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論文内容要旨

The behavior of linear dynamical systems can be understood by studying their mathematical descriptions, typically differential or difference equations. The state-space representation is a generalization of differential or difference equations for linear dynamical systems, and emphasizes the internal structure of systems as well as their input-output performance. This approach is in contrast to the Laplace-transform or z-transform description of systems, both of which represent the input-output performance alone. Since the internal structure of systems must be taken into account in many practical applications, the state-space representation of linear dynamical systems is considered as a powerful theoretical tool for analysis of linear dynamical systems, and thus achieved the most rapid growth in the linear system theory.

One of the useful and famous applications of the state-space representation is balanced model reduction, which is well-known theory of approximation of large-scale linear dynamical systems and has significant connections to many applications in the linear system theory. Furthermore, in the field of signal processing theory, the state-space representation has also been widely used to analyze quantization effects of digital filters such as roundoff noise, coefficient sensitivity, and limit cycles. Needless to say, such state-space analysis has led to many attractive methods for synthesis of digital filters of low quantization effects.

The key factors in the above-mentioned issues are the controllability/observability Gramians and the second-order modes. The Gramians exhibit structural properties of systems, and thus they are vital to analysis of the dynamics of systems. The second-order modes are the scalar quantities obtained from the controllability and observability Gramians, and they are also called the Hankel singular values in systems theory context. In the balanced model reduction, the second-order modes are used for mathematical evaluation of system approximation errors. In the analysis of quantization effects of digital filters, the second-order modes are known to characterize the minimum attainable value of roundoff noise and coefficient sensitivity of digital filters. Therefore, the study of the Gramians and second-order modes is essential to the development of practical applications of the state-space representation, as well as the understanding of the behavior of linear dynamical systems.

Although it seems that the linear system theory from the viewpoint of the state-space representation has been well established, there are still many practically important topics which have not been discussed by the state-space approach. This is especially true for the signal processing theory, including analog and digital filter theory.

The purpose of this thesis is the state-space investigation of linear systems from a signal processing point of view. The topics we are particularly interested in are power complementary systems and frequency transformation. They are well known in the signal processing theory such as multirate signal processing, filter design, and filter synthesis, whereas only a few results have been obtained on the state-space investigation of these topics. Hence this thesis aims at making further investigation of these topics through the state-space analysis and bringing fresh insights to the linear system theory. Although many of the results given in this thesis will be related to the field of signal processing such as analog and digital filters, we emphasize that our results have significant connections to many other fields, such as circuit theory, control theory, and communication theory.

Most of the material in this thesis is devoted to the above-mentioned issues. The contributions of this thesis break up into three segments and they are summarized as follows.

The first part of this thesis presents a novel algebraic analysis of power complementary systems in state-space form. Here we focus on the non-uniqueness of the description of power complementary systems, and we derive new properties of power complementary systems with respect to the zeros, coefficients, and Gramians. To achieve this, we make frequent use of the bounded-real Riccati equations, which were developed in the field of H^{∞} control theory; we discuss the connections between the state-space investigation of power complementary systems and solutions to the bounded-real Riccati equations.

The second part derives some novel properties of the second-order modes of linear dynamical systems from the viewpoint of frequency characteristics of systems. These properties are derived from the results obtained in the first part: the state-space analysis of power complementary systems. Although the second-order modes play central roles in many practical applications mentioned above, little investigation has been done to discuss physical meanings of the second-order modes. The second part of this thesis is aimed at making this point clear, and for this purpose we reveal some novel properties of the second-order modes, relating the second-order modes to the magnitude and phase responses of linear dynamical systems. Furthermore, we point out important relationships of the new properties of the second-order modes to the balanced model reduction and synthesis of digital filters of low quantization effects.

The third part is concerned with frequency transformation and divided into three topics: Gramian-preserving frequency transformation, state-space analysis of 2-D discrete frequency transformation, and state-space analysis of lossy bounded-real transformation. In the first topic of this part, we apply the existing state-space formulation of continuous/discrete-time frequency transformation to derive a new description of the frequency transformation. Our proposed description enables the

Gramians of linear dynamical systems to be invariant under frequency transformations, and thus we refer to this new description as "Gramian-preserving frequency transformation". In the second topic, we discuss the state-space analysis of 2-D discrete frequency transformation. Although the frequency transformation is defined for 2-D discrete systems in terms of the transfer functions, there has been no state-space investigation of the 2-D frequency transformation. Our discussion deals with this problem and presents novel state-space formulation of frequency transformation for 2-D discrete systems. Furthermore, we discuss invariance of the second-order modes under 2-D frequency transformations in the case when 2-D discrete systems have separable denominator polynomials. The result from this discussion shows that the second-order modes of 2-D discrete systems are not invariant under all frequency transformations, but invariant under some specific frequency transformations. In the third topic, we turn our attention again to discrete-time systems, and we discuss the property of the second-order modes under "lossy" transformations. This is the counterpart of the property discussed in the continuous-time case, where it was proved that the values of the second-order modes of continuous-time systems are decreased under any lossy transformation. We discuss this property for discrete-time systems, and derive the result which is parallel to the continuous-time case: the values of the second-order modes of discrete-time systems are decreased under any lossy transformation.

This thesis is organized as follows. Chapter 1 is the introductory part, which explains the motivations and purposes of this research work. Chapter 2 provides necessary background material for this thesis. Chapters 3, 4 and 5 present our main results. Each of these chapters respectively corresponds to each of the main three contributions explained above. Chapter 6 concludes this thesis and summarizes the main contributions of this thesis. Suggestions for future work are also included.

論文審査結果の要旨

状態空間表現は，線形動的システムの入力・出力に加えて内部状態をも記述できるという特長があり，伝達関数表現よりも広範な問題を取り扱うことができる．しかし信号処理の分野では，状態空間表現に基づく解析がまだ十分になされていない課題がある．本論文では，そのような課題として“電力相補システム”および“周波数変換”に着目し，これらを状態空間表現の観点から理論的に研究した．その結果，伝達関数表現に基づく議論では解明されなかった様々な性質を明らかにし，システムおよびフィルタの動的振る舞いに関する重要な知見を得た．本論文はこれらの成果をまとめたものであり，全編6章よりなる．

第１章は緒言である．

第２章では，本論文の基礎となる線形動的システムおよびフィルタに関する基礎的性質と状態空間表現について述べている．

第３章では，電力相補システムを状態空間表現に基づいて解析し，電力相補システムの構造に関する新しい定理を導出している．さらに，振幅・極配置および零点配置が特殊な関係にある電力相補システムが有する新しい性質を解明している．これらの結果は，今後の制御系やフィルタの高精度な設計法につながる重要な基礎理論として位置づけられる．

第４章では，システムにおいて実用上重要である“２次モード”が有する性質を議論している．本章では，２次モードの値が周波数特性に明確に関係づけられることを示し，さらにこの結果を用いて，ディジタルフィルタの量子化誤差の最小値や平衡形モデル低次数化の近似誤差の値が，周波数特性に依存するという新しい性質を導いている．これは信号処理・回路網や制御など幅広い分野において実用上重要な基礎的性質である．

第５章では，状態空間表現に基づいて周波数変換について議論している．本章では，システムの構造の性質を保存する新しい周波数変換の記述法を提案し，これによってさまざまな特性を有する高精度ディジタルフィルタが容易に実現されることを示している．さらに本章では，２次元システムの周波数変換を状態空間表現に基づいて議論し，２次元システムの構造と周波数変換との関係を簡潔な式で導いている．また，変換に用いる関数が損失性を有する場合についても議論し，この変換に対して２次モードの値は減少することを証明している．これらの結果は，線形動的システム理論における新しい知見である．

第６章は結言である．

以上要するに本論文は，状態空間表現を用いて信号処理の観点から線形動的システムとフィルタの性質を議論し，数多くの新しく重要な知見を与えたものであり，電子工学および信号処理の発展に寄与するところが少なくない．

よって，本論文は博士(工学)の学位論文として合格と認める．