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## 論文内容要旨

### 1. Introduction

Higher Functions performed by neural networks are important subject not only for application to engineering but also for understanding a brain of animals in the higher orders. The higher functions of the neural networks treated in the present thesis are associative memory, generalization ability, learning and visual selective attention and so on. The associative memory is an ability of recalling its related pattern from patterns embedded into a neural network when a pattern is presented to the neural network. The generalization ability is to extract concepts from example patterns which are related to those concepts. The learning corresponds to modification of connections between neurons in order to make a map implemented by a neural network approach to a desired map. The visual selective attention is regarded as a function by which an amount of the visual information can be reduced in order to perform effective processing of information in the brain.

Statistical physics is a systematic prescription by which we can derive macroscopic properties of the system from microscopic relations among components in a system. For example, the storage capacity of the memory or the generalization ability is a macroscopic property of a neural network, while an energy function or a cost function represents a sum of microscopic relations in the system. In the present thesis, each pattern embedded into the neural networks is assumed to be a random variable. When the energy function of a system exists, we can obtain the free energy, which is one of macroscopic quantities, by using the replica method. From the free energy, we can derive equations of order parameters such as a retrieval overlap and a generalization overlap in the equilibrium state. On the other hand, for a system in which the energy function is not defined, a generating function of path-integral representation is sometimes useful in order to derive the macroscopic property such as dynamical order parameters from microscopic relations.

In the present thesis, we investigate the storage capacity and the generalization ability in Chapters 2 and 3, the learning speed in Chapter 4 and the visual selective attention in Chapter 5. Concluding remarks are given in Chapter 6.

## 2. Recurrent Neural Networks

### 2.1 Hopfield Neural Networks

We investigate the generalization ability of a fully connected Hopfield neural network (HNN), in which a state of neurons is expressed by a spin- $S$  ( $S > 1/2$ ) Ising spin and connections are modified by the Hebbian learning rule. We obtain macroscopic properties from the energy function of the system by using the replica method; we derive equations for order parameters under the assumption of the replica symmetry. We show behavior of the generalization error as a function of the number of examples for  $S=1$ ,  $3/2$  and  $\infty$  in Fig.1. We find that the generalization ability is enhanced for a larger value of  $S$  when the number of embedded concept patterns is of a lower order than the number of neurons in the thermodynamic limit.

### 2.2 Sequence Processing Neural Networks

We investigate the storage capacity and the retrieval property of a fully connected sequence processing neural network (SPNN) with a non-monotonic transfer function (NMTF) by an analytic method and by numerical simulations. Because there is no energy function in the system due to asymmetry of connections, we use the generating function of path-integral representation for the analytic method, and obtain equations for dynamical order parameters in stationary states. We find that the system with the NMTF can retrieve more sequences of patterns than that with a monotonic transfer function (MTF) when we choose optimally a value of parameter for non-monotonicity of the transfer function. We also find that some chaotic behavior appears in the retrieval error when non-monotonicity of the transfer function increases, as shown in Fig.2. The analytic results are in excellent agreement with the results by numerical simulations.

## 3. Layered Neural Networks

### 3.1 Layered Neural Networks Using Q-States Clock Neurons

We investigate the storage capacity of a fully connected layered neural network with intra-layer connections (LNNILC), in which a state of neurons is expressed by a  $Q$ -states clock spin. We assume that the inter-layered neurons and the intra-layered neurons are updated simultaneously. In our formulation, the system becomes the HNN, the layered neural network (LNN) or the LNNILC by adjusting a parameter  $\Omega$  which represents a degree of competition between the inter-layer connections and the intra-layer connections. We obtain macroscopic properties of the system by using the replica method with the assumption of the replica symmetry. We show the storage capacity as a function of the parameter  $\Omega$  for  $Q=2, 3, 4, 5, 6, 7$  and  $\infty$ . We find that the storage capacity is maximum for  $Q=3$  and  $\Omega=-0.1$ . We also find that the storage capacity of the LNNILC is enhanced in comparison with that of the HNN and that of the LNN for  $Q=2, 3, 4$  and  $5$ . However, for  $Q > 5$ , it turns out that the storage capacity becomes maximum for the LNN.

### 3.2 Sparse Coding for Layered Neural Networks

We investigate the storage capacity for a fully connected LNN and a fully connected LNNILC in the case of sparse coding. We assume that the LNN has different MTFs for even layers and for odd layers, and that the LNNILC has different MTFs for inter-layer and for intra-layer, and the inter-layered neurons and the intra-layered neurons are updated alternately. We derive recursion relations for order parameters by using the signal-to-noise ratio method,

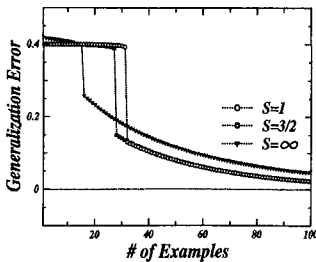


Fig. 1

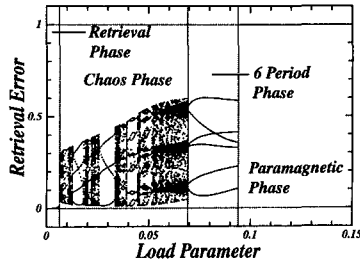


Fig. 2

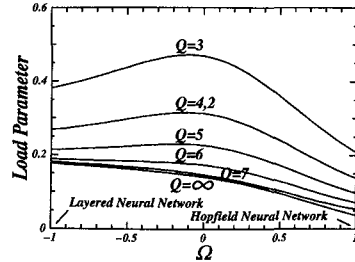


Fig. 3

and then apply the self-control threshold method. In Fig.4, we show the critical value of the storage capacity as a function of the firing rate  $a$  for the LNN and the LNNILC. We find that the critical value of the storage capacity behaves as  $0.11 / |a \ln a|$  for small values of firing rate  $a$  for both the LNN and the LNNILC. We also find that the basin of attraction becomes larger for both the LNN and the LNNILC when the self-control threshold method is applied.

#### 4. On-Line Learning of Two-Layered Neural Networks

We investigate dilution effect on an on-line learning of two-layered neural networks by using a gradient descent algorithm within the framework of the statistical physics. In the thermodynamic limit by assuming the self-averaging, we derive coupled first-order differential equations for order parameters which describe a learning process. We show the generalization error as a function of the number of examples for the non-diluted teacher network, for the symmetrically diluted teacher network and for the asymmetrically diluted teacher network in Fig.5. We find that the asymmetric dilution of the connections in the teacher network makes the learning speed faster than the symmetric dilution of the connections in the teacher network. Namely we find that the initial plateau appeared in the learning curve for the asymmetric dilution is shorter than that for the symmetric dilution. It turns out that the learning does not converge when the teacher network is diluted too much. As for the dilution of connections in the student network, the learning converges imperfectly when the dilution is strong.

#### 5. Mathematical Modeling of Visual Selective Attention

We propose a mathematical model of the visual selective attention using a two-layered neural network which consists of the layer of the hippocampus and that of the visual cortex. We use neurons described by a Hodgkin-Huxley equation, which generates spikes of an electrical signal periodically. We investigate the mathematical model by numerical calculations. We show the firing times of the neurons in both the hippocampal layer and the visual cortical layer in Fig.6. We find that synchronous phenomena occur not only for the frequency but also for the firing time. This point is a new finding different from those by the system with phase oscillator neurons investigated previously.

#### 6. Concluding Remarks

We have investigated the higher functions of the neural networks such as the storage capacity of the memory, the generalization ability, the learning and the visual selective attention and so on, within the framework of the statistical physics. We have found the increase of the storage capacity for the SPNN with the NMTF and also for the LNNILC with Q-states clock neurons, the improvement of the generalization ability for the HNN with spin-S Ising neurons and the speed-up of the learning for the two-layered neural network with randomly diluted connections. It has been pointed out that the visual selective attention is considered as the synchronous phenomena between the firing times of the neurons in the hippocampus and those in the visual cortex by using the Hodgkin-Huxley equation.

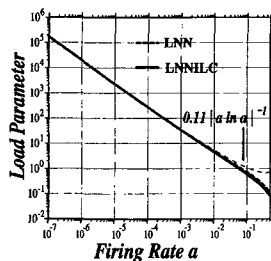


Fig. 4.

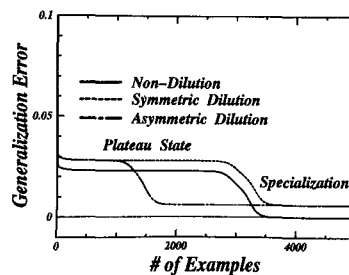


Fig. 5.

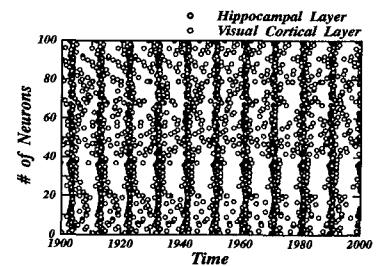


Fig. 6.

## 論文審査の結果の要旨

記憶容量、汎化能力、パターンの系列想起などをはじめとするニューラルネットワークの高次機能の研究は、ニューラルネットワークの情報処理能力の解明や高等動物の脳機能の理解のための基礎的研究として重要な課題である。これらの課題は、いろいろな学問分野の観点から研究がなされてきているが、解明すべき問題は沢山ある。著者はこれらの問題のいくつかを統計物理学の観点から研究した。本論文は、この成果をまとめたもので、全文6章よりなる。

第1章は序論であり、本研究の背景及び目的を述べている。

第2章では、再帰的ニューラルネットワークについて、はじめに多値状態をもつニューロンのHopfieldモデルの記憶容量と汎化能力について研究した。次に非単調入出力関数を用いた系列パターンの想起について研究し、単調入出力関数を用いた場合よりも記憶容量が増加することを示し、さらに想起に関してカオス的振舞いを発見した。系列パターンの想起に関するカオス的振舞いは全く新しい発見であり注目すべき結果である。

第3章では、はじめにQ状態クロックニューロンを用いた層状ニューラルネットワークの記憶容量について研究した。次に層状ニューラルネットワークのスパースコーディングについて研究し、自己制御閾値法を用いることによりベイズンが拡大することを解明した。

第4章では、2層ニューラルネットワークのオンライン学習について、教師ネットワークと生徒ネットワークにおける結合のランダム切断の効果を研究した。教師ネットワークでの結合のランダム切断により、生徒ネットワークの学習速度が改善されることを明らかにした。これは興味ある成果である。

第5章では、Hodgkin-Huxleyニューロンからなる2層ニューラルネットワークを用いて、海馬体と視覚皮質に関する視覚の選択的注意に対する数理モデルを提案した。数値計算により、2層のニューロン集団間に、発火時間に関してガンマ振動の同期現象が起こることを示した。これは視覚の選択的注意を理解するための基礎として重要な知見である。

第6章は結論である。

以上要するに本論文は、統計物理学の観点から記憶容量の増加、汎化能力の向上、学習速度の改善、また選択的注意のモデル化など、ニューラルネットワークの情報処理に関する高次機能について研究したものであり、情報基礎科学及び統計物理学の発展に寄与するところが少なくない。

よって、本論文は博士（情報科学）の学位論文として合格と認める。