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論　文　内　容　要　旨

CHAPTER I INTRODUCTION

Over the recent years, much research work has been carried out in the area of multi-dimensional (M-D) digital filters. Although the main research effort has focused on 2-D digital filters, the study on 3-D and higher dimensional digital filters is being expected in many areas such as moving image processing, geophysical data processing and so on.

One of the main obstacle in the application of M-D digital filters is the large amount of computation required in the design and implmentation of these filters. To over come this obstacle some special class of M-D digital filters have been developed.
In this paper, we will study the design problem of separable denominator (SD) M-D digital filters. Reasons for our choice of SD M-D digital filters are as follows. First of all, as will be shown in this paper, the analysis and design of SD M-D digital filters can be performed using results obtained for 1-D digital filters. At the same time, since the numerator of the transfer function of an SD M-D digital filter is a general M-D

polynomial, we can consider it as the cascade of a general FIR digital filter, and a separable all pole IIR digital filters. Thus, any specification can be approximated as closely as desired.

The prolem of designing digital filters can be divided into two stages as approximation and synthesis. Since these two interrelated stages have been studied separately until now, many redundant computation steps are involved in the traditional design procedure. To design digital filters more efficiently, this paper will propose a unified design method of 1–D digital filters in Chapter Ⅲ. The basic idea of the unified design is based on a "balanced approximation method" proposed by Kung[1] and an equivalent relation between balanced realizations and optimal realizations (with respect to quantization errors) of digital filters[2]. This method can perform the approximation and synthesis simultaneously with much less computational complexity. Resulting state–space digital filters of the unified design method are always guaranteed to be stable, nearly optimal and free of overflow oscillations.

In Chapter Ⅳ, we will introduce the concept of characteristic filters of SD M–D digital filters. By introducing characteristic filters, the relation between SD M–D digital filters and 1–D digital filters becomes very clear, and the analysis and degign of SD M–D digital filters can be performed exactly as same as those of 1–D digital filters. On the basis of the relation between SD M–D digital filters and their characteristic filters,the unified design method will be extended to SD M–D case in Chapter Ⅴ.

CHAPTER Ⅱ FUNDERMENTAL THEORY OF MULTI–DIMENSIONAL DIGITAL FILTERS

In order for this paper to be self–contained, in this chapter, we will give a brief review of some concepts about M–D digital filters such as the transfer functions, the state–space representations and so on.

CHAPTER Ⅲ UNIFIED DESGN OF 1–DIMENSIONAL DIGITAL FILTERS BASED ON SYSTEM BALANCING

Philosophy of the Unified Design

Until now, the approximation and synthesis of digital filters have been studied independently. Because relations between these two design stages have not been considered, synthesis of optimal realizations usually requires many redundant computations. These computations along with those required in the approximations stage make the design of digital filters very complex. However, the design problem can be greatly simplified if we employ an approximation method that can result in a realization from which the optimal realization can be found easily.

Thus, to design digital filters efficiently, it is necessary to consider the approximations and the synthesis simultaneously, so that the relation between optimal realizations and realizations obtained in the approximation stage can be fully utilized. Once the relation between optimal realizations and other realizations are known, we can choose a realization from which optimal realizations can be easily synthesized, and develop an approximation method that can result in such a realization. Using this approximation method, redundant computations can be reduced.

Equivalent Relation between Balanced Realizations and Optimal Realizations

Assume that the initial realization is DF(A,b,c,d) which associated controllability gramian K and observability gramian W. The equivalent transformation used to synthesize the optimal realization DF(A_0 ,b_0 , c_0, d) is given by[3]

$$T = T_0 R_1 A R_0^t \qquad (3.13)$$

and the gramians change as follows:

$$\dot{K} = T^{-1} K T T^{-t}, \quad \dot{W} = T^t W T. \qquad (3.7)$$

Substituting Eq. (3.13) into (3.7), we can get the gramians of the optimal realization as follows:

$$K_0 = (\rho^{1/2} R_0^t)^{-1} \, \theta \, (\rho^{1/2} R_0^t)^{-t} \qquad (3.16)$$

and

$$W_0 = (\rho^{1/2} R_0^t)^t \, \theta \, (\rho^{1/2} R_0^t) \qquad (3.17)$$

where ρ is a scalar given by

$$\rho = \sum_{i=1}^{n} \theta_i \, / \, n. \qquad (3.18)$$

Since Θ is the controllability or the observability gramian of the balanced realization, optimal realizations can be synthesized from balanced realizations using the following equivalent transformation:

$$T = \rho^{1/2} R_0^t \qquad (3.19)$$

Unified Design of 1-Dimensional Digital Filters

According to Kung[1], a nearly balanced realization can be synthesize from a given impulse response h(0),h(1),.......,h(N) by the following algorithm:

Step 1: form the Hankel matrix as follows:

$$\Phi = \begin{pmatrix} h(1) & h(2) & \dots & h(N) \\ h(2) & h(3) & \dots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ h(N) & 0 & \dots & 0 \end{pmatrix} \qquad (3.25)$$

Step 2: Suppose that the singular valuses of Φ are σ_i, $i=1, 2, \dots, N$, and satisfy $\sigma_i \geqq \sigma_{i+1}$. Then the singular value decomposition (SVD) of Φ can be expressed as

$$\Phi = U_1 \Sigma_1 V_1 + U_2 \Sigma_2 V_2 \qquad (3.26)$$

where

$$\Sigma_1 = \mathrm{diag}\,(\sigma_1, \sigma_2, \dots, \sigma_n) \qquad (3.27a)$$

$$\Sigma_2 = \mathrm{diag}\,(\sigma_{n+1}, \dots, \sigma_N) \qquad (3.27b)$$

and where n is an integer such that $\sigma_n \gg \sigma_{n+1}$

Step 3: Suppose that the realization which can approximately generate the given impulse response is DF(A, b, c, d), then its coefficient matrices can be obtained as follows:

$$A = (\Sigma_1^{-1/2} U_1^t)(U_1 \Sigma_1^{1/2})^{\uparrow} \qquad (3.28\,a)$$

$$b = \text{First column of } \Sigma_1^{1/2} V_1 \qquad (3.28\,b)$$

$$c = \text{First row of } U_1 \Sigma_1^{1/2} \qquad (3.28\,c)$$

$$d = h_0 \qquad (3.28\,d)$$

where $(*)^{\uparrow}$ expresses the operation of one row shift upward with the last row filled by zeros.

Thus, using the relation between balanced realizations and optimal realizations, nearly optimal realizations can be synthesized directly from given impulse responses as follows:

Step 1 and Step 2: Same as that of Kung's algorithm.

Step 3: Using Eqs. (3.20) and (3.28), the coefficient matrices of a nearly optimal realization can be calculated by

$$A = R_0 (\Sigma_1^{-1/2} U_1^t)(U_1 \Sigma_1^{1/2})\ R_0^t \qquad (3.34a)$$

$$b = \rho^{-1/2} R_0 (\text{First column of } \Sigma_1^{1/2} V_1) \qquad (3.34b)$$

$$c = (\text{First row of } U_1 \Sigma_1^{1/2})\,\rho^{1/2} R^t \qquad (3.34c)$$

$$d = h_0 \qquad (3.34d)$$

The above algorithm has the following advantages:

1) The approximation and the synthesis of digital filter design can be performed simultaneously after the desired impulse response is given.
2) Compared with conventional design method, the amount of computation is very small. Throughout the whole design, the main computation is the SVD of a symmetric matrix.

CHAPTER Ⅳ STRUCTURAL PROPERTIES AND CHARACTERISTIC FILTERS OF SEPARABLE DENOMINATOR MULTI-DIMENSIONAL DIGITAL FILTERS

Theorem 4.1: Suppose $i_{p_1}, i_{p_2}, \ldots, i_{p_k}$ $(1 \leq p_1 < p_2 < \ldots < p_k \leq M)$ are positive elements of i, then the state transition matrix A of an SD M-D digital filter can be calculated by

$$A^i = A^{i_{pk} e_{pk}} A^{i_{p\,k-1} e_{p\,k-1}} \ldots A^{i_{p1} e_{p1}} \qquad (4.1)$$

provided that $i > 0$. ///

Theorem 4.2 A realization MDDF (A, B,C, d) is locally controllable iff G(q) is full rank, where G(q) is called the controllability matrix of SDMD (A, B,C, d), and is given by

$$G(q) = [g(e_1), g(2e_1), \ldots, g(q_1 e_1), g(e_2), g(e_1+e_2), g(2e_1+e_2), \ldots, g(q_1 e_1+e_2), g(2e_2), g(e_1+2e_2), \ldots, g(q_1 e_1+q_2 e_2), \ldots, g(q)] \qquad (4.5)$$

where g(i) is the state impulse response of MDDF (A, B,C, d).

Theorem 4.3 A realization MDDF (A, B,C, d) is locally observable iff O(q) is full rank, where O(q) is called the observability matrix of SDMD(A, B, C, d), and is given by

$$O^t(q) = [f^t(0), f^t(e_M), f^t(2e_M), \ldots, f^t(q_M e_M), f^t(e_{M-1}), f^t(e_{M-1}+e_M), \ldots, f^t(e_{M-1}+q_M e_M), f^t(2e_{M-1}), f^t(2e_{M-1}+e_M), \ldots, f^t(q_{M-1} e_{M-1}+q_M e_M), \ldots, f^t(q)] \qquad (4.7)$$

where $f(i) = CA^i$.

For SD M-D digital filters, we have shown that G and O take the following forms:

$$G(q) = \begin{pmatrix} G_1(q_1) & 0 & \cdots & 0_1 \\ 0 & G_2(q_2) & \cdots & 0_2 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & G_M(q_M) \end{pmatrix} \begin{matrix})\, n_1 \\ )\, n_2 \\ \\ )\, n_M \end{matrix} \qquad (4.9)$$

where

$$G_j(q_j) = [B_j, A_{jj}B_j, \ldots, A_{jj}^{q_j-1} B_j], \; j = 1, 2, \ldots, M \qquad (4.10a)$$

$$B_1 = b_1 \qquad (4.10b)$$

$$B_j = [b_j, A_{j,1} G_1(q_1), \ldots, A_{j,j-1} G_{j-1}(q_{j-1})]$$

$$j = 2, 3, \ldots, M. \qquad (4.10c)$$

and

$$O(q) = \begin{pmatrix} & & O_M(q_M+1) \\ & O_{M-1}(q_{M-1}+1) & 0 \\ \cdots & \cdots & \cdots \\ \underbrace{O_1(q_1+1)}_{n_1} & \underbrace{0}_{n_{M-1}} & \underbrace{0}_{n_M} \end{pmatrix} \qquad (4.11)$$

where

$$O_j(q_j) = \begin{pmatrix} C_j \\ C_j A_{jj} \\ \cdots \\ C_j A_{jj}^{q-1} \end{pmatrix}, \qquad j = 1, 2, \ldots, M \qquad (4.12a)$$

$$C_M = c_M \qquad (4.12b)$$

$$C_j = \begin{bmatrix} c_j \\ O_M(q_M)A_{Mj} \\ \cdots\cdots\cdots \\ O_{j+1}(q_{j+1})A_{j+1,j} \end{bmatrix}, \quad \text{for} \quad j = 1, 2, \ldots, M-1. \qquad (4.12c)$$

In the above equations, A_{jj}, b_j and c_j are the coefficient matrices of SDMD(A, B, C, d).

Now, let us define M 1–D digital filters DF (A_{jj}, B_j, C_j) with B_j and C_j given by Eqs. (4.10) and (4.12), respectively. Them, from the theory of 1–D systems, we have the following theorems:

Theorem 4.4 A realization SDMD (A, B, C, d) is locally controllable iff DF (A_{jj}, B_j, C_j), $j = 1, 2, \ldots, M$ are separately controllable in the 1–D sence.

Theorem 4.5 A realization SDMD(A, B, C, d) is locally observable iff DF(A_{jj}, B_j, C_j), $j = 1, 2, \ldots, M$ are separately observable in the 1–D sence.

Thus, we call the 1–D digital filters DF (A_{jj}, B_j, C_j) the characteristic filters in this paper.

Theorem 4.7: The coefficient matrices of a realization SDMD(A, B, C, d) can be found from those of its characteristic filters by the following equations:

$$A_{jj}: \text{The same for} \quad j = 1, 2, \ldots, M \qquad (4.18a)$$

$$b_j = \text{First column of } B_j, \quad j = 1, 2, \ldots, M \qquad (4.18b)$$

$$c_j = \text{First row of } C_j, \quad j = 1, 2, \ldots, M \qquad (4.18c)$$

$$A_{ij} = [\text{First } \mu_j q_j \text{ columns of } \overleftarrow{B}_i^{\mu_j}]G^+(q_j) \qquad (4.18d)$$

$$= O_i^+[\text{First } \nu_{M-i} q_{M-i} \text{ rows of } C_j^{\uparrow \nu_{M-i+1}}]$$

$$\text{for } i, j = 1, 2, \ldots, M \text{ and } i > j \qquad (4.18e)$$

where

$$\mu_j = (q_{j-1} + 1)(q_{j-2} + 1) \ldots (q_1 + 1). \quad j = 2, \ldots, M \qquad (4.19a)$$

$$\mu_1 = 1 \qquad (4.19b)$$

$$\nu_{M-i} = (q_M + 1)(q_{M-1} + 1) \ldots (q_{M-i+1} + 1), \quad j = 1, \ldots, M-1 \qquad (4.19c)$$

$$\nu_M = 1 \qquad (4.19d)$$

and $(\cdot)^{+}$ expresses the pseudo-inverse of a matrix, and $(\cdot)^{\leftarrow i}$ $((\cdot)^{\uparrow i})$ expresses the operation of shifting a matrix for i columns (rows) leftward (upward), and filling the right columns (bottom rows) with zeros.

Theorem 4.8 The impulse response $h_j(k)$ $(k=0,1,\ldots)$ of the characteristic filter DF (A_{jj},B_j,C_j) is an $[(q_M+1)(q_{M-1}+1)\ldots(q_{j-1}+1)]\times[(q_{j+1}+1)\ldots(q_1+1)]$ − th order matrix, and can be formed from $h(i)$ as follows:

1) The columns of $h_j(k)$ are formed by arranging $h(i)$ $(i_j=k)$ so that i is increased in the order $i_M, i_{M-1},\ldots, i_{j+1}$;
2) The rows of $h_j(k)$ are formed by arranging $h(i)$ $(i_j=k)$ so that i is increased in in the order $i_1, i_2,\ldots, i_{j-1}$. ///

CHAPTER V UNIFIED DESIGN OF SEPARABLE DENOMINATOR MULTI-DIMENSIONAL DIGITAL FILTERS BASED ON SYSTEM BALANCING

Balanced Approximation of Separable Denominator Multi-Dimensional Digital Filters

Definition 5.1 A realization SDMD(A,B,C,d) is said to be balanced iff its characteristic filters DF (A_{ii},B_i,C_i), $i=1,2\ldots,M$, are balanced realizations, i.e.

$$K_j = W_j = \mathrm{diag}\,(\theta_{j1},\theta_{j2},\ldots,\theta_{jn_j}) \qquad (5.8)$$

where (K_j,W_j) are the controllability gramians and the observability gramians of the characteristic filters, and can be obtained by using the following Lyapunov equation:

$$K_j = A_{jj}K_jA_{jj}^t + B_jB_j^t = \sum_{i=1}^{j} A_{ji}K_iA_{ji}^t + b_jb_j^t$$

$$j=1,\ 2\ldots,M \qquad (5.5)$$

$$W_j = A_{jj}^tW_jA_{jj} + C_j^tC_j = \sum_{j=1}^{M} A_{ij}^tW_iA_{ij} + c_j^tc_j,$$

$$j=M,\ M-1,\ \ldots,1. \qquad (5.6)$$

Take SD 3−D digital filters as an example, we can state the balanced approximation as follows[4]:

Step 1: Suppose that the specification is given as a 3−D impulse response $h(i)$, $0\leqq i\leqq N$.

Step 2: Form the ideal impulse responses of the characteristic filters as follows:

$$h_1(k) = C_1A_{11}^{k-1}B_1$$

$$= [h(k,0,0), h(k,0,1),\ldots, h(k,0,N_3), h(k,1,0), h(k,1,1),\ldots, h(k,1,N_3),\ldots, h(k,N_2,N_3)] \qquad (5.9\,a)$$

$$h\ (k) = C_2 A_{22}^{k-1} B_2$$

$$= \begin{pmatrix} h(0,k,0) & h(1,k,0) & \dots & h(N_1,k,0) \\ h(0,k,1) & h(1,k,1) & \dots & h(N_1,k,1) \\ \dots & \dots & \dots & \dots \\ h(0,k,N_3) & h(1,k,N_3) & \dots & h(N_1,k,N_3) \end{pmatrix} \qquad (5.9\ b)$$

and

$$h\ (k) = C_3 A_3^{k-1} B_3$$

$$= [h(0,0,k), h(1,0,k), \dots, h(N_1,0,k), h(0,1,k), h(1,1,k), \dots, h(N_1,1,k), \dots, h(N_1,N_2,k)]. \qquad (5.9\ c)$$

Step 3: From the Hankel matrices from h_1, h_2 and h_3 as follows:

$$\Phi_j = \begin{pmatrix} h_j(1) & h_j(2) & \dots & h_j(N_j) \\ h_j(2) & h_j(3) & \dots & 0 \\ \dots & \dots & \dots & \dots \\ h_j(N_j) & 0 & \dots & 0 \end{pmatrix}, \quad j = 1,\ 2,\ 3. \qquad (5,10)$$

Step 4: From the above Hankel matrices, the characteristic filters of the desired SD 3–D digital filter can be obtained using Kung's method, as described in Chapter Ⅲ.

Step 5: Find the coefficient matrices of the desired SD M–D digital filter SDMD(A, B, C, d) using Theorem 4.7 and the following relations:

$$d = h\ (0, 0, 0) \qquad (5.15)$$

$$O_2^+ = \Sigma_{21}^{-1/2} U_{21}^t, \text{ and } O_3^+ = \Sigma_{31}^{-1/2} U_{31}^t \qquad (5.16\ a)$$

$$G_1^+ = V_{11} \Sigma_{11}^{-1/2}, \text{ and } G_2^+ = V_{21} \Sigma_{21}^{-1/2} \qquad (5.16\ b)$$

where U_{21}, V_{21} and Σ_{21} and U_{31} and Σ_{31} are obtained from the SVD of Φ_2 and Φ_3, respectively.

Analysis and Minimization of Quantization Effects in Separable Denominator Multi-Dimensional Digital Filters

Due to roundoff after multiplication, the actual SD M–D digital filter implemented by a finite wordlength mechine can be described by

$$\tilde{x}'(i) = A\tilde{x}\ (i) + B u\ (i) + \alpha\ (i) \qquad (5.19\ a)$$

$$\tilde{y}\ (i) = C\tilde{x}\ (i) + d u\ (i) + \beta\ (i) \qquad (5.19\ b)$$

where $\tilde{x}(i)$ and $\tilde{y}(i)$ are the actual state vector and the actual output, respectively, $\alpha(i)$

and $\beta(i)$ are, respectively, n–th and 1–st order error vectors generated due to roundoff after multiplication in Ax+Bu and Cx+du.

On the basis of this model, we can get the variance of the output roundoff noise as follows[5]:

$$E[\Delta y^2] = \sigma^2 tr[QW] + \sigma^2 q = \sigma^2 G^r + \sigma^2 q \qquad (5.25)$$

$$= \sigma^2 \sum_{j=1}^{M} tr[Q_j W_j] + \sigma^2 q = \sigma^2 \sum_{j=1}^{M} G_j^r + \sigma^2 q \qquad (5.28)$$

where W is the observability gramian of SDMD(A,B,C,d). In the context of digital filter design, W is usually called the noise matrix. In Eq. (5.25), since $\sigma^2 G^r$ is the dominant term of the error variance, G is called the noise power gain of SDMD(A,B, C,d).

On the other hand, due to coefficient quantization, the actual filter implemented by a finite–wordlength machine is described by

$$\tilde{x}'(i) = \tilde{A}\,\tilde{x}(i) + \tilde{B}\,u(i) \qquad (5.30a)$$

$$\tilde{y}(i) = \tilde{C}\,\tilde{x}(i) + \tilde{d}\,u(i) \qquad (5.30b)$$

where $\tilde{x}(i)$ and $\tilde{y}(i)$ are the actual state vector and the actual output, respectively, $\tilde{A} = A + \Delta A$, $\tilde{B} = B + \Delta B$, $\tilde{C} = C + \Delta C$ and $\tilde{d} = d + \Delta d$ are coefficient quantization errors.

Since the coefficient quantization errors can be assumed to be statistically independent random variables and uniformly distributed in the range $[-2^{-\ell}/2,\ 2^{-\ell}/2]$, where ℓ is the coefficient wordlength. Using this assumption, we can obtain the output error variance due to coefficient quantizations as follows[6]:

$$E[\Delta y\ ] = \sigma^2 tr[QW] + \sigma^2 q = \sigma^2 G^c + \sigma^2 q \qquad (5.42)$$

$$= \sigma^2 \sum_{j=1}^{M} tr[Q_j W_j] + \sigma^2 q = \sigma^2 \sum_{j=1}^{M} G_j^c + \sigma^2 q \qquad (5.43)$$

where

$$Q = diag\left(\sum_{j=1}^{n} r(a_{1j}) E[x_j^2], \ldots, \sum_{j=1}^{n} r(a_{nj}) E[x_j^2]\right)$$

$$+ \sigma^2 diag(r(b_1), \ldots, r(b_n)) E[u^2]. \qquad (5.39)$$

and

$$q = \sum_{j=1}^{n} r(c_j) E[x_j^2] + r(d). \qquad (5.41)$$

In the above equation, since $\sigma^2 G^c$ is the dominant term, G^c can be take as the coefficient sensitivity of SDMD(A, B, C, d).

Comparing Eq. (5.28) with (5.43), we can verify that the power gain G^r of the roundoff noise and the coefficient sensitivity G are exactly the same. Thus, the minimization of overall quantization effects in SD M−D digital filters can be performed by minimizing either the roundoff noise or the coefficient sensitivity.

Since the minimization problem is completely separated into minimizations of noise power gains or the coefficient sensitivity in M directions, the quantization errors can be minimized independently by using results obtained for 1−D digital filters.

In addition to minimum roundoff noise and minimum coefficient sensitivity, we can also prove that optimal realizations of SD M−D digital filters are also free of overflow oscillations under zero input conditions.

Unified Design of Separable Denominator Multi-Dimensional Digital Filters Based on System Balancing.

Take the 3−D case as an example, the unfied design algorithm can be described as follows:

Step 1 to Step 3 are the same as the balanced approximation given in the beginning of this chapter.

Step 4: the coefficient matrices of the desired characteristic filters are given by

$$A_{11} = U_1 (\Sigma_{11}^{-1/2} U_{11}^t)(U_{11} \Sigma_{11}^{1/2}) \uparrow^{(N_2+1)(N_3+1)} U_1^t \qquad (5.60\,a)$$

$$B_1 = \rho^{-1/2} U_1 (\text{First column of } \Sigma_{11}^{1/2} V_{11}^t) \qquad (5.60\,b)$$

$$C_1 = (\text{First } (N_2+1)(N_3+1) \text{ rows of } U_{11} \Sigma_{11}^{1/2})\, \rho^{1/2} U_1^t . \qquad (5.60\,c)$$

Coefficient matrices of the rest two characteristic filters can be found in a similar manner.

Step 5: In stead of Eq. (5.16), the relations used to found the coefficient matrices of the desired SD M−D digital filter SDMD (A,B,C,d) are given by

$$O_2^+ = \rho_2^{-1/2} U_2 (\Sigma_{21}^{-1/2} U_{21}^t) \qquad (5.61a)$$

$$O_3^+ = \rho_3^{-1/2} U_3 (\Sigma_{31}^{-1/2} U_{31}^t) \qquad (5.61b)$$

$$G_1^+ = (V_{11} \Sigma_{11}^{-1/2})\, \rho_1^{1/2}\ U_1^t \qquad (5.61c)$$

$$G_2^+ = (V_{21} \Sigma_{21}^{-1/2})\, \rho_2^{1/2}\ U_2^t . \qquad (5.61d)$$

In the above equations, ρ_1, ρ_2 and ρ_3 are scalars given by

$$\rho_j = \sum_{k=1}^{n_j} \theta_{jk} / n_j , \quad \text{for } j = 1, 2, 3 \tag{5.62}$$

and where θ_{jk}, $j = 1, 2, 3$: $k = 1, 2, \ldots, n_j$, are the second order modes of the $j-$th characteristic filters. The matrices U_1, U_2 and U_3 are orthogonal, and satisfy

$$U_j \begin{pmatrix} \theta_{j1} & & & \\ & \theta_{j2} & & \\ & & \ddots & \\ & & & \theta_{j,n_j} \end{pmatrix} U_j^t = \begin{pmatrix} \rho_j & & & \\ & \rho_j & & \\ & & \ddots & \\ & & & \rho_j \end{pmatrix}, \quad j = 1, 2, 3. \tag{5.63}$$

CHAPTER VI CONCLUSION AND REMARKS

In this paper, we have studied the class of M−D digital filters whose transfer functions are separable in denominator. As stated in Chapter I, the goal of this paper is to develop an efficient method for designing SD M−D digital filters. In Chapter III, we have studied the structural properties of optimal realizations (with respect to quantization errors), and shown that optimal realizations are in fact scaled and rotated balanced realizations. Based on this "balancing" property of optimal realizations, we have proposed a unified design method that can result in optimal realizations directly from time domain specifications. The main advantage of the unified design method is that the computational complexity is much simpler than conventional design methods. In Chapter IV, we have studied the structural properties of SD M−D digital filters, and introduced the concept of characteristic filters. As a result, the relation between SD M−D digital filters and 1−D digital filters become very clear, and the analysis and design of SD M−D digital filters can be performed in exactly the same manner as those of 1−D digital filters. In Chapter V, we have first studied the approximation of SD M−D digital filters, and proposed a balanced approximation method for SD M−D digital filters. Then, we have analyzed the quantization errors, (roundoff noise and coefficient quantization errors) in SD M−D digital filters, and proposed a method for minimizing these errors. Based on these results and those obtained in Chapter III and IV, we have finally proposed a unified design method of SD M−D digital filters.

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審　査　結　果　の　要　旨

近年，画像信号や地震波，気象データなどの膨大な多次元信号処理の必要性に伴い，多次元ディジタルフィルタの近似および量子化誤差の解析・最小化の研究が望まれていた。

著者は，分母分離形多次元ディジタルフィタの近似と量子化誤差の解析・最小化において平衡性の概念が重要な役割を担っていることに着目し，分母分離形多次元ディジタルフィルタの統一的設計法を提案すると共に，その有効性を明らかにした。本論文は，その成果をまとめたものであり，全文6章よりなる。

第1章は，緒言である。第2章では，多次元ディジタル信号処理の概要について述べると共に，多次元ディジタルフィルタの設計における近似と量子化誤差の解析・最小化に関する問題点を明確にしている。

第3章では，分母分離形多次元ディジタルフィルタ設計の基礎となる1次元ディジタルフィルタの統一的設計について考察し，その設計法を与えている。とくに，1次元ディジタルフィルタにおける近似と量子化誤差の解析・最小化に対して，平衡性の概念が統一的視点を与えることを見出しているが，これは極めて重要な知見である。

第4章では，特性フィルタの概念を提案し，分母分離形多次元ディジタルフィルタの構造上の特質が，特性フィルタの性質によって決定されることを明らかにしている。この特性フィルタを用いることによって，分母分離形多次元ディジタルフィルタの解析が，1次元ディジタルフィルタの解析の問題に帰着されるため，その解析が極めて容易になっている。

第5章では，平衡性の概念を用いて，分母分離形多次元ディジタルフィルタの統一的設計法について考察している。まず，分母分離形多次元ディジタルフィルタにおける平衡形近似法を考察している。次に，量子化誤差の解析・最小化を行い，平衡性が成立するとき，分母分離形多次元ディジタルフィルタの量子化誤差が最小となることを見出しているが，これは重要な成果である。さらに，これらの方法を総合することによって，分母分離形多次元ディジタルフィルタの統一的設計法を提案し，その方法の有効性を明らかにしている。

第6章は結言である。

以上要するに本論文は，分母分離形多次元ディジタルフィルタにおいて，平衡性という極めて重要な概念を提案し，これらを用いて分母分離形多次元ディジタルフィルタの統一的設計法を確立し，その有効性を明らかにしたものであり，電子工学および情報工学の発展に寄与するところが少なくない。

よって，本論文は工学博士の学位論文として合格と認める。