Fusion at deep subbarrier energies: potential inversion revisited

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Abstract. For a single potential barrier, the barrier penetrability can be inverted based on the WKB approximation to yield the barrier thickness. We apply this method to heavy-ion fusion reactions at energies well below the Coulomb barrier and directly determine the inter-nucleus potential between the colliding nuclei. To this end, we assume that fusion cross sections at deep subbarrier energies are governed by the lowest barrier in the barrier distribution. The inverted inter-nucleus potentials for the ^{16}O +^{144}Sm and ^{16}O +^{208}Pb reactions show that they are much thicker than phenomenological potentials. We discuss a consequence of such thick potential by fitting the inverted potentials with the Bass function.

Keywords: Heavy-ion fusion, coupled-channels method, barrier distribution, inter-nucleus potential
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INTRODUCTION

Nuclear reactions are primarily governed by the nucleus-nucleus potential. Several methods have been proposed to compute the real part of the inter-nuclear potential. Among them, the double folding model has been often employed and has enjoyed a success in describing elastic and inelastic scattering for many systems [1, 2, 3]. The Woods-Saxon form, which fits the double folding potential in the tail region, has also often been used to parametrize the inter-nuclear potential [4].

In recent years, many experimental evidences have accumulated that show that the double folding potential fails to account for the *fusion* cross sections at energies close to the Coulomb barrier [5, 6, 7, 8, 9, 10, 11]. This trend has become even more apparent in the recent measurements of fusion cross sections at extreme subbarier energies, that show a much steeper fusion excitation functions as compared with theoretical predictions [12].

The scattering process is sensitive mainly to the surface region of the nuclear potential, while the fusion reaction is also relatively sensitive to the inner part. The double folding potential and the Woods-Saxon potential are reasonable in the surface region [13]. However, it is not obvious whether they provide reasonable parametrizations inside the Coulomb barrier, where the colliding nuclei significantly overlap with each other [6, 14, 15].

In this contribution, we discuss the radial shape of the inter-nucleus potential inside the Coulomb barrier and investigate its deviation from the conventional parametrizations. To this end, we shall first determine the inter-nuclear potential directly from the

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experimental data without assuming any parametrization [16].

INTER-NUCLEUS POTENTIAL FROM FUSION DATA

There have been lots of attempts to determine an inter-nucleus potential directly from experimental fusion excitation functions [17]. In the 70's, it was fashionable to plot a fusion excitation function as a function of $1/E$[18]. Since the classical fusion cross section is given by $\sigma(E) = \pi R_b^2(1 - V_b/E)$, where R_b and V_b are the position and the height of the Coulomb barrier, respectively, the value of R_b and V_b can be read off from such plot (see also Ref. [19]). Bass analysed the first derivative of $E\sigma$, that is, $d(E\sigma)/dE$, and extracted an empirical inter-nucleus potential [20]. He fitted the deduced potential using a function

$$V(r) \propto \frac{1}{A\exp[(r-R_P-R_T)/d_1]+B\exp[(r-R_P-R_T)/d_2]}, \quad (1)$$

where A, B, d_1, and d_2 are adjustable parameters. This potential (the Bass potential) has been widely used, especially for fusion of massive systems.

A more direct way to determine an inter-nucleus potential is to use the potential inversion method based on the WKB approximation [21, 22]. For a single channel system with a potential $V(r)$, the inversion formula relates the thickness of the potential, *i.e.,* the distance between the two classical turning points at a given energy E, with the classical action S as

$$t(E) \equiv r_2(E) - r_1(E) = -\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2\mu}}\int_E^{V_b} dE'\,\frac{\left(\frac{dS}{dE'}\right)}{\sqrt{E'-E}}, \quad (2)$$

where μ is the reduced mass between the colliding nuclei. The classical action $S(E)$ is given by

$$S(E) = \int_{r_1(E)}^{r_2(E)} dr\,\sqrt{\frac{2\mu}{\hbar^2}(V(r)-E)}, \quad (3)$$

and can be obtained once the penetrability $P(E)$ is found in some way using the WKB relation $P(E) = 1/[1+e^{2S(E)}]$.

Balantekin *et al.* assumed a one-dimensional energy independent local potential, and applied this method [22]. They found that the inversion procedure leads to an unphysical multi-valued potential for heavy systems. This analysis has actually provided a clear evidence for inadequacy of the one-dimensional barrier passing model for heavy-ion fusion reactions, and has triggered to develop the coupled-channels approach.

POTENTIAL INVERSION REVISITED

The main reason why Balantekin *et al.* obtained the unphysical inter-nucleus potentials is that they did not take into account the channel coupling effect, which has by now been well understood in terms of barrier distribution [5, 23, 24, 25, 26]. We can then ask ourselves whether a well behaved potential is obtained if one explicitly takes into account the channel coupling effect. We address this question by applying the inversion procedure only to the lowest barrier in the barrier distribution.

In heavy-ion fusion reactions, it is well known that the s-wave penetrability for the Coulomb barrier can be approximately obtained from the fusion cross section $\sigma_{\rm fus}$ as [22, 23, 24, 25]

$$P(E) = \frac{d}{dE}\left(\frac{E\sigma_{\rm fus}}{\pi R^2}\right). \tag{4}$$

In the previous application of the inversion formula by Balantekin *et al.*, they assumed that the penetrability so obtained was resulted from the penetration of a one dimensional energy independent potential[22]. Instead, here we assume that the penetrability P is given as a weighted sum of contribution from many distributed barriers, where the distribution arises due to a coupling of the relative motion between the colliding nuclei to nuclear intrinsic degrees of freedoms such as collective vibrational or rotational excitations. In this eigen channel picture, the penetrability is given by, $P(E) = \sum_n w_n P_n(E)$, where P_n is the penetrability for the n-th eigen-barrier and w_n is the corresponding weight factor.

At energies below the lowest eigen barrier (*i.e.*, the adiabatic barrier) in the barrier distribution, one expects that only the lowest barrier contributes to the total penetrability, $P(E) \approx w_0 P_0(E)$. This indicates that one can apply the inversion formula to the lowest eigen barrier using fusion cross sections at deep subbarrier energies, after correcting the weight factor. The height of the lowest barrier could be estimated from the lowest peak in the fusion barrier distribution [16].

Notice that the inversion formula yields only the barrier thickness, $t(E)$, and one has to supplement either the outer or the inner turning points to determine the radial shape of the potential [22]. We estimate the *outer* turning point $r_2(E)$ using the Coulomb interaction of point charge and the Woods-Saxon nuclear potential, with the range parameter of $R_0 = \sum_{i=P,T}\left(1.233A_i^{1/3} - 0.98A_i^{-1/3}\right) + 0.29 \quad ({\rm fm})$, and the diffuseness parameter of a=0.63 fm. We adjust the depth V_0 in order to reproduce the barrier height V_b determined from the peak position of the barrier distribution. Since the Coulomb term dominates at the outer turning point, except for the region near the barrier top, the inverted potential is insensitive to the actual shape of nuclear potential employed to estimate the outer turning point. The Woods-Saxon potential determines not only the outer turning point but also the position of the potential barrier, R_b. In the actual application of the inversion formula shown below, we smooth the data points with a fifth-order polynomial fit to the function $\ln[E\sigma_{\rm fus}/\pi R^2]$ [22]. We have confirmed that the results do not significantly change even if we use a higher order polynomial fit. We also fit the lowest peak of the barrier distribution using the Wong formula [27] in order to accurately estimate the barrier height V_b.

The resultant inverted inter-nucleus potentials for the ^{16}O+^{144}Sm and the ^{16}O+^{208}Pb systems are shown in Figs. 1 and 2, respectively. We used the measured cross sections reported in Refs. [5, 28] for the inversion procedure. The uncertainty of the inverted potential is estimated in the same way as in Ref. [22]. The dashed line shows the barrier due to the Woods-Saxon potential used to estimate the outer turning points. One clearly sees that the inverted potentials are much thicker than the phenomenological potentials

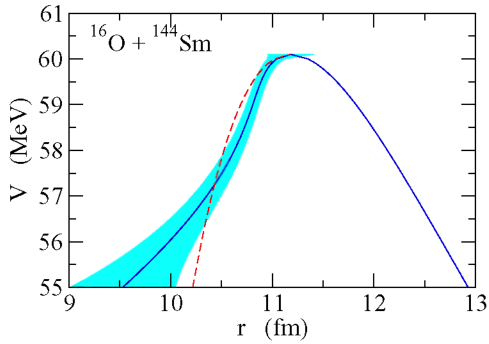

FIGURE 1. The adiabatic potential for the $^{16}O+^{144}Sm$ reaction obtained with the inversion method. The dashed line is a barrier due to a phenomenological Woods-Saxon potential.

at low energies, although they are close to the phenomenological potentials at energies close to the potential barrier. This trend is opposite to what Balantekin *et al.* found in the previous analysis. If there was an unresolved peak in the barrier distribution below the main peak, one would obtain a much thinner barrier than the phenomenological potential, as in the previous analysis. Having thick barriers, rather than thin barriers, we are convinced that the main peak of the barrier distribution for the $^{16}O+^{144}Sm$ and the $^{16}O+^{208}Pb$ reactions indeed consist of the lowest eigen barrier.

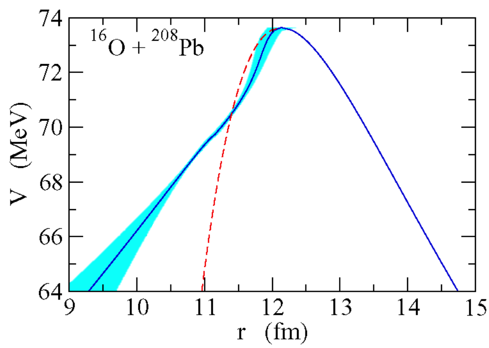

FIGURE 2. Same as Fig. 1, but for the $^{16}O+^{208}Pb$ reaction.

The thicker the potential is, the smaller the penetrability is, and also the stronger the energy dependence of the penetrability is. The thick potentials barrier obtained for the $^{16}O+^{144}Sm$ and $^{16}O+^{208}Pb$ systems are thus consistent with the recent experimental observations [12, 28] that the fusion excitation function is much steeper than theoretical predictions at deep subbarrier energies. Although the present analysis does not exclude a possibility of a shallow potential [15], the present study suggests that the origin of the steep fall-off phenomenon of fusion cross section can be at least partly attributed to the departure of inter-nuclear potential from the Woods-Saxon shape.

For the $^{16}O+^{208}Pb$ system shown in Fig. 2, the deviation of the inverted potential from the phenomenological potential starts to occur at around $E = 70.4$ MeV. It is amusing to notice that this energy is very close to the potential energy at the contact configuration estimated with the Krappe-Nix-Sierk potential[29, 30]. Inside the touching configuration, the potential represents the fission-like adiabatic potential energy surface. The effect of such one-body potential has been considered recently and is shown to account well for the steep fall-off phenomena of fusion cross sections [30]. The inverted potentials which we obtain are thus intimately related to the one-body dynamics for deep subbarrier fusion reactions.

DISCUSSION

Although the inverted potentials shown in Figs. 1 and 2 are well behaved, there remains a question whether the inter-nucleus potential in itself is actually thick or it simply mocks up some dynamical effects such as energy and angular momentum dissipations. To address this question, we fit the inverted potential with the Bass function given by Eq. (1). A motivation to use the Bass function is that it leads to a thicker potential than a double folding potential. This is demonstrated in Fig. 3 for the ^{16}O+^{208}Pb system. One can see that the Bass potential is much thicker than the Woods-Saxon potential with the surface diffuseness parameter of a=0.65 fm, although both the potentials are similar to each other in the tail region. Notice that the thickness of the Bass potential is similar to that of the Woods-Saxon potential with a=1.0 fm.

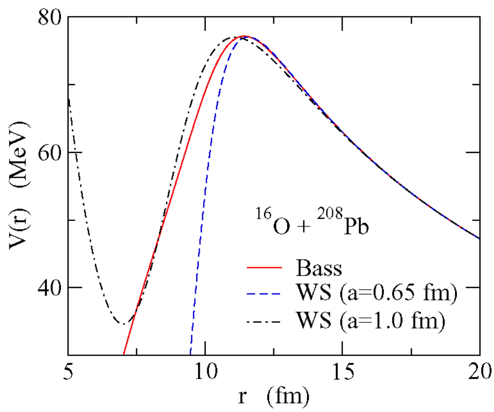

FIGURE 3. Comparison of the Bass potential with the Woods-Saxon potential with two different values of surface diffuseness parameter a for the ^{16}O+^{208}Pb system.

Figure 4 shows the result of the fitting for the ^{16}O+^{208}Pb system. In the original Bass potential, the parameters A, B, d_1 and d_2 take the value of A=0.03 MeV^{-1}, $B = 0.0061$ MeV^{-1}, d_1 =3.3 fm, and d_2=0.65 fm [20]. We slightly change the value of A to 0.05 MeV^{-1}, and adjust the depth of the potential. The dot-dashed line in Fig.4 shows the potential obtained in this way, which we use as the bare potential for the coupled-channels calculation. The lowest eigen-barrier obtained by diagonalizing the coupling matrix at each position r is denoted by the dashed line. To this end, we include the single octupole excitation in ^{16}O and the double octupole excitations in ^{208}Pb, using the CCFULL scheme [31]. The resultant eigen-potential fits well the inverted potential obtained in the previous section.

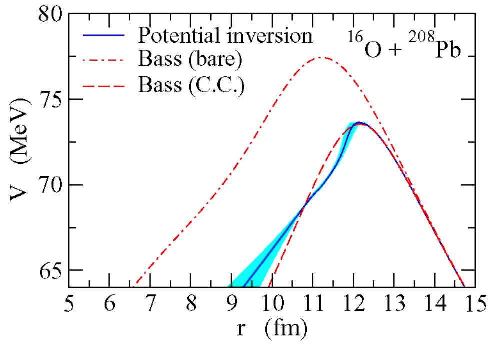

FIGURE 4. The result of the fitting of the inverted potential with the Bass function given by Eq. (1) for the $^{16}O+^{208}Pb$ system. The dot-dashed line shows a bare potential, while the dashed line is the lowest eigen-barrier obtained by including the octupole excitations in the colliding nuclei.

The result of coupled-channels calculation with this potential is shown in Fig. 5. This calculation well reproduces the steep fall-off of fusion cross sections at deep subbarrier energies. A small discrepancy around $E_{cm} \sim 70$MeV may be accounted for by including the one-neutron pick-up transfer channel, as was recently pointed out by Esbensen and Misicu [32]. However, this calculation largely underestimates the fusion cross sections at energies above the Coulomb barrier. We have checked that it is the case even when we use an internal imaginary potential for fusion, instead of the incoming wave boundary condition. Since the potential which fits the deep subbarrier data does not reproduce the measured fusion cross sections at higher energies, this may be an indication of some other missing dynamical effect, either at deep subbarrier energies or at energies above the barrier (or both). The same conclusion has been reached in Ref. [28].

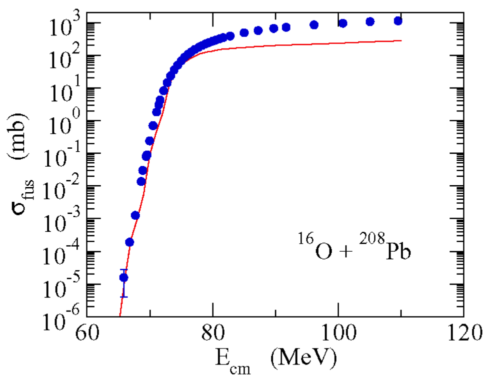

FIGURE 5. The fusion cross sections for the $^{16}O+^{208}Pb$ system obtained with the coupled-channels calculations with the potential shown in Fig. 4.

Another indication of a missing dynamical effect could be seen in the experimental barrier distributions for the $^{16}O+^{144}Sm$ system. It has been known that the fusion barrier distribution for this system has a clear double-peaked structure, whereas the higher peak is significantly smeared in the quasi-elastic barrier distribution [33]. We have recently performed the coupled-channels calculations for this system with several coupling schemes [34]. Our calculations indicate that, within the same coupling scheme, the quasi-elastic and fusion barrier distributions are always similar to each other, and the difference in the experimental barrier distributions cannot be accounted for within the standard coupled-channels approach. This indeed suggests that some physical effects, besides the collective excitations in the colliding nuclei, have to be taken into account in order to explain the fusion and quasi-elastic scattering simultaneously.

SUMMARY

We applied the potential inversion method, which relates the potential penetrability to the thickness of the potential barrier, in order to investigate the radial dependence of the inter-nucleus potential for heavy-ion fusion reactions. To this end, we assumed that the tunneling is well described by the lowest adiabatic barrier at deep subbarier energies, and extracted the penetrability by combining the experimental barrier distribution and fusion cross sections. We found that the resultant potential for the ^{16}O+^{144}Sm and ^{16}O+^{208}Pb systems is much thicker than a barrier obtained with a phenomenological Woods-Saxon potential. This indicates that the steep fall-off phenomenon of fusion cross sections recently observed in several systems can be partly accounted for in terms of a deviation of inter-nuclear potential from the Woods-Saxon shape, although some dynamical effects are also expected to play a role.

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