temperature due to the yet unsettled value of the pairing gap in large nuclear deformation. The important thing is that we should use the pairing gap at the saddle point in our calculations, because our formula for fission is intimately related to that in the transition state theory. Studying the influence of the pairing vibration on the spontaneous fission, the authors of [30] obtained a fairly large value of the pairing gap at the saddle point of the fission, which is about two times larger than the standard value. The large effective pairing gaps were also used in the time-dependent Hartree-Fock (TDHF) calculations for the induced fission of ^{236}U [31]. If we replace the pairing gap which we used to obtain Fig. 3 by such large effective pairing gaps, the sudden decrease of the fission rate due to the disappearance of the pairing gap occurs nearly at the threshold temperature found in [4]. In order to draw a definite conclusion on the connection between our critical temperature and the threshold temperature in [4] more detailed studies of the coupling form factor as well as of the temperature and the coordinate dependence of the friction constant [1,8,32–34] are required. The work toward this direction is now in progress.

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