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学位の種類　博士（経済学）

学位記番号　経博第95号博士（経済学）

学位授与年月日　平成16年3月25日

学位授与の要件　学位規則第4条第1項該当

研究科・専攻　東北大学大学院経済学研究科（博士課程後期3年の課程）
経営学専攻

論文題目　Essays on Knightian Uncertainty
（ナイト流不確実に関する研究）

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論文内容の要旨

Abstract

Chapter 1 reviews developments of expected utility theories focusing on *Choquet Expected Utility Theory* and *Maximin Expected Utility Theory*. Choquet Expected Utility Theory states that if a decision maker satisfies some set of axioms, then her belief is captured by a *non-additive probability measure (*or *a capacity)* and her preference is represented by *Choquet integrals*. This theory is axiomatized in the frameworks of Anscombe and Aumann (1963) and Savage (1954) by Schmeidler (1989), and Gilboa (1987) and Sarin and Wakker (1992), respectively. Maximin Expected Utility Theory states that if she satisfies some set of axioms, then her belief is captured by the *set of probability measures* and her preference is represented by the *minimum of expected utilities over the set of probability measures*. This theory is also axiomatized in the frameworks of Anscombe and Aumann (1963) and Savage (1954) by Gilboa and Schmeidler (1989) and Casadesus-Masanell, Klibanoff and Ozdenoren (2000), respectively.

Chapter 2 analyzes individual's portfolio selection problems under *ambiguity*. Introducing the concept of ambiguity, we show the existence of portfolio inertia under the assumptions that decision

maker's beliefs are captured by an inner measure, and that her preferences are represented by the Choquet integral with respect to the inner measure. Recently, Epstein (1999) casts doubt on the usage of the convexity of non-additive measures as *uncertainty aversion*, and introduces the concept of ambiguity. Epstein (1999) defines *unambiguous events* as those events to which a decision maker can assign probabilities, and *ambiguous events* as the ones to which she cannot. Zhang (2002) proposes a new axiomatization of Choquet Expected Utility. This states that if a certain set of axioms is satisfied, then decision maker's beliefs are captured by an inner measure and her preferences are represented by the Choquet integral with respect to the inner measure. Zhang's axiomatization is more restrictive than Schmeidler's one (1989). However, adopting Zhang's approach, we can incorporate the concept of ambiguity into Choquet Expected Utility, and shed light on the possibility of its applications to other economic problems. In this chapter, based on Zhang (2002), we show that there exists portfolio inertia under Choquet Expected Utility with an inner measure, which is neither additive nor convex. This approach is one of the starkest contrasts to the one that depends on the convexity of non-additive measures. Furthermore, we study the difference between ambiguity and uncertainty by considering in vestors' behavior.

Chapter 3 analyzes individual's portfolio selection problems under *uncertainty*. By introducing the concept of *Knightian uncertainty*, Dow and Werlang (1992) first account for the existence of portfolio inertia, which has not been accounted for under the concept of *risk*. We have witnessed a lot of research that analyzed how the degree of *risk aversion* proposed by Arrow (1965) and Pratt (1964) affects decision making in the *non-deterministic situation*. To the best of our knowledge, however, there has been no dominant parameterization that measures the degree of *uncertainty aversion*. Thus, it has not been clear how the degree of uncertainty aversion will affect decision makers' behavior in optimization problems, for example, consumption-saving problems or portfolio selection problems, and so forth. Ozaki and Streufert (1999, 2001) propose some parameter that measures the convexity of non-additive measures that is considered to capture the attitude toward Knightian uncertainty. Their parameter is as tractable as Arrow/Pratt measure of absolute risk aversion. Thus, in this chapter, based on their approach, we analyze how the degree of Knightian uncertainty aversion affects the range of price that investors never change their positions. The purpose of this chapter is to show that an increase (a decrease) in Knightian uncertainty aversion measured by the parameter proposed by Ozaki and Streufert (1999, 2001) will expand (shrink) portfolio inertia.

Chapter 4 analyzes investors' portfolio selection problems in a *two-period dynamic model of Knightian uncertainty*. While the existence of portfolio inertia cannot be accounted for under the standard expected utility theory, it can be explained under non-expected utility theories. In this chapter, we extend their static frameworks to a two-period dynamic framework in order to incorporate decision maker's updating behavior. Within such a framework, we can consider the following question: Does

obtaining new information affect her portfolio inertia, which is derived from her optimization behavior? Consider the following situation: investors are divided into two groups. One group of them is always well informed of economic conditions in details. The other is provided no such information. A question occurs. Which groups do better performance of investment? In the literature on the behavioral finance, it has been often reported that the latter group outperforms the former. Can such an observation be theoretically accounted for under expected (or non-expected) utility theories that also incorporate some decision maker's updating behavior? The purpose of this chapter is to provide an explanation for such results reported in the literature on the behavioral finance from a theoretical point of view.

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論文審査結果の要旨

本論文は、非期待効用理論を適用して、証券市場において観察される Portfolio Inertia すなわち証券に値が付かなかったり取り引きされない時間帯がある現象が存在することを説明し、さらに不確実性と、投資家の主観的な売値と買値の差（Portfolio Inertia の幅）の大きさとの関係を理論的に解明した論文である。

非期待効用理論とは、期待効用理論では説明できない人間の経済行動や、期待効用理論の公理を満たさない環境であるナイト的不確実性（Knightian Uncertainty：確率も付与できない状況）下での人々の選好順序を説明する理論であり、近年、研究が進んでいる。

1章では、ナイト的不確実性下での人間の選好順序を表す理論として、ショケ期待効用理論（Choquet Expected Utility Theory）とマクシミン期待効用理論（Maximin Expected Utility Theory）に関する研究成果を的確かつ簡潔に紹介している。

2章では、「曖昧さ」（ambiguity）という概念を採用し、投資家の不確実性に対する信念が非加法的測度である内測度（inner measure）で表明され、またその選好がショケ期待効用で表されるときに Portfolio Inertia が存在する可能性があることと、「曖昧さ」の増加が Portfolio Inertia の幅を拡大することを示している。「曖昧さ」とは確率を付与できない事象があることを意味する。ここでは測度に関して凸性を条件としない点が新しい。

3章では、2章とは異なり「ナイト的不確実性の回避」という概念を採用している。前章と同様に、投資家の選好がショケ期待効用で表されるとき、投資家が「不確実性回避的」であると Portfolio Inertia が存在する可能性があることと、「不確実性回避」の程度が大きいと Portfolio Inertia の幅が拡大することを示している。ここでは「不確実性回避」の程度は非加法的測度の凸性の大きさと正の関係があるが、新しく提唱された「不確実性回避」の程度を表すパラメータを用いて Portfolio Inertia に関する結果を導いている点が新しい。

4章では、2期間という設定で、1期後に情報を得られる投資家とまったく情報を得られない投資家の二つの投資家グループに関して Portfolio Inertia の分析を行っている。ここではナイト的不確実性を表すのに ε-contamination という概念を用い、また、1期目に新しい情報を得た投資家が当初の確率信念をベイズ流に改訂するルールとしてよく知られた2つのルールを適用する。投

資家の確率信念がε-contaminationから導かれる非加法で凸である測度で表明され、投資家の選好順序がショケ期待効用で表されるとき、1期に情報を得られる投資家にも得られない投資家にもPortfolio Inertiaが存在することを示している。さらに興味深い結論として、ナイト的不確実性の程度がある水準を超えて大きい場合には、1期に情報を得られる投資家のPortfolio Inertiaの幅の大きさが、まったく情報を得られない投資家よりも大きくなる、という逆説的な結論を得ている。

ナイト的不確実性を扱うアプローチは複数存在し、2章と3章は異なるアプローチにより分析しているが互いの比較の考察がないこと、4章では0期にしか証券の取引が行われず、1期には取引が行われないという設定に関する欠点があること、情報の多い投資家の方がPortfolio Inertiaの幅が拡大し、より証券取引に消極的になることの経済的な意味付けがないことなど、いくつかの問題は残されているが、これらは今後の研究課題としてもらいたい。

本論文は、従来の理論では説明できなかった証券市場におけるPortfolio Inertiaという現象をナイト的不確実性下での選好理論という最新かつ高度な数学を扱う理論を適用して解明し、また不確実性の程度とPortfolio Inertiaの幅の関係の分析に成功している。これらの成果は、ファイナンス、およびミクロ経済学の分野での貴重な貢献と考えられる。

よって本論文は博士（経済学）論文として「合格」であると判定する。