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論 文 内 容 要 旨

Failure of solids has been extensively investigated to characterize its mechanism. When solids undergo failure, they display characteristic geometrical patterns. For instance, steel specimens subjected to tensile loading present necking; shear bands are formed in the triaxial compression specimens of soils until reaching final failure; an initially straight column subjected to large axial compression displays a characteristic buckling mode.

Since patterns formed prior to the failure vary with materials, the mechanism of the generation of the patterns has been studied individually for particular materials and for particular boundaries, in various fields of mechanics. However, since the failure pattern itself is fairly dependent on the boundaries of the system, the study of the failure pattern of a particular system often loses the generality. Moreover, characteristic geometrical deformation often develops prior to the formation of the final failure pattern. Although such deformation is not discernible at first, it grows and localizes by changing its appearance recursively and finally induces such final pattern and failure. For metal, the dislocations increase and move to accumulate at the boundary, and, as a result, cause the localization of the deformation. For soil, fracture and re-formation of the soil structure form shear bands.

Periodic and symmetric patterns, such as the diamond pattern, are observed when the specimens are homogeneous. The echelon mode can be found in various materials such as soils, rocks and metals. The cross-checker pattern is often found in metals. Rocks often display characteristic geometrical patterns, such as joint or texture. For instance, basalt shows regularly-arranged hexagonal joint and another joint perpendicular to the hexagonal one. This joint structure was generated by the Benard convection and by the shrinkage during the cooling. Furthermore, minerals such as calcite that construct rocks have joints due to its composition of crystal. The triaxial compression specimen of sand sometimes shows the diamond pattern that is known as a typical buckling pattern of shell. Patterns are observed not only in solids but also in fluids. The Taylor--Couette flow in a hollow cylinder, which is a rotating annular of fluids, displays various wave patterns. The convective motion of fluid, the so-called Benard problem, displays a regularly arrayed hexagonal cellular structure. Moreover, the scale of the patterns varies broadly from the size of atoms to the size of the earth, or even of the galaxy. The Zebra patterns on the ocean floors serve as an example of a pattern of thousand-kilometer scale. Furthermore, patterns govern their final failure, regardless of the size of deformation.

For its importance and universality, pattern formation has been studied intensively in experimental mechanics, theoretical and applied mechanics, applied mathematics, and so on.

In applied mechanics, the study of the uniqueness of solution of elastic-plastic solids is one of the origins of the study of instability of materials and pattern formation. As for this approach, numerical simulations of bifurcation and post bifurcation behavior are reported considerably. On the contrary, parametric experimental studies have been done for some particular specimens under various boundaries. In computational mechanics, finite element method is mainly employed for the simulation and interpretation of preceding experiment.

The aforementioned numerical approach and experimental one focus on the simulation and quantitative prediction of a particular phenomenon occurring in a particular system and on the constitutive characteristics of a particular material. On the contrary, geometrical patterns have also been investigated in other fields of research. At the expense of beauty and orderliness, uniform systems often display complex physical behavior that is called the "symmetry-breaking" bifurcation, or, in other words, the "pattern selection". In fluid mechanics and nonlinear mathematics, it is well known that such patterns are selected through recursive bifurcation which "breaks" symmetry. For instance, wave patterns with various kinds of symmetries in the Taylor-Couette flow appear as a result of pattern selection. Moreover, in view of recent developments of group-theoretic bifurcation theory in nonlinear mathematics, rules of symmetry-breaking bifurcation can be obtained in a systematic manner. The group-theoretic bifurcation theory, thus, aims at qualitative investigation for pattern formation of general symmetric systems. For another approach for systematic study of patterns, we merely refer to fractal which address self-similarity and scale invariance.

With resort to the analogy between the patterns in flows and in materials, bifurcation has come to be acknowledged to be the underlying mechanism to form patterns of materials, such as cracks and shear zones. These studies are helpful in the qualitative prediction, classification and interpretation of results of experiments, while FEM analysis, for example, gives us quantitative information. Furthermore, effectively combined with experiment and numerical simulation, the group-theoretic bifurcation theory can present perspective on the pattern forming deformation and failure.

Yet, studies of pattern formation based on the group-theoretic bifurcation theory have been restricted to plane problems up to now. This is partly because that it is not easy indeed to measure the property, such as displacement, strain, temperature, velocity of elastic wave and so on in a three-dimensional domain. In any case, conventional studies of pattern formation cannot explain three-dimensional patterns. We aim, in this thesis, at investigation of the bifurcation hierarchy and patterns generated by bifurcation of a three-dimensional domain by means of the group-theoretic bifurcation theory. Since the group theory has been generally employed for the description of the symmetry of crystal lattice, it is destined to be applicable to the description of the symmetry appearing in materials. Furthermore, the validity of the mechanism of pattern formation derived in this manner is assessed by the following two numerical analyses. One is a three-dimensional pattern simulation, which is the extension of the image simulation of a two-dimensional uniform domain, for the process of evolution of joint structure of rocks. The other is the multi-scale modeling of material instability and pattern formation with non-convex homogenization.

In this context, instability and pattern formation are discussed from three standpoints and this thesis is organized as follows.

In Chapter 2, first, assumption and limitation of this thesis are clarified. Then notions of the group equivariance and of the group-theoretic bifurcation analysis of a uniform rectangular domain are summarized.

To better express the local uniformity at the sacrifice of the consistency with the boundary conditions, the infinite-periodic-domain approximation, which assumes that the domain is periodically extended in each of the three directions, is employed. Since real material properties manifest itself sufficiently away from the boundaries and usually form some characteristic patterns, the use of periodic boundaries is essential in the simulation of true material properties.

The underlying mechanism of the pattern formation for the rectangular parallelepiped uniform domain has been clarified by extending the preexisting group-theoretic bifurcation theory for a uniform rectangular domain. The proper modeling of the symmetry of the rectangular parallelepiped uniform domain as $O(2) \times O(2) \times O(2)$-invariant system has led to the successful simulation of patterns of interest, including: oblique layered pattern, column pattern and diamond pattern. It is vital in the study of pattern formation of (three-dimensional) uniform materials to exploit translational symmetries with the use of the periodic boundaries and to have the knowledge on the bifurcation of an $O(2) \times O(2) \times O(2)$-invariant system presented in this chapter. This bifurcation analysis literally exhausts all possible deformation patterns formed in real (three-dimensional) uniform materials.

The characterization of the real three-dimensional phenomena by two-dimensional ones must conserve the symmetry with respect to another direction. Such characterization ignores bifurcation in the excluded direction. The above conclusion is also accepted by interpretation that conventional two-dimensional patterns are regarded as the projection of the three-dimensional patterns investigated in this chapter. Therefore, the three-dimensional model discussed here has the essential necessity and the primary importance to analyze the real pattern observed in nature.

In Chapter 3, a systematic procedure for the image simulation of progress of deformation patterns of uniform materials is proposed by highlighting recursive symmetry-breaking bifurcation as the fundamental mechanism to generate patterns. We here focus on a rectangular domain with periodic boundaries. It should be emphasized again that the use of periodic boundaries is essential in the simulation of true material properties.

Rules of the recursive bifurcation, which are expressed in terms of a hierarchy of subgroups labeling the symmetries of deformation patterns, are constructed by extending the preexisting group-theoretic studies for this domain. The use of periodic boundaries has led to the emergence of the subgroups labeling stripe and echelon symmetries that disappear if these boundaries are not used. These rules of bifurcation are interpreted in terms of the double Fourier series to prepare for image analysis of deformations in a rectangular domain. The use of the Fourier series has physical necessity in that the direct bifurcation modes of uniform domains are always harmonic and that periodic properties are better expressed in the frequency domain. Mode interference with high frequencies after bifurcation is advanced as the mechanism of localization of deformations.

The procedure for image simulation is applied to a few uniform materials, including kaolin and steel specimens. The intensity of the digital images of the deformation patterns of these specimens in the frequency domain is successfully classified with the use of the rules of recursive bifurcation. As a result of these, the transient process of deformations, which was not discernible by the mere visual observations and was less understood up to now, is identified based on a firm theoretical basis. The recursive bifurcation has thus been acknowledged to be the underlying mechanism of pattern formation of uniform materials.

Image simulation is also extended to three-dimensional pattern simulation by exploiting the mechanism clarified in Chapter 2. The procedure for pattern simulation developed in this manner is applied to the idealized schematic model of the cleavage of calcite and of the columnar joint of basalt.

In Chapter 4, in contrast with the preceding mathematical discussion, we have originally developed a characterization of the material instability by the microstructural instability for a concrete mechanical example of material instability characterized by micro (or local) structural bifurcation. This approach gives an innovative methodology for logical or deductive characterization of the material instability that is conventionally modeled by only a sort of phenomenological or experimental way. This two-scale characterization of material instability employs the theory of non-convex homogenization, which is an extension of multi-scale homogenization method. The infinite-periodic-domain approximation employed in the preceding chapters for expressing local uniformity is consistent with the periodicity that is required in the homogenization method.

The main difficulties, for the multi-scale modeling implementing instability, are due to loss of convexity of total potential energy and due to the determination of the pertinent representative volume element (RVE) that contains multiple unit cells. The variational formulation is achieved with the help of Γ-convergence theory within the framework of the non-convex homogenization method, while the number of unit cells in an RVE, which is mandatory in the construction of an appropriate RVE, is determined by the block-diagonalization method of the group-theoretic bifurcation theory. The latter method enables us to identify the most critical bifurcation mode that minimizes the total potential energy among all possible bifurcation patterns for an assembly of arbitrary numbers of periodic microstructures. Thus, the number of unit cells to be employed in the RVE for the micro-scale analysis of the derived two-scale boundary value problem can be determined. The block-diagonalization method of the group-theoretic bifurcation theory plays a crucial role to overcome the problem of the determination of micro-structural periodicity that hitherto has remained open. In this context, we also emphasize the importance of the consideration of the pattern formation (selection) from the standpoint of the symmetry-breaking bifurcation.

By virtue of this two-scale modeling, representative numerical examples for a cellular solid show the feasibility of the proposed method and illustrate the material instability at a macroscopic point that is induced by the geometrical instability in a microscale.

審査結果の要旨

巨視的に均質とみなせる材料には、特徴的な幾何学パターンがしばしば観察される。それらのパターンは、材料の破壊形態を支配していると考えられる。個別の材料の破壊に関する研究は、多岐に渡り行われているが、破壊に至るまでに現れるパターンと破壊の因果関係に関する一般論については、十分な研究が行われていない。材料の破壊現象の根本原理の解明のためには、上記のような一般論が必要である。

本論文は、材料の不安定とそれに伴うパターン形成のメカニズムに関する一般論、およびパターン形成に伴う材料特性の変化をモデル化するための手法を構築したものであり、全編5章からなる。

第1章は序論である。

第2章では、三次元的に均質な材料を $O(2)\times O(2)\times O(2)$-同変系としてモデル化することにより、この領域において起こり得る分岐の仕組みを群論的分岐理論により明らかにし、分岐により発生可能な幾何学パターンを分類している。これらは、これまで材料固有のものとして捉えられていた破壊形態に対して、一般理論を提供するものであり、材料の破壊現象を根本的に明らかにする上で重要な成果である。

第3章では、対称性破壊分岐の繰り返しをパターン形成の根本的なメカニズムとみなすことによって、均質材料の変形パターンの進展を推定するパターンシミュレーションの方法を提案している。繰り返し分岐の法則は、変形パターンの対称性を表す(部分)群の階層構造によって表され、領域のパターン形成に関する群論的考察によって構成される。これらは、単なる目視では識別することが困難で、これまで見落とされがちであった変形パターンの遷移を、理論的裏付けのもとに推定することを可能にしたものであり、有用な成果である。

第4章では、マルチスケール法に基づいて、材料不安定現象をミクロ構造の不安定現象によりモデル化する方法論を確立している。ミクロ構造の不安定現象を定式化するために、非凸ポテンシャル汎関数のための一般化収束論の数学的概念を適用した。また、代表体積要素に含まれるべき単位周期構造の数を系のポテンシャルが最小となるように決定する必要があったが、本論文ではこの問題を群論的分岐理論から導かれるブロック対角化法によって解決する方法を構築している。これらは、これまで現象学的方法、もしくは実験的方法によってのみモデル化されてきた材料不安定現象の論理的、もしくは演繹的モデル化を可能としたものであり、実用上きわめて重要な成果である。

第5章は結論である。

以上要するに本論文は、材料の不安定により誘起されるパターン形成の一般論を構築するとともに、それを用いた材料の変形履歴を推定する手法およびパターン形成により誘導される材料の非線形性・異方性のモデル化を行う手法を構築し、それらの有用性を明らかにしたもので、土木工学ならびに固体力学の発展に寄与するところが少なくない。

よって、本論文は博士(工学)の学位論文として合格と認める。