博士論文

Applications of non-perturbative techniques in QCD and supersymmetric theories

（QCDと超対称理論における非摂動的方法の応用）

中村 充利

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Applications of non-perturbative techniques in QCD and supersymmetric theories

Mitsutoshi Nakamura

Department of Physics, Tohoku University, Sendai 980-77, Japan

Abstract

The sum rules in QCD and dualities in supersymmetric theories are well-known non-perturbative techniques. The sum rules can be derived for theories whose global symmetry is dynamically broken. They provide us with non-trivial relations among the physical quantities. In supersymmetric gauge theories, the Seiberg duality can often be used to analyze the strongly coupled gauge theories by using their weakly coupled dual descriptions. Although the sum rules and dualities are powerful techniques for strongly coupled theories in general, sum rules for supersymmetric theories or dualities in non-supersymmetric theories are not widely used for their analysis. Since one may obtain new insights from such trials, we study sum rules for dynamical supersymmetry breaking and also try to apply the idea of Seiberg duality to chiral symmetry breaking in non-supersymmetric QCD. We report the results of those analyses in this article.

Contents

Chapter 1

Introduction and Overview

1.1 Introduction

Many quantum field theories have symmetries. Those symmetries play important roles for physics. For example they constrain interactions among the particles and the Lagrangian of the low energy effective theories. The local symmetries explain fundamental interactions, such as electromagnetic, weak, and strong interactions. Even if the some of symmetries are spontaneously broken, they provide us with non-trivial imprints, such as the Nambu-Goldstone theorem which claims that there exists the same number of massless particles as that of broken generators.

Global symmetries sometimes can be realized as dynamically broken ones as a consequence of strong interactions. There are two particularly important examples of dynamical symmetry breaking in phenomenology of particle physics. One is chiral symmetry breaking in QCD. The QCD describes the inter-quarks interactions and at low energy, the strong force between quarks causes a spectacular phenomenon, the quark confinement. This strong interaction also breaks chiral symmetry, $SU(N_f)_L \times SU(N_f)_R$, to diagonal subgroup, $SU(N_f)_V$ and pions appear as Nambu-Goldstone bosons, the massless particle required in Nambu-Goldstone theorem, of spontaneous broken chiral symmetry. The other interesting example is dynamical supersymmetry (SUSY) breaking. SUSY is the only possible extension of space-time symmetry consistent with Poincaré symmetry and relates fermionic degrees of freedom and bosonic ones. Low energy SUSY has been considered as a solution to explain the light Higgs mass compared with the scale of Grand Unified Theory. Also SUSY is required in superstring theory, a candidate of fundamental theory of physics. Since we have not observed SUSY yet, SUSY should be realized as spontaneously broken symmetry. Many dynamical SUSY breaking models have been proposed in the literature.

Although the theories with dynamical symmetry breaking are important in particle physics, their analyses are difficult since many important parts of physics cannot be accessed by the perturbation theory. Therefore we need non-perturbative methods to analyze theories with dynamical symmetry breaking. For example, Weinberg has derived non-perturbative results in QCD using spontaneously broken chiral symmetry and its algebra [1]. His results are called as the Weinberg sum rules. These are relations among spectral functions, and can be derived if the ultraviolet (UV) theory is asymptotically free and the symmetry is broken by a vacuum expectation value (VEV) of an operator whose mass dimension is high enough. The ingredients for deriving the Weinberg sum rules are (well-defined) operators such as currents and their transformation laws under the broken symmetry. In the case of chiral symmetry breaking in QCD, by using the charge-current algebra, Weinberg has derived two sum rules. Once the spectral functions are approximated by summation of one-particle states of hadrons, the rules reduce to relations among hadron masses and decay constants: $f_\pi^2 - f_\rho^2 + f_{a_1}^2 \simeq 0$ and $m_\rho^2 f_\rho^2 - m_{a_1}^2 f_{a_1}^2 \simeq 0$. They catch qualitative features of hadron properties correctly.

Dualities are another class of non-perturbative method to analyze the dynamical system, which is often used in SUSY theories. For example, the Seiberg duality [2] is known to be the electric-magnetic duality in $\mathcal{N} = 1$ supersymmetric gauge theories and is often used to analyze strongly coupled supersymmetric gauge theories. The most characteristic feature of Seiberg duality is to relate the gauge theories with different gauge groups. It is a duality between $SU(N_c)$ gauge theory with quarks have $SU(N_f)$ flavor symmetry and $SU(N_f - N_c)$ gauge theory with gauge singlet mesons and dual quarks. This duality passes the nontrivial consistency checks, anomaly matching and the existence of mapping between their moduli spaces.

The sum rules and dualities are well-known strong methods in non-SUSY and SUSY theories, respectively, for the analysis of strongly coupled systems. However, the sum rules for SUSY theories or dualities in non-SUSY theories are not widely used as tools for the analyses, although in principle one may be able to obtain new insights from such trials. In this article we report the results of such trials; we exchange the techniques to analyze chiral symmetry breaking and dynamical SUSY breaking. We apply the idea of Seiberg duality [2] to chiral symmetry breaking in non-SUSY QCD, and we derive Weinberg sum rules to dynamical SUSY breaking from the SUSY current algebra [3]. Indeed, we obtain various new insights as explained below.

We propose the low energy effective model of QCD to analyze the chiral symmetry breaking through the idea of Seiberg duality. Since Seiberg duality is electric-magnetic duality of supersymmetric gauge theory, we interpret the model as the magnetic dual picture of QCD. If magnetic description is realized in QCD, the dual Meissner effect [4, 5] is plausible explanation of QCD confinement. The condensation of the magnetic monopole, $\langle m \rangle \neq 0$, makes the QCD vacuum to be in the dual superconducting phase, where color fluxes sourced by quarks are squeezed into tubes, explaining the linear potential between quarks.

The magnetic description provides us not only the physical description of color confinement but also the vector mesons as the dual gauge bosons. It has been known that the masses and interactions of the vector mesons, ρ and ω, and π are well described by spontaneous broken $U(N_f)$ gauge theory [6]. In the model in Ref. [6], the vector mesons obtain their masses through the VEV of scalar particles which have both gauge and flavor symmetries. Interpreting the vector mesons as dual gauge bosons, such scalar particles are magnetic monopoles since they have the quantum number of dual gauge symmetry. The condensation of the monopole with flavor symmetry describes the both color confinement and chiral symmetry breaking. Similar phenomena have been observed in supersymmetric gauge theories [7, 8, 9, 10].

We identify the $U(N_f)$ gauge symmetry as the magnetic gauge symmetry and the magnetic gauge bosons as vector mesons. The identification of the vector mesons as magnetic gauge bosons is motivated by recent discussions in supersymmetric gauge theories [11, 12, 13]. (See also [14] for an earlier discussion.) One can derive the $U(N_f)$ gauge theory from the SUSY regularized QCD with massive auxiliary fields [12]. The model contains the vector and scalar fields as well as the string as a solitonic object. Since string carries a magnetic flux in the magnetic picture, it can be naturally identified as the confining string via electric-magnetic duality. We examine whether such identification works at the quantitative level [15]. By using hadron masses and coupling constants as inputs, one can calculate the string tension and the Coulomb force between static quarks. We obtain values which are consistent with those inferred from the quarkonium spectrum and the Regge trajectories in the hadron spectrum.

We also consider the Weinberg sum rules in dynamical SUSY breaking models. We find that sum rules can be derived for even "incalculable models", such as models proposed in Refs. [16, 17] and for the models we have not found yet. When we apply the analogy to the Hidden Local Symmetry in QCD to SUSY breaking dynamics, the rho meson counterpart in SUSY breaking dynamics should be a spin-2 resonance. If there is a direct interaction between the Standard Model particles and SUSY breaking dynamics, the sum rules among higher spin resonances may be observed as the first sign of the dynamical SUSY breaking.

In next section we overview the chiral symmetry breaking and dynamical SUSY breaking models. We construct and analyze the low energy effective theory of chiral symmetry breaking as magnetic description of QCD in next chapter. In construction of our model we require $U(N_f)$ gauge group as magnetic gauge symmetry of $SU(N_c)$ and it contains the magnetic monopoles with dual gauge and flavor symmetry as Higgs fields of dual theory. Those feature can be seen in the Seiberg duality of supersymmetric models. In Section. 2.1, we write down the Lagrangian of that model and discuss the particles which we can explain through the model. Higgsing $U(N_f)$ gauge model has vortex solution [18, 19, 20, 21] and it can be interpreted as confinement string in our model since dual charge of our model can be interpreted as ordinary color charge. The vortex solution and its comparison to QCD data are discussed in Section. 2.2 and 2.3 respectively. The chiral phase transition at finite temperatures is discussed in Section. 2.4. The detailed discussion of relation between our model and the Seiberg duality is in Section. 2.5. In Chapter. 3 we discuss the dynamical SUSY breaking. The Weinberg sum rules of dynamical symmetry breaking are derived using the correlators of component currents in the supercurrent multiplet. The supercurrent multiplet is known to be well-defined in wide class of SUSY theories. From the transformation laws of the component fields, a set of sum rules can be derived involving states with spins 0, 1/2, 1, 3/2, and 2. Their correlators and derived sum rules are discussed in Section. 3.2 and Section. 3.3 describes relationship between the sum rules and symmetry breaking models.

1.2 Overview

The chiral symmetry breaking in QCD and dynamical SUSY breaking are the most interesting and attractive example of dynamical symmetry breaking in particle physics. We analyze those dynamical symmetry breaking in this article. In this chapter we overview those dynamical symmetry breaking.

1.2.1 Chiral symmetry breaking

In this article we discuss the dynamical symmetry breaking. The most famous example of dynamical symmetry breaking is chiral symmetry breaking. The QCD with massless quarks have chiral symmetry, $SU(N_f)_L \times SU(N_f)_R$ and it is dynamically broken to diagonal subgroup, $SU(N_f)_V$. Although the quarks have masses, the chiral symmetry breaking approximately explain hadron physics. For example pions can be interpreted as its Nambu-Goldstone boson (their masses are 139 and 135 MeV for $\pi^\pm$ and π^0 respectively) and hadrons can be classified through remaining $SU(N_f)_V$ symmetry.

Hidden Local Symmetry model

The non-linear realization of chiral symmetry is most straightforward model describing low energy physics of QCD and contains only pions. We introduce the light hadrons such as vector particles, rho mesons whose masses are 770 MeV, into this model as extension. Hidden Local Symmetry (HLS) model is such model [6] which describes not only the pions but also rho mesons as gauge bosons of hidden local symmetry.

In order to explain the HLS model, we first comment on non-linear realization of chiral symmetry:

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right), \tag{1.1}$$

$$U = \exp\left[2i\pi(x)/f_\pi\right], \quad \pi \equiv \pi^a T^a. \tag{1.2}$$

where f_π is pion decay constant and T^a are the generators of flavor symmetry normalized as $\mathrm{Tr}\, T^a T^b = \frac{1}{2}\delta^{ab}$. The transformation low of $U(x)$ under flavor symmetry is

$$U(x) \to g_L U(x) g_R^\dagger, \tag{1.3}$$

where g_L and g_R are elements of $SU(N_f)_L$ and $SU(N_f)_R$ respectively. This Lagrangian (1.1) is non-linear realization of chiral symmetry and describes only pions and their interactions. As next step we introduce rho mesons into this model as gauge bosons and derive HLS model [6].

We divide $U(x)$ into two new fields, $\xi_L(x)$ and $\xi_R(x)$, as following:

$$U(x) = \xi_L^\dagger(x)\xi_R(x), \tag{1.4}$$

where $\xi_L(x)$ and $\xi_R(x)$ transform under $SU(N_f)_L$ and $SU(N_f)_R$ respectively. Using this notation the theory have additional $SU(N_f)_V$ symmetry and their transformation laws are

$$\xi_L(x) \to h\xi_L(x)g_L^\dagger, \quad \xi_R(x) \to h\xi_R(x)g_R^\dagger, \tag{1.5}$$

where h is element of $SU(N_f)_V$. In order to localize this additional symmetry we introduce the gauge fields $V_\mu$which transform as

$$V_\mu \to ih(x)\partial_\mu h(x)^\dagger + h(x)V_\mu h(x)^\dagger, \tag{1.6}$$

and covariant derivatives as $D_\mu \xi_{R,L} = [\partial_\mu - iV_\mu(x)]\xi_{R,L}$. The Lagrangian is

$$\mathcal{L} = -\frac{1}{4g^2}F_{\mu\nu}^2 + \mathcal{L}_A + a\mathcal{L}_V, \tag{1.7}$$

$$\mathcal{L}_A = -\frac{f_\pi^2}{4}\mathrm{Tr}\left(D_\mu\xi_L \cdot \xi_L^\dagger - D_\mu\xi_R \cdot \xi_R^\dagger\right)^2, \tag{1.8}$$

$$\mathcal{L}_V = -\frac{f_\pi^2}{4}\mathrm{Tr}\left(D_\mu\xi_L \cdot \xi_L^\dagger + D_\mu\xi_R \cdot \xi_R^\dagger\right)^2, \tag{1.9}$$

$$\tag{1.10}$$

where a and g are model parameters and $F_{\mu\nu}$ are field strength derived from $V_\mu(x)$. If we fix the gauge as $\xi \equiv \xi_L^\dagger = \xi_R$, $\mathcal{L}_A$ is identical to the Lagrangian (1.1) and $\mathcal{L}_V$ is

$$\mathcal{L}_V = f_\pi^2\mathrm{Tr}\left\{V_\mu - \frac{1}{2i}\left(\partial_\mu\xi \cdot \xi^\dagger + \partial_\mu\xi^\dagger \cdot \xi\right)\right\}^2, \tag{1.11}$$

which vanishes when we submit solution of V_μ equation of motion.

The Lagrangian (1.7) describes the interactions between pions and rho mesons. After rescaling $g^{-1}V_\mu \to V_\mu$ we obtain

$$g_{\rho\pi\pi} = \frac{1}{2}ag, \tag{1.12}$$

$$m_\rho^2 = ag^2 f_\pi^2. \tag{1.13}$$

The observed value of $g_{\rho\pi\pi}$, m_ρ and f_π are

$$m_\rho = 770\mathrm{MeV}, \quad g_{\rho\pi\pi} = 6.03, \quad f_\pi = 92.4\mathrm{MeV}. \tag{1.14}$$

If we take the model parameter as,

$$a \simeq 2, \quad g \simeq 6, \tag{1.15}$$

this model well describes those masses and interactions.

1.2.2 Supersymmetry and its breaking

The dynamical SUSY breaking is another interesting example of dynamical symmetry breaking. We derive the relationship of parameters in low energy description of dynamical SUSY breaking through the approach of the Weinberg sum rules [1] which is traditionally used for chiral symmetry breaking. In this subsection, we explain basic idea of supersymmetry and its breaking.

SUSY is the only possible extension of symmetry of S-matrix consistent with relative quantum field theories and relates the fermionic degrees of freedom and bosonic ones. This relation derives attractive feature, the cancellation of quantum corrections between fermion loops and boson loops, for supersymmetric theories. This is the main motivation of SUSY at low energy since the expected value of higgs boson mass is too large without any cancellation of quantum loop corrections.

If SUSY is realized in nature, there should be additional particles as the partners of ordinary Standard Model particles which we have already observed since SUSY requires same numbers of fermionic and bosonic degrees of freedom. For example the supersymmetric partners of gauge bosons are gauge adjoint Majorana fermions called as gaugino and that of quarks are the complex scalar particles. Although the SUSY has attractive feature to explain light higgs mass, we have not observed such supersymmetric partner particles yet. Therefore we expect SUSY is realized as spontaneous broken symmetry. If SUSY is broken, SUSY algebra requires that the VEV of the Hamiltonian takes the non-zero positive value, *i.e.* $\langle H \rangle > 0$. Therefore the Hamiltonian and hence the diagonal components of energy-momentum tensor become order parameters of SUSY breaking. In discussion of SUSY breaking, we attention this property, $\langle H \rangle \neq 0$, and consider the scalar potential takes non-zero value or not since the non-zero expectation values of kinetic terms of Hamiltonian and non-zero spin particles break the Poincaré symmetry.

Since we discuss the concrete model of dynamical SUSY breaking in next subsection, we comment how to construct the SUSY models. First of all we consider the multiplets of SUSY. The SUSY multiplets contain same numbers of fermionic and bosonic degrees of freedom. For example chiral multiplets contain two components Weyl spinor and a complex scalar field and vector multiplets contain two components Weyl spinor and vector fields with ± 1 helicities. The superfields describe those multiplets using additional coordinates, θ_α and $\bar{\theta}^{\dot{\alpha}}$, which are transformed like Weyl spinor under Poincaré symmetry. The chiral superfields are the superfields which depend on only the coordinates $y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$ and θ where x^μ are ordinary space-time coordinates and the superfields satisfied real condition are called as real superfields.

Using the transformation laws of those superfields, we can construct SUSY invariant Lagrangian. We extract the components fields which are transformed to the total derivative terms or zero. We find that $\theta\theta$ components of chiral superfields and $\theta\theta\overline{\theta\theta}$ components of real superfields have such transformation laws. Therefore we can define SUSY theory using two combinations of superfields and covariant derivatives. One of them is Kähler potential which is combinations satisfied real conditions and contains kinetic parts of Lagrangian. The remaining one is superpotential satisfied chiral superfields conditions.

Since Hamiltonian is the order parameter of SUSY breaking, we consider the scalar potential to check whether the model breaks SUSY or not. The concrete formula of scalar potential is

$$V = \sum_{\text{for each fields}} FF^* + \sum_{\text{for each fields}} \frac{1}{2} D^a D^a \tag{1.16}$$

where F and D^a are $\theta\theta$ and $\theta\theta\overline{\theta\theta}$ components of chiral and real superfields respectively and index a describes gauge index of vector field, if the theory has canonical Kähler potential which does not derive non renormalizable interactions. The auxiliary fields F can be determined by superpotential W as following,

$$F = -\frac{\partial W^*}{\partial \phi^*} \tag{1.17}$$

where ϕ is the complex scalar component of chiral superfield corresponding with F component. From this scalar potential (1.16), we immediately conclude that the non-zero expectation value of F imply the spontaneous broken SUSY. Therefore we analyze the superpotential in the investigation of many SUSY breaking models.

In the last part of this subsection, we also comment on R-symmetry which plays an important role in our discussion in Chapter. 3. The R-symmetry is the phase rotation symmetry of the generators of SUSY. Since the component fields in superfields transform each other under SUSY transformation, each components in superfields are assigned different R-charges. This fact also remarks that the new fermionic coordinates, θ_α and $\bar{\theta}^{\dot{\alpha}}$, have the R-charges, -1 and $+1$ respectively.

The R-charge of real superfields are zero since real conditions determine its phase. However those of chiral superfields are determined by the concrete formula of superpotential. The R-charge of the superpotential is 2, since the $\theta\theta$ component of superpotential appears in Lagrangian.

1.2.3 SUSY breaking models

There are many kinds of dynamical SUSY breaking model, such as IYIT model, "incalculable model" and so on. In order to image the mechanism of dynamical SUSY breaking, we briefly overview the dynamical SUSY breaking models and simplest linear model in this subsection.

O'Raifeartaigh model

At first we comment on linear sigma model such as O'Raifeartaigh model [22], though it is not the dynamical symmetry breaking model. The simplest example of O'Raifeartaigh model has superpotential,

$$W = -k\Phi_1 + m\Phi_2\Phi_3 + \frac{y}{2}\Phi_1{\Phi_3}^2, \tag{1.18}$$

where Φ's are chiral superfields and k, m and y are model parameters. Deriving the scalar potential through this superpotential, we get

$$\begin{aligned} &V = |F_1|^2 + |F_2|^2 + |F_3|^2, \\ &F_1 = k - \frac{y}{2}\phi_3^{*2}, \quad F_2 = -m\phi_3^*, \quad F_3 = -m\phi_2^* - y\phi_1^*\phi_3^*. \end{aligned} \tag{1.19}$$

F_1 and F_2 cannot be zero simultaneously and the R-charge of both F terms are 0. Therefore in this model, the SUSY breaking operator is dimension 2 operator F and the R-symmetry is unbroken.

IYIT model and vector-like models

The simplest mechanism of dynamical SUSY breaking is conflicts between the minimum of the scalar potential derived from superpotential at classical level and the constraint of moduli space which comes from quantum effect. This mechanism breaks SUSY even if theories are vector-like such as IYIT model [23, 24] and models in Refs. [25, 26, 27].

IYIT models [23, 24] are examples of dynamical SUSY breaking without R-symmetry breaking. The simplest IYIT model is $SU(2)$ SQCD with $SU(4)_F$ flavor symmetry. The matter fields are $SU(2)$ doublet chiral superfields, Q, and singlet superfield, S. There representation under $SU(4)_F$ flavor symmetry are **4** and **6** corresponding with Q and S respectively. Its classical superpotential is

$$W_{\text{classical}} = \lambda S^{ij} Q_i Q_j, \tag{1.20}$$

where λ is model parameter and i, j are $SU(4)_F$ indices. Indices of S^{ij} are antisymmetric, such that $S^{ij} = -S^{ji}$, since S belongs to **6** representation of $SU(4)_F$. The effective superpotential, which takes into account the full non-perturbative effects, are written down by gauge-invariant low energy degrees of freedom $M_{ij} = Q_i Q_j$. The concrete form of it is

$$W_{\text{IYIT}} = \lambda S^{ij} M_{ij} + X \left(\text{Pf} M - \Lambda^4 \right), \tag{1.21}$$

where X is new chiral superfield, $\text{Pf}M \equiv \epsilon^{ijkl} M_{ij} M_{kl}$ are Pfaffian of antisymmetric matrix M_{ij} and Λ is dynamical scale of $SU(2)$ gauge interaction. The additional part of superpotential comes from the constraint, $\text{Pf}M = \Lambda^4$, of quantum moduli space. This is O'Raifeartaigh type superpotential and breaks SUSY.

ISS model

ISS model [28] has metastable SUSY breaking vacuum. This model is $SU(N_c)$ gauge theory with N_f flavor. Its superpotential is

$$W = m_i Q_i \bar{Q}_i, \tag{1.22}$$

where Q_i and $\bar{Q}_i$ are quarks and antiquarks, m_i are their mass parameters which are assumed to be much smaller than the dynamical scale Λ, and index i takes the integer value between 1 and N_f. We assume $N_c < N_f < \frac{3}{2} N_c$, where there is weakly coupled description of the theory below the dynamical scale Λ. This weakly coupled description has gauge group $SU(N_f - N_c)$ and meson fields $M_{ij} \sim Q_i Q_j$ and dual quarks q_i and $\bar{q}_i$ as law energy degrees of freedom. The superpotential is

$$W = m_i M_{ii} - \frac{1}{\hat{\Lambda}} q_i M_{ij} \bar{q}_j. \tag{1.23}$$

The F_{Mij} are

$$F_{Mij} = -m_i^* \delta_{ij} + \frac{1}{\hat{\Lambda}} q_i^\dagger \bar{q}_j^\dagger. \tag{1.24}$$

The $F_M = 0$ condition for all components of M_{ij} cannot be satisfied. The matrix $q_i \bar{q}_j$ has at most the rank $N_f - N_c$ since the q_i ($\bar{q}_j$) are (anti-)fundamental field of $SU(N_f - N_c)$. The rank of mass matrix $m_i \delta_{ij}$ is maximum rank, N_f. Therefore this superpotential breaks SUSY. Once we take into account the non perturbative effect, true supersymmetric vacuum appear far away from the origin of the meson fields, M. The life time of fake vacuum can be arbitrarily long if $m_i \ll \Lambda$.

incalculable model

It is suggested that SUSY is dynamically broken in the theory which has no flat direction at the fundamental level and spontaneous broken global symmetry. If global symmetry is spontaneously broken in SUSY theory and SUSY is not broken at low energy, massless chiral multiplet arises since it must contain Goldstone boson, and if the superpartner of the Goldstone boson is not itself a Goldstone boson, this massless scalar boson reflects the existence of flat direction at low energy effective theory. It seems quite implausible that the low energy effective theory has flat direction if the classical theory at fundamental level does not have flat direction. This means SUSY is not preserved at low energy.

"Incalculable models" are this kinds of dynamical SUSY breaking models such as chiral gauge theory in Ref. [16, 17]. In this kind of model one cannot obtain the low energy modes and its superpotential via direct calculations. An example of incalculable model is $SU(5)$ SUSY gauge theory with two chiral multiplets whose representations are $\bar{\mathbf{5}}$ and $\mathbf{10}$. Since we cannot construct gauge invariant polynomial from those two chiral multiplets, this model does not have superpotential. After some discussion we conclude that this model has no flat direction. We explain the explicit proof of this fact according to the discussion in Ref. [17].

Since this model does not have superpotential, the classical scalar potential is determined by D^a in Eq. (1.16). The auxiliary fields D^a are proportional to $\phi^\dagger T^a \phi$, where ϕ contains all scalar fields and the T^a are the generators of gauge group in the (generally reducible) representation to which ϕ belongs. The necessary condition to vanish scalar potential is

$$\phi^\dagger T^a \phi = 0, \tag{1.25}$$

for any values of a. We consider the linear combination of the generator matrices of fundamental representation of $SU(5)$ and the unit matrix with complex coefficient and introduce new basis in the space expanded by them. This new basis contain real orthogonal matrices $({a^i}_j)$ for $(i, j = 1, .., 5)$ and real antisymmetric matrices $({a^i}_i - {a^j}_j)$ for $(i, j = 1, .., 5, i \neq j)$ where $({a^i}_j)_k{}^l = {\delta^i}_k {\delta_j}^l$. Using this new basis the condition (1.25) are modified as

$$\phi^\dagger {A^i}_j \phi = \lambda {\delta^i}_j, \tag{1.26}$$

where matrix ${A^i}_j$ is the tensor product of ${a^i}_j$ corresponding with the representations of ϕ and λ is (real) constant. Since λ comes from the unit matrix part and it is contributed to ${a^i}_i$ with same coefficient for each value of i, Eq. (1.26) takes same value, λ, for any values of i, j. We compute the condition (1.26) in case of $SU(5)$ with a $\bar{\mathbf{5}}$ (F^i) and a $\mathbf{10}$ (T_{ij}). We obtain

$$2(T^\dagger T)^i{}_j - F^i F_j^\dagger = \lambda {\delta^i}_j. \tag{1.27}$$

To satisfy this condition we simultaneously diagonalize the both hermitian matrices, $T^\dagger T$ and $FF^\dagger$. Since T is antisymmetric, the eigenvalue of $T^\dagger T$ are of the form $(a, a, b, b, 0)$ with $a, b \geq 0$. However that of $FF^\dagger$ are single positive value. Therefore the condition (1.27) cannot be satisfied, unless $T = F = 0$. This implies that this model has no flat direction.

This model also has two non anomalous $U(1)$ symmetries. The first, we denote $U(1)_A$ symmetry, is transforms two chiral superfields as

$$F \to \exp(3i\alpha)F, \quad T \to \exp(-i\alpha)T. \tag{1.28}$$

The second symmetry is R-symmetry whose transformation laws are

$$\begin{aligned} &W_\alpha(\theta) \to \exp(-i\alpha) W_\alpha(\theta e^{i\alpha}), \\ &F(\theta) \to \exp(9i\alpha) F(\theta e^{i\alpha}), \\ &T(\theta) \to \exp(-i\alpha) T(\theta e^{i\alpha}), \end{aligned} \tag{1.29}$$

where W_α is the gauge adjoint chiral multiplet which contains the gaugino and field strength of gauge bosons. (The gaugino is contained in the lowest component of W_α and the fermionic components of other two chiral multiplets are the θ component of them.) If those two non anomalous $U(1)$ symmetries are unbroken at low energy, 't Hooft anomaly match conditions must be satisfied. There are four anomalies in this model. Using those charge assignments* one can easily compute them:

$$\begin{aligned} &\sum R^3 = 4976, && \sum R^2 A = 1500, \\ &\sum R A^2 = 450, && \sum A^3 = 125. \end{aligned} \tag{1.30}$$

Those anomalies should be reproduced by low energy modes if those two $U(1)$ symmetries are unbroken. Therefore we discuss the $U(1)_A$ charges, A, and R-charges of the chiral superfields describing the low energy modes. There are no massless fields with conserved $U(1)$ charge and spins greater than 1/2 because of theorem of Case, Gasiorowicz, Weinberg, and Witten [29]. Therefore the low energy modes can be described by the spin less chiral superfields or spin 1/2 chiral superfields with $R = -1$ and $A = 0$. At first we discuss the possibility of zero spin chiral superfields. From the gauge invariance we conclude that the $A = 5n$, where n is some integer. Using this integer n we also constraint the R-charges as $(n + R) \mod 2 = 0$, since the chiral superfields at low energy can be constructed by the chiral superfields, F, T, and W^α, as well as even numbers of covariant derivative to satisfy Lorentz invariance. (This implies that $R + A$ is odd for fermions.) Considering the fact that $\sum (R + A)^3$ is odd, we cannot reproduce anomalies (1.30) with even number of fermions. In case of other possibility of chiral superfields, A is zero and R-charge of fermion is -1. Those charges satisfy the same condition, $R + A$ is odd, of fermionic components of chiral superfields.

Affleck *et al.* compute for the solutions of anomaly equations. There are no solutions with three particles and $-500 \le A_i, R_i \le 500$. With five particles and $-50 \le A_i, R_i \le 50$, there are three:

$$\begin{aligned} &\text{Solution 1:} \quad (-5,-26),\ (5,20),\ (5,24),\ (0,-1),\ (0,9), \\ &\text{Solution 2:} \quad (15,-6),\ (-15,-4),\ (10,13),\ (-10,11),\ (5,12), \\ &\text{Solution 3:} \quad (-5,-10),\ (5,12),\ (5,16),\ (0,3),\ (0,5). \end{aligned} \tag{1.31}$$

Clearly the simplest sets of chiral superfields with these quantum numbers are extremely complicated. It seems quite implausible that the theory realizes the fermions with those complicated quantum numbers at low energy.

This results suggest that the R-symmetry and/or $U(1)_A$ symmetry are spontaneously broken at low energy. If the R-symmetry is spontaneously broken, there is a simple order

* The charge assignments of each constituent fermions are 10, 0, and -1 for F, T, and W_α respectively.

parameter to describe the breaking of this symmetry: $\lambda^\alpha \lambda_\alpha$. If gaugino condensation, $\langle \lambda^\alpha \lambda_\alpha \rangle \neq 0$, happens, R-symmetry is spontaneously broken and the Konishi anomaly [30] implies SUSY is also broken. Since this discussion is independent on whether $U(1)_A$ is broken or not, we consider the case that $U(1)_A$ is broken and R-symmetry is unbroken as next step. In this case R^3 anomaly matching requires at least three massless fermion fields. The simplest solution is three fermions with R charges $(17, 4, -1)$. Such a complicated spectrum already seems implausible. Even if those fermions are realized at low energy, spontaneously broken $U(1)_A$ suggests that SUSY is spontaneously broken following the first part of this discussion.

Chapter 2

Application of Seiberg duality to chiral symmetry breaking

The chiral symmetry is dynamically broken in QCD which confines the quarks into hadrons at low energy. We consider the low energy description of QCD. In supersymmetric gauge theory we obtain the low energy descriptions through the duality called as Seiberg duality [2]. We adopt the characteristic features of Seiberg duality to construct low energy effective model of QCD.

2.1 Magnetic linear sigma model

Seiberg duality relates infrared limits of two supersymmetric gauge theories. The gauge theory describing magnetic description has the different gauge group from original one. This suggest us to consider that $U(N_f)$ gauge theory of HLS can be interpreted as magnetic description of QCD. Therefore we construct $U(N_f)$ gauge theory. This model is Higgs model of $U(N_f)$ gauge theory and describes Higgsing of the magnetic gauge group as well as chiral symmetry breaking. (See the Section 2.5 for concrete relation our model and Seiberg duality.)

2.1.1 Lagrangian

We propose the following Lagrangian to describe the magnetic picture of QCD. It is a $U(N_f)$ gauge theory, and the Lagrangian possesses the $U(N_f)_L \times U(N_f)_R$ chiral symmetry. The vacuum expectation values (VEVs) of the Higgs fields, H_L and H_R, break the chiral symmetry down to the diagonal subgroup, $U(N_f)_V$, providing massless Nambu-Goldstone bosons identified as pions and η. The η meson (or η' in the three-flavor language) can obtain a mass through a term which breaks axial $U(1)$ symmetry explicitly such as $\det(H_L H_R)$ although we ignore it in this paper. The VEVs of the Higgs fields give masses to $U(N_f)$ gauge bosons. We identify these massive gauge bosons as the ρ and the ω mesons*. The

*In the three-flavor language, one should include $K^*(892)$ and $\phi(1020)$ in the vector mesons.

Lagrangian is given by

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F^{(\omega)}_{\mu\nu}F^{(\omega)\mu\nu} - \frac{1}{4}F^{(\rho)a}_{\mu\nu}F^{(\rho)\mu\nu a} \\ &+\frac{f_\pi^2}{2}\mathrm{Tr}\left[|D_\mu H_L|^2 + |D_\mu H_R|^2\right] \\ &-V(H_L, H_R). \end{aligned} \tag{2.1}$$

The first and the second terms represent the kinetic terms of the $U(1)$ and the $SU(N_f)$ parts of the $U(N_f)$ gauge bosons: ω_μ and ρ^a_μ, respectively. The Higgs fields H_L and H_R are $N_f \times N_f$ matrices which transform as

$$H_L \to g_L H_L g_H^{-1}, \quad H_R \to g_H H_R g_R^{-1}, \tag{2.2}$$

under the $U(N_f)_L$, the gauged $U(N_f)$, and the $U(N_f)_R$ group elements, g_L, g_H, and g_R, respectively. The covariant derivatives are, therefore, given by

$$D_\mu H_L = \partial_\mu H_L + ig_2 H_L \rho^a_\mu T^a + ig_1 Q\omega_\mu H_L, \tag{2.3}$$

$$D_\mu H_R = \partial_\mu H_R - ig_2 \rho^a_\mu T^a H_R - ig_1 Q\omega_\mu H_R. \tag{2.4}$$

Here, we normalized the $SU(N_f)$ generators in the fundamental representation, T^a, and the $U(1)$ charge, Q, such that

$$\mathrm{Tr}\left(T^a T^b\right) = \frac{1}{2}\delta^{ab}, \tag{2.5}$$

and

$$Q = \sqrt{\frac{1}{2N_f}}. \tag{2.6}$$

The most general potential terms consistent with the symmetries are given by

$$\begin{aligned} V(H_L, H_R) &= f_\pi^4\Bigg[\frac{\lambda_0 - \lambda_A}{8N_f}\left(\mathrm{Tr}(H_L H_L^\dagger) + \mathrm{Tr}(H_R^\dagger H_R) - 2N_f\right)^2 \\ &+\frac{\lambda_A}{8}\left\{\mathrm{Tr}\left[(H_L^\dagger H_L + H_R H_R^\dagger)^2\right] - 4\left(\mathrm{Tr}(H_L H_L^\dagger) + \mathrm{Tr}(H_R^\dagger H_R)\right)\right\} \\ &+\frac{\lambda' - \lambda''}{8N_f}\left(\mathrm{Tr}(H_L H_L^\dagger) - \mathrm{Tr}(H_R^\dagger H_R)\right)^2 \\ &+\frac{\lambda''}{8}\mathrm{Tr}\left[(H_L^\dagger H_L - H_R H_R^\dagger)^2\right]\Bigg], \end{aligned} \tag{2.7}$$

where we assumed the parity invariance under $H_L \leftrightarrow H_R$. This potential stabilizes H_L and H_R at

$$\langle H_L \rangle = \langle H_R \rangle = \mathbf{1}. \tag{2.8}$$

At the vacuum, $4N_f^2$ degrees of freedom in H_L and H_R break up to N_f^2 massless Nambu-Goldstone bosons, N_f^2 longitudinal modes of the gauge bosons, N_f^2 massive scalar particles, and N_f^2 massive pseudoscalar particles. The decay constant of the Nambu-Goldstone particles is given by f_π at tree level. The gauge group is completely broken and the unbroken global symmetry is vectorial $U(N_f)_V$.

The physical modes at the vacuum can be classified by the representations of $U(N_f)_V$, the spin and the parity. The masses of the physical modes are given by

$$\text{singlet Nambu-Goldstone boson } (\eta): \quad m_\eta = 0, \tag{2.9}$$

$$\text{adjoint Nambu-Goldstone boson } (\pi): \quad m_\pi = 0, \tag{2.10}$$

$$\text{singlet vector } (\omega): \quad m_\omega^2 = g_1^2 f_\pi^2, \tag{2.11}$$

$$\text{adjoint vector } (\rho): \quad m_\rho^2 = g_2^2 f_\pi^2, \tag{2.12}$$

$$\text{singlet scalar } (f_0): \quad m_S^2 = 2\lambda_0 f_\pi^2, \tag{2.13}$$

$$\text{adjoint scalar } (a_0): \quad m_A^2 = 2\lambda_A f_\pi^2, \tag{2.14}$$

$$\text{singlet pseudoscalar:} \quad m_{PS}^2 = 2\lambda' f_\pi^2, \tag{2.15}$$

$$\text{adjoint pseudoscalar:} \quad m_{PA}^2 = 2\lambda'' f_\pi^2, \tag{2.16}$$

at tree level. Terms with λ' and λ'' are not very important in the following discussion†.

Hereafter, we take

$$g_1 = g_2 \equiv g, \tag{2.17}$$

as the ρ and ω mesons have similar masses.

† The pseudoscalar particles are, in fact, CP even, and thus they are exotic states which are absent in the hadron spectrum. One should take large λ' and λ'' to make the exotic states heavy so that the model can be a low-energy effective theory of QCD. We thank M. Harada, V.A. Miransky, and K. Yamawaki for discussion on this point.

2.1.2 Vector mesons and pions

When we integrate out the massive scalar and pseudoscalar mesons, the model reduces to a non-linear sigma model of Ref. [6]. The matching at tree level gives $a = 1$, where a is a parameter in the low-energy Lagrangian:

$$\mathcal{L} \ni \frac{(1-a)f_\pi^2}{4}\mathrm{Tr}\left[|\partial_\mu(U_L U_R)|^2\right]. \tag{2.18}$$

The unitary matrices U_L and U_R are fields to describe the Nambu-Goldstone modes including the ones eaten by the gauge bosons. The transformation properties of U_L and U_R under the gauge and flavor groups are the same as H_L and H_R, respectively. From the low energy data, the preferred value of a is estimated to be $a \sim 2$ with an error of 15% [31]. Although there is a factor of two difference from the prediction, this discrepancy can be explained by including quantum corrections and/or higher dimensional operators. As discussed in Ref. [31], the quantum correction makes the Lagrangian parameter $a(\Lambda)$ approaches to unity when we take Λ to be large, such as $a(\Lambda) \simeq 1.33 \pm 0.28$ for $\Lambda = 4\pi f_\pi \sim 1$ GeV. Moreover, the quantum corrections from the scalar loops give positive contributions to the gauge boson masses, that further reduces the $a(\Lambda)$ parameter. Therefore, one can think of the Lagrangian in Eq. (2.1) as the one defined at a high energy scale such as the mass scale of the scalar mesons.

However, the large quantum corrections result in predictions which depend on the choice of input physical quantities when we work at tree level, although the differences should be canceled after including quantum corrections. In this case, one should choose a set of physical quantities which gives small enough coupling constants so that the use of the perturbative expansion is valid and the tree-level results are reliable.

The Lagrangian has four parameters relevant for the discussion: g, f_π, λ_0, and λ_A. The λ_0 and λ_A parameters can be obtained from the scalar masses as we discuss later. The gauge coupling constant g and the f_π parameter can be estimated from two of physical quantities. The well-measured physical quantities which can be used as input parameters are [31]:

$$g_\rho = (340 \text{ MeV})^2, \quad g_{\rho\pi\pi} = 6.0, \quad F_\pi = 92 \text{ MeV}, \quad m_\rho = 770 \text{ MeV}, \tag{2.19}$$

where g_ρ and $g_{\rho\pi\pi}$ are the decay constant and the coupling to two pions of the ρ meson measured by $\rho \to e^+e^-$ and $\rho \to \pi\pi$ decays, respectively, and F_π is the decay constant of the pion. The relations to the Lagrangian parameters at tree level are given by

$$g_\rho = g f_\pi^2, \quad g_{\rho\pi\pi} = \frac{g}{2}, \quad F_\pi = f_\pi, \quad m_\rho = g f_\pi. \tag{2.20}$$

Among them, the pair to give the smallest gauge coupling is g_ρ and m_ρ such as

$$g = \frac{m_\rho^2}{g_\rho} = 5.0, \quad f_\pi = \frac{g_\rho}{m_\rho} = 150 \text{ MeV}. \tag{2.21}$$

The value $g = 5.0$ means that the loop expansion parameter, $g^2 N_f/(4\pi)^2$, is of order 30% whereas other choices of input quantities give $90 - 210\%$ for $N_f = 2$. Therefore, the choice above is unique to make a quantitative prediction. Indeed, the values in Eq. (2.21) are close to the ones evaluated at one-loop level. In Ref. [31], the parameters at a scale $\Lambda \sim 1$ GeV is obtained to be $g(\Lambda) \sim 3.3-4.2$, $f_\pi(\Lambda) \sim 130-150$ MeV, and $a(\Lambda) \sim 1.0-1.5$, which reproduce all the physical quantities in Eq. (2.19). We use the values of g and f_π in Eq. (2.21) in the following discussion. However, we should bear in mind that there are theoretical uncertainties at the level of a factor of two in the results obtained at the classical level.

2.1.3 Scalar mesons

In the hadron spectrum, there are light scalar mesons, such as σ, κ, $f_0(980)$ and $a_0(980)$, which have not been understood as $q\bar{q}$ states in the quark model since they are anomalously light. We propose to identify them as the Higgs bosons in this linear sigma model. We do not consider heavier scalar mesons as candidates since otherwise the formulas in Eqs. (2.13) and (2.14) indicate that the coupling constants are large and the perturbation theory would not be applicable.

By taking the masses of $f_0(980)$ and $a_0(980)$ as input quantities[‡], *i.e.*,

$$m_S = m_A = 980 \text{ MeV}, \tag{2.22}$$

Eqs. (2.13) and (2.14) give the coupling constants λ_0 and λ_A as

$$\sqrt{\lambda_0} = \sqrt{\lambda_A} = 4.6, \tag{2.23}$$

at tree level, where f_π in Eq. (2.21) is used. We use these values of coupling constants for later calculations.

2.2 Vortex strings

Since the model has a spontaneously broken gauged $U(1)$ factor, there is a vortex string as a classical field configuration. The string carries a quantized magnetic flux. Below we construct a solution with a unit flux, which will be identified as the confining string.

There have been similar approaches to the confinement in QCD. The Ginzburg-Landau models (the magnetic Higgs models) are constructed from phenomenological approaches [19, 32, 33] or based on the QCD Lagrangian [34, 35] through the abelian projection [36], and the stable vortex configurations are identified as the confining string. In supersymmetric theories, there have been numbers of discussion on the vortex configurations [20, 21, 37, 38, 39, 40, 41, 42]. In particular, the non-abelian string [20, 21], which we discuss shortly, has been extensively studied as a candidate of the confining string.

Our model combines Higgsing of the magnetic gauge group and chiral symmetry breaking. As discussed in the previous section, the model parameters are fixed by physical quantities such as masses and couplings of hadrons. Therefore, the properties of the strings such as the string tension can be evaluated quantitatively. Below, we explicitly construct a classical field configuration of the vortex string.

[‡]Since σ and κ are quite broad resonances, we do not use their masses as inputs.

2.2.1 Non-abelian vortex solutions

In this model, there are string configurations called the non-abelian vortices which carry the minimal magnetic flux. By defining the following gauge field,

$$A_\mu^{ij} = \sqrt{2}\left(Q\omega_\mu \delta_{ij} + T_{ij}^a \rho_\mu^a\right), \tag{2.24}$$

there is a vortex configuration made of, *e.g.,* the $i = j = 1$ component rather than the overall $U(1)$ gauge field ω_μ. Compared to the string solution made of ω_μ, this non-abelian string carries only $1/N_f$ of the magnetic flux and thus it is stable.

In constructing the vortex configurations, we follow the formalism and numerical methods of Ref. [43], where the potential between a monopole and an anti-monopole is evaluated numerically in the abelian-Higgs model. Classical field configurations are constructed by numerically solving field equations while imposing the gauge field to behave as the Dirac monopoles [44] as approaching to their locations.

We consider a non-abelian vortex solution, where the magnetic flux is sourced by a Dirac-monopole and a Dirac-antimonopole configurations of the A_μ^{ij} gauge field with $i = j = 1$, representing non-abelian monopole configurations. These monopole and anti-monopole are not present as physical states in the model of Eq. (2.1), and we introduce them as field configurations with an infinite energy, *i.e.,* static quarks[§]. The object we construct here, therefore, corresponds to a bound state of heavy quarks such as the charmonium and the bottomonium. In order to describe light mesons, the light quarks should be present somewhere in the whole framework. We discuss a possible framework in Section 2.5.

In the cylindrical coordinate, (ρ, φ, z), where the monopole and the antimonopole located on the z-axis at $z = \pm R/2$, we denote $(A_D)_\mu^{ij}$ as the configuration to describe the monopole-antimonopole system. They are given by

$$(A_D)_0^{ij} = 0, \tag{2.25}$$

$$\boldsymbol{A}_D^{ij} = 0, \quad \text{except for} \;\; i = j = 1, \tag{2.26}$$

and

$$\boldsymbol{A}_D^{11} = a_D \hat{\boldsymbol{\varphi}} = -\frac{N_{\text{flux}}}{\sqrt{2}g}\frac{1}{\rho}\left[\frac{z - R/2}{[\rho^2 + (z - R/2)^2]^{1/2}} - \frac{z + R/2}{[\rho^2 + (z + R/2)^2]^{1/2}}\right]\hat{\boldsymbol{\varphi}}. \tag{2.27}$$

The number of the flux, N_{flux}, is quantized as $N_{\text{flux}} \in \mathbb{Z}$ by the Dirac quantization condition [44]. Equivalently, the magnetic charge of the monopole is quantized as

$$q_m = \frac{4\pi N_{\text{flux}}}{\sqrt{2}g}. \tag{2.28}$$

[§]In $U(N)$ gauge theories with Higgs fields in the adjoint representation, there are monopoles as solitonic objects which are identified as junctions of vortices [45, 46] rather than the endpoints. The monopoles we are considering should not be confused with such configurations.

The gauge field is well-defined everywhere except for the interval $-R/2 \leq z \leq R/2$ on the z-axis. The Dirac quantization condition ensures that the interval is covered in a different gauge. For constructing a vortex configuration, the following ansatz are taken:

$$A_\mu^{ij} = A_\mu^i \delta^{ij}, \quad A_\mu^i = (A_D)_\mu^{ii} + a_\mu^i, \tag{2.29}$$

$$a_0^i = 0, \quad \boldsymbol{a}^i = a^i(\rho, z)\hat{\boldsymbol{\varphi}}, \tag{2.30}$$

$$(H_L)_{ij} = (H_R)_{ij} = \phi_i(\rho, z)\delta_{ij}, \quad \phi_i = \phi_i^*. \tag{2.31}$$

With the ansatz, the Lagrangian is reduced to

$$\begin{aligned}
\mathcal{L} \;\; = \;\; & -\frac{1}{4}\sum_i F^{i\mu\nu}F^i_{\mu\nu} \\
& + f_\pi^2 \sum_i (\partial_\mu \phi_i)^2 + \frac{f_\pi^2}{2} g^2 \sum_i \phi_i^2 (A_\mu^i)^2 \\
& - \frac{\lambda_0}{2N_f} f_\pi^4 \left(\sum_i \phi_i^2 - N_f \right)^2 \\
& - \frac{\lambda_A}{2N_f} f_\pi^4 \left(N_f \sum_i \phi_i^4 - \left(\sum_i \phi_i^2 \right)^2 \right),
\end{aligned} \tag{2.32}$$

and the field equations are obtained as

$$\nabla^2 \phi_i - \frac{g^2}{2}(a^i + a_D \delta^{i1})^2 \phi_i = \frac{\lambda_0}{2N_f}\left(\sum_j \phi_j^2 - N_f \right)\phi_i + \frac{\lambda_A}{2N_f}\left(N_f \phi_i^2 - \sum_j \phi_j^2 \right)\phi_i, \tag{2.33}$$

$$\left(\nabla^2 - \frac{1}{\rho^2} \right) a^i = \frac{g^2}{2}(a^i + a_D \delta^{i1})\phi_i^2, \tag{2.34}$$

where we take the unit of

$$\sqrt{2} f_\pi = 1. \tag{2.35}$$

For $i \neq 1$, $a^i = 0$ is the solution.

The potential energy between the monopole and the anti-monopole is given by

$$\begin{aligned}
V(R) &= -\frac{2\pi N_{\text{flux}}^2}{g^2 R} \\
&\quad + \int d^3x \left[-\frac{g^2}{4}\phi_1^2(a^1 + a_D)a^1 - \frac{\lambda_0}{8N_f}\left(\left(\sum_i \phi_i^2\right)^2 - N_f^2 \right) \right. \\
&\qquad \left. - \frac{\lambda_A}{8N_f}\left(N_f \sum_i \phi_i^4 - \left(\sum_i \phi_i^2\right)^2 \right) \right]. \qquad (2.36)
\end{aligned}$$

The first term comes from the magnetic Coulomb potential, $V_{\text{Coulomb}} = -q_{\text{mag}}^2/4\pi R$. The second term is the contribution from the non-trivial field configurations, and gives the linear potential between a monopole and an antimonopole for a large R. The self-energies of the Dirac monopoles are subtracted, and thus this expression provides a finite quantity.

For $\lambda_0 = \lambda_A$, which is the case as in Eq. (2.23), the problem simplifies to the case of the abelian string. The field equations gives

$$\phi_i = 1, \quad \text{for } i \neq 1, \qquad (2.37)$$

as solutions and the equations for ϕ_1 and a^1 becomes

$$\nabla^2\phi_1 - \frac{g^2}{2}(a^1 + a_D)^2\phi_1 = \frac{\lambda_0}{2}\left(\phi_1^2 - 1\right)\phi_1, \qquad (2.38)$$

$$\left(\nabla^2 - \frac{1}{\rho^2}\right) a^1 = \frac{g^2}{2}(a^1 + a_D)\phi_1^2. \qquad (2.39)$$

The potential energy is in this case given by

$$V(R) = -\frac{2\pi N_{\text{flux}}^2}{g^2 R} + \int d^3x \left[-\frac{g^2}{4}\phi_1^2(a^1 + a_D)a^1 - \frac{\lambda_0}{8}\left(\phi_1^4 - 1\right) \right]. \qquad (2.40)$$

The N_f dependence disappears from the potential energy.

2.2.2 Numerical results

We numerically solve Eqs. (2.38) and (2.39) by following the procedure explained in Ref. [43]. The partial differential equations are solved by using the Gauss-Seidel method. The obtained field configurations are used to evaluate the potential energy in Eq. (2.40).

In the unit of Eq. (2.35), the potential energy $V(R)$ times the gauge boson mass m_ρ can be obtained as a function of $m_\rho R$. In this normalization, we have a single parameter κ defined by

$$\kappa = \frac{m_S}{\sqrt{2}m_\rho} = \frac{\sqrt{\lambda_0}}{g} = \frac{\sqrt{\lambda_A}}{g}. \qquad (2.41)$$

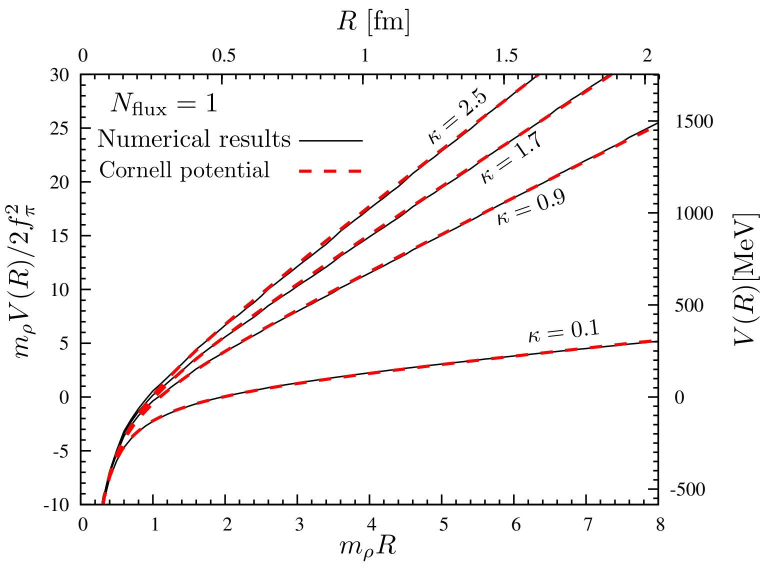

Figure 2.1: Potential energy of the monopole-antimonopole system for $\kappa = 0.1$, 0.9, 1.7 and 2.5. The fittings with the Cornell potential are superimposed (dashed lines).

This corresponds to the Ginzburg-Landau parameter of superconductors. The numerical results are shown in Fig. 2.1, where the potential energies for $N_{\text{flux}} = 1$ are drawn with four choices of parameters, $\kappa = 0.1$, 0.9, 1.7, and 2.5. We see a linear potential in a large R region. By fitting the slope of the linear regime, one can extract the string tension $\hat{\sigma}$ in the unit of Eq. (2.35). We show in Fig. 2.2 the tension $\hat{\sigma}$ as a function of κ. These results are all consistent with Ref. [43], except that the unit of the flux is different due to the non-abelian feature of the vortex. For $\kappa = 1/\sqrt{2}$, the field equations reduce to a set of first order differential equations whose solutions are known as the BPS state. In this case, the tension is simply given by $\hat{\sigma} = \pi$, which we have confirmed with an accuracy of $0.1 - 0.2$ percent.

2.3 Comparison to QCD data

Now we compare the numerical results with data from experimental measurements. We identify the non-abelian Dirac monopoles with the minimal magnetic charge, $N_{\text{flux}} = 1$, as static quarks, since otherwise the string with $N_{\text{flux}} = 1$ is stable and such a stable string is absent in QCD. The potential between a quark and an antiquark with a distance R can be parametrized by the following form:

$$V(R) = -\frac{A}{R} + \sigma R. \tag{2.42}$$

This potential, called the Cornell potential, well fits the quarkonium spectrum with parameters:

$$A \sim 0.25 - 0.5, \quad \sqrt{\sigma} \sim 430 \text{ MeV}. \tag{2.43}$$

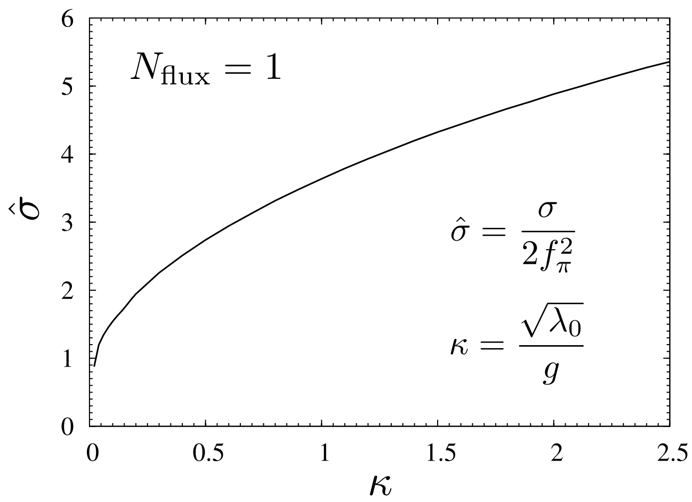

Figure 2.2: The string tension $\hat{\sigma}$ in the unit of $2f_\pi^2$ as a function of the Ginzburg-Landau parameter κ.

A similar value of the string tension σ is obtained from the Regge trajectories of the hadron spectrum. The lattice simulations also reproduce the shape of the potential with $A \sim 0.25 - 0.4$ [47, 48, 49] and $\sqrt{\sigma}/m_\rho \sim 0.50 - 0.55$ [50]. In perturbative QCD, at tree level, the Coulomb part $V \sim -A/R$ is obtained from the one-gluon exchange between quarks. At a higher loop level, the shape of the potential approaches to the form in Eq. (2.42) [51]. Computations at three-loop level have been performed recently in Refs. [52, 53], and it is reported that the result is in good agreement with lattice simulations up to a distance scale $R \lesssim 0.25$ fm [52]. See, for example, Ref. [54] for a review of the static QCD potential.

The Cornell potential also well fits the numerically obtained potential in the previous section. We superimpose the fittings with the Cornell potential in Fig. 2.1 as dashed lines.

2.3.1 Coulomb potential

In the electric picture, *i.e.,* in QCD, the Coulomb part $V \sim -A/R$ is obtained with

$$A = \frac{N_c^2 - 1}{2N_c} \frac{g_s^2}{4\pi}, \tag{2.44}$$

where the strong gauge coupling g_s depends on R through renormalization.

By duality, in the magnetic picture, the Coulomb term is accounted by a magnetic Coulomb force between monopoles. By using the magnetic charge in Eq. (2.28) with $N_{\rm flux} = 1$, the coefficient is given by

$$A = \frac{q_m^2}{4\pi} = \frac{2\pi}{g^2}. \tag{2.45}$$

This corresponds to the first term in Eq. (2.40). Using the value of g in Eq. (2.21), we obtain

$$A = 0.25. \tag{2.46}$$

The value is consistent with Eq. (2.43). This is already an interesting non-trivial test of the hypothesis that the vector mesons are the magnetic gauge fields.

Note here that the Coulomb term in Eq. (2.40) arises from a solution of the classical field equations with boundary conditions given by the Dirac monopoles. Although the vacuum is in a Higgs phase, the Coulomb force dominates when the distance R is small compared to the inverse of the gauge boson mass. In the world-sheet theory of the string, it has been known that the Coulomb force can be reproduced as the Lüscher term which stems from the boundary conditions of the string world sheet [55]. Interestingly, the Lüscher term gives $A = \pi/12 \sim 0.26$ which is pretty close to the above estimation.

2.3.2 Linear potential

As we have seen already, the linear potential is obtained as in Fig. 2.1. The normalized string tension $\hat{\sigma}$ is shown in Fig. 2.2 as a function of κ. From Eqs. (2.21), (2.23) and (2.41), the κ parameter is given by

$$\kappa = 0.90. \tag{2.47}$$

With this value, we obtain from Fig. 2.2,

$$\hat{\sigma} = 3.5. \tag{2.48}$$

By using f_π in Eq. (2.21) to recover the mass dimension, we obtain

$$\sqrt{\sigma} = 400 \text{ MeV}. \tag{2.49}$$

This is close to $\sqrt{\sigma}$ in Eq. (2.43). The prediction is not very sensitive to κ. For example, $\kappa = 0.6 - 1.2$ gives $\sqrt{\sigma} = 360 - 420$ MeV. Although we expect a large theoretical uncertainty from quantum corrections, it is interesting to note that the estimated string tension is in the right ballpark. The hypothesis that the ρ and ω mesons as magnetic gauge bosons and light scalar mesons as the Higgs bosons is found to be consistent with the experimental data.

It is important to notice that there is no dependence on N_f in the field equations (2.38), (2.39) or in the expression of the QCD potential (2.40). It is essential to have this property that the string is non-abelian. The dimensionless quantity $\sqrt{\sigma}/m_\rho$ is, in this case, predicted to be N_f independent, which is consistent with the results from the lattice QCD [50].

2.4 QCD phase transition

At a finite temperature, QCD phase transition takes place. The lattice simulations support that deconfinement and chiral symmetry restoration happen at similar temperatures. The chiral transition temperature has been computed in lattice simulations, and found to be $T_c \sim 150 - 160$ MeV [56, 57] for physical quark masses.

A simple estimate of the transition temperature is possible in the magnetic model in Eq. (2.1). The deconfinement and the restoration of the chiral symmetry both correspond to the phase transition to the vacuum with $H_L = H_R = 0$, which is stabilized by thermal masses at a finite temperature. When we define the transition temperature T_c to be the one at which the Higgs fields become non-tachyonic at the origin, the temperature is obtained to be [58]

$$T_c = \sqrt{\frac{8}{\eta N_f}} f_\pi, \tag{2.50}$$

where the factor η is a dimensionless quantity given by

$$\eta = 1 + \frac{2m_\rho^2}{m_S^2} + \frac{2m_{PS}^2 + m_S^2}{3m_S^2}, \tag{2.51}$$

at the lowest level of perturbation. Each term in the η parameter represents the contribution to the thermal masses of the Higgs fields from different particles. The first term, the unity, is the contribution from the scalar mesons. One should add up all the particles which obtain masses from the VEVs of H_L and H_R. The estimation of η is quite non-trivial since there are particles which we did not consider, such as nucleons, and also the summation should be weighted by the abundance in the thermal bath, which may be affected by their large thermal masses, *i.e.,* there may be large higher order corrections.

By putting f_π in Eq. (2.21), we obtain

$$T_c = \begin{cases} 170 \text{ MeV} \times \left(\frac{\eta}{3}\right)^{-1/2}, & (N_f = 2), \\ 140 \text{ MeV} \times \left(\frac{\eta}{3}\right)^{-1/2}, & (N_f = 3). \end{cases} \tag{2.52}$$

The value $\eta \sim 3$ seems to give temperatures consistent with ones from lattice simulations. It is interesting that $\eta \sim 3$ is obtained from Eq. (2.51) when we take m_{PS} around the cut-off scale, $\Lambda \sim 1$ GeV.

The formula in Eq. (2.50) predicts that the transition temperature is inversely proportional to $\sqrt{N_f}$. This is numerically consistent with the flavor dependence of T_c studied in Ref. [50] for two and three flavors in the chiral limit. There, T_c is obtained to be 173 ± 8 MeV and 154 ± 8 MeV for two and three flavors, respectively. A simulation with a larger number of N_f should be able to test this prediction.

2.5 Non-supersymmetric duality from the Seiberg duality

The assumption in the whole framework is the electric-magnetic duality between the $SU(N_c)$ gauge theory with N_f massless quarks and $U(N_f)$ gauge theory with bosonic Higgs fields.

	$SU(N_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$SU(N_c)_V$	$U(1)_{B'}$	$U(1)_R$
Q	N_c	N_f	1	1	1	0	$(N_f - N_c)/N_f$
$\overline{Q}$	$\overline{N_c}$	1	$\overline{N_f}$	-1	1	0	$(N_f - N_c)/N_f$
Q'	N_c	1	1	0	$\overline{N_c}$	1	1
$\overline{Q}'$	$\overline{N_c}$	1	1	0	N_c	-1	1

Table 2.1: Quantum numbers in the electric picture.

The replacement of N_c in the gauge group with N_f is familiar in supersymmetric gauge theories. For example, the Seiberg duality in the $\mathcal{N} = 1$ supersymmetric theories replaces $SU(N_c)$ gauge group by $SU(N_f - N_c)$ in the magnetic picture. We explain here a possible connection between the Lagrangian in Eq. (2.1) and the Seiberg duality, which is discussed in Ref. [12]. We extend the discussion of Ref. [12] regarding the vortex string and interpretations of constituent quarks.

It is obvious that the non-supersymmetric QCD can be obtained from supersymmetric QCD's by adding masses to superpartners and send them to infinity. What is non-trivial is if a vacuum in the theory with small masses of superpartners is continuously connected to the non-supersymmetric theory when we send the masses to large values. Such a continuous path may or may not exist depending on the space of parameters defined by a supersymmetric theory to start with. Recently, it is found in Ref. [12] that there is an explicit model which reduces to QCD in a limit of parameters and has a vacuum with the same structure as the low energy QCD in a region of parameters where the Seiberg duality can be used. By hoping that the region extends to the QCD limit, one can study non-perturbative features of QCD, such as strings, at the classical level in the dual picture.

The proposed mother theory is $\mathcal{N} = 1$ supersymmetric QCD with N_c colors and $N_f + N_c$ flavors. By giving supersymmetric masses to the extra N_c flavors and soft supersymmetry breaking masses for gauginos and scalar quarks, one obtains non-supersymmetric QCD with N_c colors and N_f flavors. The global symmetries and quantum numbers are listed in Table 2.1, where $SU(N_c)$ is the gauge group. The $U(1)_{B'}$ symmetry is absent in the actual QCD, and will be spontaneously broken in the vacuum we discuss later. In order to avoid the appearance of the unwanted Nambu-Goldstone mode associated with this breaking, we gauge $U(1)_{B'}$. The $SU(N_c)_V$ group is also an artificially enhanced symmetry, and thus we gauge it. Since the added gauge fields only interact with extra flavors, the limit of large mass parameters still gives the non-supersymmetric QCD we wanted.

The magnetic picture of the mother theory is an $SU(N_f)$ gauge theory with $N_f + N_c$ flavors and meson fields. The particle content and the quantum numbers are listed in Table 2.2. It was found in Ref. [12] that there can be a stable vacuum outside the moduli space by the help of the soft supersymmetry breaking terms. The vacuum is at $\langle q \rangle = \langle \bar{q} \rangle \neq 0$, where $SU(N_f) \times SU(N_f)_L \times SU(N_f)_R$ is spontaneously broken down to a single vectorial $SU(N_f)_V$ symmetry, that is the isospin symmetry. The symmetry breaking provides massless pions and simultaneously gives masses to the $SU(N_f) \times U(1)_{B'}$ gauge fields. Those massive gauge fields can be identified as the vector mesons, ρ and ω.

	$SU(N_f)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$SU(N_c)_V$	$U(1)_{B'}$	$U(1)_R$
q	N_f	$\overline{N_f}$	1	0	1	N_c/N_f	N_c/N_f
$\overline{q}$	$\overline{N_f}$	1	N_f	0	1	$-N_c/N_f$	N_c/N_f
Φ	1	N_f	$\overline{N_f}$	0	1	0	$2(N_f - N_c)/N_f$
q'	N_f	1	1	1	N_c	$-1 + N_c/N_f$	0
$\overline{q}'$	$\overline{N_f}$	1	1	-1	$\overline{N_c}$	$1 - N_c/N_f$	0
Y	1	1	1	0	1 + Adj.	0	2
Z	1	1	$\overline{N_f}$	-1	$\overline{N_c}$	1	$(2N_f - N_c)/N_f$
$\overline{Z}$	1	N_f	1	1	N_c	-1	$(2N_f - N_c)/N_f$

Table 2.2: Quantum numbers in the magnetic picture.

Although the deformation with massive N_c flavors provides us with a QCD-like vacuum, there are several unsatisfactory features as noted in Ref. [12]. Here we discuss those issues and consider a possible interpretation. In the above discussion, it sounds somewhat strange that the $U(1)_{B'}$ gauge field is identified as the ω meson which is in the same nonet as the ρ meson, whereas the $U(1)_{B'}$ seems to have a completely different origin from the $SU(N_f)$ magnetic gauge group. Second, in the particle content in Table 2.2, there are fields which have $U(1)_B$ charges ± 1, *i.e.,* "quarks." These degrees of freedom do not match the picture of confining since they look like free quarks. Finally, there is a vortex string associated with the spontaneous breaking of $U(1)_{B'}$, which we would like to identify as the QCD string. However, since the stability of the string is ensured by topology, it is stable even in the presence of the massless quarks. The real QCD string should be unstable since a pair creation of the quarks can break the string.

A possible interpretation is emerged from the consideration of the origin of $U(1)_{B'}$ in the magnetic picture. As one can notice from the quantum numbers, $U(1)_{B'}$ in the electric and magnetic pictures look different. In particular, the gauged global symmetry in the electric picture is $U(N_c) \simeq (SU(N_c) \times U(1))/\mathbb{Z}_{N_c}$ whereas one cannot find a $U(N_c)$ gauge group in the magnetic picture. This leads us to consider a possibility that there is an additional $U(1)$ factor as a part of the magnetic gauge group. The actual magnetic gauge group is $U(N_f)$, and it is broken by a VEV of a field with the quantum number of $Q'^{N_c} q^{N_f}$ so that $U(1)_{B'}$ in the magnetic picture is an admixture of two $U(1)$'s. Namely, the duality of the gauge group goes through an intermediate step:

$$
\begin{aligned}
SU(N_c) \times U(N_c) \ \ \text{(electric)} \ \ &\rightarrow \ \ U(N_f) \times U(N_c) \ \ \text{(magnetic)} \\
&\rightarrow \ \ SU(N_f) \times SU(N_c)_V \times U(1)_{B'} \ \ \text{(magnetic)}. \qquad (2.53)
\end{aligned}
$$

Under this assumption, when we send the gauge coupling of $U(1) (\subset U(N_c))$ in the electric picture to be a large value, the gauge boson of the $U(1)_{B'}$ factor in the magnetic picture is mostly the one from the $U(N_f)$ magnetic gauge group. The identification of the ω meson becomes reasonable since the origin is now the same as the ρ meson.

Since $U(1)_{B'}$ is spontaneously broken by $\langle q \rangle = \langle \bar{q} \rangle \neq 0$, there is a stable vortex string which can be explicitly constructed as a classical field configuration in the magnetic picture.

	(electric charges)/e	(magnetic charges)/$(2\pi/e)$
q_1	1	0
$\bar{q}_1$	-1	0
q'_1	$1-1/N_c$	1
$\bar{q}'_1$	$-1+1/N_c$	-1
$q'_{I\neq 1}$	$-1/N_c$	1
$\bar{q}'_{I\neq 1}$	$1/N_c$	-1
Z	$1/N_c$	-1
$\bar{Z}$	$-1/N_c$	1

Table 2.3: Electric and magnetic charges under a $U(1)$ factor in $SU(N_f) \times U(1)_{B'}$.

The duality steps (2.53) imply that there is another string in the magnetic picture: one associated with $U(N_f)$ and another with $U(N_c)$. However, if we go back to the electric picture, there is only a single $U(1)$ factor in $U(N_c)$, which can only give a single kind of string. This sounds like a mismatch of two descriptions.

We propose here that the $U(N_f)$ string, made of q, $\bar{q}$, ρ, and ω, is in fact unstable since the "quarks" can attach to the endpoints, and thus that is the one which should be identified as the QCD string. The $U(N_c)$ string is stable, but should decouple in the QCD limit. As mentioned already, there are "quarks" in the magnetic picture, q', $\bar{q}'$, Z and $\bar{Z}$. They are natural candidates of the "quarks" which attach to the $U(N_f)$ string. In turn, if they are the degrees of freedom at the string endpoints, a linear potential prevents them to be in the one-particle states. Therefore, the "quarks" disappear from the spectrum. This interpretation seems to give resolutions to all the unsatisfactory features raised before: the nature of ω, free quarks, and the stable string.

For this interpretation to be possible, q', $\bar{q}'$, Z and $\bar{Z}$ should carry magnetic charges of $U(N_f)$ in addition to the quantum numbers listed in Table 2.2. Since we assume the electric-magnetic duality between the $SU(N_c)$ and the $U(N_f)$ gauge groups, it is equivalent to say that q', $\bar{q}'$, Z and $\bar{Z}$ should be colored under $SU(N_c)$, *i.e.,* Z and $\bar{Z}$ are the quarks (the non-abelian monopoles in the magnetic picture) and q' and $\bar{q}'$ are non-abelian dyons. It is interesting to notice that they indeed have N_c degrees of freedom.

In the $SU(N_f) \times U(1)_{B'}$ magnetic gauge group, there is a $U(1)$ factor which rotates a particular component of q_I and $\bar{q}_I$, where I is the index of the $SU(N_f)$ gauge group. The vortex string associated with such a $U(1)$ factor is called the non-abelian string and the one with the minimal magnetic flux is stable. Therefore, the "quarks" should attach to this string. When we take q_1 is the one which rotates under the $U(1)$ factor and normalize the charge of it as unity, the charges of other charged fields are listed in the left column of Table 2.3. By assuming that q and $\bar{q}$ have no magnetic charges, the Dirac-Schwinger-Zwanziger condition [59, 60] allows the magnetic charges listed in the right column of Table 2.3 as the minimal magnetic charges divided by $(2\pi/e)$ with e being the gauge coupling constant. Interestingly, they agree with the "color charge" of $SU(N_c)_V$ up to a normalization, which may be indicating that a part of $SU(N_c)_V$ in the magnetic picture descends from the electric gauge group, $SU(N_c)$. For dynamical fields with both electric and magnetic quantum numbers, we loose the standard Lagrangian description of the model. However, since the sector of q, $\bar{q}$ (and Φ) is all singlet under $SU(N_c)_V$ and is decoupled from the colored sector, there can be a Lagrangian to describe it, and we assume that is the model in Eq. (2.1).

It is amusing to see that many ingredients to describe the hadron world are present in this model, such as the vector mesons, the pions, the light scalar mesons, the QCD string, and the constituent quarks. This is somewhat surprising since the Seiberg duality is supposed to describe only massless degrees of freedom. The non-trivial success of the model may be indicating that the addition of N_c massive quarks is a right direction to fully connect the electric and magnetic pictures of $\mathcal{N} = 1$ supersymmetric QCD.

Chapter 3

The Weinberg sum rules for dynamical SUSY breaking

In previous chapter we discuss the dynamical chiral symmetry breaking and construct the effective model of QCD as the magnetic description. In this chapter we consider the dynamical SUSY breaking, the other interesting example of dynamical symmetry breaking. In particular, we derive the Weinberg sum rules among particles in the dynamical sector.

3.1 Weinberg sum rules

In this section, we explain the procedure of deriving sum rules rather in detail. The starting point is the symmetry transformation of current correlator* as

$$\Pi^{\mu\nu}(x-y) \equiv \left\langle \delta_Q \left[j_1^\mu(x) j_2^\nu(y) \right] \right\rangle, \tag{3.1}$$

where δ_Q implies the symmetry transformation corresponding with spontaneous broken symmetry. This current correlator should vanish if the symmetry is unbroken.†

We use the momentum space in later discussions and we simply describe $\Pi^{\mu\nu}(k)$ as the Fourier transformation of $\Pi^{\mu\nu}(x-y)$. We extract the Lorentz structure and define the scalar part of this correlator as

$$\Pi^{\mu\nu}(k) = k^\mu k^\nu \Pi_1(k^2) - k^2 \eta^{\mu\nu}\, \Pi_2(k^2), \tag{3.2}$$

where $\Pi_1(k^2)$ and $\Pi_2(k^2)$ are only dependent on k^2 since they are Lorentz scalar.

We extend the function $\Pi_1(s)$ to a complex plane; it has a branch cut on the real and positive value of s. By the Cauchy integral theorem, we obtain the following identity:

$$0 = \int_{C_A} ds\ s^n \Pi_1(s) + \int_{C_B} ds\ s^n \Pi_1(s), \tag{3.3}$$

*We define $\langle \cdots \rangle$ by the path integral, and thus they are Lorentz covariant.

†In original Weinberg sum rules, we use axial part of chiral symmetry transformation as δ_Q and vector and axial vector currents as j_1^μ and j_2^μ respectively.

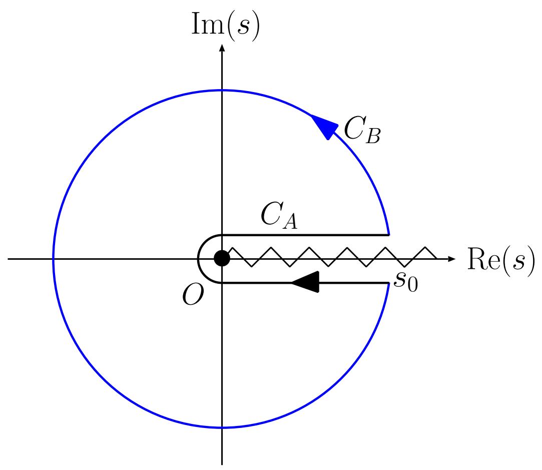

Figure 3.1: Contour of the integral.

where n is an integer. The paths C_A and C_B are shown in Fig. 3.1 where s_0 is an arbitrary real and positive number.

The Weinberg sum rules can be obtained by using the operator product expansion (OPE) for the second integral. We here consider an asymptotically free theory where the OPE at a UV scale can be done perturbatively. Let $\mathcal{O}$ be the lowest dimensional operator whose VEV breaks symmetry we consider, and d be the mass dimension (defined by the classical scaling in the UV theory) of $\mathcal{O}$. If we determine d_Π as the dimension of Π_1, it can be expanded as

$$\Pi_1(s) \simeq \frac{c_{\mathcal{O}}\langle\mathcal{O}\rangle}{(-s)^{(d-d_\Pi)/2}} + \cdots, \tag{3.4}$$

where $\cdots$ are higher order terms in the $1/(-s)$ expansion and $c_{\mathcal{O}}$ is a dimensionless coefficient. Here $(d-d_\Pi)/2$ should be an integer since it can be obtained by a calculation of Feynman diagrams. If $(d-d_\Pi)/2$ is not an integer, such an operator either does not contribute or should be supplied by some dimensionful parameter in the Lagrangian. The second integral in Eq. (3.3) vanishes for $n < (d-d_\Pi)/2 - 1$.

On the other hand, the function $\Pi_1(s)$ for the real and positive s can be expressed in terms of a spectral function as follows:

$$\Pi_1(s) = -\int_0^\infty d\sigma^2 \frac{\rho(\sigma^2)}{s-\sigma^2+i\epsilon} + \Delta(s), \tag{3.5}$$

where $\Delta(s)$ represents contact terms which are regular everywhere. By using the expression in Eq. (3.5), the first integral in Eq. (3.3) reduces to

$$2\pi i \int_0^{s_0} ds\ s^n \rho(s) \tag{3.6}$$

for $n \geq 0$. For $n < 0$, the integral depends on $\Delta(s)$.

In asymptotically free theories, the use of the OPE is justified when $(-s)$ is sufficiently large. Therefore, the quantity (3.6) should asymptotes to zero for $s_0 \to \infty$ if n is within the window:

$$0 \leq n < \frac{d - d_\Pi}{2} - 1. \tag{3.7}$$

In this window, we obtain,

$$\int_0^\infty ds \; s^n \rho(s) = 0, \tag{3.8}$$

for each integer value of n satisfied the condition (3.7). Approximating the spectrum function, $\rho(s)$, as the sum of low energy effective modes, we obtain the sum rules which relates among the physical quantities of such low energy effective modes.

To obtain concrete image of this derivation, we explain the derivation of the sum rules in QCD as example. In QCD case, the current correlator is

$$\Pi^{\mu\nu\;abc}_{\rm QCD}(x-y) \equiv \left\langle \delta^a_{Q_A} \left[\; j^{\mu b}_V(x) \; j^{\nu c}_A(y) \; \right] \right\rangle, \tag{3.9}$$

where δ_{Q_A} implies the transformation of axial part of chiral symmetry, j^μ_V and j^μ_A are vector and axial vector currents respectively, and a, b, and c, denote the flavors indices. From the current algebra, we easily derive the following formula,

$$\Pi^{\mu\nu\;abc}_{\rm QCD}(x-y) = i f^{abd} \left\langle \; j^{\mu d}_A(x) \; j^{\nu c}_A(y) \; \right\rangle + i f^{acd} \left\langle \; j^{\mu b}_V(x) \; j^{\nu d}_V(y) \; \right\rangle, \tag{3.10}$$

where f^{abc} is structure constant of flavor symmetry group and each current correlators proportional to δ^{dc} and δ^{bd} respectively. Extracting Lorentz and group structure we obtain $k^\mu k^\nu$ part of this current correlator as,

$$\Pi_{\rm QCD\,1}(k^2) = \Pi_{AA}(k^2) - \Pi_{VV}(k^2), \tag{3.11}$$

where the Π_{AA} and Π_{VV} correspond with first term and second term of Eq. (3.10) respectively and negative sign of second term reflects the antisymmetry property of structure constant, $f^{acb} = -f^{abc}$. The mass dimension of Eq. (3.11) is zero (*i.e.* $d_\Pi = 0$). The lowest dimension operator appeared in the OPE of Eq. (3.11) is $(\bar{q}q)^2$ whose dimension is six (*i.e.* $d = 6$). Therefore we obtain two sum rules as

$$\int_0^\infty ds \;\; (\rho_{AA}(s) - \rho_{VV}(s)) = 0, \tag{3.12}$$

$$\int_0^\infty ds \; s\,(\rho_{AA}(s) - \rho_{VV}(s)) = 0, \tag{3.13}$$

where $\rho_{AA}(s)$ and $\rho_{VV}(s)$ are spectrum functions corresponding with $\Pi_{AA}(s)$ and $\Pi_{VV}(s)$ respectively. As final step we approximate those spectrum functions as sum of the low energy modes as following:

$$\rho_{AA}(s) = f_\pi^2 \delta(s) + f_{a_1}^2 \delta(s - m_{a_1}^2), \tag{3.14}$$

$$\rho_{VV}(s) = f_\rho^2 \delta(s - m_\rho^2). \tag{3.15}$$

Using this approximate formula of spectrum functions, we obtain the Weinberg sum rules,

$$f_\pi^2 - f_\rho^2 + f_{a_1}^2 \simeq 0, \tag{3.16}$$

$$m_\rho^2 f_\rho^2 - m_{a_1}^2 f_{a_1}^2 \simeq 0. \tag{3.17}$$

3.2 Supercurrent and sum rules

In this section we derive the sum rule for physical quantities in the low energy effective theory of dynamical SUSY breaking using the algebra of the supercurrent multiplet. Same analysis can be applied to the General Gauge Mediation (GGM) [61] formalism which describe the soft SUSY breaking terms as the current correlators. The sum rules for GGM are discussed in Appendix. A.

3.2.1 Supercurrent and correlators

In a wide class of supersymmetric field theories, one can define a real supermultiplet called the supercurrent (J^μ) [62] (See [63] for a recent discussion). It is composed of the SUSY current (S^μ_α), the symmetric energy momentum tensor ($T^{\mu\nu}$), the R-current (j^μ), and a scalar operator x. The θ and $\bar\theta$ expansion of supercurrent are as following:

$$\begin{aligned} J_\mu(x,\theta,\bar\theta) =& j_\mu + i\theta\left(S_\mu + \frac{i}{\sqrt{2}}\sigma_\mu\bar\psi\right) - i\bar\theta\left(\bar S_\mu + \frac{i}{\sqrt{2}}\bar\sigma_\mu\psi\right) \\ &+ \frac{i}{2}\theta\theta\partial_\mu x^\dagger - \frac{i}{2}\overline{\theta\theta}\partial_\mu x + \theta\sigma^\nu\bar\theta\left[2T_{\mu\nu} - \frac{2}{3}T_\rho{}^\rho\eta_{\mu\nu} - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\rho j^\sigma\right] \\ &+ i\theta\theta\left[-\frac{i}{2}\partial_\nu S_\mu\sigma^\nu + \frac{1}{2\sqrt{2}}\partial^\nu\bar\psi\bar\sigma_\nu\sigma_\mu\right]\bar\theta \\ &- i\overline{\theta\theta}\theta\left[\frac{i}{2}\sigma^\nu\partial_\nu\bar S_\mu + \frac{1}{2\sqrt{2}}\sigma_\mu\bar\sigma_\nu\partial^\nu\psi\right] \\ &+ \theta\theta\overline{\theta\theta}\left[-\frac{1}{2}\partial_\mu\partial^\nu j_\nu + \frac{1}{4}\partial^2 j_\mu\right]. \end{aligned} \tag{3.18}$$

The R-current defined in this way is not conserved unless the theory is conformal. The transformation laws of those component fields under SUSY are given by

$$\delta_Q j_\mu = -i\eta\left(S_\mu - \frac{1}{3}\sigma_\mu\bar\sigma^\nu S_\nu\right), \tag{3.19}$$

$$\delta_Q x = -\frac{2}{3}i\eta\sigma^\mu\bar S_\mu, \tag{3.20}$$

$$\delta_Q x^\dagger = 0, \tag{3.21}$$

$$\delta_Q S_{\mu\alpha} = 2(\sigma_{\mu\nu}\eta)_\alpha\partial^\nu x^\dagger, \tag{3.22}$$

$$\delta_Q \bar S^{\dot\alpha}_\mu = i(\bar\sigma^\nu\eta)^{\dot\alpha}\left[2T_{\mu\nu} + i\partial_\nu j_\mu - i\eta_{\mu\nu}\partial\cdot j - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\rho j^\sigma\right], \tag{3.23}$$

$$\delta_Q T^{\mu\nu} = -\frac{1}{2}\left[\eta\sigma^{\rho\mu}\partial_\rho S^\nu + \eta\sigma^{\rho\nu}\partial_\rho S^\mu\right]. \tag{3.24}$$

Correlators	R-charge	Dim. of $\mathcal{O}$
$C_1,\ C_6,\ C_7$	0	d_0
$C_2,\ C_3,\ C_5,\ C_8$	2	d_2
C_4	4	d_4

Table 3.1: R-charges associated with each correlator. d_0, d_2 and d_4 denote the dimension of the lowest-dimension SUSY breaking operator which contribute to the OPE of correlators with $R = 0$, 2 and 4.

By using the above component fields, we define the following set of current correlators:

$$C_1^{\mu\nu\rho\sigma}(x,y) \equiv \left\langle \delta_Q^\alpha \left[\bar{S}^{\mu\dot{\alpha}}(x)T^{\rho\sigma}(y)\right] \right\rangle(\sigma^\nu)_{\alpha\dot{\alpha}}, \tag{3.25}$$

$$C_2^{\mu\nu\rho\sigma\kappa}(x,y) \equiv \left\langle \delta_Q^\alpha \left[S^\mu_\gamma(x)T^{\rho\sigma}\right] \right\rangle(\sigma^{\nu\kappa})_\alpha^{\ \gamma}, \tag{3.26}$$

$$C_3^{\mu\nu}(x,y) \equiv \left\langle \delta_Q^\alpha \left[\bar{S}^{\mu\dot{\beta}}(x)x^\dagger(y)\right] \right\rangle(\sigma^\nu)_{\alpha\dot{\beta}}, \tag{3.27}$$

$$C_4^{\mu\nu\kappa}(x,y) \equiv \left\langle \delta_Q^\alpha \left[S^\mu_\gamma(x)x^\dagger(y)\right] \right\rangle(\sigma^{\nu\kappa})_\alpha^{\ \gamma}, \tag{3.28}$$

$$C_5^{\mu\nu}(x,y) \equiv \left\langle \delta_Q^\alpha \left[\bar{S}^{\mu\dot{\beta}}(x)x(y)\right] \right\rangle(\sigma^\nu)_{\alpha\dot{\beta}}, \tag{3.29}$$

$$C_6^{\mu\nu\kappa}(x,y) \equiv \left\langle \delta_Q^\alpha \left[S^\mu_\gamma(x)x(y)\right] \right\rangle(\sigma^{\nu\kappa})_\alpha^{\ \gamma}, \tag{3.30}$$

$$C_7^{\mu\nu\rho}(x,y) \equiv \left\langle \delta_Q^\alpha \left[\bar{S}^{\mu\dot{\beta}}(x)j^\rho(y)\right] \right\rangle(\sigma^\nu)_{\alpha\dot{\beta}}, \tag{3.31}$$

$$C_8^{\mu\nu\rho\kappa}(x,y) \equiv \left\langle \delta_Q^\alpha \left[S^\mu_\gamma(x)j^\rho(y)\right] \right\rangle(\sigma^{\nu\kappa})_\alpha^{\ \gamma}. \tag{3.32}$$

If SUSY is unbroken, all of them are vanishing.

Since it will become important when we derive sum rules, let us here discuss R-charges associated with the above correlators. The R-symmetry plays a crucial role for SUSY breaking [64], and in most cases, it is assumed that UV theories of SUSY breaking models are R-symmetric. Therefore, in the present study, we assume that UV theories, from which OPE of the correlators are calculated, have R-symmetry. The R-charges associated with each correlator are uniquely fixed since the components of the supercurrent have R-charges determined from the SUSY algebra. Those are summarized in Table 3.1, and operators that appear in the OPE of each correlator should have the same R-charges as corresponding correlators. If the R-symmetry is not broken spontaneously, correlators with non-zero R-

charges should vanish identically, and only correlators with zero R-charge, namely C_1, C_6 and C_7, would provide non-trivial sum rules. Meanwhile, if the R-symmetry is spontaneously broken, correlators with non-zero R-charges are also non-vanishing, and further sum rules can be derived. For later convenience, we introduce d_0, d_2 and d_4 to denote the dimension of the lowest-dimension SUSY breaking operator which contribute to the OPE of correlators with $R = 0$, 2 and 4. (See Table 3.1.) The number of sum rules we can derive from each correlator depends on values of $d_{0,2,4}$ as we will discuss in detail later.

3.2.2 Sum rules in effective theories

An explicit form of sum rules can be derived by approximating the spectral function by one-particle states of hadrons. Such an approximation is valid when there is a weakly coupled description of hadrons at low energy. We assume that there is such an effective description. As hadronic degrees of freedom, we introduce fields with spins from 0 to 2 as follows:

- ϕ (massive or massless spin 0 (scalar)),
- π (massive or massless spin 0 (pseudoscalar)),
- λ (the Goldstino, spin 1/2, massless),
- χ (massive spin 1/2 (Majorana)),
- v_μ (massive spin 1 (real)),
- ψ_μ (massive spin 3/2),
- $h^{\mu\nu}$ (massive spin 2).

Except for λ, there can exist multiple particles with the same spin and parity. In the following, we suppress the indices associated with such multiple particles. The sum rules we obtain below should be understood as the one with summations of these indices.

One particle parts of the supercurrent multiplet can be parametrized as follows:

$$S^\mu_\alpha = if^4\sigma^\mu\bar{\lambda} - 2f^2f'\sigma^{\mu\nu}\partial_\nu\lambda - 2m_\psi f_\psi\sigma^{\mu\nu}\psi_\nu - 2f_\chi\sigma^{\mu\nu}\partial_\nu\chi + \cdots , \tag{3.33}$$

$$\bar{S}^{\mu\dot{\alpha}} = if^4\bar{\sigma}^\mu\lambda - 2f^2f'^*\bar{\sigma}^{\mu\nu}\partial_\nu\bar{\lambda} - 2m_\psi f^*_\psi\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu - 2f^*_\chi\bar{\sigma}^{\mu\nu}\partial_\nu\bar{\chi} + \cdots , \tag{3.34}$$

$$T_{\mu\nu} = -\frac{1}{2}m_{\rm P}^2m_h^2h_{\mu\nu} - \frac{f_\phi}{2}(\eta_{\mu\nu}\Box - \partial_\mu\partial_\nu)\phi + \cdots , \tag{3.35}$$

$$x = c_\phi^2\phi + ic_\pi^2\pi + \cdots , \tag{3.36}$$

$$j^\mu = m_vf_vv^\mu + f_\pi\partial^\mu\pi + \cdots , \tag{3.37}$$

where $\cdots$ are terms which are not linear in fields. The normalization of the fields are such that the propagators are given in Appendix B. We have implicitly assumed the CP invariance, *i.e.,* the absence of the mixing between ϕ and π, for simplicity. By using the above parametrizations and the propagators in Appendix B, one can explicitly calculate the correlators Eqs. (3.25)-(3.32) as a sum over the contributions from hadrons.

Following the same procedure in Section 3.1, one can derive the sum rules from $C_1 - C_8$ using the effective theory. For example, we obtain

$$|f'|^2 + |f_\chi|^2 + \frac{2}{3}|f_\psi|^2 = f_\phi^2 + \frac{8}{3}m_{\rm P}^2 \tag{3.38}$$

from the correlator C_1. This rule applies to the models with $d_0 = 3$ and $d_0 = 4$. To derive this rule, we use two approximations; one is the tree level approximation in the effective theory and the other is the perturbative calculation of the OPE for the correlator. The effective theory should have a UV cut-off, $\Lambda_{\rm eff}$, below which the picture of the hadron exchange (tree-level approximation) is justified. On the other hand, the OPE is a good expansion at a sufficiently short distance, $(-s) > \Lambda_{\rm OPE}$, where $\Lambda_{\rm OPE}$ is a typical scale where the UV description breaks down. Therefore, the above sum rule gives a good approximation if $\Lambda_{\rm eff} \gg \Lambda_{\rm OPE}$ and if one takes s_0 in Fig. 3.1 within the window, $\Lambda_{\rm OPE} < s_0 < \Lambda_{\rm eff}$. In the case of QCD, this condition, $\Lambda_{\rm eff} > \Lambda_{\rm OPE}$, seems to be marginally satisfied, therefore the Weinberg's sum rules are satisfied in the real world to a good accuracy. The hadron summation in the sum rules should be taken while masses exceed $\Lambda_{\rm OPE}$ [65, 66].

Repeating the same discussion for the rest of the correlators, $C_2 - C_8$, we obtain sum rules:

- Boson sum rule ($d_0 = 3$ and 4)

$$f_\phi^2 + \frac{8}{3}m_{\rm P}^2 = f_\pi^2 + f_v^2, \tag{3.39}$$

- Scalar sum rule ($d_2 = 4$)

$$f_\phi c_\phi^2 = 0, \quad f_\pi c_\pi^2 = 0, \tag{3.40}$$

- Fermion sum rule ($d_2 = 4$)

$$f^2 f' = m_\psi f_\psi^2 = -\frac{3}{4}m_\chi f_\chi^2. \tag{3.41}$$

The correlator C_4 does not lead any sum rule for $d_4 \leq 4$. For $d_2 > 4$ and $d_4 > 4$, there can be more sum rules. However, we do not try to derive those in this paper since we are not aware of such models.

3.2.3 Improvement of currents and sum rules

The entries in the sum rules, such as f', f_χ, f_ϕ, and f_π, depend on the definition of the currents in the UV theory. In deriving the sum rules, we have defined the currents as components of the supercurrent multiplet, J^μ. Moreover, we have implicitly assumed that the current does not contain parameters with negative mass dimensions, otherwise the dimension of $\mathcal{O}$ can be arbitrarily small.

If such a supercurrent is uniquely defined, there is no ambiguity for f's. If it is not uniquely defined, the sum rules should hold for any choice of the supercurrents. The supercurrent J^μ has in general a freedom of the improvement,

$$J_\mu \to J_\mu - \partial_\mu(\Omega + \bar{\Omega}), \tag{3.42}$$

where Ω is a chiral superfield. Therefore, the improvement is possible when there is a gauge-invariant chiral superfield with a mass dimension less than or equal to two in the UV theory.

For example, if there is a chiral operator M with dimension two and R-charge zero, such as a meson operator, M can be the operator Ω. In the same way as the currents, we parametrize the one-particle parts of the operator M by low energy variables as

$$m = -\frac{i}{\sqrt{2}}\left(\frac{F_\phi}{\sqrt{2}}\phi - \frac{iF_\pi}{\sqrt{2}}\pi\right) + \cdots, \tag{3.43}$$

$$\psi_{M\alpha} = -\frac{i}{\sqrt{2}}\left(F'\lambda_\alpha + F_\chi\chi_\alpha\right) + \cdots, \tag{3.44}$$

$$F_M = -i(C_\phi^{*2}\phi - iC_\pi^{*2}\pi) + \cdots, \tag{3.45}$$

where

$$M(y,\theta) = m(y) + \sqrt{2}\theta\psi_M(y) + \theta\theta F_M(y). \tag{3.46}$$

With these parametrizations, the improvement in Eq. (3.42) with $\Omega = cM$, with c a real dimensionless parameter, shifts the decay constants as

$$f' \to f' + cF', \tag{3.47}$$

$$f_\chi \to f_\chi + cF_\chi, \tag{3.48}$$

$$f_\phi \to f_\phi + cF_\phi, \tag{3.49}$$

$$f_\pi \to f_\pi + cF_\pi, \tag{3.50}$$

$$c_\phi^2 \to c_\phi^2 + cC_\phi^2, \tag{3.51}$$

$$c_\pi^2 \to c_\pi^2 + cC_\pi^2. \tag{3.52}$$

The constants f, f_v, f_ψ, and $m_{\rm P}$ are unchanged by the improvement.

When $d_0 \geq 3$, sum rules in Eqs. (3.38) and (3.39) should hold for any choice of c.

Therefore, we obtain the following relations:

$$|F'|^2 + |F_\chi|^2 = F_\phi^2, \tag{3.53}$$
$$\mathrm{Re}[f'^* F'] + \mathrm{Re}[f_\chi^* F_\chi] = f_\phi F_\phi, \tag{3.54}$$
$$F_\phi^2 = F_\pi^2, \tag{3.55}$$
$$f_\phi F_\phi = f_\pi F_\pi, \tag{3.56}$$

in addition to Eqs. (3.38) and (3.39). As a trivial example, the effective theory described by a single chiral superfield,

$$M \propto \phi + i\pi + \sqrt{2}\theta(\lambda \text{ or } \chi) + \theta\theta F, \tag{3.57}$$

satisfies the sum rules in Eqs. (3.53)–(3.56).

3.3 UV models and sum rules

In this section, we consider the explicit models of dynamical SUSY breaking and discuss which sum rules in Eqs. (3.38)–(3.41) apply to them. Here, we classify those models by whether R-symmetry is spontaneously broken, and by dimensions of the SUSY breaking operators.

3.3.1 Models with unbroken R-symmetry

We first discuss the models without spontaneous R-symmetry breaking. In this case, the correlators with non-vanishing R-charges identically vanish, and thus only Eqs. (3.38) and (3.39) can apply. Since R-symmetry is not broken, $f' = 0$ in this case. In most models, $d_0 = 4$ (except for the model with non-vanishing D-term for a $U(1)$ factor), and therefore both sum rules apply.

A famous example is the O'Raifeartaigh model [22].[‡] However, in this case, the sum rules do not give new information since one can explicitly derive the low energy models. Examples of dynamical SUSY breaking models are the IYIT model [23, 24] and the ISS model [28] where the ISS model has unbroken discrete R-symmetry. Both of the examples have calculable IR descriptions which reduces to the O'Raifeartaigh models.

3.3.2 Models with spontaneous R-symmetry breaking ($d_2 \leq 3$)

When R-symmetry and SUSY are both broken by an operator with $R = 2$ and dimension less than four, those models predict the sum rules in (3.38) and (3.39).

Examples are incalculable models such as chiral gauge theories in Ref. [16, 17]. There are also possibilities that the incalculable Kähler potential can produce a non-trivial R-symmetry breaking vacuum in the vector-like theories such as in [25, 26, 27], although there are known effective descriptions in these cases.

[‡]There are also the O'Raifeartaigh models with broken R-symmetry [67].

In models of Ref. [16, 17], it is suggested that the gaugino condensation, which has dimension three, breaks both SUSY and the R-symmetry through the Konishi anomaly [30]. In the vector-like models in Ref. [25, 26, 27], a dimension-three operator, $\delta_{\bar{Q}\dot{\alpha}}(\bar{\psi}_S^{\dot{\alpha}} S)$, is the one which breaks both SUSY and R-symmetry, where ψ_S and S are the fermionic and the bosonic components of a gauge singlet chiral superfield.

3.3.3 Models with spontaneous R-symmetry breaking ($d_2 \geq 4$)

Possibly some gauge theory without a matter field can be of this type, although there is no known example. In this case, all the sum rules in Eqs. (3.38)–(3.41) can be derived.

Since R-symmetry is spontaneously broken, one can say $f_\pi \neq 0$. This implies that the left-hand side of Eq. (3.39) is non-vanishing and therefore the left-hand side of Eq. (3.38) is also non-vanishing. Together with Eq. (3.41), $m_\psi f_\psi^2$ is non-vanishing (unless there is a cancellation among same-spin fermions). Therefore, this type of model generally involves massive spin-3/2 field.

If one finds that the sum rules in Eqs. (3.40) and (3.41) apply in some hadronic models of SUSY breaking such as the dual gravity constructions [68, 69, 70], it may be suggesting that the microscopic description is in this category.

Chapter 4

Summary and discussion

In this article we discuss the techniques of dynamical symmetry breaking. The interesting examples of dynamical symmetry breaking are the chiral symmetry breaking in QCD and dynamical SUSY breaking. The quite different approaches are adopted to analyze those dynamical symmetry breaking. For analysis of chiral symmetry breaking, Weinberg derived powerful non-perturbative result called as Weinberg sum rules [1]. For analysis of dynamical symmetry breaking, we often use the Seiberg duality [2], the electric-magnetic duality of supersymmetric gauge theories, since the dynamical symmetry breaking are considered in the context that supersymmetric gauge theories are strongly coupled. We exchange those analyzing techniques each other, *i.e.* we use the idea of Seiberg duality to construct effective model containing chiral symmetry breaking and derive the sum rules for dynamical SUSY breaking.

Since Seiberg duality relates the gauge theory with different gauge symmetries, we consider the gauge theories with non-$SU(N_c)$ gauge symmetry and interpret that gauge theory as magnetic description of QCD. It is known that the higgsing $U(N_f)$ gauge theory well describes the vector mesons and pions which are the lightest hadrons appeared as a consequence of the color confinement in QCD. Therefore we construct higgsing $U(N_f)$ gauge theories as the magnetic description of QCD and examine whether this interpretation works well or not.

Our effective model contains the vector mesons and Nambu-Goldstone bosons as well as the scalar particles. Those particles can be interpreted as the hadrons realized in nature. For example the magnetic gauge bosons are interpreted as the vector mesons and scalar particles are the scalar mesons, *i.e.* f_0 and a_0 corresponding with flavor singlet and adjoint scalar respectively. It is the most interesting point of interpretation of such $U(N_f)$ gauge theory as magnetic description of QCD that the string solution as a solitonic object of $U(N_f)$ gauge theory can be identified as the confinement string since such vortex strings carry magnetic charge of our model and the magnetic charge of our model can be interpreted as the color charge in original electric description of QCD. To examine this identification we compare the energy of vortex string and the potential energy between the static quark and antiquark. Our model reproduces the qualitative feature of the potential energy, although our analysis is at tree level. Therefore our model and interpretation work well at least qualitative level.

We also consider the other interesting dynamical symmetry breaking, the dynamical SUSY breaking. To approach what happen in low energy description of dynamical SUSY breaking models, we derive the sum rules, which have been derived for chiral symmetry breaking in QCD, for dynamical SUSY breaking models. The sum rules involve massive fields with spin 3/2 and 2. It is interesting to note here that there is an analogy of this situation in QCD.

The Nambu-Goldstone bosons (pions) associated with the chiral symmetry breaking are described non-linear sigma model which has UV cut off scale. The cut-off scale can be pushed higher by including massive hadrons. The simplest possibility is to promote the non-linear sigma model to a linear-sigma one by introducing a scalar field (which is usually called the sigma meson). The sum rules for chiral symmetry breaking require the existence of the spin-1 hadrons instead of massive scalar particle. Indeed, a vector meson (the rho meson) appeared as the next lightest in actual hadronic world. The HLS model [6] contains the vector meson (the rho meson) and well describes its interactions and mass.

In SUSY breaking case, the low-energy effective Lagrangian is formulated by Volkov and Akulov in Ref. [71], where the Nambu-Goldstone fermion, the Goldstino, is introduced as non-linearly transforming field under SUSY. The simplest possibility for the next lightest mode is the superpartner of the Goldstino, formulating the low-energy effective model with a chiral supermultiplet. This is analogous to the linear sigma model realization of the chiral symmetry case. As in QCD, it is worth considering an alternative realization, namely the SUSY breaking model equivalent of the HLS realization. Such a realization is achieved by introducing the massive spin-2 field, as discussed in Ref. [72].

Another realization of the massive higher spin states in SUSY gauge theories is related to the gauge/gravity correspondence. For example, in the Holographic QCD model [73, 74, 75, 76], the HLS naturally emerges and the rho meson appears as a "Kaluza-Klein (KK)" excitation mode of the five-dimensional gauge field in the holographic dual. In the context of the gauge/gravity duality, the possibility of the dynamical SUSY breaking has been discussed [68, 69, 70]. If the gravity dual of the dynamical SUSY breaking model is successfully constructed, the Goldstino should be identified with a normalizable zero mode of the KK modes of the bulk gravitino [77]. Furthermore, massive spin-3/2 and massive spin-2 modes also appear from gravitino and graviton in the dual supergravity. In this sense, our effective theory with the hidden local SUSY can be related to the dual supergravity.

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Appendix A

Direct gauge mediation and sum rules

In this section, by using the language of the general gauge mediation [61], we derive sum rules which are related to the current correlators in the hidden sector. Then, with those sum rules, we show that the sfermion mass squared can be expressed in terms of masses of the spin 0, 1/2 and 1 particles in the SUSY breaking sector.

A.1 Current multiplet and correlators

We introduce the current superfield $\mathcal{J} = \mathcal{J}(x, \theta, \bar{\theta})$. It is defined as a real linear superfield which satisfies the current conservation conditions, $\bar{D}^2\mathcal{J} = D^2\mathcal{J} = 0$. In components, it can be expressed as

$$\mathcal{J} = J + i\theta j - i\bar{\theta}\bar{j} + \theta\sigma^\mu\bar{\theta}j_\mu - \frac{1}{2}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu j + \frac{1}{2}\bar{\theta}\bar{\theta}\theta\sigma^\mu\partial_\mu\bar{j} - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box J. \tag{A.1}$$

Transformation laws of these component fields under SUSY are given by

$$\delta_Q J = -i\eta j, \tag{A.2}$$
$$\delta_Q j_\alpha = 0, \tag{A.3}$$
$$\delta_Q \bar{j}^{\dot{\alpha}} = i(\bar{\sigma}^\mu\eta)^{\dot{\alpha}}(j_\mu + i\partial_\mu J), \tag{A.4}$$
$$\delta_Q j_\mu = -\eta\partial_\mu j. \tag{A.5}$$

Here, η is a parameter of the SUSY transformation, and we defined $\delta_Q\mathcal{O} = -\eta_\alpha\delta_Q^\alpha\mathcal{O}$.

Now, we consider the following current correlators*:

$$D_1^{\mu\nu}(x, y) \equiv \left\langle \delta_Q^\alpha \left[\bar{j}^{\dot{\alpha}}(x) j^\mu(y)\right] \right\rangle (\sigma^\nu)_{\alpha\dot{\alpha}}, \tag{A.6}$$
$$D_2^\mu(x, y) \equiv \left\langle \delta_Q^\alpha \left[\bar{j}^{\dot{\alpha}}(x) J(y)\right] \right\rangle (\sigma^\mu)_{\alpha\dot{\alpha}}. \tag{A.7}$$

*We define $\langle \cdots \rangle$ by the path integral, and thus they are Lorentz covariant.

These D's should vanish if SUSY is unbroken. For later convenience, we rewrite Eqs. (A.6) and (A.7) in terms of the Fourier transformed functions, $\tilde{C}$'s, introduced in Ref. [61]:

$$\tilde{C}_0(k^2) = \int \frac{d^4x}{i(2\pi)^4} \Big\langle J(x)J(y) \Big\rangle e^{ik\cdot(x-y)}, \tag{A.8}$$

$$-(\bar{\sigma}_\mu)^{\dot{\alpha}\alpha} k^\mu \tilde{C}_{1/2}(k^2) = \int \frac{d^4x}{i(2\pi)^4} \Big\langle j^\alpha(x)\bar{j}^{\dot{\alpha}}(y) \Big\rangle e^{ik\cdot(x-y)}, \tag{A.9}$$

$$-(k^2\eta^{\mu\nu} - k^\mu k^\nu)\tilde{C}_1(k^2) = \int \frac{d^4x}{i(2\pi)^4} \Big\langle j^\mu(x)j^\nu(y) \Big\rangle e^{ik\cdot(x-y)}, \tag{A.10}$$

$$\epsilon_{\alpha\beta} M \tilde{B}_{1/2}(k^2) = \int \frac{d^4x}{i(2\pi)^4} \Big\langle j_\alpha(x)j_\beta(y) \Big\rangle e^{ik\cdot(x-y)}, \tag{A.11}$$

where M is a characteristic mass scale of the theory. Using those $\tilde{C}$'s, we can write down the $k^\mu k^\nu$ part (k^μ part) of D_1 (D_2) as follows:

$$D_1^{\mu\nu}|_{k^\mu k^\nu} = -2i \int \frac{d^4k}{i(2\pi)^4} k^\mu k^\nu \left(\tilde{C}_1(k^2) - \tilde{C}_{1/2}(k^2)\right) e^{-ik\cdot(x-y)}, \tag{A.12}$$

$$D_2^{\mu}|_{k^\mu} = -2i \int \frac{d^4k}{i(2\pi)^4} k^\mu \left(\tilde{C}_0(k^2) - \tilde{C}_{1/2}(k^2)\right) e^{-ik\cdot(x-y)}. \tag{A.13}$$

A.2 Sum rules

In this section, we derive the sum rules through the currents correlators (A.6) and (A.7). We define,

$$\Pi_{D_1}(s) \equiv \tilde{C}_1(s) - \tilde{C}_{1/2}(s), \tag{A.14}$$

$$\Pi_{D_2}(s) \equiv \tilde{C}_0(s) - \tilde{C}_{1/2}(s), \tag{A.15}$$

as $\Pi_1(k^2)$ in the deriving procedure in Section. 3.1. The mass dimension of both functions are zero. Therefore we take $d_\Pi = 0$ and obtain the window of Eq. (3.7) as,

$$0 \le n < \frac{d}{2} - 1. \tag{A.16}$$

In general $d \le 4$ since $T^\mu{}_\mu$ can always be the SUSY breaking operator.

From $d \le 4$, such n can only be zero. For $d = 3$ or 4 the sum rules we obtained from D_1 and D_2 are

$$\int_0^\infty ds \left(\rho_1(s) - \rho_{1/2}(s)\right) = 0, \tag{A.17}$$

$$\int_0^\infty ds \left(\rho_0(s) - \rho_{1/2}(s)\right) = 0, \tag{A.18}$$

where

$$\tilde{C}_a(s) = -\int_0^\infty d\sigma^2 \frac{\rho_a(\sigma^2)}{s - \sigma^2 + i\epsilon}. \quad (a = 0,\ 1/2,\ 1) \tag{A.19}$$

No sum rule for $\tilde{B}_{1/2}$ is obtained from other correlators.

A.3 Low energy models and sum rules

Let us assume that the SUSY breaking model is a confining theory and its low-energy physics is well described by the lowest modes *à la* Weinberg [1]:

$$\rho_a(s) = f_a^2 \delta(s - m_a^2). \tag{A.20}$$

In this case, the sum rules Eqs. (A.17) and (A.18) suggest

$$f_0^2 = f_{1/2}^2 = f_1^2 \equiv f_h^2. \tag{A.21}$$

It states that the decay constants are the same even though the masses can split.

By using the formula of the general gauge mediation [61], the scalar masses via gauge mediation are given by

$$\begin{aligned} m_s^2 &= g^4 c_2 \int \frac{d^4k}{i(2\pi)^4} \frac{1}{k^2} \left(3\tilde{C}_1(k^2) - 4\tilde{C}_{1/2}(k^2) + \tilde{C}_0(k^2) \right) \\ &= \frac{g^4 c_2 f_h^2}{(4\pi)^2} \log \frac{m_0^2 m_1^6}{m_{1/2}^8}. \end{aligned} \tag{A.22}$$

Here, m_0, $m_{1/2}$, m_1 are masses of the particles with spin 0, 1/2, and 1 in the hidden sector, respectively, and c_2 is the quadratic Casimir invariant. A finite result is obtained due to the sum rules. (Similar to the $\pi^+ - \pi^0$ mass splitting by QED. See [78].) Interestingly, in Ref. [79], the same expression for the sfermion mass squared was derived in a model with gauge messengers.

From Eqs. (A.20) and (A.21), the gaugino masses can also be calculated as

$$m_\lambda = \frac{g^2 f_h^2}{m_{1/2}}. \tag{A.23}$$

In summary, by using the sum rules, one can express the sfermion and gaugino masses by hadron masses in the hidden sector.

Appendix B

Propagators

$$\langle \lambda_\alpha(x)\bar{\lambda}_{\dot{\beta}}(y)\rangle = \frac{1}{f^4}\ (\sigma^\rho)_{\alpha\dot{\beta}} \int \frac{d^4k}{i(2\pi)^4}\frac{k_\rho}{-k^2-i\epsilon}e^{-ik\cdot(x-y)}. \tag{B.1}$$

$$\langle \chi_\alpha(x)\bar{\chi}_{\dot{\beta}}(y)\rangle = (\sigma^\rho)_{\alpha\dot{\beta}} \int \frac{d^4k}{i(2\pi)^4}\frac{k_\rho}{m_\chi^2-k^2-i\epsilon}e^{-ik\cdot(x-y)}. \tag{B.2}$$

$$\langle \chi_\alpha(x)\chi^\beta(y)\rangle = \delta_\alpha^\beta \int \frac{d^4k}{i(2\pi)^4}\frac{m_\chi}{m_\chi^2-k^2-i\epsilon}e^{-ik\cdot(x-y)}. \tag{B.3}$$

$$\langle \psi_{\mu\alpha}(x)\bar{\psi}_{\nu\dot{\beta}}(y)\rangle = \left(P_L\langle\Psi_\mu(x)\bar{\Psi}_\nu(y)\rangle P_R\right)_{\alpha\dot{\beta}}. \tag{B.4}$$

$$\langle \psi_{\mu\alpha}(x)\psi_\nu^\beta(y)\rangle = \left(P_L\langle\Psi_\mu(x)\bar{\Psi}_\nu(y)\rangle P_L\right)_\alpha^{\ \beta}. \tag{B.5}$$

$$\langle \Psi_\mu(x)\bar{\Psi}_\nu(y)\rangle = \int \frac{d^4k}{i(2\pi)^4}\frac{P_{\mu\nu}(k)}{m_\psi^2-k^2-i\epsilon}e^{-ik\cdot(x-y)}. \tag{B.6}$$

$$P_{\mu\nu} = -\left(\eta_{\mu\nu}-\frac{k_\mu k_\nu}{m_\psi^2}\right)(\not{k}+m_\psi) - \frac{1}{3}\left(\gamma_\mu+\frac{k_\mu}{m_\psi}\right)(\not{k}-m_\psi)\left(\gamma_\nu+\frac{k_\nu}{m_\psi}\right). \tag{B.7}$$

$$\langle h^{\mu\nu}(x)h^{\rho\sigma}(y)\rangle = \frac{2}{m_{\rm P}^2}\int \frac{d^4k}{i(2\pi)^4}\frac{B^{\mu\nu;\rho\sigma}}{m_h^2-k^2-i\epsilon}e^{-ik\cdot(x-y)}. \tag{B.8}$$

$$
\begin{aligned}
B_{\mu\nu;\rho\sigma} &= \left(\eta_{\mu\rho} - \frac{k_\mu k_\rho}{m_h^2}\right)\left(\eta_{\nu\sigma} - \frac{k_\nu k_\sigma}{m_h^2}\right) + \left(\eta_{\mu\sigma} - \frac{k_\mu k_\sigma}{m_h^2}\right)\left(\eta_{\nu\rho} - \frac{k_\nu k_\rho}{m_h^2}\right) \\
&\quad -\frac{2}{3}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_h^2}\right)\left(\eta_{\rho\sigma} - \frac{k_\rho k_\sigma}{m_h^2}\right). \qquad \text{(B.9)}
\end{aligned}
$$

$$
\langle\phi(x)\phi(y)\rangle = \int \frac{d^4k}{i(2\pi)^4}\frac{1}{m_\phi^2 - k^2 - i\epsilon}e^{-ik\cdot(x-y)}. \qquad \text{(B.10)}
$$

$$
\langle\pi(x)\pi(y)\rangle = \int \frac{d^4k}{i(2\pi)^4}\frac{1}{m_\pi^2 - k^2 - i\epsilon}e^{-ik\cdot(x-y)}. \qquad \text{(B.11)}
$$

$$
\langle v^\mu(x)v^\nu(y)\rangle = \int \frac{d^4k}{i(2\pi)^4}\frac{\eta^{\mu\nu} - k^\mu k^\nu/m_v^2}{k^2 - m_v^2 + i\epsilon}e^{-ik\cdot(x-y)}. \qquad \text{(B.12)}
$$

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