Fusion and Quasi-elastic scattering around the Coulomb barrier: determination of inter-nucleus potential

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Abstract. We invert experimental data for heavy-ion fusion cross sections at energies well below the Coulomb barrier in order to directly determine the internucleus potential between the colliding nuclei. In contrast to the previous applications of the inversion formula, we explicitly take into account the effect of channel couplings on fusion reactions, by assuming that fusion cross sections at deep subbarrier energies are governed by the lowest barrier in the barrier distribution. The surface region of the internuclear potential is determined from quasi-elastic scattering at deep subbarrier energies, while the inner part is determined with the WKB formula. We apply this procedure to the ^{16}O +^{144}Sm and ^{16}O +^{208}Pb reactions, and find that the inverted internucleus potential are much thicker than phenomenological potentials.

Keywords: Heavy-ion fusion, coupled-channels method, barrier distribution, inter-nucleus potential
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INTRODUCTION

A standard tool to analyze heavy-ion reactions at energies around the Coulomb barrier is the coupled-channels approach [1]. In addition to excitation properties of colliding nuclei, such as the excitation energy and deformation parameter, an inter-nucleus potential is one of the most important inputs for coupled-channels calculations. This is so, because nuclear reactions at subbarrier energies are primarily governed by the height, the position, and the curvature of the Coulomb barrier. Also, the coupling form factors are generated from the inter-nucleus potential.

Several methods have been proposed to compute the real part of the inter-nuclear potential. Among them, the double folding model has been often employed and has enjoyed a success in describing elastic and inelastic scattering for many systems [2, 3, 4]. The Woods-Saxon form, which fits the double folding potential in the tail region, has also often been used to parametrize the inter-nuclear potential [5].

In recent years, many experimental evidences have accumulated that show that the double folding potential fails to account for the *fusion* cross sections at energies close to the Coulomb barrier [6, 7, 8, 9, 10, 11, 12]. This trend has become even more apparent in the recent measurements of fusion cross sections at extreme subbarier energies, that show a much steeper fusion excitation functions as compared with theoretical predictions [13].

The scattering process is sensitive mainly to the surface region of the nuclear potential, while the fusion reaction is also relatively sensitive to the inner part. The double folding

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potential and the Woods-Saxon potential are reasonable in the surface region [14]. However, it is not obvious whether they provide reasonable parametrizations inside the Coulomb barrier, where the colliding nuclei significantly overlap with each other [7, 15, 16].

In this contribution, we discuss the radial shape of the inter-nucleus potential inside the Coulomb barrier and investigate its deviation from the conventional parametrizations. For a single potential barrier, the barrier penetrability can be inverted based on the WKB approximation to yield the barrier thickness. We shall apply this method to heavy-ion fusion reactions at energies well below the Coulomb barrier and directly determine the inter-nucleus potential between the colliding nuclei [17].

INTER-NUCLEUS POTENTIAL FROM FUSION DATA

There have been lots of attempts to determine an inter-nucleus potential directly from experimental fusion excitation functions [18]. In the 70's, it was fashionable to plot a fusion excitation function as a function of $1/E$[19]. Since the classical fusion cross section is given by $\sigma(E) = \pi R_b^2(1 - V_b/E)$, where R_b and V_b are the position and the height of the Coulomb barrier, respectively, the value of R_b and V_b can be read off from such plot (see also Ref. [20]). Bass analysed the first derivative of $E\sigma$, that is, $d(E\sigma)/dE$, and extracted an empirical inter-nucleus potential [21]. The Bass potential has been widely used, especially for fusion of massive systems.

A more direct way to determine an inter-nucleus potential is to use the potential inversion method based on the WKB approximation [22, 23]. For a single channel system with a potential $V(r)$, the inversion formula relates the thickness of the potential, *i.e.,* the distance between the two classical turning points at a given energy E, with the classical action S as

$$t(E) \equiv r_2(E) - r_1(E) = -\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2\mu}}\int_E^{V_b} dE'\,\frac{\left(\frac{dS}{dE'}\right)}{\sqrt{E'-E}}, \tag{1}$$

where μ is the reduced mass between the colliding nuclei. The classical action $S(E)$ is given by

$$S(E) = \int_{r_1(E)}^{r_2(E)} dr\sqrt{\frac{2\mu}{\hbar^2}(V(r)-E)}, \tag{2}$$

and can be obtained once the penetrability $P(E)$ is found in some way using the WKB relation $P(E) = 1/[1+e^{2S(E)}]$.

Balantekin *et al.* assumed a one-dimensional energy independent local potential, and applied this method [23]. They found that the inversion procedure leads to an unphysical multi-valued potential for heavy systems. This analysis has actually provided a clear evidence for inadequacy of the one-dimensional barrier passing model for heavy-ion fusion reactions, and has triggered to develop the coupled-channels approach.

The main reason why Balantekin *et al.* obtained the unphysical inter-nucleus potentials is that they did not take into account the channel coupling effect, which has by now been well understood in terms of barrier distribution [6, 24, 25, 26, 27]. We can then ask ourselves whether a well-behaved potential is obtained if one explicitly takes into account the channel coupling effect. We address this question by applying the inversion procedure only to the lowest barrier in the barrier distribution.

In heavy-ion fusion reactions, it is well known that the *s*-wave penetrability for the Coulomb barrier can be approximately obtained from the fusion cross section $\sigma_{\rm fus}$ as [23, 24, 25, 26]

$$P(E) = \frac{d}{dE}\left(\frac{E\sigma_{\rm fus}}{\pi R^2}\right). \tag{3}$$

In the previous application of the inversion formula by Balantekin *et al.*, they assumed that the penetrability so obtained was resulted from the penetration of a one dimensional energy independent potential[23]. Instead, here we assume that the penetrability P is given as a weighted sum of contribution from many distributed barriers, where the distribution arises due to a coupling of the relative motion between the colliding nuclei to nuclear intrinsic degrees of freedoms such as collective vibrational or rotational excitations. In this eigen channel picture, the penetrability is given by, $P(E)=\sum_n w_n P_n(E)$, where P_n is the penetrability for the n-th eigen-barrier and w_n is the corresponding weight factor.

At energies below the lowest eigen barrier (*i.e.*, the adiabatic barrier) in the barrier distribution, one expects that only the lowest barrier contributes to the total penetrability, $P(E)\approx w_0 P_0(E)$. This indicates that one can apply the inversion formula to the lowest eigen barrier using fusion cross sections at deep subbarrier energies, after correcting the weight factor. The height of the lowest barrier could be estimated from the lowest peak in the fusion barrier distribution [17].

In order to demonstrate how this works, Figs. 1(a) and 1(b) show the second and the first derivatives of the measured $E\sigma_{\rm fus}$ [6] for the ^{16}O+^{144}Sm reaction, respectively. The former quantity is usually referred to as the fusion barrier distribution [6, 25]. For this system, one can clearly recognize that the barrier distribution has a double peaked structure. Correspondingly, the first derivative $d(E\sigma_{\rm fus})/dE$ appears to have two steps as a function of energy. Assuming that the main peak of the barrier distribution around $E_{\rm c.m.}\sim$60 MeV consists only of the contribution from the lowest eigen barrier, we scale the first derivative $d(E\sigma_{\rm fus})/dE$ so that it has a value of 0.5 at the peak energy, which we assume to be identical to the position of the lowest barrier, V_b. The function thus obtained is shown by the filled circles in Fig. 1(c). This function can be interpreted as the penetrability for the lowest barrier, to which one can apply the inversion formula to determine the radial shape.

Notice that the inversion formula yields only the barrier thickness, $t(E)$, and one has to supplement either the outer or the inner turning points to determine the radial shape of the potential [23]. We estimate the *outer* turning point $r_2(E)$ using the Coulomb interaction of point charge and the Woods-Saxon nuclear potential, with the diffuseness parameter of a=0.63 fm. This value is estimated from analyses of quasi-elastic scattering

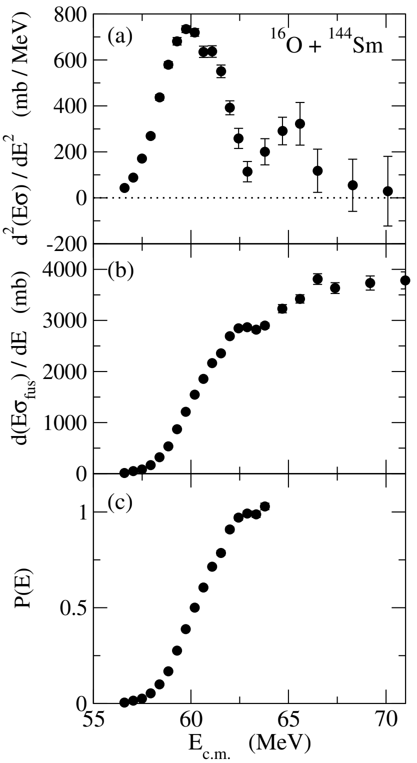

FIGURE 1. Fig. 1(a): The experimental fusion barrier distribution for the ^{16}O+^{144}Sm reaction defined as $d^2(E\sigma_{\rm fus})/dE^2$. The experimental data are taken from Ref. [6]. Fig. 1(b): The first derivative of $E\sigma_{\rm fus}$ for the ^{16}O+^{144}Sm reaction. Fig. 1(c): The same as Fig.1 (b), but normalized so that it is 0.5 at the energy of the lower peak in the barrier distribution shown in Fig. 1(a).

at deep subbarrier eneriges, at which the scattering cross sections are sensitive mainly to the value of surface diffuseness parameter [14, 28].

The resultant inverted inter-nucleus potentials for the ^{16}O+^{144}Sm and the ^{16}O+^{208}Pb systems are shown in Fig. 2. We used the measured cross sections reported in Refs. [6, 29] for the inversion procedure. The dashed line shows the barrier due to the Woods-Saxon potential used to estimate the outer turning points. One clearly sees that the inverted potentials are much thicker than the phenomenological potentials at low energies, although they are close to the phenomenological potentials at energies close to the potential barrier. This trend is opposite to what Balantekin *et al.* found in the previous analysis. If there was an unresolved peak in the barrier distribution below the main peak, one would obtain a much thinner barrier than the phenomenological potential, as in the previous analysis. Having thick barriers, rather than thin barriers, we are convinced that the main peak of the barrier distribution for the ^{16}O+^{144}Sm and the ^{16}O+^{208}Pb reactions indeed consist of the lowest eigen barrier.

The thicker the potential is, the smaller the penetrability is, and also the stronger

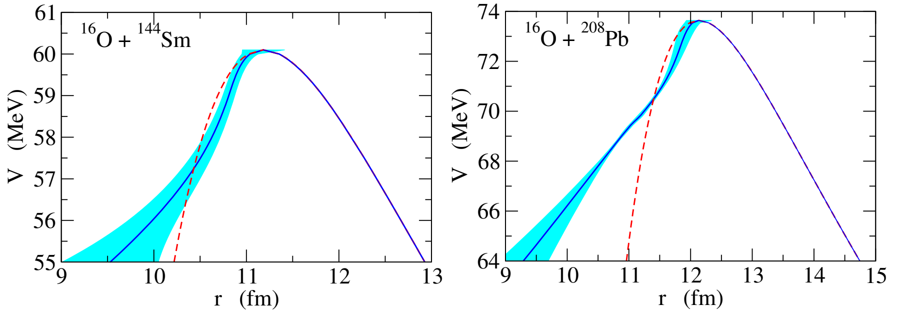

FIGURE 2. The adiabatic potential for the ^{16}O+^{144}Sm (the left panel) and for the ^{16}O+^{208}Pb (the right panel) reactions obtained with the inversion method. The dashed line is a barrier due to a phenomenological Woods-Saxon potential.

the energy dependence of the penetrability is. The thick potentials barrier obtained for the ^{16}O+^{144}Sm and ^{16}O+^{208}Pb systems are thus consistent with the recent experimental observations [13, 29] that the fusion excitation function is much steeper than theoretical predictions at deep subbarrier energies. Although the present analysis does not exclude a possibility of a shallow potential [16], the present study suggests that the origin of the steep fall-off phenomenon of fusion cross section can be at least partly attributed to the departure of inter-nuclear potential from the Woods-Saxon shape.

For the ^{16}O+^{208}Pb system shown in Fig. 2, the deviation of the inverted potential from the phenomenological potential starts to occur at around $E = 70.4$ MeV. It is amusing to notice that this energy is very close to the potential energy at the contact configuration estimated with the Krappe-Nix-Sierk potential[30, 31]. Inside the touching configuration, the potential represents the fission-like adiabatic potential energy surface. The effect of such one-body potential has been considered recently and is shown to account well for the steep fall-off phenomena of fusion cross sections [31]. The inverted potentials which we obtain are thus intimately related to the one-body dynamics for deep subbarrier fusion reactions.

SUMMARY

We applied the potential inversion method, which relates the potential penetrability to the thickness of the potential barrier, in order to investigate the radial dependence of the inter-nucleus potential for heavy-ion fusion reactions. To this end, we assumed that the tunneling probability is well described only by the lowest adiabatic barrier at deep subbarier energies, and extracted the penetrability by combining the experimental barrier distribution and fusion cross sections. We found that the resultant potential for the ^{16}O+^{144}Sm and ^{16}O+^{208}Pb systems is much thicker than a barrier obtained with a phenomenological Woods-Saxon potential. This indicates that the steep fall-off phenomenon of fusion cross sections recently observed in several systems can be partly accounted for in terms of a deviation of inter-nuclear potential from the Woods-Saxon shape, although some dynamical effects are also expected to play a role.

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REFERENCES

1. K. Hagino, N. Rowley, and A.T. Kruppa, Comp. Phys. Comm. **123**, 143 (1999).
2. G.R. Satchler and W.G. Love, Phys. Rep. **55**, 183 (1979).
3. M.E. Brandan and G.R. Satchler, Phys. Rep. **285**, 143 (1997).
4. D.T. Khoa and G.R. Satchler, Nucl. Phys. **A668**, 3 (2000).
5. R.A. Broglia and A. Winther, *Heavy Ion Reactions*, Vol. 84 in Frontiers in Physics Lecture Note Series (Addison-Wesley, Redwood City, CA, 1991).
6. J.R. Leigh, M. Dasgupta, D.J. Hinde, J.C. Mein, C.R. Morton, R.C. Lemmon, J.P. Lestone, J.O. Newton, H. Timmers, J.X. Wei, and N. Rowley, Phys. Rev. C**52**, 3151 (1995).
7. J.O. Newton, R.D. Butt, M. Dasgupta, D.J. Hinde, I.I. Gontchar, C.R. Morton, and K. Hagino, Phys. Lett. B**586**, 219 (2004); Phys. Rev. C**70**, 024605 (2004).
8. J.O. Newton, C.R. Morton, M. Dasgupta, J.R. Leigh, J.C. Mein, D.J. Hinde, H. Timmers, and K. Hagino, Phys. Rev. C**64**, 064608 (2001).
9. K. Hagino, M. Dasgupta, I.I. Gontchar, D.J. Hinde, C.R. Morton, and J.O. Newton, *Proceedings of the Fourth Italy-Japan Symposium on Heavy-Ion Physics, Tokyo, Japan* (World Scientific, Singapore, 2002), pp. 87-98; nucl-th/0110065.
10. K. Hagino, N. Rowley, and M. Dasgupta, Phys. Rev. C**67**, 054603 (2003).
11. I.I. Gontchar, D.J. Hinde, M. Dasgupta, and J.O. Newton, Phys. Rev. C**69**, 024610 (2004).
12. M. Dasgupta, D.J. Hinde, J.O. Newton, and K. Hagino, Prog. Theo. Phys. Suppl. **154**, 209 (2004).
13. C.L. Jiang *et al.*, Phys. Rev. Lett. **89**, 052701 (2002); Phys. Rev. C**69**, 014604 (2004); Phys. Rev. Lett. **93**, 012701 (2004); Phys. Rev. C**71**, 044613 (2005); Phys. Rev. C**73**, 014613 (2006).
14. K. Washiyama, K. Hagino, and M. Dasgupta, Phys. Rev. C**73**, 034607 (2006).
15. C.H. Dasso and G. Pollarolo, Phys. Rev. C**68**, 054604 (2003).
16. S. Misicu and H. Esbensen, Phys. Rev. Lett. **96**, 112701 (2006); Phys. Rev. C**75**, 034606 (2007).
17. K. Hagino and Y. Watanabe, Phys. Rev. C**76**, 021601(R) (2007).
18. L.C. Vaz, J.M. Alexander, and G.R. Satchler, Phys. Rep. **69**, 373 (1981).
19. W. Scobel, H.H. Gutbrod, M. Blann, and A. Mignerey, Phys. Rev. C**14**, 1808 (1976).
20. N. Rowley, A. Kabir, and R. Lindsay, J. of Phys. **G15**, L269 (1989).
21. R. Bass, Phys. Rev. Lett. **39**, 265 (1977).
22. M.W. Cole and R.H. Good, Phys. Rev. A**18**, 1085 (1978).
23. A.B. Balantekin, S.E. Koonin, and J.W. Negele, Phys. Rev. C**28**, 1565 (1983); M. Inui and S.E. Koonin, Phys. Rev. C**30**, 175 (1984).
24. N. Rowley, G.R. Satchler, and P.H. Stelson, Phys. Lett. **B254**, 25 (1991).
25. M. Dasgupta, D.J. Hinde, N. Rowley, and A.M. Stefanini, Annu. Rev. Nucl. Part. Sci. **48**, 401 (1998).
26. A.B. Balantekin and N. Takigawa, Rev. Mod. Phys. **70**, 77 (1998).
27. K. Hagino and A.B. Balantekin, Phys. Rev. A**70**, 032106 (2004).
28. K. Hagino, T. Takehi, A.B. Balantekin, and N. Takigawa, Phys. Rev. C**71**, 044612 (2005).
29. M. Dasgupta *et al.*, Phys. Rev. Lett. **99**, 192701 (2007).
30. H.J. Krappe, J.R. Nix, and A.J. Sierk, Phys. Rev. C**20**, 992 (1979).
31. T. Ichikawa, K. Hagino, and A. Iwamoto, Phys. Rev. C**75**, 057603 (2007); Phys. Rev. C**75**, 064612 (2007).